G. NELEMANS, S.F. PORTEGIES ZWART AND F. VERBUNT
Astronomical Institute “Anton Pannekoek”, University of Amsterdam
Boston University/MIT
Astronomical Institute, Utrecht University

Double white dwarfs could be important sources for space based gravitational wave detectors like OMEGA and LISA. We use population synthesis to predict the current population of double white dwarfs in the Galaxy and the gravitational waves produced by this population. We simulate a detailed power spectrum for an observation with an integration time of \(10^6\) s. At frequencies below \(\sim 3\) mHz confusion limited noise dominates. At higher frequencies a few thousand double white dwarfs are resolved individually. Including compact binaries containing neutron stars and black holes in our calculations yields a further few hundred resolved binaries and some tens which can be detected above the double white dwarf noise at low frequencies. We find that binaries in which one white dwarf transfers matter to another white dwarf are rare, and thus unimportant for gravitational wave detectors. We discuss the uncertainties and compare our results with other authors.

1 Introduction

Since most stars evolve into a white dwarf and since most stars are members of binaries, close binaries containing two white dwarfs are expected to be very numerous in our Galaxy. Such binaries may dominate the gravitational wave signal detected by space based detectors like LISA and OMEGA. The expected numbers of these binaries could be so large that they form a confusion limited noise in the detector.

Figure 1 shows a typical example of a binary evolution leading to a double white dwarf. In this example the first phase of mass transfer is stable and the second mass transfer phase results in a common envelope and a dramatic decrease of the orbital period. Further evolution of double white dwarfs is driven by the emission of gravitational waves and may lead to systems in which the lighter white dwarf transfers matter to the more heavy white dwarf. This is only stable when the mass ratio of the two white dwarfs is smaller than 2/3. Double white dwarfs with stable mass transfer are called interacting double white dwarfs or AM CVn stars.

Until recently very few double white dwarfs were known. However, improved observations have lead to a dramatic increase in the number of observed systems (see Table 1). In section 2 we describe a detailed calculation of the number of double white dwarfs in the Galaxy and compare their properties with those of the observed systems. In section 3 we compute the gravitational wave signal of these binaries and in section 4 we compare our results with those of other authors.

---

*SPZ is a Hubble Fellow*
Figure 1: Example of the formation of a double white dwarf. The masses of both stars and the orbital period at each evolutionary phase are indicated. The initial binary consists of a 2.0 $M_\odot$ star and a 1.9 $M_\odot$ star in a 232 day orbit (a semi-major axis of 250 $R_\odot$) (a). The more massive star evolves first into a giant and starts transferring matter to its companion (b). Most of the transferred matter leaves the binary and takes angular momentum with it. The orbit first shrinks but expands when the donor has become (much) less massive than its companion. When the donor has lost all its envelope it becomes a 0.42 $M_\odot$ white dwarf (c). Subsequently the companion evolves and engulfs the white dwarf. The core of the companion and the white dwarf move in a common envelope in a rapidly shrinking orbit ("spiral-in") until the envelope is ejected (d - e). A close double white dwarf results (e). The last phase is plotted again on a 100 times larger scale.

<table>
<thead>
<tr>
<th>$M_1$ ($M_\odot$)</th>
<th>$M_2$ ($M_\odot$)</th>
<th>$P_\text{b}$ (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.00</td>
<td>1.90</td>
<td>232.1</td>
</tr>
<tr>
<td>1.95</td>
<td>1.87</td>
<td>241.8</td>
</tr>
<tr>
<td>0.42</td>
<td>1.92</td>
<td>161.2</td>
</tr>
<tr>
<td>0.42</td>
<td>1.90</td>
<td>168.1</td>
</tr>
<tr>
<td>0.43</td>
<td>0.41</td>
<td>1.2</td>
</tr>
</tbody>
</table>

2 Population Synthesis

To compute the current population of close double white dwarfs we use population synthesis as follows. We initialize a large number of ‗zero-age‘ binaries and evolve these binaries according to simplified prescriptions for stellar and binary evolution, including stellar wind, mass transfer (which may involve loss of mass and angular momentum from the binary), common envelope and supernovae. The code is improved with respect to an earlier version in a more accurate treatment of the formation of white dwarfs.

