Experimental results on the high-field thermopower of a two-dimensional electron gas in a GaAs-Ga$_{1-x}$Al$_x$As heterojunction

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We report experimental results on the thermopower $S_{xx}$ and Nernst-Ettingshausen coefficient $S_{yx}$ for a GaAs-Ga$_{1-x}$Al$_x$As heterojunction both at zero field and at high magnetic fields. Although the high-field data are qualitatively in agreement with the theoretical models based on diffusion effects, the experimental magnitudes are much larger than those predicted, with the discrepancy in the high-field limit ($\nu = \frac{1}{2}$) being about two orders of magnitude. It seems probable that phonon drag is responsible, but detailed predictions are not yet available. Although $S_{xx}$ resolves spin splitting with a clarity similar to that seen in $\rho_{xx}$, the fractional quantum Hall effect (FQHE) is not visible in $S_{xx}$ at $\nu = \frac{1}{2}$, even though it is clearly seen in $\rho_{xx}$. The FQHE is observed in $\rho_{xx}$, but we argue that this is not as significant as its absence in $S_{xx}$.

There have been a number of theoretical studies of the thermoelectric properties of a two-dimensional (2D) electron gas in a magnetic field, but as yet only a few experimental results have been reported. The interest of such studies lies in the unique situation of complete quantization of the carriers under these circumstances, which in the case of the resistivities has led to the discovery of the normal and the fractional quantum Hall effect (FQHE).

The thermoelectric tensor is defined by the relation $E = \mathbf{S} \cdot \nabla T$, where $E$ is the measured electric field produced by the temperature gradient $\nabla T$; and, for an isotropic gas, there exist only two independent components, the thermopower $S_{xx} = S_{yy}$ and the Nernst-Ettingshausen coefficient $S_{yx} = -S_{xy}$. Theoretical results for these coefficients based upon carrier diffusion driven by the thermal gradient have shown that in the disorder-free limit $S_{yx} = 0$, and $-eS_{xx}$ measures the entropy per electron, with $-e$ the electronic charge. Within this framework $-eS_{xx}$ should show positive maxima at half-filled Landau levels and minima at filled Landau levels. At the maxima $S_{xx}$ is given by

$$S_{xx} = -(k/e)\ln 2/(n+\frac{1}{2}) ,$$

provided $k_0 \gg kT$ and $kT \gg \Gamma$, the Landau-level broadening (the other factors have their usual significance). If spin splitting is resolved, Eq. (1) becomes

$$S_{xx} = -(k/e)\ln 2/(2n+1 \pm \frac{1}{2}) .$$

In the quantum limit, i.e., when all electrons occupy the last level, $S_{xx}$ increases monotonically with $B$, but Eq. (1) or (2) can be used to evaluate $S_{xx}$ when the level is just half full.

For moderate disorder these results for $S_{xx}$ are unchanged. However, $S_{yx}$ becomes finite and resembles the negative of the derivative of the conductivity $\sigma_{yx}$ with respect to the chemical potential $\mu$. It shows alternating peaks and negative values as a function of $B$, each having similar magnitude (roughly 10–30 $\mu$V K$^{-1}$) independent of $n$. Finally, in the quantum limit $-S_{yx}$ should saturate at a similar level, or perhaps a somewhat higher level than the peak values.

It should be made clear that the above theories do not take into account the possible effects of phonon drag. Recently Nicholas has pointed out that phonon drag might strongly influence the thermopower, just as in the case of bulk semiconductors and metals.

The present experimental investigations were made to check the predictions of Eqs. (1) and (2), particularly in the quantum limit. Although we have detailed results for 4 GaAs-Ga$_{1-x}$Al$_x$As heterojunctions with various electron densities, this paper concentrates on the data for that sample with the lowest carrier density (which enabled the quantum limit to be readily attained). The behavior of the other samples is consistent with the present sample.

