Precise measurement of the top quark mass in dilepton decays using optimized neutrino weighting

D0 Collaboration

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**Abstract**

We measure the top quark mass in dilepton final states of $t\bar{t}$ events in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV, using data corresponding to an integrated luminosity of 9.7 fb$^{-1}$ at the Fermilab Tevatron Collider. The analysis features a comprehensive optimization of the neutrino weighting method to minimize the statistical uncertainties. We also improve the calibration of jet energies using the calibration determined in $t\bar{t} \rightarrow$ lepton + jets events, which reduces the otherwise limiting systematic uncertainty from the jet energy scale. The measured top quark mass is $m_t = 173.32 \pm 1.36$ (stat) $\pm 0.85$ (syst) GeV.

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1. Introduction

The discovery of the top quark in 1995 [1,2] completed the three quark families of the standard model (SM). Since then, the top quark has been one of the focal points of the Fermilab Tevatron and of the CERN LHC programs. The top quark stands out because of its large mass, $m_t$, which is a fundamental parameter in the SM. Its Yukawa coupling to the Higgs boson, $Y_t = \sqrt{2}m_t/\nu$, where $\nu$ is the vacuum expectation value of the Higgs field, is close to unity, implying that the top quark may play a special role in electroweak symmetry breaking. In addition, $m_t$ is linked to the $W$ and Higgs boson masses, $M_W$ and $M_H$, through radiative corrections [3]. Following the Higgs boson discovery [4,5], a precise measurement of $m_t$ provides a test of the electroweak sector of the SM and information on whether our universe resides in a stable or metastable region of that theory [6–8]. The short lifetime of the top quark prevents its confinement in the strong color field, since top quarks decay before hadronizing. This allows a particularly precise study of pure quantum chromodynamic (QCD) effects. A comparison of the measured $m_t$ and the $m_t$ extracted from cross section mea-

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**Notes:**

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measurements [9–12] may provide a probe of higher order and soft QCD corrections to the observed mass [13].

Assuming the SM branching ratio of $t \to Wb \approx 100\%$, $t\bar{t}$ decays yield distinct final state categories according to the number of charged leptons with high transverse momentum ($p_T$) from $W$ boson decays. Dilepton $(2\ell$, $\ell = e$ or $\mu$) events, such as $e\mu$, $e\mu$, and $\mu\mu$, with neutrinos from two $W \to \ell\nu$ decays, are relatively rare but have low background. We present a measurement of $m_t$ using $p\bar{p}$ collider data collected with the D0 detector at the Fermilab Tevatron collider, corresponding to an integrated luminosity of 9.7 fb$^{-1}$, in events with two high-$p_T$ electrons or muons of opposite electric charge. Two high-$p_T$ jets must also be observed, one of which must be identified as being consistent with originating from a $b$ quark. This analysis is based on our previous dilepton measurement [14], but with increased integrated luminosity and multiple optimizations to improve the precision of $m_t$.

We reduce the dominant statistical contribution to the uncertainty on $m_t$ through an optimization of the methods for kinematic reconstruction and statistical analysis. Lacking a dijet signature from $W \to q\bar{q}'$, which is present in $t\bar{t}$ → lepton + jets ($\ell + $jets) events and was used to improve the precision of jet energy calibration with a $W$ mass constraint [15], previous dilepton analyses at the Tevatron have reached a sensitivity limit imposed by standard jet calibration methods [16,17]. Progress in calibrating jet energies in the dilepton channel [14] provides improved cross-checks across different channels and a more significant contribution from the dilepton channel to the world average $m_t$ [18]. For comparison, the most recent measurements of $m_t$ in the dilepton channel from CDF, ATLAS, and CMS are, respectively, $m_t = 171.5 \pm 1.9$(stat) ± 2.5(syst) GeV [19], $m_t = 173.79 \pm 0.54$(stat) ± 1.30(syst) GeV [20], and $m_t = 172.50 \pm 0.43$(stat) ± 1.46(syst) GeV [21]. In this analysis, we substantially reduce the otherwise dominant uncertainty in the jet energy scale by applying the methods of Ref. [14].