For each initial binary the mass of the more massive component ($M$), the mass ratio ($q \equiv m/M \leq 1$, where $m$ is the mass of the less massive component), the period ($P$) and eccentricity ($e$) are chosen randomly from distributions given by

\[
\begin{align*}
\text{Prob}(M) & \propto M^{-2.5} \quad \text{for } 0.8 \leq M \leq 100 M_\odot \\
\text{Prob}(q) & \propto \text{cnst} \quad \text{for } 0 \leq q \leq 1 \\
\text{Prob}(P) & \propto P^{-1} \quad \text{for } 1 \leq \log P \leq 10^6 \\
\text{Prob}(e) & \propto 2e \quad \text{for } 0 \leq e \leq 1
\end{align*}
\]

(1)

We initialized 100,000 binaries and computed their evolution. Almost 27,500 of them became double white dwarfs with periods below 100 days. All these binaries have circular orbits due to the strong tidal forces during mass transfer phases. Most double white dwarfs contain two helium white dwarfs (49%), 25% contains two C/O white dwarfs, 22.5% has both varieties and 3.5% contains an O/Ne/Mg white dwarf. 52% of all systems are close enough that mass transfer
Figure 2: Left: predicted mass ratio distribution of the current double white dwarf population. We define the mass ratio for the double white dwarf as the mass of the last formed (i.e. the potentially visible) white dwarf over the mass of the first formed white dwarf. Right: gray scale plot of distribution over orbital period and mass of the visible component. Grey scale gives the expected number of visible systems with magnitude smaller than 15. Points with error bars indicate observed systems given in Table 1.

Table 1: Parameters of known double white dwarfs (for references see Maxted & Marsh, 1999 and Moran, 1999), with \( m \) denoting the mass of the visible (i.e. last formed) white dwarf. The mass ratio for the double white dwarfs is defined as the mass of the last formed (i.e. the potentially visible) white dwarf over the mass of the first formed white dwarf.

<table>
<thead>
<tr>
<th>WD</th>
<th>( P(\text{d}) )</th>
<th>( q )</th>
<th>( m/\text{M}_\odot )</th>
<th>WD</th>
<th>( P(\text{d}) )</th>
<th>( q )</th>
<th>( m/\text{M}_\odot )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0135−052</td>
<td>1.556</td>
<td>0.90</td>
<td>0.47</td>
<td>0136+768</td>
<td>1.407</td>
<td>1.31</td>
<td>0.34</td>
</tr>
<tr>
<td>0957−666</td>
<td>0.061</td>
<td>1.14</td>
<td>0.37</td>
<td>1022+050</td>
<td>1.157</td>
<td>0.35</td>
<td></td>
</tr>
<tr>
<td>1101+364</td>
<td>0.145</td>
<td>0.87</td>
<td>0.31</td>
<td>1202+608</td>
<td>1.493</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>1204+450</td>
<td>1.603</td>
<td>1.00</td>
<td>0.51</td>
<td>1241−010</td>
<td>3.347</td>
<td>0.31</td>
<td></td>
</tr>
<tr>
<td>1317+453</td>
<td>4.872</td>
<td>0.33</td>
<td>0.33</td>
<td>1713+332</td>
<td>1.123</td>
<td>0.38</td>
<td></td>
</tr>
<tr>
<td>1824+040</td>
<td>6.266</td>
<td>0.39</td>
<td>0.39</td>
<td>2032+188</td>
<td>5.084</td>
<td>0.36</td>
<td></td>
</tr>
<tr>
<td>2331+290</td>
<td>0.167</td>
<td>0.39</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

starts within a Hubble time. In 1.4% of the cases the mass transfer is stable and an AM CVn system is formed. The remaining systems merge.

