Electronic subbands of a $\delta$ doping layer in GaAs in a parallel magnetic field

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The $B_{||}$ magnetoconductivity in the V-shaped potential well of $\delta$-doped GaAs(Si) shows oscillatory behavior along the $B_{||}$ axis at constant $N_z$. At finite field $B_{||}$ the subband levels are raised by a diamagnetic energy shift and may be pushed over the Fermi level. Due to the formation of magnetoelectric subbands the $E(k)$ dispersions are distorted. According to a self-consistent model calculation the features of $\sigma(B_{||})$ are explained in terms of the oscillating density of states at the Fermi level.

I. INTRODUCTION

The electronic states of a sheet of Si donors in molecular-beam-epitaxy (MBE)-grown GaAs form a set of two-dimensional (2D) subbands in a V-shaped potential well. The 2D character of the electronic levels has been demonstrated in Shubnikov–de Haas magnetoresistance experiments for donor sheet densities in the range of $10^{12}–10^{13}$ cm$^{-2}$. Typically several subbands are occupied. The occupancies agree very well with the results of a self-consistent subband calculation.

Here we consider the effect of a $B_{||}$ field in the $\hat{z}$ direction in the plane of the donor atoms in Fig. 1. Due to a diamagnetic energy the bottom of each subband level is shifted upward. Higher energy levels are eventually pushed out of the Fermi surface and the carriers redistribute between the remaining subbands. As a function of $B_{||}$ an oscillatory variation of the conductivity is expected. In Ref. 2, the conductivity oscillation is named the “diamagnetic” Shubnikov–de Haas effect. For the usual Shubnikov–de Haas effect, with increasing magnetic field, the free electron levels are swept out of the Fermi energy.

Previous $B_{||}$ magnetotransport experiments have been done for 2D electrons in InAs$^{3,4}$ InSb$,^5$ and Hg$_{1-x}$Cd$_x$Te$.^5$ For each of these several subbands are occupied in the asymmetric, triangular potential at the insulator-semiconductor interface for $B_z=0$. When the latter are successively emptied with rising $B_{||}$, pronounced minima in $d\sigma/dN_z$ are found. In Ref. 5 it is shown that the dips in $d\sigma/dN_z$ relate to subband onsets with increasing $N_z$ as observed for $B_{||}=0$. Fan charts of the structures in a $B_{||}$-$N_z$ diagram give nearly straight lines. In Ref. 2 this behavior has been explained in terms of a “universal” plot with dimensionless fields and densities.

The present $\delta$ layers differs from the MIS (metal-insulator-semiconductor) systems in two respects. For one, the potential is symmetric in $z$. The $k_x=k_y=0$ state remains the lowest level in the subband. It experiences a purely diamagnetic shift with $B_{||}$. The second difference is that the layer density cannot be tuned without disturbing the symmetry of the potential. Thus each $N_z$ point for data in the $B_{||}$-$N_z$ fan chart requires a new sample. Data must be taken in a sweep of $B_{||}$.

Section II below gives a few relevant, experimental details. In Sec. III we present results on measurements of the conductivity oscillations for several different $\delta$ layers. Section IV is concerned with the analysis and discussion of the data.

II. EXPERIMENTAL NOTES

The samples are MBE grown, lightly $n$-type GaAs on a semi-insulating substrate. The growth is interrupted to introduce the Si donors. The deposition rate of the Si is $7.02 \times 10^{10}$ sec$^{-1}$ cm$^{-2}$. The layer thicknesses on the semi-insulating substrate are 5000 Å $n$-GaAs, Si doping sheet, 3000 Å $n$-GaAs. The nominal sheet dopings are 4.3, 8.6, and $17.2 \times 10^{12}$ Si atoms/cm$^2$ for samples 1, 2,
TABLE I. Nominal Si doping, \( N_i \), and occupied bands from a calculation: For samples 1 and 2, oscillations have been identified from the \( n = 0, 1, \) and 2 subbands and densities \( N_i^0, N_i^1, \) and \( N_i^2 \). For sample 3, we use high donor concentration (density above \( \sim 10^{13} \text{cm}^{-2} \)).

<table>
<thead>
<tr>
<th>Sample</th>
<th>Nominal Si doping (( 10^{12} \text{ cm}^{-2} ))</th>
<th>( N_i = \sum N_i )</th>
<th>Occupied subbands</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.3</td>
<td>3.7</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>8.6</td>
<td>7.4</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>17.2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

and 3. Sample 1 is from the batch described in previous work. Measurements are made in the Corbino-disc geometry of the capacitive-coupling arrangement. All data are for 4.2 K. Magnetic fields up to 23 T are provided by the high-field facility in Grenoble. The conductivity is measured after illuminating the samples with band-gap radiation at 4.2 K.