## 2. Detector and data sample

### 2.1. Detector

The D0 detector [22,23] has a central-tracking system, consisting of a silicon microstrip tracker and a central fiber tracker, both located within a 1.9 T superconducting solenoidal magnet, with designs optimized for identification of the $p\bar{p}$ collision vertex and track reconstruction at pseudorapidities [24] of $|\eta| < 3$ and $|\eta| < 2.5$, respectively. The liquid-argon/uranium calorimeter has a central section covering $|\eta| \leq 1.1$, and two end sections that extend coverage to $|\eta| \approx 4.2$, with all three housed in separate cryostats. An outer muon system, covering $|\eta| < 2$, consists of a layer of tracking detectors and scintillation trigger counters in front of 1.8 T iron toroids, followed by two similar layers after the toroids.

### 2.2. Object reconstruction

We require electrons to satisfy an identification criterion based on boosted decision trees [25] using calorimeter and tracking information. Muons must satisfy requirements that match hits in the muon system to a track in the central tracking detector that is required to have a small distance of closest approach to the beam axis [26]. We require hits in the muon layers inside and outside the toroid. Muons and charged hadron momenta are measured in the central tracking detector, while electron, photon ($\gamma$), jet, and charged hadron energies are measured in the calorimeters. Muons must be isolated from jets and from nearby tracks. Electrons and muons must have their extrapolated track trajectories isolated from calorimeter energy depositions greater than an energy threshold. Electrons and muons must have $p_T > 15$ GeV, and $|\eta| < 2.5$ and $< 2.0$, respectively. We reconstruct jets using an iterative, midpoint-seeded cone algorithm with a cone parameter of $R_{cone} = 0.5$ [27]. Jets with embedded muons from the decay of $b$-hadrons require an additional correction to jet energy to account for the associated neutrino. A multivariate discriminant [28] is used to identify jets that contain a $b$-hadron (i.e., $b$ jets) from a vertex displaced from the interaction point. We define the missing transverse momentum ($\vec{E}_T$) attributed to the escaping neutrinos as the negative of the vector sum of all transverse components of calorimeter cell energies, corrected for the measured muon momenta and the response of the calorimeter to electrons. We also correct $\vec{E}_T$ for detector response in the jet energy calibration, as described below. Details of object reconstruction are provided in Ref. [29].

### 2.3. Standard jet energy calibration

We calibrate the energy of jets to be the energy of the particle jets reconstructed using the midpoint algorithm [27]. We correct for the effects of the calorimeter response to particle constituents of jets, energy leaking into the cone from particles directed outside it, as well as energy deposits outside the cone from particles inside it [30]. Charged hadrons have an energy-dependent response that is lower than that of electrons and photons. We therefore apply corrections obtained from $\gamma +$jet events to account for the energy dependence of the jet response in the central $|\eta|$ region. We also apply a relative $\eta$-dependent correction obtained from $\gamma +$jet and dijet events. We employ the same methods to calibrate jet energies independently in the Monte Carlo (MC) simulation and in data. The MC is used to help study potential biases in the data. We incorporate a correction for jets in the MC simulation that accounts for the difference in single-particle response between data and MC. This procedure ensures that the flavor dependence of the jet response in data is replicated in MC. In the MC we account for multiple $p\bar{p}$ interactions by correcting the jet energy to the particle level of only those particles that are directed within the jet cone at particle level. The typical systematic uncertainty in the energy calibration of each jet in the dilepton sample is 2%. This precision is limited by systematic uncertainties of the $\gamma +$jet method in the $p_T$ range of jets in $t\bar{t}$ events. Details about this “standard jet energy scale” calibration can be found in Ref. [30]. We require that jets have $p_T > 20$ GeV and $|\eta| < 2.5$ after calibration, but before applying additional corrections from the $W \to q\bar{q}'$ constraint in the $\ell +$jets channel discussed below.

