Letter to the Editor

Formation of undermassive single white dwarfs and the influence of planets on late stellar evolution

G. Nelemans & T. M. Tauris
Astronomical Institute, “Anton Pannekoek”, University of Amsterdam, Kruislaan 403, NL-1098 SJ Amsterdam, The Netherlands

Abstract. We propose a scenario to form low-mass, single, slow rotating white dwarfs from a solar-like star accompanied by a massive planet, or a brown dwarf, in a relatively close orbit (e.g. HD 89707). Such white dwarfs were recently found by Maxted & Marsh (1998). When the solar-like star ascends the giant branch it captures the planet and the subsequent spiral-in phase expels the envelope of the giant leaving a low-mass helium white dwarf remnant. In case the planet evaporizes, or fills its own Roche-lobe, the outcome is a single undermassive white dwarf. The observed distribution of planetary systems supports the applicability of this scenario.

Key words: stars: giant, mass-loss, planetary systems – binaries: evolution – white dwarfs: formation

Send offprint requests to: gijsn@astro.uva.nl

<table>
<thead>
<tr>
<th>Name</th>
<th>Mass ($M_\odot$)</th>
<th>$v_{\text{rot}} \sin i$ (km s$^{-1}$)</th>
<th>d (pc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WD 1353+409</td>
<td>0.40</td>
<td>$&lt; 50$</td>
<td>130</td>
</tr>
<tr>
<td>WD 1614+136</td>
<td>0.33</td>
<td>$&lt; 50$</td>
<td>180</td>
</tr>
</tbody>
</table>

In this letter we suggest a different, simple solution to the formation of these single, low-mass (undermassive) white dwarfs by investigating the influence of massive planets, or brown dwarfs, in relatively close orbits around solar-like stars (Sect. 2). A short discussion of the consequences of our planetary scenario is given in Sect. 3.

2. Planets around solar-like stars

2.1. Introduction

We propose a scenario in which a solar-like star is surrounded by a massive planet, or a brown dwarf, in a relatively close orbit. When the star evolves on the giant branch it will become big enough to capture its planet via tidal forces (cf. Rasio et al. 1996; Soker 1996). The planet spirals into the envelope of the giant and a so-called common envelope phase is initiated. The frictional drag on the planet, arising from its motion through the common envelope, will lead to loss of its orbital angular momentum (spiral-in) and deposit of orbital energy in the envelope. The orbital energy is converted into thermal and kinetic energy of the envelope which is therefore being ejected. The result of this common envelope evolution is determined by the energy balance and the fate of the planet. As a result of friction, and the large temperature difference between the envelope of the giant and the equilibrium temperature of the planet, low-mass planets evaporize due to heating. In case the planet evaporizes completely, the outcome will be a single star with a rotating and reduced envelope – otherwise we end up with a planet orbiting the naked core of a giant. The destiny of this white dwarf-planet system is determined by the orbital separation.
In this letter we first present the expected outcome of a common envelope evolution between a giant and a planet; thereafter we look at the important question of the onset of this evolution. We will closely follow the treatment of Soker (1996; 1998), focusing on the cases where (most of) the envelope is lost in a common envelope, leaving a undermassive white dwarf. Research in this field has been carried out to explain elliptical and bipolar planetary nebulae (Soker 1996) and the morphology of the Horizontal Branch in clusters (Soker 1998).

2.2. The outcome of the common envelope phase

Below we outline our scenario in somewhat more detail. By simply equating the difference in orbital energy to the binding energy of the envelope of the giant we can compute the ratio of final to initial separation (Webbink 1984). Let $\eta_{eq}$ describe the efficiency of ejecting the envelope, i.e. of converting orbital energy into the kinetic energy that provides the outward motion of the envelope: $\Delta E_{bind} = \eta_{eq} \Delta E_{orb}$ or (using $m_p \ll M_{env}$):

$$a_f \simeq \frac{\eta_{eq} \lambda}{2} \frac{M_{core} m_p}{M_{env}} R_g = \frac{f}{1-\chi} \frac{m_p}{M} R_g$$

(1)

where $R_g$ is the radius of the giant star at the onset of the spiral-in phase, $\lambda$ is a weighting factor ($< 1.0$) for the binding energy of the core and envelope of the giant star, $\chi = M_{core}/M$, $m_p$ is the planetary mass and $M_{core, env}$ and $a_f$ are the mass of the helium core and hydrogen-rich envelope of the evolved star ($M = M_{core} + M_{env}$), and the final separation after all the envelope is expelled, respectively. In our calculations we chose $\lambda = 0.5$ and $\eta_{eq} = 4$ (cf. Tauris 1996; Portegies Zwart & Yungelson 1998) and hence $f = 1$.

