

Reconstructing the evolution of double helium white dwarfs: envelope loss without spiral-in

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Abstract. The unique core-mass – radius relation for giants with degenerate helium cores enables us to reconstruct the evolution of three observed double helium white dwarfs with known masses of both components.

The last mass transfer phase in their evolution must have been a spiral-in. In the formalism proposed by Webbink (1984), we can constrain the efficiency of the deposition of orbital energy into the envelope to be $1 \lesssim \alpha \lesssim 6$, for an envelope structure parameter $\lambda = 0.5$. We find that the two standard mass transfer types (stable mass transfer and spiral-in) are both unable to explain the first phase of mass transfer for these three binaries.

We use a parametric approach to describe mass transfer in low-mass binaries, where both stars are of comparable mass and find that the orbital characteristics of the observed double helium white dwarfs can be well reproduced if the envelope of the primary is lost with ~ 1.5 times the specific angular momentum of the initial binary. In this case no substantial spiral-in occurs.

Key words: stars: white dwarfs – stars: mass loss – binaries: close – stars: evolution

1. Introduction

The long lasting problem that we observe many double stars which are expected to form close pairs of white dwarfs, but yet that of the observed white dwarfs not one seemed to have a close white dwarf companion, was solved by the discovery of such pairs, starting with L870-2 (= WD 0135+052) in 1988 (Saffer et al. 1988). In total 14 close detached binary white dwarfs are known at present, see Table 1. The fact that six of these systems have their orbital period and the masses of both components determined provides an opportunity to test binary evolution theory in detail.

Models for the formation of close double white dwarfs envision two standard scenarios to produce these systems (Tutukov & Yungelson 1981; Iben & Tutukov 1984; Webbink 1984; Tutukov & Yungelson 1988; Han et al. 1995; Iben et al. 1997; Han 1998). In the first scenario, two low-mass ($M \lesssim 2.3 M_{\odot}$) stars evolve through two stages of spiral-in. The first spiral-in will shrink the orbit, so the second spiral-in happens in a binary with a much smaller orbital separation. The Roche lobe filling giant (secondary) now has a small radius and therefore a small core. The white dwarf that is formed last is thus less massive than its companion, with a mass ratio $m_{\text{bright}}/m_{\text{dim}} \lesssim 0.55$ (see e.g. Sandquist et al. 2000).

In the second scenario, the first phase of mass transfer is stable; the second phase of mass transfer is again a spiral-in. The evolution of the orbit and the growth of the core during the first, slow phase of mass transfer depend on the amount of mass and angular momentum that is lost from the system. If the evolution in this phase is conservative, the expected final mass ratio $m_{\text{bright}}/m_{\text{dim}} \approx 1.14 - 1.18$ (Tutukov & Yungelson 1988; Sandquist et al. 2000).

All white dwarfs in close pairs known today have low masses ($M \lesssim 0.5 M_{\odot}$). Note, however, that the inaccuracy of the mass determinations is as large as $\sim 0.05 M_{\odot}$ due to uncertainties in model atmospheres and cooling curves for white dwarfs (see e.g. Napiwotzki et al. 1999). These low masses suggest they are helium white dwarfs, but it cannot be excluded *a priori* that white dwarfs with masses $\gtrsim 0.35 M_{\odot}$ are so called hybrid white dwarfs, i.e. having small CO cores and relatively thick ($\sim 0.1 M_{\odot}$) helium envelopes (Iben & Tutukov 1985). For the most massive ones ($M \gtrsim 0.45 M_{\odot}$), this is even the only option, since helium white dwarfs must have a mass below $0.46 M_{\odot}$ (Sweigart et al. 1990). For the less massive ones the probability to form hybrid white dwarfs is 4 – 5 times lower than to form helium white dwarfs (Nelemans et al. 2000).

Table 1. Parameters of known close double white dwarfs with m_{WD} denoting the mass of the brightest white dwarf and M_{WD} denoting its companion. For references see Maxted & Marsh (1999)¹ and Maxted et al. (2000).

WD	$P(\text{d})$	m_{WD}/M_{\odot}	M_{WD}/M_{\odot}
0135-052	1.556	0.47	0.52
0136+768	1.407	0.44	0.34
0957-666	0.061	0.37	0.32
1101+364	0.145	0.31	0.36
1204+450	1.603	0.51	0.51
1704+481	0.145	0.39	0.56
1022+050	1.157	0.35	
1202+608	1.493	0.40	
1241-010	3.347	0.31	
1317+453	4.872	0.33	
1713+332	1.123	0.38	
1824+040	6.266	0.39	
2032+188	5.084	0.36	
2331+290	0.167	0.39	

¹ For WD 0136+768 we give the masses of components after Bergeron et al. (1992), correcting a misprint in Maxted & Marsh (1999).