### 3. Absolute jet calibration from a $W \to q\bar{q}'$ constraint

As in Ref. [14], we apply a multiplicative correction factor to the energy of jets in data based on an analysis of $t\bar{t} \to \ell +$jets events using the $W \to q\bar{q}'$ decays as a constraint. Application of this factor, $1.0250 \pm 0.0046$(stat) [15], improves the agreement between MC and data and allows us to use its uncertainty to reduce the uncertainty on the absolute energy scale by a factor of $\approx 4$ relative to the standard jet energy scale, while retaining its $\eta$ and $p_T$ dependence. To apply this scale, which comes from light-quark jets, to the dilepton sample, which has $b$ jets, it is important to ensure that the variation in the ratio of data over MC jet response between different flavors be placed on an equal footing. The standard jet energy scale [30] achieves this on a jet-by-jet basis by using single particles in MC jets to correct the simulation so that it has the same kinematic and flavor-dependent jet response as that in data. This ensures that the energies of $b$ jets in dilepton simulated samples agree with those of $b$ jets in the dilepton data sample at the same level as light-quark jets. Aside from fragmentation differences between data and MC which are discussed below, this
approach justifies the use of the $\ell +$ jets constraint in the dilepton channel.

4. Event selection

The $t\bar{t}$ candidate events in the $ee$ and $\mu\mu$ channels are required to pass single-lepton triggers. The full suite of triggers is used for selecting $e\mu$ events. The dilepton event selection before optimization is described in Ref. [29]. We optimize the selection based on MC events to provide the smallest expected statistical uncertainty in $m_\ell$. We require two isolated leptons with opposite electric charge. We require at least two jets, where at least one of the two jets with highest $p_T$ must be identified as a $b$ jet. For the $e\mu$ channel, our selections have an efficiency for tagging $b$ jets of 72%, and a light-quark mistag rate of 12% in the central region in $y$. The same-flavor channels employ slightly tighter $b$ tagging requirements and thus have a few percent lower efficiency, and 30% lower mistag rate. We require events in the $e\mu$ channel to have $E_T > 40$ GeV. This $E_T$ selection is also applied to $ee$ events when the dielectron invariant mass is between 70 and 100 GeV, to reduce the $Z \to ee$ background contribution. We define a $E_T$ significance variable, $S$, which measures the likelihood for the observed $E_T$ to be a fluctuation from $E_T = 0$ GeV. We require $S > 3.5$ (4) for the $ee$ ($e\mu\mu$) channel. We require $e\mu$ events to have $H_T > 100$ GeV, where $H_T$ is the scalar sum of the $p_T$ of the two highest-$p_T$ jets and of the lepton with highest $p_T$. The $H_T$, $b$ tagging, and $E_T$-based requirements are optimized to minimize the expected statistical uncertainty in $m_\ell$ in each channel. The expected signal-to-background (S/B) ratio is $\approx 7$ for these channels. These requirements yield a 3% improvement in statistical precision in $m_\ell$ relative to the selections in Ref. [14]. After implementing all these selections, we obtain 340, 115, and 110 events in the $e\mu$, $ee$ and $\mu\mu$ channels, respectively.