Each of the samples employed here has been investigated previously by measuring the oscillatory magnetoconductivity in a perpendicular field. From this Shubnikov-de Haas effect period the subband occupancies \( N_i^k \) are determined. For samples 1 and 2, oscillations have been identified from the \( n = 0, 1, \) and 2 subbands and densities \( N_i^0, N_i^1, \) and \( N_i^2 \) are known with good accuracy. A calculation that matches these occupations is used to give the total \( N_i \) and the number of occupied bands. These are listed in Table I below. For densities above \( \sim 10^{13} \text{cm}^{-2} \) we have difficulty in assigning the observed Shubnikov-de Haas periods. Moreover, a noticeable saturation of the sheet conductivity with increasing Si concentration suggests that the number of mobile carriers does not increase linearly with the doping. This problem is to be discussed elsewhere, but we include here as sample 3, one of these with high donor concentration, with unknown total \( N_i \) and number of occupied bands.

III. RESULTS

In Fig. 2 the conductance \( \sigma(B_\parallel) \) and its derivative \( d\sigma/dB_\parallel \) are shown for each of the three samples. The variation of \( \sigma \) with \( B_\parallel \) is typically 10% near \( \sigma_0 \). The oscillations extend to values above and below \( \sigma_0 \). The conductivity does not show the decrease characteristic of a perpendicular field. The oscillations, even though they remind one of the Shubnikov-de Haas effect, are not periodic in \( 1/B_\parallel \). In analogy with previous experiments on MIS structures and the model calculation, the oscillations represent an emptying of subbands with rising \( B_\parallel \).

In order to relate the \( d\sigma/dB \) oscillations in Fig. 2 to the previous experiments on the density-tuned MIS structures, it is necessary to identify the phase that represents the subband cutoff with rising \( B_\parallel \). It is known that the \( d\sigma/dN_i \) dip feature marks the onset of occupation of a new subband with rising \( N_i \) in the MIS system. We argue as follows for the \( d\sigma/dB_\parallel \) data. The increment \( \Delta B_\parallel \) leads to cutoff for a subband in much the same way as a decrement \( \Delta N_i \). Thus the maxima of \( d\sigma/dB_\parallel \) relate directly to the minima of the \( d\sigma/dN_i \) data. The maxima have been marked in Fig. 2 and labeled with an index that at this stage represents an educated guess.

IV. ANALYSIS AND DISCUSSION

We analyze the sweep out of the subbands with rising \( B_\parallel \) by solving the Schrödinger equation:

\[
\left[ \frac{\Psi''}{2m^*} + \frac{\Psi'}{2m^*} + \frac{1}{2m^*} (p_x + ezB_\parallel)^2 + V(z) \right] \Psi = E \Psi
\]

FIG. 2. \( B_\parallel \) magnetoconductivity and derivative \( d\sigma/dB_\parallel \) for three different samples. Maxima in \( d\sigma/dB_\parallel \) mark the cutoff condition at which the \( n \)th subband level coincides with \( E_F \).

FIG. 3. Calculated cutoff condition for the \( n \)th subband as a function of \( N_i \) and \( B_\parallel \). The dots are experimental results.
in the self-consistent and, because of the redistribution of carriers, \( B_\parallel \)-dependent accumulation layer potential \( V(z) \). The predicted \( B_\parallel \) versus \( N_\parallel \) for this case is given in terms of dimensionless, normalized field and density parameters in Ref. 2. In Fig. 3 the full lines give the result for the range of \( B_\parallel \) and \( N_\parallel \) relevant for the present experiments. The lines are labeled with the subband index \( n \). When the positions of the \( d\sigma/dB_\parallel \) maxima marked in the data of Fig. 2 for samples 1 and 2 are entered for the respective values of \( N_\parallel \), we note good agreement and unambiguous identification of the subband index.

With the consideration of the conductivity data from sample 3, we come to face a central problem in the characterization of the \( \delta \)-layer subbands for densities above \( 1 \times 10^{13} \text{ cm}^{-2} \). The energy of the subband levels and their filling to \( E_F \) is so high that nonparabolicity of the \( \Gamma \)-point conduction band cannot be ignored. For the \( N_\parallel \) of sample 2 the \( E_F \) in the \( n=0 \) subband according to a "parabolic" model calculation is 164 meV with \( B_\parallel =0 \). At such energy according to Ref. 8 there is a considerable change of \( m^* \). The subband structure and partial filling of subbands are modified by nonparabolicity. They depend substantially on the energy position of subbands derived from the \( L \)- and \( X \)-point minima of the conduction bands. These questions are to be treated in Ref. 7. For present purposes we note only that the calculation of Ref. 2 ignores nonparabolicity and that its effect on the interpretation of the \( d\sigma/dB_\parallel \) data cannot be estimated reliably. It has been shown in Ref. 5 with work on the extremely nonparabolic subbands of \( \text{Hg}_1-x\text{Cd}_x\text{Te} \) that the cutoff points in \( B_\parallel \) versus \( N_\parallel \) follow a straight line that is approximately given by the calculation in Ref. 2. It is in this sense that we fit for sample 3 the \( d\sigma/dB_\parallel \) maxima at an \( N_\parallel \) value of \( 1.2 \times 10^{13} \text{ cm}^{-3} \) and assign the corresponding subband indices. Compared to the design density of \( 1.72 \times 10^{13} \text{ Si} \) donors per cm\(^2\), this is a remarkable reduction. The effect may be linked with the conductivity and density saturation cited in Ref. 1, but further clarification is needed with regard to the nonparabolicity question.