The results are normalised to the estimated star formation history of the Galaxy (4 \( \text{M}_\odot \text{yr}^{-1} \)), assuming all stars are formed in binaries. With this normalization the birth rate for double white dwarfs is 0.1 \( \text{yr}^{-1} \) in the galaxy and the merger rate is \( 5 \times 10^{-2} \text{yr}^{-1} \). Of these mergers about 10% has a total mass of more than the Chandrasekhar limit (1.44 \( \text{M}_\odot \)), yielding a merger rate of \( 5 \times 10^{-3} \text{yr}^{-1} \). Since some models for type Ia supernovae involve merging of these white dwarfs this can be compared to the estimated supernova Ia rate of \( 3-4 \times 10^{-3} \text{yr}^{-1} \). The birth rate of interacting white dwarfs is only \( 1.3 \times 10^{-3} \text{yr}^{-1} \).

We assume a constant star formation history in the galaxy for the last 10 Gyr. This gives a current population of \( \sim 3 \times 10^8 \) double white dwarfs in the galaxy at present. Figure 2 shows the mass ratio distribution and the distribution over period and mass of the last formed (i.e. potential visible) white dwarf. The systems currently known are indicated. We also list the properties of these systems in Table 1.

We see that the predicted distributions match the (small) observed sample well. However, since we have not modelled the selection effects of the observed sample, we can not yet compare the number of detected systems with the predictions.
Figure 3: Gravitational wave signal for a $10^6$ s observation. Left: power spectrum of double white dwarfs, Right: power spectrum for all compact binaries in the Galaxy. The planned sensitivity for LISA for an $10^6$ s observation is indicated with the dashed line.

3 Gravitation waves

To compute the gravitational wave signal for the population of double white dwarfs, we distribute our ($\sim 3 \times 10^8$) double white dwarfs in the Galaxy according to a disk distribution with radius of 15 kpc, with the sun at 8.5 kpc from the center. The height of the disk is 200 pc. For each of these double white dwarfs we compute the strength of the gravitational wave amplitude from

$$h = 5.1 \times 10^{-22} \left( \frac{M^2}{M + m} \right) \left( \frac{M + m}{M_\odot} \right)^{2/3} \left( \frac{P_{orb}}{1 \text{ hr}} \right)^{-2/3} \left( \frac{d}{1 \text{ kpc}} \right)^{-1} \quad (2)$$

The frequency of the wave is given by $f = 2/P_{orb}$. We add the amplitudes together in frequency bins of $\Delta f = 1/T$ ($T =$ integration time) to simulate the power spectrum for this population of binaries as would be detected by gravitational wave detectors in space like LISA and OMEGA.

The result for $T = 10^6$ s is shown in Figure 3 for the population of double white dwarfs, together with the planned sensitivity for LISA. For frequencies below $\sim 3$ mHz, the number of double white dwarfs is so large that the signal is confusion limited. This means that in all frequency bins a large number of double white dwarfs is added randomly, making it impossible to resolve the individual binaries and producing a broad noise component. Only above a few mHz is the number density of systems sufficiently small that systems can be resolved individually. Some double white dwarfs are so close to the Earth, that their signal is actually strong enough to be detected above the noise. The AM CVn systems do not contribute significantly either to the noise or to the detectable sources.

When we compute the gravitational wave signal for all binaries, there is no qualitative difference. We still have the double white dwarf ‘noise’ dominant at low frequencies, but we have some 10 - 100 other resolved binaries and especially more strong signals above the noise.

4 Uncertainties and comparison with other authors

4.1 Uncertainties

Our computations are affected by uncertainties in our assumptions about the parameters of the initial binaries and about the details of the binary evolution.
Table 2: Birth rates for double white dwarfs by different groups

<table>
<thead>
<tr>
<th></th>
<th>birth rate Dwd</th>
<th>birth rate AM CVn’s</th>
</tr>
</thead>
<tbody>
<tr>
<td>this work</td>
<td>0.1</td>
<td>0.0013</td>
</tr>
<tr>
<td>Han 1998</td>
<td>0.03</td>
<td>0.026</td>
</tr>
<tr>
<td>Iben et al. 1997</td>
<td>0.09</td>
<td>0.02</td>
</tr>
</tbody>
</table>

The most uncertain of our assumed initial distributions (equation 1) is that of the mass ratio. In our calculation more than 50% of all double white dwarfs is formed from an initial binary with mass ratio larger than 0.6. An initial distribution that is more skewed towards higher mass ratios will therefore produce more double white dwarfs. This difference can be estimated from the work by Yungelson et al. who use a mass ratio distribution $\text{Prob}(q) \propto 2^q$ giving a higher merger rate for heavy white dwarfs by almost a factor 2.