5. Modeling signal and background

The $t\bar{t}$ events are simulated at 15 mass points over the range $130 \leq m^{MC}_{t\bar{t}} \leq 200$ GeV using the tree level generator ALPGEN 2.11 [31] with up to 2 additional light partons and PYTHIA 6.409 [32] with modified underlying event Tune A for parton showering and hadronization. Here, $m^{MC}_{t\bar{t}}$ refers to the input mass in ALPGEN. An additional, larger sample is generated at $m^{MC}_{t\bar{t}} = 172.5$ GeV to study systematic uncertainties. We normalize the $t\bar{t}$ production cross section to $\sigma_{t\bar{t}} = 7.24 \pm 0.23$ pb [33], which is calculated at next-to-next-to-leading order with a next-to-next-to-leading logarithm soft gluon resummation. The main backgrounds arise from three sources: $Z/\gamma^* \to \ell^+\ell^-$, diboson ($WW$, $WZ$, and $ZZ$) processes, and instrumental effects. We model the $Z/\gamma^*$ background using ALPGEN with up to 2 light partons and PYTHIA for showering and hadronization. We employ PYTHIA for the diboson background. The instrumental background arises from $W +$ jets, multijet, or $\ell +$ jets $t\bar{t}$ events where one or two jets are either mis-identified as electrons, or they contain a hadron decaying to a non-isolated lepton that passes our selection. This background is estimated from data as in Ref. [29]. We apply a full detector simulation based on GEANT 3.14 [34] for all simulated events. The objects reconstructed in simulation are smeared to ensure that their resolutions reflect those in data. Scale factors in object efficiencies are applied to improve agreement between data and MC.

6. Kinematic reconstruction

6.1. Neutrino weighting

The presence of two neutrinos in the $t\bar{t}$ decay makes it impossible to fully constrain the kinematics and thus extract a unique $m_\ell$ measurement from each event. Given the measured momenta of leptons, jets and $E_T$, the available constraints from $M_{W_t}$ and the condition $m_t = m_\ell$, we are missing one constraint to provide full $t\bar{t}$ reconstruction in dilepton events. We integrate over the phase space of neutrino rapidities for chosen values of hypothesized $m_\ell$ ($m^{calc}_{\ell}$) [35], and compare $E_T^{calc}$, the vector sum of neutrino momentum solutions at each chosen point of phase space, to the observed $E_T^{obs}$ to determine a “weight” $\omega$ characterizing the level of agreement:

$$\omega = \frac{1}{N} \sum_{i=1}^{N} \prod_{j=x,y} \exp \left( -\frac{(E_{T,calc}^{i,j} - E_{T,obs}^{i,j})^2}{2\sigma_{E_T}^2} \right)$$

where $i$ runs over all neutrino solutions for any two possible jet-lepton assignments in the $t\bar{t}$ final state (up to $N = 8$), $j$ stands for the two orthogonal coordinates in the transverse plane $(x$ and $y$), and $\sigma_{E_T}$ is a parameter representing the RMS of the difference between the transverse components of the measured $E_T$ and the sum of the solved neutrino transverse momenta. The parameter $\sigma_{E_T}$ is taken to be the same in both $x$ and $y$ directions. We perform this calculation over a range of $m^{calc}_{\ell}$, integrating $\omega$ over the neutrino phase space, to yield a distribution of $\omega(m_\ell)$ versus $m^{calc}_{\ell}$. Prior studies [36] have shown that the first two moments ($\mu_\omega$, $\sigma_\omega$) of this distribution extract most of the information about $m_\ell$. The analysis of Ref. [14] used the range of $m^{calc}_{\ell}$ values between 80 and 330 GeV in 1 GeV steps and a $\sigma_\omega$ of 7 GeV in the weight calculation. The new optimized determination of these parameters is briefly summarized below.

6.2. Optimization of weight calculation parameters

After applying the methods described above to improve the jet energy calibration, the statistical contribution is the dominant source of measurement uncertainty on $m_\ell$ in the dilepton channel. We therefore examine the parameters used for the kinematic reconstruction of $t\bar{t}$ events and for the maximum likelihood fit to reduce the expected statistical uncertainty. At each step, we verify through MC simulations that the optimization does not increase the systematic uncertainty.