In order to relate the \( d\sigma/dB_\parallel \) maxima marked as dots in Fig. 3 to the density of states for the magnetoelectric subbands, we show explicitly in Fig. 4 the energy bands versus \( k_x \) and the corresponding density of states \( D(E) \) for sample 2 in a field of 12 T. The effect of the field has been to raise the band minimum and to distort the parabolic dispersion (dashed lines in Fig. 4). The distortion results in a distinct increase of the density of states for the subbands near \( E_F \). It follows that the cutoff effect in a field is stronger than that which would be found for \( B_\parallel =0 \) when reducing \( N_\parallel \). This fact is evident in the experiment of Refs. 3—5.

Repeating the calculation for Fig. 4 at several values of \( B_\parallel \) allows us to construct the density of states variation at \( E_F \). The function \( D(E_F) \) versus \( B_\parallel \) is given in Fig. 5 for each of the subbands and as a sum. The density of states decreases abruptly with the emptying of a subband. The position of the \( d\sigma/dB_\parallel \) maxima on the \( B_\parallel \) axis coincides nicely with the decrease of the density of states, thus confirming the qualitative arguments made in the Introduc-

![FIG. 4. Magnetoelectric bands and density of states at \( B_\parallel =12 \text{ T} \). The dashed lines apply for the \( E(k) \) dispersion at \( B_\parallel =0 \text{ T} \).](image)

![FIG. 5. Calculated \( D(E_F) \) versus \( B_\parallel \) for \( N_\parallel =7.4 \times 10^{12} \text{ cm}^{-2} \). Both, individual subband contributions and the sum over all subbands are shown together with the experimental \( d\sigma/dB_\parallel \).](image)

![FIG. 6. Calculation of the subband level shift versus \( B_\parallel \). The dots represent results of first-order perturbation theory.](image)
tion. Moreover the fact that \( D(E_F) \) oscillates, rising above and falling below the value at \( B_{||} = 0 \), leads us to expect that \( \sigma(B_{||}) \) will rise and fall relative to the zero-field value. The modulation depth of the conductivity is comparable to the modulation of \( D(E_F) \) in the calculation. Such a result would be expected for the dominant ionized impurity scattering. Because the calculation shows the self-consistent electrostatic potential to be remarkably constant with \( B_{||} \), the carriers experience an unchanging strength of interaction with the ions. It follows that the scattering rate varies directly with the final density of states, namely \( D(E_F) \).

As a final point of our discussion we return to the "diamagnetism" mentioned in the introduction. To the extent that the electrostatic binding of the \( k_x = 0 \) nth subband state is unchanged in the field, it follows that \( \langle z_n^2 \rangle \) is a constant. The diamagnetic shift of the bottom of the subband is then well described as \( e^2 \langle z_n^2 \rangle B_{||}^2 / 2m^* \), the standard expression for a diamagnetic energy. Figure 6 makes clear how well this is satisfied for the uppermost levels \( n = 3 \) and 4 by comparing it with the result of the self-consistent calculation. If the occupancies of such levels are known from the conventional Shubnikov—de Haas effect measurement, the "diamagnetic" expulsion from the Fermi distribution at field \( B_{||} \) allows one to experimentally determine \( \langle z_n^2 \rangle \). The prominent peaks in Fig. 2, especially those at the lower values of \( B_{||} \), could easily have been interpreted in this simple manner. In this sense, the \( B_{||} \) experiment provides an additional simple way to explore the bound states of the \( \delta \)-layer potential. By comparison with the usual Shubnikov—de Haas formula \( \Delta(1/B_4) = e\hbar/mE_R^F \) for the \( n \)th subband period in \( 1/B_4 \), the diamagnetic variant of the Shubnikov—de Haas effect measures \( \langle z_n^2 \rangle \) according to \( E_R^F = e^2 \langle z_n^2 \rangle B_{||}^2 / 2m^* \).

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