An other uncertainty lies in the normalization of the number of binaries and single stars in our Galaxy. The star formation history is inferred from the enrichment of the interstellar gas and involves detailed modeling of stellar evolution in low metallicity environments. Note that we also assume a star forming rate which is constant in time. The value of $4 \, M_\odot \, \text{yr}^{-1}$ that we use is therefore uncertain. Since at least 50% off all stars are in binaries, the assumption that all stars are formed in binaries introduces at most a relatively small error. We estimate that the uncertainty in the total number of binaries formed is about a factor of 2.

The uncertainties in the binary evolution are large especially in the description the mass transfer and of the spiral-in process in a common envelope. Note that at least one common envelope during the evolution of the binary is necessary for the formation of a double white dwarf that is close enough to be a significant source of gravitational waves.

In a common envelope the companion is engulfed by the envelope of the donor. Frictional forces slow down its motion and heat the envelope causing it to be expelled. The details of this process involve 3D hydrodynamics, so we can only guess the efficiency with which the heating process expels the envelope. Varying this efficiency by a factor 4 results in birth rates of double white dwarfs which differ by a factor 2.

When stars transfer matter, but do not get into a common envelope phase some of the transferred material may be lost, because the accretor can not accrete all the matter. The angular momentum that is dragged along with this matter is uncertain. Varying this angular momentum by a factor 2 results in birth rates of double white dwarfs which differ by a factor 5. So this is the most important uncertainty. In the computations presented here we used a rather high loss of angular momentum with the transferred matter (three times the mean specific angular momentum of the orbit).

4.2 Comparison with other authors

We compare our results with the work of Han and Iben et al. Han uses a grid of stellar evolution models, computed with the Eggleton code. He then models the binary evolution separately and picks models from the grid. Iben et al. use the binary evolution program developed by Tutukov and Yungelson and essentially use a similar approach to ours. They make different assumptions about the mass transfer phases in the binary evolution, especially when mass is lost from the system.

As can be seen from Table the predicted birth rates for the double white dwarfs differ by a factor of 5. In part these differences are due to different assumed normalization of the initial binary population; in part due to different assumptions about the loss of angular momentum from the binary.
The discrepancies are even worse for the birth rate of AM CVn systems, which differ by a factor of 20. The formation of a stable AM CVn system requires the mass ratio to be less than 2/3. Both Han and Iben et al. derive a mass ratio distribution of the current double white dwarf population peaked at 0.5, whereas we find the peak around 0.9 (Figure 2). Thus according to Han and Iben et al. a much larger fraction of double white dwarfs evolves into stable AM CVn systems. At this moment we do not fully understand why the mass ratio distributions peak at different values.

5 Conclusions

We conclude that the expected population of double white dwarfs produces a confusion limited noise component in space based gravitational wave detectors like OMEGA and LISA at frequencies below $\sim 3$ mHz. Above this limit we expect $\gtrsim 2000$ resolved binaries, of which the majority are double white dwarfs, but with a few hundred binaries containing neutron stars and/or black holes. About 10 - 100 compact binaries (mainly with neutron star or black hole components) can be detected individually above the noise at low frequencies.

We further conclude that AM CVn systems are not important sources for space based detectors, neither as detectable individual sources, nor as confusion limited noise.

Note that our computations do not include the formation of binaries in globular clusters (reviewed in Hut et al.\textsuperscript{[6]}), nor do they include very exotic binaries which are too rare to be formed in our limited computed sample of 100,000 binaries.

Finally we stress that uncertainties in population synthesis lead to different results by different groups. Especially the amount of angular momentum that is dragged along with matter that leaves the binary system is uncertain and introduces large differences in the results. We hope to investigate these differences in the future, leading to a better understanding of binary evolution and better predictions of the current population of various types of (compact) binaries.

Acknowledgments

This work was supported by NWO Spinoza grant 08-0 to E.P.J. van den Heuvel and by NASA through a Hubble Fellowship grant awarded by the Space Telescope Science Institute, which is operated by AURA Inc., for NASA under contract NAS 5-26555.