All neutrino solutions and jet assignments yield mass estimators such as $\mu_\omega$ that are correlated with $m_\ell$. However, the correlation is substantially greater, and $\mu_\omega$, $\sigma_\omega$ values are less biased, when the correct jet assignments and solutions of neutrino momenta are chosen. Since now $m_\ell$ has been measured with high precision [18], we can optimize the range of $m^{calc}_{\ell}$ based on known values of $m_\ell$. Considering a wide range in $m^{calc}_{\ell}$ causes incorrect configurations to overwhelm the correct configuration, thereby worsening the mass resolution. Likewise, scanning over too narrow a range biases the background and worsens the mass sensitivity by causing $t\bar{t}$ and background distributions to be similar. Examination of a two-dimensional grid of upper and lower limits of the mass range yields the optimal range of $m^{calc}_{\ell} = 115$ to 220 GeV in 1 GeV steps. The value of $\sigma_{E_T}$ also has a noticeable impact on the expected precision of the analysis. This was not the case in Ref. [14], mainly because the final top quark mass measurement was less precise. In Ref. [14], the value of 7 GeV for $\sigma_{E_T}$ was obtained as the unclustered $E_T$ resolution in an earlier dataset [36], where the unclustered $E_T$ is the magnitude of the vector sum of all energy depositions in the calorimeter that are not included in lepton or jet reconstruction. However, accounting only for the unclustered energy resolution as the origin of the difference between the calculated and measured $E_T$ ignores the effect of assumptions that go into the kinematic reconstruction. For instance, the finite binning of the neutrino rapidities discretizes the solved neutrino momenta and therefore the solved $E_T$. Also, the solved $E_T$ does not include
7. Extracting the top quark mass

7.1. Maximum likelihood

We perform a binned maximum likelihood fit to the extracted moment distributions \( \{ \mu_{\omega}, \sigma_{\omega} \} \) in data. Expected probability densities are calculated using the MC samples for each of the 16 \( m_t \) points, yielding a two-dimensional probability density \( h_{\text{MC}}(\mu_{\omega}, \sigma_{\omega}) \) distribution parametrized by \( m_t \). Background samples are used to construct a background template for each channel, \( h_B(\mu_{\omega}, \sigma_{\omega}) \), with each background contributing according to its expected yield. Bins in signal templates with no events are given a weighted value corresponding to a single signal MC event to ensure that the log of likelihood is not infinite. The likelihood is given by:

\[
L(\mu_{\omega}[1..N], \sigma_{\omega}[1..N], N | n_S, n_B, m_t) = \prod_{i=1}^{N} \frac{n_S \cdot h_S(\mu_{\omega}, \sigma_{\omega})}{n_S + n_B} + \frac{n_B \cdot h_B(\mu_{\omega}, \sigma_{\omega})}{n_S + n_B},
\]

where \( N \) is the number of observed events in data, \( n_S \) is the expected number of \( tt \) events (for \( m_t = 172.5 \) GeV), and \( n_B \) is the expected total number of background events. We fit \(-\ln L\) versus \( m_t \) to a parabola in a window of \( m_t \) that is iteratively varied until a stable minimum is found. We take the minimum of the final parabola to be the fitted top quark mass, \( m_t^{\text{fit}} \). The uncertainty on the fitted mass is obtained by considering the \( m_t \) range over which the fit function increases by 0.5 units in \(-\ln L\) above this minimum. Using pseudo-experiments, we optimize the template binning of each channel separately in a two-dimensional grid that lets \( \mu_{\omega} \) and \( \sigma_{\omega} \) bin sizes vary independently. Finer binning in \( \mu_{\omega} \) and \( \sigma_{\omega} \), especially for the \( e\mu \) channel, improves the expected statistical precision in \( m_t^{\text{fit}} \) by 5%. The fitted mass window is optimized to \( \pm 15 \) GeV for all channels. Taking all the optimizations together, including event selection, weight calculation, and maximum likelihood fitting, the statistical sensitivity of this analysis is improved relative to Ref. [14] by 20% beyond the 35% gain expected from increased integrated luminosity.