2. Reconstructing the binary evolution

Because of the unique core-mass – radius relation for giants with degenerate helium cores (Refsdal & Weigert 1970), we can reconstruct the mass transfer phases in which helium white dwarfs are formed. The mass of the brightest star in WD 1101+364 ($0.31 M_{\odot}$) indicates that it is a helium white dwarf. In WD 136+768 and WD 0957-666 it cannot be excluded that the brightest stars are hybrid white dwarfs. The low mass of the dimmer companions in these three systems indicates that those are all helium white dwarfs. The white dwarfs in WD 0135-052 and WD 1204+450 are formally inconsistent with being helium white dwarfs, so we will not include them in the discussion anymore. WD 1704+481 probably consists of a helium white dwarf and a dimmer CO white dwarf, but because CO white dwarfs cool faster than helium white dwarfs (Driebe et al. 1998), it is not clear which of the white dwarfs was formed most recent so we cannot use this system in our present study.

Assuming that the mass of the white dwarf is equal to the mass of the core of the giant at the onset of the mass transfer, the radius of the progenitor of a helium white dwarf can be calculated from the core-mass – radius relation given by Iben & Tutukov (1985):

$$R \approx 10^{3.5} M_c^4 \quad (1)$$

(R and M_c in solar units). This equation is in good agreement with other equations describing this dependence for giants (e.g. Webbink et al. 1983). The *mass of the white dwarf progenitor* is however not known, since the above relation is independent of the total mass of the star. However the mass of the giant must be in the range 0.8 - 2.3

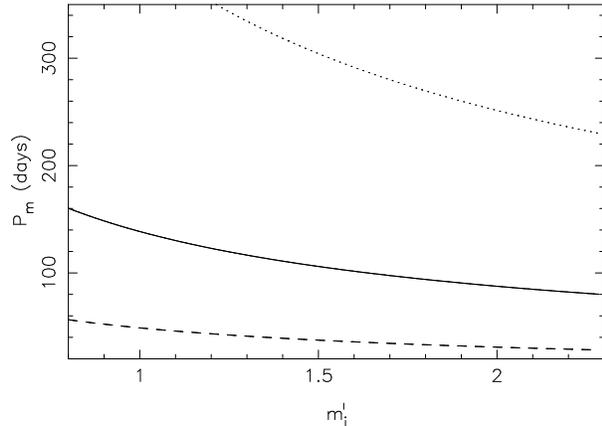


Fig. 1. Periods before the spiral-in phase in which the younger white dwarf was formed as function of the mass of its giant progenitor. Lines from bottom to top: WD 1101+346, 0957-666 and 0136+768

M_{\odot} . For less massive stars the main-sequence life time is larger than the age of the Galaxy. More massive stars do not form degenerate helium cores.

For the remainder of this article we use the following notation: M_i and M_{WD} indicate the initial mass of the original primary and the mass of the white dwarf that it forms, m_i and m_{WD} represent the same for the original secondary. If the secondary accretes mass during the first phase of mass transfer, we represent its new mass with m'_i . For the radii of the stars when they become giants we use R_g and r_g for the original primary and secondary respectively. With P_i , P_m and P we indicate the initial period, the period after the first phase of mass transfer, and the current period of the binary.

3. Last mass transfer: spiral-in

Using Eq. (1) we calculate the radii of the progenitors of the brightest white dwarfs for the three double helium white dwarfs. Since we know the mass of the white dwarf that orbited this giant and may reasonably assume that it did not accrete anything during the spiral-in phase, we can calculate the orbital separation at the onset of the spiral-in, as function of the mass m'_i of the giant,

$$a_m(m'_i) = \frac{r_g(m_{\text{WD}})}{r_L(M_{\text{WD}}/m'_i)}, \quad (2)$$

where $r_L \equiv R_L/a$ is the dimensionless Roche lobe radius, given e.g. by Eggleton (1983) and we assume $r_g = R_L$. This is shown in Fig. 1, where we use Keplers 3rd law to compute the period from the orbital separation. Comparing the periods in Fig. 1 with the observed periods in Table 1, we see that in the last mass transfer phase the orbital separation must have reduced dramatically. This

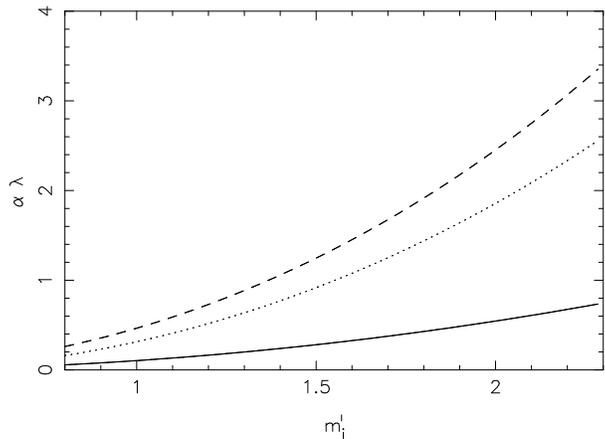


Fig. 2. The parameter $\alpha\lambda$ for WD 0957-666, 1101+364 and 0136+768 assuming that the brightest component is a helium white dwarf and their orbital periods did not change since the end of the spiral-in stage. Lines from bottom to top are for WD 0957-666, 0136+768 and 1101+364.

can only be accomplished if the last mass transfer was a spiral-in.

