Calculation of the Anisotropic Hyperfine Coupling in Cu(II) Bis(dithiocarbamate) and Cu(II) Bis(diselenocarbamate). A Formula for the Anisotropic Hyperfine Coupling Tensor

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For a molecule with one unpaired electron a general formula is derived for the elements of the anisotropic hyperfine coupling tensor, taking into account all possible electron excitations. For a check of the formula the hyperfine coupling tensors are calculated for the copper atom in bis(N,N-diethyl dithiocarbamate) copper(II) and for the copper and selenium atoms in bis(N,N-diethyl diselenocarbamate) copper(II), using the iterative extended Hückel method. Both the calculated principal values of these tensors and the direction cosines of the principal axes are in good agreement with the experimentally observed ones.

I. INTRODUCTION

Already in 1958 Maki and McGarvey expressed the spin Hamiltonian parameters in terms of atomic orbital coefficients of molecular orbital wavefunctions, describing an unpaired electron in a transition-metal complex. By experimental determination of the spin Hamiltonian they got expressions for the molecular orbital (MO) wavefunctions.

This procedure necessitates either a high symmetry of the molecule studied, or a strong simplification of the expressions for the spin Hamiltonian parameters. In a previous paper we showed that these simplifications are not allowed for g-value calculations. In this paper a general formula is derived for the elements of the anisotropic hyperfine coupling tensor. We show which excited states contribute and which do not. The resulting formula has been applied to the calculation of central atom and ligand hyperfine couplings in bis[N,N-diethyldithiocarbamato] copper(II) and bis[N,N-diethyldiselenocarbamato] copper(II), yielding values which are in very good agreement with the experimentally observed ones.

II. DEFINING THE PROBLEM

The system under consideration is a molecule containing $N$ nuclei and 2$n+1$ electrons, one of the electrons is unpaired. With the aid of second order perturbation theory, the hyperfine coupling of the electronic spin $S$ with the spin $I$ of the nucleus $T$ will be calculated. The perturbing Hamiltonian is:

$$3C_{\text{pert}} = \sum_i [3C_{LS}^i + 3C_{IL}^i + 3C_{DD}^i],$$

where the summation $i$ runs over all electrons, and

$$3C_{LS}^i = \sum_k \zeta(r_k)iL^k\cdot S,$$

with $\zeta(r_k)$ being the one-electron spin–orbit coupling operator, and $L^k$ the angular momentum of the $i$th electron around the nucleus $k$. The summation $k$ runs over all nuclei. The dipolar operator is

$$3C_{DD}^i = (P/r_i^3)[(3(r_i\cdot S^i)(r_i\cdot I)/r_i^3) - I\cdot S^i],$$

with $P = g_\epsilon g_\mu \beta N$, $g_\epsilon$ being the free electron $g$ value, $\beta$ the Bohr magneton, $g_\mu$ the $g$ value of the nucleus $T$, and $\beta_N$ the nuclear magneton. The radius vector from the nucleus $T$ towards the electron $i$ is $r_i$. Also,

$$3C_{IL}^i = (P/r_i^3)I\cdot L^i,$$

where $L^i$ is the angular momentum of the $i$th electron around the nucleus $T$. The dipolar operator $3C_{DD}^i$ may be rewritten as

$$3C_{DD}^i = (P/r_i^3)\left(\frac{4\pi}{5}\right)^{1/4}\left[4\left(2S_z^i(3Z_z^2 - 5) + Z_x^i + Z_y^i\right)I_2S_z^i + 2\left(2Z_z^i + I_2S_x^i + I_2S_y^i\right)\left(3Z_z^2 - 5\right)\right],$$

where each $Z$ function is a normalized real combination

Table I. Experimentally determined and calculated values for $\Delta g_{ii}, A_{ii}$, and $a_{ii}$ for Cu(II) (N,N-diethyldithiocarbamate) doped in the corresponding Ni(II) complex.