7.2. Ensemble testing and data results

We obtain a linear relationship between \( m_t^{\text{fit}} \) and \( m_t^{\text{MC}} \) by performing randomized pseudo-experiments using all signal mass points. The numbers of signal and background events in the pseudo-experiments are allowed to fluctuate within their Poisson uncertainties around their expected values. We require that the total number of events matches that observed in data. To minimize the effect of statistical fluctuations on our systematic uncertainties, we optimize the number of pseudo-experiments by dividing the MC sample into five subsamples, and measure systematic uncertainties with each subsample. We calculate the RMS of the five uncertainties, average over all systematic effects, and divide by \( \sqrt{5} \) to estimate the statistical component of systematic uncertainties. The average RMS decreases until we oversample, or reuse, the \( tt \) MC events by roughly a factor of three. This corresponds to 3000 pseudo-experiments. We perform a linear fit of \( m_t^{\text{fit}} \) versus \( m_t^{\text{MC}} \) to obtain a calibration slope and offset for \( m_t^{\text{fit}} \) using 3000 pseudo-experiments:

\[
m_t^{\text{fit}} = \text{Slope} \cdot (m_t^{\text{MC}} - 170) + \text{Offset} + 170.
\]

We account for oversampling by increasing the statistical uncertainties at each mass point by the appropriate oversampling factor. Likewise, we compute the pull, or the ratio of \( m_t^{\text{fit}} - m_t^{\text{MC}} \) over the average estimated uncertainty at each mass point. The slopes
Table 1
Slopes, offsets, and pull widths of the $m_t$ calibration and the expected statistical uncertainties in the mass ($\sigma_{m_t}$) for the $e\nu$, $e\mu$, and $\mu\mu$ channels, and their combination.

<table>
<thead>
<tr>
<th>Source</th>
<th>Offset [GeV]</th>
<th>Pull width</th>
<th>$\sigma_{m_t}$ [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jet energy calibration</td>
<td>$0.98 \pm 0.01$</td>
<td>$0.67 \pm 0.04$</td>
<td>$0.99$</td>
</tr>
<tr>
<td>Flavor dependence</td>
<td>$0.98 \pm 0.01$</td>
<td>$0.54 \pm 0.06$</td>
<td>$0.99$</td>
</tr>
<tr>
<td>Residual scale</td>
<td>$0.98 \pm 0.01$</td>
<td>$0.31 \pm 0.04$</td>
<td>$0.99$</td>
</tr>
<tr>
<td>b quark fragmentation</td>
<td>$0.98 \pm 0.01$</td>
<td>$0.61 \pm 0.03$</td>
<td>$0.99$</td>
</tr>
</tbody>
</table>

Table 2
Systematic uncertainties on $m_t$ for the combined dilepton measurement using $9.7 \text{ fb}^{-1}$ of integrated luminosity. For symmetrized uncertainties, the ± symbol indicates that the corresponding systematic parameters in MC are positively correlated with $m_t$ in data, and the “−” symbol indicates an anticorrelation. The uncertainties shown as ± or − only are computed by comparing a standard choice with an alternate, but are symmetrized in calculating the total uncertainty.

<table>
<thead>
<tr>
<th>Source</th>
<th>$\sigma_{m_t}$ [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jet energy calibration</td>
<td>±0.47</td>
</tr>
<tr>
<td>Flavor dependence</td>
<td>±0.27</td>
</tr>
<tr>
<td>Residual scale</td>
<td>±0.16</td>
</tr>
<tr>
<td>b quark fragmentation</td>
<td>±0.15</td>
</tr>
<tr>
<td>Object reconstruction</td>
<td>±0.10</td>
</tr>
<tr>
<td>Signal modeling</td>
<td>±0.33</td>
</tr>
<tr>
<td>Higher-order effects</td>
<td>±0.15</td>
</tr>
<tr>
<td>ISR/FSR</td>
<td>±0.07</td>
</tr>
<tr>
<td>Hadronization</td>
<td>±0.11</td>
</tr>
<tr>
<td>Color reconnection</td>
<td>±0.01</td>
</tr>
<tr>
<td>Multiple $p\bar{p}$ Interactions</td>
<td>±0.06</td>
</tr>
<tr>
<td>PDF uncertainty</td>
<td>±0.09</td>
</tr>
<tr>
<td>Method</td>
<td>±0.04</td>
</tr>
<tr>
<td>Total systematic uncertainty</td>
<td>±0.08</td>
</tr>
</tbody>
</table>