In a spiral-in, the envelope of the giant is expelled at the expense of the orbital energy of the binary. Balancing the binding energy of the envelope of the giant with the difference in orbital energy (Webbink 1984) one finds

$$\frac{M_{\text{WD}} (m'_i - m_{\text{WD}})}{\lambda r_g} = \alpha \left[\frac{M_{\text{WD}} m_{\text{WD}}}{2 a_f} - \frac{M_{\text{WD}} m'_i}{2 a_m} \right]. \quad (3)$$

The parameter λ depends on the structure of red giant envelope. The usual assumption is that $\lambda = 0.5$ (De Kool et al. 1987). The parameter α represents the efficiency of the deposition of orbital energy into the common envelope. To reduce the number of parameters, the product $\alpha\lambda$ is treated as a single parameter in the remainder of this article, but it should be noted that both λ and α will in reality be functions of the evolutionary stage of the stars.

Applying Eq. (3) and Eq. (2), we find $\alpha\lambda$ as a function of m'_i for the three systems considered. We plot this in Fig. 2, where we assume that the current periods are equal to the post spiral-in periods. This may not be the case in general since close orbits like the those of WD 0957-666 and WD 1101+364 will decay due to the loss of angular momentum by gravitational radiation. However, the estimated ages for these white dwarfs ($\sim 10^7$ yr (Moran et al. 1997) and $\sim 10^9$ yr (using the cooling curves of Driebe et al. (1998)) respectively) are short compared to the orbital decay time scale.

For the remaining white dwarf pairs listed in Table 1 the mass of only one component is known. We assume here that it is the last formed component we observe. Low-mass white dwarfs may have thick hydrogen envelopes which make them cool very slowly (Driebe et al. 1998) and the situation in which the older white dwarf is really observed

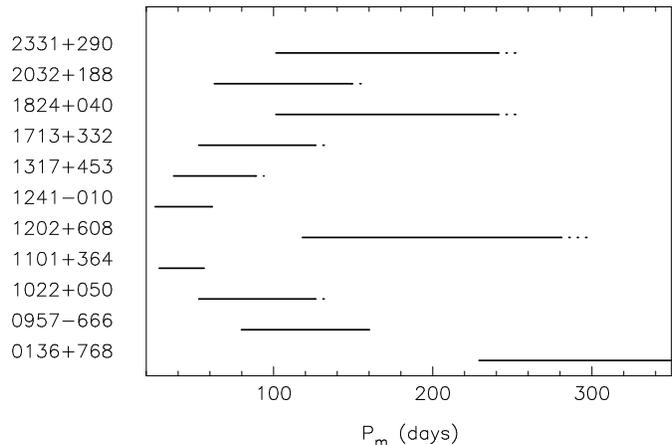


Fig. 3. Periods before the spiral-in for all double white dwarfs in which the last formed is a helium white dwarf. Limits allow for the (unknown) mass of the progenitor of this white dwarf and the mass of the unseen companion. Solid lines are for a companion mass between 0.2 and $0.65 M_{\odot}$. Dotted line gives the limit for a companion mass of $0.65 - 1.4 M_{\odot}$.

cannot be excluded *a priori*. However, as we show in an forthcoming paper (Nelemans et al. 2000), in the majority of binary white dwarfs we indeed observe the youngest of the two dwarfs.

From Figs. 1 and 2 we see that we find a range of P_m 's and $\alpha\lambda$'s for WD 0957-666, 1101+364 and 0136+768 where the mass of the second white dwarf is known. For the remaining systems we can also compute a range of P_m 's and $\alpha\lambda$'s by determining the ranges for all possible masses of the unseen companion. We do know that the companion almost certainly is another white dwarf, so the mass of this object must be between 0.2 and $1.4 M_{\odot}$ and most probably even below $0.65 M_{\odot}$ (Iben et al. 1997; Han 1998; Nelemans et al. 2000). Having also in mind that the mass of white dwarf progenitor is in the $0.8 - 2.3 M_{\odot}$ range we can derive the possible ranges of intermediate periods and $\alpha\lambda$'s for all double white dwarfs, as is shown in Figs. 3 and 4.