The sample was grown on a GaAs plate of dimensions $5 \times 11 \times 0.36$ mm$^3$. Selective etching of the junction produced a 2D sample suitable for the measurement of the electrical and Hall resistivities $\rho_{xx}$ and $\rho_{yx}$ (Fig. 1). The electron density was measured to be $1.78 \times 10^{15}$ m$^{-2}$ with a low-field mobility of 227 000 cm$^2$ V$^{-1}$ sec$^{-1}$. Contacts were made by diffusing Sn disks into the surface, their resulting...
diameters being about 0.3 mm. To calculate the relevant electric fields $E_x, E_y$ in the case of the thermoelectric coefficients, we have used $E_x = -\Delta V_x / l_x$ and $E_y = -\Delta V_y / l_y$, where the potential difference $\Delta V_x$ is measured between any pair of contacts parallel to the length of the sample and $l_x$, the distance between the midpoints of the same contacts, and similarly for $\Delta V_y$ and $l_y$ using any perpendicular pair (cf. Fig. 1). Providing the 2D gas is homogeneous, the etching should have no effect on the thermoelectric voltages between contacts because the temperature gradient is established by the substrate and not by the 2D layer (in contrast to the case of $\rho_{xx}$, where the distance between the sample limbs is the relevant quantity). For this sample, because $l_x (= 0.98 \text{ mm})$ and $l_y (= 1.29 \text{ mm})$ are of the same order as the diameters of the Sn contacts, large uncertainties in $S_{xx}$ and $S_{xy}$ are inevitably produced ($\approx \pm 30\%$). Temperature gradients were measured primarily with a pair of carbon resistors which were calibrated at several values of the magnetic field $B$. At zero field the gradients could also be measured independently with a Au+0.03% Fe versus Manganin thermocouple; the results of the two different methods agreed to within 6% Volages were determined by a Keithley 155 microvoltmeter, and all data were taken at $\pm B$ to eliminate the admixture of unwanted components.

Figure 2 shows the behavior of $S_{xx}, S_{xy}$, and $\rho_{xx}$ as a function of $B$ and $T$. At the lowest $T$ (1.68 K) spin splitting is completely resolved for $n=0$ and partially resolved for $n=1$. Both $S_{xx}$ and $S_{xy}$ behave qualitatively as expected from the theories. Thus $-S_{xx}$ shows maxima at half filling factors, goes to zero at integer filling factors, and increases monotonically in the quantum limit. Similarly, $S_{xy}$ shows distinct negative and positive peaks. However, the agreement with theory is only superficial and one notes the following major discrepancies:

1. The peaks of $S_{xx}$ are not of constant amplitude and this coefficient shows no clear tendency to saturate in the quantum limit.

2. The amplitudes of the peaks for both $S_{xx}$ and $S_{xy}$ (Table I) are much larger than those predicted (in fact by a factor of 100 for $S_{xx}$ in the quantum limit).

3. The relative amplitudes of the peaks of $S_{xy}$ for successive occupation of Landau levels do not follow those predicted by Eqs. (1) and (2). In particular, the ratio of the magnitudes of $S_{xy}$ at $\nu = \frac{3}{2}$ and $\frac{1}{2}$, which should be 3, is actually about 17 for $T \geq 4 \text{ K}$.

It is important to stress that these discrepancies cannot in any way be explained by experimental error because of the magnitude of the discrepancies, and no reasonable error source can be envisaged which could alter the relative magnitudes of the peaks. Remounting the sample in the cryostat had no significant effect on the results.

![Figure 2](image)

**FIG. 2.** Experimental data. (a) $\rho_{xx}$ at 1.68 K (right-hand scale). (b) $S_{xx}$ at various temperatures. Notice the scale change of $\times 10$ between 8 and 10 T. (c) $S_{xy}$ at two temperatures. Above about 4 K, $S_{xy}$ changes relatively little with temperatures at high fields. Notice the scale change of $\times 3$ for the data at 6.1 K above 10 T.

### Table I. Amplitude of the peaks for $-S_{xx}$.

<table>
<thead>
<tr>
<th>$\nu$</th>
<th>Calculated peak heights ($\mu$V K$^{-1}$)</th>
<th>Experimental peak heights ($\mu$V K$^{-1}$) at $T$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.68</td>
<td>3.8</td>
</tr>
<tr>
<td>0.5</td>
<td>120</td>
<td>2300</td>
</tr>
<tr>
<td>1.5</td>
<td>40</td>
<td>410</td>
</tr>
<tr>
<td>2.5</td>
<td>24</td>
<td>151$^b$</td>
</tr>
<tr>
<td>3.5</td>
<td>17</td>
<td>129$^b$</td>
</tr>
<tr>
<td>4.0</td>
<td>40</td>
<td>536</td>
</tr>
</tbody>
</table>

$^a$Half-integer values correspond to spin-resolved levels [Eq. (1)]. Integer values correspond to spin-unresolved levels [Eq. (1)].

$^b$Levels only partially resolved.
PREVIOUS EXPERIMENTAL data on $S_{\text{FQHE}}$ at lower $B$ for GaAs-Ga$_{1-x}$Al$_x$As heterojunctions gave absolute values in agreement with theoretical predictions. More recent results by the same authors have indicated that higher-mobility samples may have a much higher thermopower, in accordance with the present data.