of $m_t^{\text{MC}}$ versus $m_t^{\text{MC}}$ are close to 1, and pull widths are consistent with unity, as shown in Table 1. We calculate the final $m_t$ by correcting $m_t^{\text{MT}}$ from a given measurement by the slope and offset. We correct the statistical uncertainty using the slope and the pull width. The expected corrected statistical uncertainties for each channel are given in Table 1. In data, we obtain corrected, fitted $m_t$ values of $m_t = 171.86 \pm 1.71$ (stat), $173.99 \pm 3.04$ (stat), and $178.58 \pm 3.56$ (stat) GeV for the $e\mu$, $ee$, and $\mu\mu$ channels respectively, and $m_t = 173.32 \pm 1.36$ (stat) GeV for the combined channels.

8. Systematic uncertainties

Systematic uncertainties summarized in Table 2 arise from jet energy calibration, object reconstruction, modeling of $t\bar{t}$ and background events, and the mass-extraction method. The energies of jets are shifted up and down by the uncertainty on the absolute energy scale, which is taken from $t\bar{t}$ jets events, thereby providing shifts in $m_t$. This scale is appropriate for light-quark jets, which, after correcting for jet flavors to improve the agreement between data and MC, have different kinematic distributions than $b$ jets from $t\bar{t}$ decays. We calculate a residual uncertainty due to the kinematic differences between the $t\bar{t}$ and dilepton samples of $b$ jets. We use separate up and down estimates to extract the energy- and $\eta$-dependent shifts in $m_t$ based on uncertainties in the standard jet energy scale relative to their average value in the $t\bar{t}$+jets calibration sample. We cross-check this with an alternative method that applies shifted light-quark jet energy scales to $b$ jets in the $t\bar{t}$+jets channel [15]. These methods agree, and thereby validate the use of the $t\bar{t}$+jets scale as a jet calibration. We also cross-check using a jet-energy-dependent linear parameterization of the residual jet energy scale as in Ref. [15], obtaining results that do not exceed our estimate of uncertainties from the jet energy scale. To estimate the uncertainty corresponding to possible differences in the flavor dependence of the MC scale relative to data, we change the single-particle responses up and down by their uncertainties and obtain the shift in $m_t$. To estimate the possible dependence on the $b$ quark fragmentation in the MC, we replace the PYTHIA $b$ quark fragmentation function with the Bowler scheme [37], and compare $m_t$ with the Bowler free parameters tuned to LEP (ALEPH, OPAL, and DELPHI) or SLD data [38].

The systematic uncertainty due to the trigger efficiency is estimated by applying the ratio of single-lepton trigger efficiency parameterization in data divided by the MC parameterization to the $ee$ and $\mu\mu$ channels. The uncertainties in the modeling of the energy and momentum resolutions of electrons, muons, and jets are applied independently of each other, and the shifts in $m_t$ are extracted as uncertainties on $m_t$. Lepton energy or momentum scales and their uncertainties are extracted from $Z \rightarrow t\bar{t}$ events in data. An additional uncertainty is estimated for jet identification by shifting the jet identification efficiency within its uncertainty in MC samples to estimate their effect on $m_t$. The uncertainty from modeling $b$ tagging is evaluated by changing within their uncertainties the corrections that account for the agreement between data and MC in $b$ tagging efficiency.