Two conclusions can be drawn from Figs. 1 to 4:

1. The efficiency of the energy deposition into the common envelopes α must be high. From model calculations of stellar structure we know that $\lambda \sim 0.5 - 1.0$. If we assume that α does not depend on the evolutionary state of the giant or the combination of the masses of the giant and the white dwarf, the parameters of all double white dwarfs with two observed helium components can be reproduced with the same $\alpha \sim 4$. The only exception is WD 0957-666 for which the efficiency appears to be much lower (see Sect. 6.2 for a different solution). Note that an error in the masses of the white

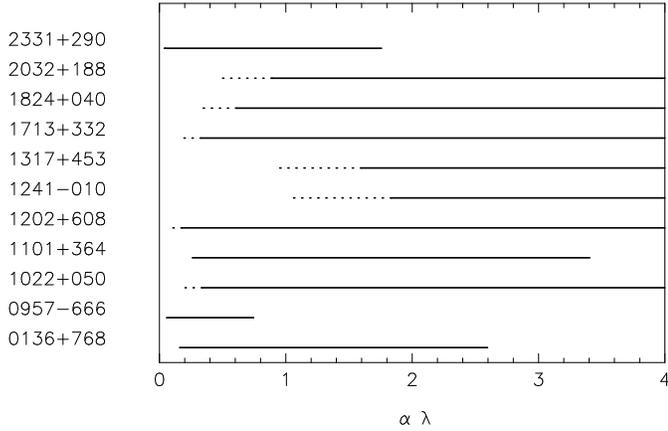


Fig. 4. The parameter $\alpha\lambda$ for the same cases as in Fig. 3

dwarfs of $0.05 M_{\odot}$ translates to an error in the value of $\alpha\lambda$ of a factor ~ 1.5 . However even this does not bring the value of α to 4 for WD 0957-666.

Since Eq. (3) only considers a rough energy budget, the conclusion $\alpha > 1$ could simply mean that we do not calculate the energy accurately. It does mean that the orbital energy deposition into common envelope has to be highly efficient. It could also mean that sources other than the orbital energy contribute to the process of common envelope expulsion (Iben & Livio 1993). E.g. it is possible that the envelope is partially removed before Roche lobe contact by an enhanced stellar wind due to tidal interaction between the giant and the companion (Tout & Eggleton 1988), yielding a lower value of $\alpha\lambda$.

2. The immediate progenitors of the known close double white dwarfs (i.e. the white dwarf + giant binaries) all had rather wide orbits (25 – >500 days). This has important consequences for the understanding of the first phase of mass transfer.

4. The first mass transfer

We compute the evolution of the binary parameters in the first mass transfer, where we start from the initial binary and evolve it forward according to the two standard scenario's. The resulting periods should be equal to the intermediate periods we reconstructed in the previous section.

4.1. Spiral-in

In the case when the first mass transfer was also a spiral-in, we can compute the period after the spiral-in by making a (standard) assumption that $\alpha\lambda$ is constant (i.e. here, as derived above $\alpha\lambda = 2$). The initial separation at which the primary fills its Roche lobe is again determined by Eq. (1) and a value for M_i between 0.8 and $2.3 M_{\odot}$. Applying

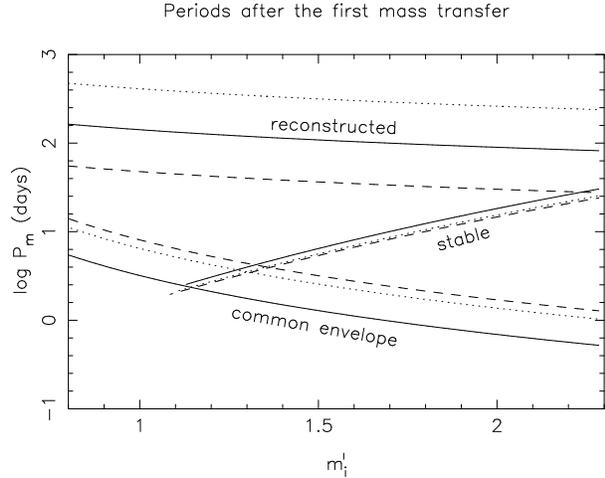


Fig. 5. Periods after the first phase of mass transfer (P_m) as function of the mass of secondary at this time (m_i^1). Top three lines are periods needed to explain the mass of the second formed white dwarf (see Fig. 1). Middle three lines give the maximum period would the first phase of mass transfer be an Algol phase, lower three ones are for the case the first mass transfer phase was a spiral-in. Solid lines for WD 0957-666, dashed lines for WD 1101+346 and dotted lines for WD 0136+768.

Eq. (3), we find a_m (and thus P_m) as a function of M_i and m_i . The maximum period after the spiral-in is given for the case $M_i = m_i$, which we plot in Fig. 5 (bottom three lines). These periods are clearly much smaller than the periods derived in the previous section (see Fig. 1). We reproduce the latter on a logarithmic scale as the top three lines in Fig. 5. In a recent article Sandquist et al. (2000) came to the same conclusion, based on the observed mass ratios and also concluded that it is very hard for binaries containing two low-mass stars to survive two spiral-in phases.