To model the effect of planetary evaporation we follow Soker (1998) and equate the local sound speed in the giants envelope to the escape velocity from the (gaseous) planet surface in order to find the approximate location of evaporation:

$$c_s^2 \simeq v_{esc}^2 \iff \frac{k_B T}{\mu m_a} = \frac{2 G m_p}{\alpha r_p}$$

(2)

We use a temperature profile for evolved solar-like stars (cf. Fig. 1) of $T \approx 1.76 \times 10^6 (r/R_\odot)^{-0.85} K$, in the entire interval of $R_{core} < r < R_g$, where $R_{core}$ is the radius of the He-core. During the spiral-in the radius of a giant-gas planet, $r_p$, may expand slightly ($\alpha r_p, \alpha > 1$) even though only a small amount of mass ($< 0.1 m_p$) is believed to be accreted (Hjellming & Taam 1991).

Solving Eq. (2), with the temperature dependence given above and assuming $\gamma = 5/3$ and Pop.I chemical abundances ($X=0.7; Z=0.02$), yields the location of the evaporation:

$$a_{evap} = \left[ 10 \alpha \left( \frac{M_J}{m_p} \right) \right]^{1.18} R_\odot$$

(3)

where $M_J = 0.001M_\odot$ (\approx a Jupiter mass) and we have assumed $r_p = 0.1R_\odot$, which is a reasonable assumption for all planets and brown dwarfs in the mass range $0.0001 < m_p/M_\odot < 0.08$ (Hubbard 1994).

*Fig. 1.* The temperature profiles for $1 M_\odot$ evolved stars on the Red Giant Branch. Notice, that $log T$ is approximately a linear function of $log r$ at all evolutionary stages from the beginning until the tip of the Red Giant Branch (just before the helium flash).

For a given stellar structure (i.e. core and envelope mass and radius) the final outcome of the common envelope phase is determined only by the mass of the planet. We can easily compute the critical planetary mass for which the planet evaporizes just at the moment the envelope is completely expelled, i.e. when $a_{evap} = a_f$. The mass associated with this critical mass ($m_{crit}$) is found from eqs. 1 and $\beta (\alpha = 1)$:

$$m_{crit} = 10 \left( \frac{1-\chi}{\chi} \frac{M}{M_\odot} \frac{R_g}{100 R_\odot} \right)^{0.46} M_J$$

(4)

Plants more massive than $m_{crit}$ survive the spiral-in. However, in order to avoid a destructive mass transfer to the white dwarf after the spiral-in, it must have a radius smaller than its Roche-lobe given by (Paczynski 1971):

$$a_{RLO} = \frac{\alpha r_p}{0.462} \left( \frac{M_{WD}}{m_p} \right)^{1/3} R_\odot$$

(5)

where $M_{WD} = M_{core}$.

If $a_f > a_{evap}$ and $a_f > a_{RLO}$, the planet will survive and the entire envelope is lost from the giant leaving a low-mass helium white dwarf remnant with a planetary companion. However, if the final separation is small enough, the planetary orbit will decay due to emission of gravitational waves on a timescale given by:

$$\tau_{gw} \approx \frac{(a_f/60 R_{WD})^4}{(M_{WD}/M_\odot)^2 (m_p/M_J)} \approx 5.0 \times 10^9 \text{ yr}$$

(6)

Hence, also in this case the final outcome of the evolution might eventually be a single undermassive white dwarf.

Planets less massive than $m_{crit}$ will evaporate (or overflow their Roche-lobe if $a_{RLO} > a_{evap}$) before the envelope is expelled completely. However heavy planets deposit significant
 orbital angular momentum in the envelope of the giant, causing enhanced mass loss due to rotation. This could lead to ejection of the envelope by planets somewhat less massive than $m_{ \text{crit}}$.

The change in structure of the star may alter the further evolution of the giant considerably. Soker (1998) suggests that such an evolution could explain the morphology of the Horizontal Branch in clusters.

For the evolution of the giant we used the relations of Iben & Tutukov (1984) for the structure of a (Pop.I) giant on the RGB: $R_g = 10^{3.5} M_{\text{core}}^4$, $L = 10^{5.6} M_{\text{core}}^5$, $M_{\text{core}} = 10^{-5.36} M_{\odot}^{6.6}$. These equations are valid on the RGB for a low-mass star $(0.8 \leq M/M_{\odot} \leq 2.2)$.

### 2.3. The onset of the common envelope phase: tidal forces and mass loss on the RGB

The moment the common envelope starts is determined by tidal forces. In the absence of any significant tidal interaction the donor star is only able to capture planets, via Roche-lobe overflow, out to a distance, $a_{\text{max}} \approx 1.6 R_g$. Taking tidal effects into account using the equilibrium tide model (Zahn 1977; Verbunt & Phinney 1995) we find, following Soker (1996):

$$a_{\text{max}} \approx 2.4 R_g \left( \frac{1 - \frac{\Delta M}{M}}{\chi} \right)^{1/12} \left( \frac{M}{M_{\odot}} \right)^{-11/12} \left( \frac{m_p}{10 M_1} \right)^{1/8}$$

where we have used the equations for the structure of the giant as given above. In our calculations (see below) we have also included mass loss, which amounts to as much as $|\Delta M|/M \approx 0.20$ at the tip of the RGB. The mass loss is lost as a fast isotropic wind with the specific angular momentum of the giant causing the orbital separation of the planet to increase by the same ratio as the total mass of the system decreases. We modeled this effect according to the Reimers formula (Kudritzki & Reimers 1978) with $\eta = 0.6$ (cf. Rasio et al. 1996).