Higher-order virtual corrections to $m_t$ are absent in the ALPGEN used to generate our standard $t\bar{t}$ samples. We therefore compare an ensemble of pseudo-experiments using MC@NLO 3.4 [39] $t\bar{t}$ events with one using ALPGEN events, where both employ HERWIG 6.510 [40] for modeling of hadronization. To evaluate the uncertainty associated with the modeling of initial and final-state radiation (ISR/FSR), we compare ALPGEN+PYTHIA with the renormalization and factorization scale changed up and down by a factor of 1.5 [15]. The $t\bar{t}$+jets analysis exhibits a discrepancy in the shape of the $p_T$ distribution of the $t\bar{t}$ system, which, although the dilepton statistics are limited, may be present in the dilepton sample. We evaluate the uncertainty in the modeling of the $t\bar{t}$ $p_T$ distribution by reweighting MC events to make them match the data. The observed shift in $m_t$ is taken as the uncertainty. Since the hadronization in our standard $t\bar{t}$ sample is modeled with PYTHIA, we estimate a hadronization uncertainty on $m_t$ by performing pseudo-experiments using an ALPGEN+HERWIG sample. We evaluate the effect of color reconnection by comparing $m_t$ measurements in ALPGEN+PYTHIA samples with two PYTHIA tunes: the Perugia2011 tune that incorporates an explicit color-reconnection scheme, and the Perugia2011 NOCR tune that does not [41]. Data and MC may have different distributions in instantaneous luminosity after event selection. This uncertainty due to multiple $p\bar{p}$ interactions is estimated by reweighting the distribution of instantaneous luminosity to make MC agree with the data for respective data-taking epochs, and then take the shift in $m_t$ with respect to the default value. The uncertainty due to the proton structure is obtained from the 20 sets of CTEQ6.1 parton distribution functions (PDF) reweighted
to CTEQ6M, where the deviations in \( m_t \) for the 20 eigenvectors sets are added in quadrature [42].

We estimate the effect of the uncertainty on the fraction of signal or background by changing the expected \( \bar{t}t \) event yields (\( n_\bar{t} \)) up and down and the expected background yields (\( n_b \)) down and up within their total uncertainties. The heavy-flavor scale factor, which is applied to the \( Z \rightarrow 2\ell \) cross section to correct the heavy-flavor content, is also changed up and down within its uncertainty to estimate its systematic effect on \( m_t \).

Our templates are constructed from MC samples for \( \bar{t}t \), \( Z \rightarrow 2\ell \), and diboson backgrounds, as well as data samples for instrumental background, yielding statistical uncertainties on their bin contents. We use Poisson distributions to modify bin contents within their statistical uncertainties to obtain 1000 new templates. We measure \( m_t \) in data using these templates, and the RMS of the measured top quark mass is taken as its uncertainty. Our method of \( m_t \) extraction relies on the correction of the fit to \( m_t \) to the input MC mass. The uncertainties from this calibration are applied to provide the uncertainty in \( m_t \). The uncertainty is reduced substantially from Ref. [14] due primarily to the reduction in the uncertainty in jet energy calibration and the optimizations for improvements in statistical uncertainty. Larger MC samples also contribute by lowering statistical fluctuations on systematic uncertainties, or reducing statistically limited systematic uncertainties.

9. Conclusions

We have measured the top quark mass in the combined dilepton channels (\( e\mu, ee, \mu\mu \)):

\[
\begin{align*}
\langle m_t \rangle &= 173.32 \pm 1.36 \text{(stat)} \pm 0.85 \text{(syst)} \text{ GeV} \\
&= 173.32 \pm 1.60 \text{ GeV}.
\end{align*}
\]

This measurement is consistent with the current world average value of \( m_t \) [18]. Our measurement is the most precise dilepton result from the Tevatron, and is competitive with the most recent LHC dilepton measurements. The systematic uncertainty of 0.49% is the smallest of all dilepton measurements.

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References

[21] G. Aad, et al., ATLAS Collaboration, Measurement of the top quark mass in the \( t\bar{t} \) \( \rightarrow \) lepton + jets and \( t\bar{t} \) \( \rightarrow \) dilepton channels using \( \sqrt{s} = 7 \) TeV ATLAS data, arXiv:1503.05427.
[25] The pseudorapidity is defined as \( \eta = -\ln \tan(\theta/2) \), where \( \theta \) is the polar angle relative to the proton beam direction.