It may be argued that $\alpha\lambda$ is different in the first mass transfer phases, because the companion now is a main-sequence star instead of a white dwarf. We can, just as in the case of the second mass transfer determine the value of $\alpha\lambda$ that is required to get the right period after the mass transfer. *If we do this, however, we find $-15 \lesssim \alpha\lambda \lesssim -5$ clearly out of the allowed range. This means that a spiral-in phase in the first mass transfer phase for these systems is ruled out.*

4.2. Stable mass transfer

For the alternative scenario, to start with a short period zero-age binary and evolve through a phase of stable mass transfer, the zero-age orbital period should be in a narrow interval, such that the primary fills its Roche lobe at the moment it still has a radiative or at least shallow convective envelope. For these stars mass transfer in which the donor stays in thermal equilibrium is possible if the ini-

tial mass ratio $q_i \equiv m_i/M_i \gtrsim 0.8$ (Tutukov et al. 1982). For more extreme mass ratios; $q_i \gtrsim 0.3$, mass transfer can proceed on a timescale in between the thermal and the dynamical timescale of the donor (e.g. trial computations for a system with $M_i = 2.3M_\odot$, $m_i = 0.8M_\odot$ and $a_i = 12.8R_\odot$ show that $\log \dot{M}_i \lesssim -5.5M_\odot \text{ yr}^{-1}$; A. Fedorova, private communication). However for mass ratios $q_i \lesssim 0.5$, the thermal timescale of the accretor is much longer than the mass transfer timescale and the accretor is expected to expand, leading to a contact configuration which is unstable (e.g. Kippenhahn & Meyer-Hofmeister 1977), unless the secondary is a convective low-mass main-sequence star (Webbink 1977).

Since we want to explore the limits of the Algol scenario, we allow all values of $q_i > 0.3$ and calculate the possible periods after the Algol phase.

We assume the mass transfer is conservative and for a given total mass of the system $M_{\text{tot}} = M_{\text{wd}} + m'_i$, the initial masses are $M_i = M_{\text{tot}}/(1 + q_i)$ and $m_i = q_i M_i$. We consider systems in which the primary fills its Roche lobe just before the star develops a relatively deep convective envelope (the outer 50% of the mass is convective). The radii of such stars are obtained by a fit to stellar models by Mengel et al. (1979): $R_{\text{max}} \approx 2.4 M_i^{1.56}$ (solar units). Using the approximation to the Roche lobe given by Paczyński (1967) and the equation for the change in period for conservative mass transfer we find the period after the Algol phase is

$$P_m \approx 1.38 \frac{M_{\text{tot}}^{7.84}}{(M_{\text{wd}} m'_i)^3} \frac{q_i^3}{(1 + q_i)^{7.84}} \text{ days.} \quad (4)$$

This equation has a maximum for $q_i = 0.62$. We compute these maximal periods as function of m'_i , where of course M_i and m_i are limited by $M_i > 0.8M_\odot$, because the primary has to evolve off the main-sequence and $m'_i < 2.3M_\odot$, because the secondary must still develop a degenerate helium core.

The resulting periods for $q = 0.62$ are shown as the middle set of lines in Fig. 5, where we actually used the equation for the Roche lobe given by Eggleton (1983), which is better in this mass ratio regime. The obtained periods are clearly not long enough to explain the origin of WD 0957-666, 1101+364 and 0136+768.

One could argue that the assumption of conservative evolution is not correct. Han (1998), for example, in his ‘best’ model assumes 50% of the mass is lost with the specific angular momentum of the donor. This could yield wider orbits than obtained above. Looking at his Fig. 7 one sees that he can indeed explain WD 1101+364, but not WD 0957-666 and especially not WD 0136+768, because for these systems the masses of the last formed white dwarfs (i.e. the periods after the Algol phase) are too large. To get higher masses one needs to lose less angular momentum with the mass, leading to even wider orbits. However, analysis of observed low-mass Algols, which are binaries currently in this stage of stable mass exchange

Table 2. Parameters of wide binaries with giant and white dwarf components. Masses in solar units.

Name	P (d)	M	m_{WD}	ref
S1040	42.8	1.5	0.22	Landsman et al. (1997)
AY Cet	56.8	2.2	0.25	Simon et al. (1985)

(e.g. Refsdal et al. 1974; Masevich & Yungelson 1975; Giannuzzi 1981; Iben & Tutukov 1984; Kraicheva et al. 1986; Maxted & Hilditch 1996) shows that their periods are smaller than would be expected with Eq. (4). This suggests that descendants of Algol-type systems have orbital separations even smaller than in the case of conservative evolution, which we assumed for Fig. 5.

We conclude that with the above assumptions Algol evolution in the first mass transfer stage for WD 0136+768, 0957-66 and 1101+364 is ruled out also.