<table>
<thead>
<tr>
<th></th>
<th>Obs</th>
<th>Calc</th>
<th>Cu</th>
<th>Obs</th>
<th>Calc</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta g_{xx}$</td>
<td>0.0177±0.0010</td>
<td>0.0183</td>
<td>$A_{xx}$</td>
<td>43.0±2.8</td>
<td>40.5</td>
</tr>
<tr>
<td>$\Delta g_{yy}$</td>
<td>0.0227±0.0010</td>
<td>0.0227</td>
<td>$A_{yy}$</td>
<td>37.0±2.8</td>
<td>39.5</td>
</tr>
<tr>
<td>$\Delta g_{zz}$</td>
<td>0.0817±0.005</td>
<td>0.0767</td>
<td>$A_{zz}$</td>
<td>-80.0±1.6</td>
<td>-80.0</td>
</tr>
<tr>
<td>$a_{xx}$</td>
<td>-79.0±1.2</td>
<td></td>
<td>$a_{xx}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


* Values for $A_{ii}$ and $a_{ii}$ in $10^{-4}$ cm$^{-1}$. 

* $A_{ii}$ are the principal values of the traceless $A$ tensor.

* $a_{ii}$ is the mean of the principal values of the nontraceless $A$ tensor.
of spherical harmonics \( Y_{2,m} \), centered on the nucleus \( T \). By equating (2a) and (2b), the definition of \( F_{2,m} \) is obvious.

Because \( \mathcal{C}_{\text{pert}} \) contains only one-electron operators, all states with two or more excited electrons do not contribute to the perturbation energy.

For the eigenfunctions \( \Psi_{n}^{\sigma} \) of the unperturbed Hamiltonian, antisymmetrized functions are taken, consisting of one or more Slater determinants. The upper index of \( \Psi \) is the expectation value of \( S_{z} \), with respect to this function, the lower denotes the type of function. The Slater determinants are composed of one-electron spin orbitals \( \psi(\sigma) \). The spatial part \( \psi \) of a spin orbital is a MO consisting of a linear combination of atomic orbitals \( \phi \).

Omitting a normalization factor \( \mathcal{N} = \sqrt{(2n - j - 1)!}^{-1/2} \), the unperturbed functions of the ground and excited states are:

(a) Ground state,
\[
\Psi_{0}^{\sigma} = | \psi_{\alpha}(1) \psi_{\beta}(2) \cdots \psi_{\alpha}(2n-1) \psi_{\beta}(2n) \psi_{n+1}^{\sigma}(2n+1) |
\]

(b) Excited states with an electron excited towards the MO of the unpaired electron,
\[
\Psi_{l}^{\sigma} = | \psi_{\alpha}(1) \psi_{\beta}(2) \cdots \psi_{\sigma}(2l-1) \cdots \psi_{n+l}^{\sigma}(2n) \psi_{n+1}^{\sigma}(2n+1) |, \quad l < n+1.
\]

(c) Excited states with the unpaired electron excited towards the initially empty MO,
\[
\Psi_{l}^{\sigma} = | \psi_{\alpha}(1) \psi_{\beta}(2) \cdots \psi_{\sigma}(2l-1) \cdots \psi_{n+l}^{\sigma}(2n) \psi_{0}(2n+1) |, \quad h > n+1.
\]

(d) Excited states with an electron excited from a doubly occupied MO towards an empty one. These excitations give rise to two doublet and one quartet state,
\[
\Psi_{d}^{\sigma} = 2^{-1/2} | \psi_{\alpha}(1) \psi_{\beta}(2) \cdots \psi_{\sigma}(2l-1) \cdots \psi_{n+l}^{\sigma}(2n) \psi_{0}(2n+1) |
\]
\[
\Psi_{d}^{\sigma} = 6^{-1/2} | 2 | \psi_{\alpha}(1) \psi_{\beta}(2) \cdots \psi_{\sigma}(2l-1) \cdots \psi_{n+l}^{\sigma}(2n) \psi_{0}(2n+1) |
\]
\[
\Psi_{d}^{\sigma} = 3^{-1/2} | 3 | \psi_{\alpha}(1) \psi_{\beta}(2) \cdots \psi_{\sigma}(2l-1) \cdots \psi_{n+l}^{\sigma}(2n) \psi_{0}(2n+1) |
\]