### 2.4. Results

We will now demonstrate an approximate picture for the fate of stars with planets of different masses and separations to illustrate the applicability of this scenario for producing undermassive single white dwarfs as observed in nature. For the evolution of a solar-like star on the Red Giant Branch we will investigate to which separation (or alternatively which orbital period) a given planet will be captured by the star and compute the outcome of the spiral-in process for different planetary masses.

In Fig. 2 we have plotted the different critical separations discussed above as a function of planetary mass. Our example is based on a 1.0 $M_{\odot}$ star with a core-mass of 0.33 $M_{\odot}$ (cf. WD 1614+136 in Table 1). We find $m_{\text{crit}} = 21 M_1$. Less massive planets expel only part of the envelope (e.g. a planet with $m_p = 15 M_1$ will only expel half of the envelope, neglecting enhanced mass loss of the giant due to the spin-up of the envelope). Planets with masses between 15 and $25 M_1$ are presumably disrupted as they fill their Roche-lobe during/after the spiral-in. Planets more massive than $25 M_1$ survive the spiral-in and will eject the entire envelope. However, for $m_p < 25 M_1$ the envelope will be disrupted as a result. Planets more massive than $25 M_1$ survive the common envelope phase but will later spiral in due to gravitational wave radiation (shaded area indicates a spiral-in timescale of less than 5 Gyr). Above 0.08 $M_{\odot}$ (80 $M_1$), the companions are heavy enough to ignite hydrogen as stars (hatched region).

![Fig. 2. Separations of interest (in units of $R_{\text{WD}} = 10000$ km) after the spiral-in phase for a 1 $M_{\odot}$ star with a core of 0.33 $M_{\odot}$ as a function of planetary mass. The solid line gives the separation for which the liberated orbital energy is equal to the binding energy of the envelope (dotted line for ejecting half of the envelope). The dashed line gives the separation below which the planet fills its Roche-lobe. The dash-dotted line gives the separation at which the planet is evaporated. A minimum planetary mass of $\sim 21 M_1$ is needed to expel the entire envelope. Planets lighter than this value are seen to be evaporated. However, for $15 < m_p/M_1 < 25$ the planet fills its Roche-lobe and is likely to be disrupted as a result. Planets more massive than $\sim 25 M_1$ survive the common envelope phase but will later spiral in due to gravitational wave radiation (shaded area indicates a spiral-in timescale of less than 5 Gyr). Above 0.08 $M_{\odot}$ (80 $M_1$), the companions are heavy enough to ignite hydrogen as stars (hatched region).](image-url)
3. Discussion

We must bear in mind the uncertainties at work in our scenario, and it is possible that future detailed studies of the interactions between a planet and a common envelope may change the mass limits derived in this letter. Also notice that the two undermassive white dwarfs in Table 1 might very well have substellar companions (brown dwarfs or planets) below the observational threshold mass of $\sim 0.1 M_\odot$.

3.1. The final mass of the white dwarf

In the lower panel of Fig. 3 we give the final white dwarf mass in case all of the envelope is expelled. We see that white dwarfs with masses between 0.20 – 0.45 $M_\odot$ can in principle be formed with this scenario. If the common envelope phase initiates while the donor is on the Asymptotic Giant Branch, a C-O white dwarf will be formed.

3.2. Rotation of the white dwarf

The final rotational period of the white dwarf is essentially determined only by the rotation of the core of the giant: the planet transfers almost all of its angular momentum to the giant’s envelope which is expelled. The rotation of the core strongly depends on the coupling between the core and the envelope of the giant (Spruit 1998), but is in any case in agreement with the measured upper-limits for the white dwarfs as given in Table 1.

3.3. A white dwarf ejected from a binary?

An other possibility for the formation of single undermassive white dwarfs is a binary origin. Consider a compact system with a giant star (the progenitor of the undermassive white dwarf) and a normal white dwarf companion. When the giant fills its Roche-lobe it transfers its envelope to the companion leaving a low-mass helium white dwarf as a remnant. The companion may be subsequently lost either because it exploded as a type Ia SNe, or formed a (high velocity) neutron star from an accretion-induced collapse – also leading to disruption of the binary.

Acknowledgements. We would like to thank Frank Verbunt for discussions and Bart Bisscheroux for providing Fig. 1. This research was supported in part by the NWO Spinoza-grant SPI 78-327. T.M.T. acknowledges the receipt of a Marie Curie Research Grant from the European Commission.

References

Paczyński B., 1971, ARA&A 9, 183