There are some other observed systems with white dwarfs which probably could not be formed through Algol-type evolution (Table 2), since white dwarf + giant binaries with $M_{\text{WD}} \lesssim 0.25M_\odot$ end their Algol phase with periods below ~ 30 d. (Kraicheva et al. 1986). In another system, HD 185510, with $P_{\text{orb}}=20.7$ day, a giant has a hot companion which is classified as a $0.3 M_\odot$ sdB star from its temperature and gravity (Frasca et al. 1998). However, it also fits the range of temperatures and gravities for $\sim 0.25M_\odot$ helium white dwarfs (Driebe et al. 1998) and the system may be a viable Algol descendant. If it is an sdB star, which *a priori* is less likely, it actually matches a scenario similar to one for WD 0957-666 rather well (see Sect. 6.2).

5. Unstable mass transfer revised

Since both standard scenarios for the first phase of mass transfer appear to be ruled out, the situation apparently is more complex. Tout & Eggleton (1988) (see also Han 1998) assume that due to tidal effects of the companion, the star can lose up to 150 times more mass in the stellar wind than without companion. This has of course the desired effect that the orbit widens before the mass transfer starts, and that there is less envelope mass left to be expelled, leading to a less dramatic spiral-in. However, if we recompute the lines for a spiral-in in Fig. 5 with reduced envelopes such that they overlap with the reconstructed lines, we find that for WD 1101+364 we need to reduce the envelope by 70% and for the others by even more than 90%.

In searching for a different solution we start by noting that the original spiral-in picture (Paczyński 1976) considers a companion which orbits inside the envelope of the giant. The companion experiences drag forces while moving in the envelope and frictional effects brake the companion. In this process orbital energy is transformed into heat and motion of the gas and finally into kinetic energy

that causes the envelope of the giant to be expelled. This picture is very much based on the situation where there is a tidal instability which causes the decay of the orbit of the companion in systems with a high mass ratios of the components (Darwin 1908; Counselman 1973). In the case that the common envelope is caused by a runaway mass transfer, the common envelope will not look much like the equilibrium envelope of the star and worse, the angular momentum of the orbit is so large that the common envelope is in principle easily brought into co-rotation with the orbit. At that moment there are no drag forces anymore.

Since for stars with deep convective envelopes mass loss on a dynamical time scale seems, in the current state of the art in stellar evolution modelling, inevitable, we have to assume that in the progenitors of the observed double white dwarfs some kind of common envelope engulfing the whole system forms. The parameters of the observed close binary white dwarfs suggest that this envelope is subsequently lost without much spiral-in. The energy to expel the envelope may be supplied by the luminosity of the giant or by tidal heating, or a combination of both. In absence of a detailed physical description we will describe the effects of this mechanism in terms of the angular momentum balance.

We compare the pre- and post-mass transfer binaries, under the assumption that the envelope of the giant is lost completely from the binary (i.e. $m'_i = m_i$), and that this mass loss reduces the angular momentum of the system in a linear way, as first suggested for the general case of non-conservative mass transfer by Paczyński & Ziolkowski (1967)

$$J_i - J_m = \gamma J_i \frac{\Delta M}{M_{\text{tot}}}, \quad (5)$$

where J_i is the angular momentum of the pre-mass transfer binary and M_{tot} is the total mass of the binary. The change of the orbital period as function of the initial and final masses of the components then becomes

$$\frac{P_m}{P_i} = \left(\frac{M_{\text{WD}} m'_i}{M_i m_i} \right)^{-3} \left(\frac{M_{\text{WD}} + m'_i}{M_i + m_i} \right) \left(1 - \gamma \frac{\Delta M}{M_i + m_i} \right)^3. \quad (6)$$

We can estimate γ for the three double helium white dwarf systems, just as for the case of $\alpha\lambda$. It turns out that all three systems are consistent with a value of γ between ~ 1.4 and ~ 1.7 (Fig. 6).

Thus the mass transfer from a giant to a main-sequence star may in general either be stable (in the case where the giant still has a radiative, or at least not too deep convective envelope), unstable, leading to a spiral-in, or a process in which the envelope is lost without much of a spiral-in. Which systems do and which do not experience a spiral-in is related to the mass ratio of the components. As can be seen from Eq. (6) for systems with $\gamma \Delta M \approx M_i (1 + q)$ the periods already become very small and the effect is essentially the same as in the case of a spiral-in.

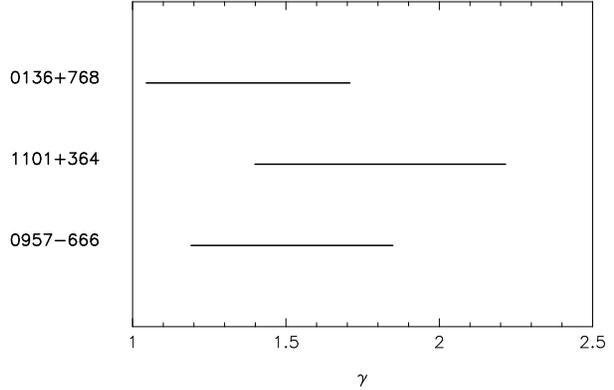


Fig. 6. Derived range of possible values of γ , for the three double helium white dwarfs.