III. THE HYPERFINE COUPLING TENSOR

Defining a spin-Hamiltonian \( \mathcal{C}_{\sigma} = S \mathbf{A} \mathbf{I} \), with \( \mathbf{A} \) being the anisotropic hyperfine coupling tensor, we require \( \mathcal{C}_{\sigma} \) and \( \mathcal{C}_{\text{pert}} \) to yield the same perturbation energy. Applying second order perturbation theory for degenerate states, the requirement is
\[
\langle \Psi_{0}^{\sigma} | \mathcal{C}_{\sigma} | \Psi_{0}^{\sigma} \rangle = \langle \Psi_{0}^{\sigma} | \mathcal{C}_{\text{pert}} | \Psi_{0}^{\sigma} \rangle + \sum_{\sigma' \neq \sigma} \sum_{\pi} \langle \Psi_{\sigma}^{\pi} | \mathcal{C}_{\text{pert}} | \Psi_{0}^{\sigma'} \rangle \langle \Psi_{\sigma'}^{\sigma'} | \mathcal{C}_{\text{pert}} | \Psi_{0}^{\pi'} \rangle / (E_{0} - E_{n}),
\]
where \( \sigma \) and \( \sigma' \) and the nuclear spin functions \( \pi \) and \( \pi' \) are arbitrary and may be equal. The summations run over all possible excited states, \( E_{0} - E_{n} \) being the excitation energy.

A. First Order Contribution

\( \mathcal{C}_{LL} \) and \( \mathcal{C}_{II} \) do not contribute in first order. \( \mathcal{C}_{DD} \) yields a traceless, symmetric tensor with elements
\[
A_{\nu \mu} = \sum_{\pi} \sum_{\sigma' \neq \sigma} \frac{\langle \Psi_{\sigma}^{\pi} | \mathcal{C}_{\text{pert}} | \Psi_{\sigma'}^{\sigma'} \rangle \langle \Psi_{\sigma'}^{\sigma'} | \mathcal{C}_{\text{pert}} | \Psi_{\sigma}^{\pi} \rangle}{E_{0} - E_{n}},
\]
where the symbols \( \nu \) and \( \omega \) are \( x, y, \) or \( z \).

B. Second Order Contribution

In second order all terms are neglected which explicitly contain \( \langle \sigma^{2} \rangle \) twice. The energies of the states are calculated by summing one-electron MO energies. The remaining terms have the form
\[
\langle \Psi_{\sigma}^{\pi} | \mathcal{C}_{LL} | \Psi_{\sigma}^{\pi'} \rangle \langle \Psi_{\sigma}^{\pi'} | \mathcal{C}_{DD} | \Psi_{\sigma}^{\pi} \rangle
\]

(5a)
and
\[
\langle \Psi_0 | 3c_{DD} | \Psi_n'''> \langle \Psi_n''' | 3c_{LS} | \Psi_0 '\rangle
\]  
(5b)
and the similar products with the operators interchanged.

It may be deduced that second order contributions from terms of the type \(5a\) yield a nontraceless tensor
\[
A_{\nu\omega}^{2a} = 2P \sum_{\text{uptrac}} \sum_{\nu} (E_{\Psi_{n+1}} - E_{\Psi_n})^{-1} \sum_{k=1}^{N} \langle \Psi_{n+1} | \xi(r_k) L^k_b | \psi_m \rangle \langle \psi_m | L_{\omega/r^5} | \psi_{n+1} \rangle,
\]
where \(L^k_b\) is the \(b\)th component of the angular momentum operator \(L_k\) centered on the nucleus \(k\).

This contribution is due to excited states \(\Psi_{n+1}'\) and \(\Psi_n''\); the other excited states do not contribute.

Also the terms \(5b\) give a contribution which is due only to the excited states \(\Psi_{n+1}'\) and \(\Psi_n''\)
\[
A_{\nu\omega}^{2b} = P \sum_{\text{uptrac}} \sum_{\nu} \sum_{u,t} \left[ i e_{u\nu} \langle E_{\Psi_{n+1}} - E_{\Psi_n} \rangle \langle \psi_{n+1} | L^k_b | \psi_m \rangle \langle \psi_m | F_{\omega/r^5} | \psi_{n+1} \rangle \right],
\]
with \(\varepsilon\) being the Levi–Civita symbol. The summations \(u\) and \(t\) run over the Cartesian coordinates \(x, y,\) and \(z\).