Another important experimental observation concerns the FQHE. As can be seen from Fig. 2, minima are observed in $\rho_{xx}$ at $\frac{1}{2}$ and $\frac{3}{4}$ occupancy. In the latter case the minimum is roughly a half of the resistivity value at slightly higher or lower fields, yet no minimum is observed in $S_{\text{FQHE}}$. There is also structure in $\rho_{xx}$ near $\nu = \frac{1}{2}$, but none visible in $S_{\text{FQHE}}$. A resistivity minimum due to partially resolved spin splitting can be seen at about 2.8 T, and this is reflected in $S_{\text{FQHE}}$ with about the same relative magnitude. These observations show that the gaps in the energy spectrum produced by electron-electron interaction (at $\nu = \frac{1}{2}$ or $\frac{3}{4}$) and those resulting from single-particle interactions (spin splitting) have a different effect on $S_{\text{FQHE}}$ compared to $\rho_{xx}$. The fact that a dip appears in $S_{\text{FQHE}}$ at $\nu = \frac{1}{2}$ is perhaps not unexpected because $S_{\text{FQHE}}$ is sensitive to $\rho_{xx}$, being given by $S_{\text{FQHE}} = \rho_{xx} \varepsilon_{\text{FQHE}} + \rho_{xy} \varepsilon_{\text{FQHE}}$, where $\varepsilon$ is the thermoelectric tensor defined via $J = \tau \cdot E - \tau \cdot \nabla T$, with $\tau = \tau_{\text{el}}$, and $J$ the electric current density. Indeed, since $S_{\text{FQHE}} = \rho_{xx} \varepsilon_{\text{FQHE}} - \rho_{xy} \varepsilon_{\text{FQHE}}$, then one expects a contribution to $S_{\text{FQHE}}$ of $\rho_{xx} \varepsilon_{\text{FQHE}} \approx - (\rho_{xx}/\rho_{xy}) S_{\text{FQHE}}$, which will show a dip of about the observed magnitude.

To explain the difference in magnitude between the experimental and theoretical values of $S_{\text{FQHE}}$, we suggest that phonon drag strongly influences the observed thermopower. Very recently Nicholas has used a formula derived by Herr to estimate this contribution. The result for $B = 0$ is

$$S^d = - fL\nu/\mu T,$$

where $L$ and $\nu$ are the mean free path and velocity of the phonons, $\mu$ the electron mobility of the electrons, and $f$ a factor which takes into account the efficiency of transferring momentum between the electron and phonon systems; $f$ can be approximated by $\mu/\mu_p$, where $\mu_p$ is the mobility that the electrons would have if they were scattered by phonons only. For GaAs-Ga$_{1-x}$Al$_x$As structures, we expect $L$ to be similar to the smallest dimension of the GaAs crystal (0.36 mm), so taking $\mu_p = 3000 T m^2 V^{-1} sec^{-1}$ and $\nu \sim 4000$ m/sec, $S^d \sim -470 \mu V K^{-1}$. For comparison purposes, Fig. 3 shows the zero-field thermopower $S$ of the sample. The diffusion term is estimated to be

$$S^d = - \pi^2/3(k/e)(kT/\mu_p)(\nu + 1),$$

where $\mu_e$ is the chemical potential ($\sim 75$ K) and $\nu \sim 1$; thus $S^d \approx 7.6 T \mu V K^{-1}$. Subtracting $S^d$ from the observed $S$ (Fig. 3) leaves a contribution which peaks near 13 K with a magnitude of $-250 \mu V K^{-1}$, which is similar to that expected from phonon drag. The decrease at high $T$ is consistent with other phonon scattering mechanisms, but the decrease below $\sim 10$ K appears to be too rapid to be accounted for by the increasing inefficiency of electron-phonon scattering at low temperatures.

The extension of the above results to high $B$ seems to be less reliable than the calculation at zero $B$. If one uses the argument of Nicholas with our sample, then one expects peak values of $S_{\text{FQHE}} \sim 0.16/(\nu + 1/2) V K^{-1}$, a result which does not give the correct field dependence, and which is at least an order of magnitude too large even for the case of $\nu = 1$. Although this estimate is unreliable, the idea that phonon drag is significant appears to be valid.

In conclusion we note the following points:

1. Both $S_{\text{FQHE}}$ and $S_{\text{FQHE}}$ behave qualitatively as predicted, especially at lower $T$, but their experimental magnitudes are much larger than expected, particularly in the quantum limit at $T \geq 4$ K.

2. The FQHE is not visible in $S_{\text{FQHE}}$ at temperatures where it is clear in $S_{\text{FQHE}}$. Although it is observed in $S_{\text{FQHE}}$, we argue that this is probably not particularly significant.

Data at lower $T$ would be of great interest with regard to the magnitudes of $S_{xx}$ and $S_{\text{FQHE}}$, and also with respect to the FQHE.

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