6. Formation of observed systems

From Figs. 2 and 4 it's clear that WD 0136+768, WD 0957-666 and WD 1101+364 could be formed with $\alpha\lambda$ below 0.8. However from Fig. 4 it looks like the extremely short-period system WD 0957-66 falls out of the sample in the sense that the other systems, as well as systems with unobserved companions, are compatible with a value of $\alpha\lambda \sim 2$.

6.1. Formation of helium white dwarf pairs

In Fig. 7 we show evolutionary scenarios for WD 0136+768 (left) and WD 1101+364 (middle) in which they consist of two helium white dwarfs. We included the effect of stellar winds which was not taken into account in the preceding discussion. Therefore we use slightly different values for γ (1.75 and 1.85 respectively) than the values derived in Fig. 6. The difference in the initial mass ratio of the components results in dramatically different orbital periods after the envelope of the primary is lost: almost tripling in the first case and decrease by half in the second. As a result, after the second mass transfer episode the second white dwarf is the more massive member of the pair in WD 0136+768 but the less massive in WD 1101+364. The large difference in periods and especially masses after the first phase of mass transfer, results in rather different final orbital periods.

For WD 0957-666 a scenario in which both components are helium white dwarfs can also be constructed but then one has to assume that $\alpha\lambda$ is atypically small. However we suggest a different scenario.

6.2. An alternative scenario for WD 0957-666:

carbon-oxygen white dwarf with helium companion

There is another solution which allows us to explain the origin of WD 0957-666 using $\alpha\lambda \approx 2$ and $\gamma \approx 1.75$, like for the other two systems.

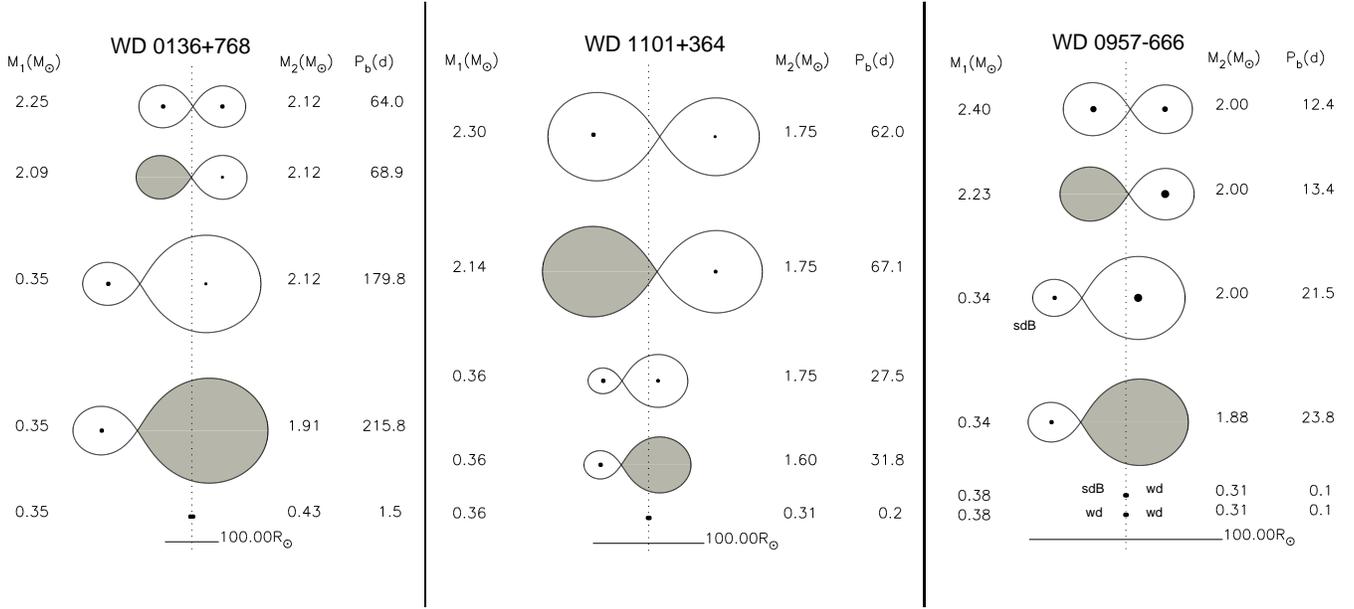


Fig. 7. Evolutionary scenarios for the formation of WD 0136+768, WD 1101+364 and WD 0957-666 (left to right). In all scenario’s the primaries lose their envelope after filling their Roche lobe, causing a change in the orbital period described by Eq. (6). Whether the orbit widens (WD 0136+768 and WD 0957-666) or shrinks (WD 1101+364) depends on the mass ratio. The second mass transfer always results in a spiral-in. For WD 0957-666 we present a scenario in which in the first mass transfer a helium star (sdB star) is formed which becomes a hybrid white dwarf only after the companion has become a helium white dwarf in the second phase of mass transfer.