The excited states \(\Psi_{n+1}'\) do not give a contribution, while the contributions from the states \(\Psi_{n+1}'\), \(\Psi_n''\), and \(\Psi_n''\) cancel each other.

Summing up, the expression for a general tensor element is:
\[
A_{\nu\omega} = P \{ \langle \psi_{n+1} | F_{\omega/r^5} | \psi_{n+1} \rangle + \sum_{\text{uptrac}} \sum_{\nu} (E_{\Psi_{n+1}} - E_{\Psi_n})^{-1} \sum_{k=1}^{N} \left[ 2 \langle \psi_{n+1} | \xi(r_k) L^k_b | \psi_m \rangle \langle \psi_m | L_{\omega/r^5} | \psi_{n+1} \rangle \right] \}
\]
(8)

IV. APPLICATIONS

A. Cu(II)(N,N-Diethyldithiocarbamate)\(_2\); Cu(dtc)\(_2\)

As is reported elsewhere, we performed iterative extended Hückel calculations on the compound

![Cu complex structure](image)

taking into account all valence orbitals. Using the structure of the host lattice [that is the corresponding Ni(II) complex\(^a\)] and employing values for the empirical parameters which are accepted as reasonable in the literature, the \(g\) values and anisotropic hyperfine couplings were calculated, taking into account only one-center integrals. The first order contributions \(A_{\nu\omega}^1\) (Formula 4) were: \(A_{xx} = 48.5 \times 10^{-4}\) cm\(^{-1}\), \(A_{yy} = 48.5 \times 10^{-4}\) cm\(^{-1}\), and \(A_{zz} = -97.0 \times 10^{-4}\) cm\(^{-1}\). The second order contributions were calculated with approximate formulas\(^b\):

\[
A_{xx} = P' \left[ (\Delta g_{xx} - (3/14) \Delta g_{yy}) \right],
\]
\[
A_{yy} = P' \left[ (\Delta g_{yy} - (3/14) \Delta g_{xx}) \right],
\]
\[
A_{zz} = P' \left[ (\Delta g_{xx} + (3/14) \Delta g_{yy} + (3/14) \Delta g_{zz}) \right],
\]

where \(P' = g_e \beta g_C (\text{Cu}) (\tau^{-1}) = 315.98 \times 10^{-4}\) cm\(^{-1}\).

These formulas yield a pseudo contact interaction of \(12.9 \times 10^{-4}\) cm\(^{-1}\) and anisotropic interactions: \(A_{xx} = -8.8 \times 10^{-4}\) cm\(^{-1}\), \(A_{yy} = -6.9 \times 10^{-4}\) cm\(^{-1}\), and \(A_{xx} = +15.7 \times 10^{-4}\) cm\(^{-1}\). Adding these values to the first order contributions, we obtained: \(A_{xx} = 39.7 \times 10^{-4}\) cm\(^{-1}\), \(A_{yy} = 41.6 \times 10^{-4}\) cm\(^{-1}\), and \(A_{zz} = -81.3 \times 10^{-4}\) cm\(^{-1}\). These values agree rather well with the observed ones, but the anisotropy in the \(xy\) plane is too small and the order of \(A_{xx}\) and \(A_{yy}\) is even reversed.

It turns out, however, that a calculation with the formula derived in this paper (Eq. 8) yields much better results, taking into account only one-center contributions. It is expected that more-center integrals may be neglected, because of the \(r^{-3}\) dependency of the

| Table II. Experimentally determined\(^a\) and calculated values for \(g_{ii}, A_{ii}\)^b,c,d for Cu(II) (N,N-diethylselenocarbamate)\(_2\) doped in the corresponding Ni(II) complex. |
|-------------|----|-------------|----|-------------|----|-------------|----|-------------|----|-------------|
|             | Obs | Cu          | Calc | Se\(_i\) | Obs | Calc | Se\(_i\) | Obs | Calc | Se\(_i\) | Obs | Calc |
| \(g_1\)     | 2.0511±0.0001 | 36.4±0.5 | 33.4 | \(A_{xx}\) | 57.4±0.3 | 54.6 | \(A_{yy}\) | -26.4±0.3 | -26.3 | \(A_{zz}\) | -31.0±0.6 | -28.3 |
| \(g_2\)     | 2.0021±0.0002 | 28.8±0.5 | 28.5 | \(A_{yy}\) | -25.5±0.7 | -26.4 | \(A_{zz}\) | -32.3±1.7 | -28.2 |
| \(g_3\)     | 1.9943±0.0002 | -65.2±0.5 | -62.0 | \(A_{zz}\) | 48.0±0.2 | \(g_{iso}\) | 45.6±0.3 |

\(^a\) See Ref. 8. Footnotes b, c, and d have the same meanings as in Table I.
interactions. The resulting values are listed in Table I, after subtraction of a pseudo contact interaction of $8.2 \times 10^{-4}$ cm$^{-1}$. Although the anisotropy in the $xy$ plane is still not big enough, the order of $A_{xx}$ and $A_{yy}$ is right.

B. Cu(II)($N,N$-Diethyldiselenocarbamate)$_2$; Cu(dsc)$_2$

After the EPR investigations on the $n$-butyl compound$^7$ we did also single crystal EPR measurements on the complex

![Complex Structure]

doped in the Ni(II) compound.$^8$ The central part of the latter compound is planar, and has a nearly $D_{3h}$ symmetry.$^9$ The measured principal values of the $g$-tensor, and the hyperfine coupling tensors of the central copper atom and the ligand selenium atoms are listed in Table II. The principal axes of the $A_{Cu}$ tensor coincide with the $x$, $y$, and $z$ axes (the $x$ and $y$ axis being the bisectors of the angles Se$_1$-Cu-Se$_2$); the angles of the other principal axes relative to these are listed in Table III.

The approximate equations (9) are derived under the assumption that the principal axes of the $g$ and $A$ tensor coincide. Table III illustrates that this is no longer the case for the tensors in Cu(dsc)$_2$, and hence Eq. (9) cannot be applied. With Eq. (8), the anisotropic hyperfine couplings of the copper atom as well as of the selenium atoms have been calculated, again using the iterative extended Hückel method and taking into account only one center integrals. The resulting principal values are listed in Table II, the angles of the principal axes relative to those of the $A_{Cu}$ tensor in Table III. Calculated $g$ values are not listed, because they are affected very much by neighboring molecules, as will be published later. These molecules do not affect the hyperfine couplings, because of the $r^{-3}$ dependency of these interactions. The principal values are in very good agreement with the experiment and the order of all couplings is right. The average difference between the observed and calculated directions of the principal axes is 4.9°, a very satisfactory result, keeping in mind the approximations of the extended Hückel method.

For the radial part of the atomic wavefunctions in the extended Hückel calculations, we used Slater-type orbitals from Refs. 10–12.

The required valence state ionization energies were taken from Ref. 13 or calculated from the data in Ref. 14.

The radial parts of the integrals containing the spin-orbit operator are approximated by the atomic spin-orbit coupling constants: $\lambda(Cu\ 3d)^{15}=828$ cm$^{-1}$; $\lambda(Cu\ 4p)^{16}=925$ cm$^{-1}$; $\lambda(S\ 3p)^{17}=382$ cm$^{-1}$; $\lambda(C\ 2p)^{17}=28$ cm$^{-1}$; $\lambda(N\ 2p)^{17}=76$ cm$^{-1}$; and $\lambda(Se\ 4p)^{18}=1690$ cm$^{-1}$. For calculating the expectation values of $r^{-3}$, use has been made of the Hartree–Fock functions published by Clementi.$^{19}$

The calculations were performed at the University Computing Center on the IBM 360/50 computer.

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HYPERFINE COUPING IN Cu(II) COMPLEXES


An elaborate report will be published soon.


Calculated from Ref. 14.