The mass of the observable white dwarf in WD 0957-666 system allows it to be a hybrid white dwarf (Iben & Tutukov 1985). Such white dwarfs descend from stars with initial mass $2.3 M_\odot - 5 M_\odot$, which fill their Roche lobes in the stage of hydrogen burning in a shell around a non-degenerate helium core, become hot subdwarfs in the core helium-burning stage, but don’t experience envelope expansion after the formation of a degenerate carbon-oxygen core. Their masses are between 0.33 and $0.8 M_\odot$. The formation of hybrid white dwarfs was considered in the study of the population of white dwarfs by Tutukov & Yungelson (1993, see their scenario 3) and all their following studies.

In a scenario shown in Fig. 7 (right) we start with a system of $2.4 M_\odot$ and $2.0 M_\odot$ in a relatively close orbit ($a_0 \approx 37 R_\odot$). At the instant of Roche lobe overflow the primary has a deep convective envelope and we apply Eq. (6) to compute the change in the orbital period. The primary becomes a compact helium star. A peculiarity of low-mass helium stars is their long life time, $\sim 1.1 \times 10^7 M^{-3.75}$ yr for $0.33 \lesssim M/M_\odot \lesssim 0.7$ (Pols et al. 1991), comparable to the lifetime of their main-sequence progenitors. As a consequence, the initially slightly less massive secondary fills its Roche lobe and becomes a helium white dwarf while the former primary still burns helium in its core. For some 250 Myr the system could be observed as a hot subdwarf with a companion unseen due to the difference in luminosities. After core helium exhaustion the subdwarf cools and becomes a “hybrid” white dwarf. Moran et al. (1997) estimate from its T_{eff} that the

cooling age of this white dwarf is only $\sim 10^7$ yr and that the ratio of the luminosities of components is close to 5. This is compatible with the age of about 250 Myr expected in our scenario for the $0.31 M_\odot$ companion.

7. Conclusion

We followed the binary evolution for three double helium white dwarfs backwards and came to the following conclusions.

1. The last phase of mass transfer (the primary has already become a white dwarf and the secondary fills its Roche lobe) was a spiral-in, for which we can constrain α , which describes the efficiency of orbital energy deposition into the common envelope, to lie between 1 and 6, assuming a structure parameter $\lambda = 0.5$. This efficiency value may be an overestimate since λ may increase towards 1 at the end of the first red giant stage. Our result is in agreement with values of $\alpha \sim 4$ found in population synthesis studies of low-mass X-ray binaries (Tauris 1996), double neutron stars (Portegies Zwart & Yungelson 1998) and double white dwarfs (Nelemans et al. 2000).
2. The parameters of all observed double helium white dwarfs may be reproduced with the same $\alpha\lambda \approx 2$.
3. WD 0957-666 is the only system for which $\alpha\lambda$ appears to be lower if both components are helium dwarfs. However, this system might have been formed with

- $\alpha\lambda \approx 2$ if the immediate precursor of the currently observed white dwarf was a non-degenerate helium star and now it is a hybrid white dwarf.
4. In order to explain the relatively high masses of the observed white dwarfs in close pairs, their direct progenitors (i.e. white dwarf + giant binaries) must have had relatively wide orbits (between 25 and >500 days). The standard cases of mass transfer (Algol evolution and spiral-in) applied to the first phase of mass transfer, can not explain these intermediate wide orbits. Only if the masses of the observed white dwarfs are (much) lower than the current estimates (i.e. below $0.3 M_{\odot}$) they could be formed through a phase of stable mass transfer.
 5. We suggest that in the first mass transfer phase for low-mass binaries with similar masses of the two stars, most of the mass of the envelope of the evolved star is lost without a significant spiral-in. This suggestion is supported by the fact that the original reasoning for spiral-in (drag forces in the envelope) is not applicable here, because the envelope can easily be spun up to corotate with the binary.
In the absence of a physical picture for the removal of the envelope, we introduce a simple parameter $\gamma = (\Delta J / \Delta M_{\text{tot}}) (M_{\text{tot}} / J)$ to describe the loss of the angular momentum of the system as in the early work of Paczyński & Ziolkowski (1967). Our analysis of the observed parameters for all observed double helium white dwarfs shows that the material of the envelope of the giant is expelled with 1.4 to 1.7 times the specific angular momentum of the initial binary. The details of this kind of mass transfer should be investigated using 3D gas-dynamical calculations, which are becoming available (e.g. Bisikalo et al. 1998), but are not yet accurate enough to make predictions.
 6. With some well-constrained assumptions for the masses of the unseen companions in the other 8 double white dwarfs, we find similar results as for the double white dwarfs with two helium components.
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