Flavour tagged time dependent angular analysis of the $B_s^0 \to J/\psi \phi$ decay and extraction of $\Delta \Gamma_s$ and the weak phase $\phi_s$ in ATLAS

The ATLAS Collaboration

Abstract

A measurement of the $B_s^0 \to J/\psi \phi$ decay parameters, updated to include flavour tagging is reported using 4.9 fb$^{-1}$ of integrated luminosity collected by the ATLAS detector from $\sqrt{s} = 7$ TeV $pp$ collisions recorded in 2011 at the LHC. The values measured for the physical parameters are:

$$\phi_s = 0.12 \pm 0.25 \text{ (stat.)} \pm 0.05 \text{ (syst.) rad}$$

$$\Delta \Gamma_s = 0.053 \pm 0.021 \text{ (stat.)} \pm 0.010 \text{ (syst.) ps}^{-1}$$

$$\Gamma_s = 0.677 \pm 0.007 \text{ (stat.)} \pm 0.004 \text{ (syst.) ps}^{-1}$$

$$|A_\parallel(0)|^2 = 0.220 \pm 0.008 \text{ (stat.)} \pm 0.009 \text{ (syst.)}$$

$$|A_0(0)|^2 = 0.529 \pm 0.006 \text{ (stat.)} \pm 0.012 \text{ (syst.)}$$

$$\delta_\perp = 3.89 \pm 0.47 \text{ (stat.)} \pm 0.11 \text{ (syst.) rad}$$

where the parameter $\Delta \Gamma_s$ is constrained to be positive. The $S$-wave contribution was measured and found to be compatible with zero. Results for $\phi_s$ and $\Delta \Gamma_s$ are also presented as 68% and 95% likelihood contours, which show agreement with the Standard Model expectations.
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The ATLAS Collaboration

(Dated: July 8, 2014)

A measurement beyond the predictions of the Standard Model (SM) may alter CP violation in $B$-decays. A channel that is expected to be sensitive to new physics contributions is the decay $B_s^0 \rightarrow J/\psi\phi$. CP violation in the $B_s^0 \rightarrow J/\psi\phi$ decay occurs due to interference between direct decays and decays with $B_s^0 - \bar{B}_s^0$ mixing. The oscillation frequency of $B_s^0$ meson mixing is characterized by the mass difference $\Delta m_s$ of the heavy ($B_{LH}$) and light ($B_{LL}$) mass eigenstates. The CP violating phase $\phi_s$ is defined as the weak phase difference between the $B_s^0 - \bar{B}_s^0$ mixing amplitude and the $b \rightarrow c\bar{s}d$ decay amplitude. In the absence of CP violation, the $B_{LH}$ state would correspond to the CP odd state and the $B_{LL}$ to the CP even state. In the SM the phase $\phi_s$ is small and can be related to CKM quark mixing matrix elements via the relation $\phi_s \simeq -2\beta_s$, with $\beta_s = \arg[-(V_{ts}V_{sb}^*)/(V_{cs}V_{cb}^*)]$; a value of $\phi_s \simeq -2\beta_s = -0.037 \pm 0.002 \text{ rad}$ [1] is predicted in the SM. Many new physics models predict large $\phi_s$ values whilst satisfying all existing constraints, including the precisely measured value of $\Delta m_s$ [2] [3].

Another physical quantity involved in $B_s^0 - \bar{B}_s^0$ mixing is the width difference $\Delta\Gamma_s = \Gamma_s - \Gamma_{\bar{s}}$, which is predicted to be $\Delta\Gamma_s = 0.087 \pm 0.021 \text{ ps}^{-1}$ [4]. Physics beyond the SM is not expected to affect $\Delta\Gamma_s$ as significantly as $\phi_s$ [5]. Extracting $\Delta\Gamma_s$ from data is nevertheless useful as it allows theoretical predictions to be tested [5].

The decay of the pseudoscalar $B_s^0$ to the vector–vector final-state $J/\psi\phi$ results in an admixture of $CP$ odd and $CP$ even states, with orbital angular momentum $L = 0$, 1 or 2. The final states with orbital angular momentum $L = 0$ or 2 are $CP$ even while the state with $L = 1$ is $CP$ odd. Flavour tagging is used to distinguish between the initial $B_s^0$ and $\bar{B}_s^0$ states. The $CP$ states are separated statistically using an angular analysis of the final-state particles.

In this paper, an update to the previous measurement [6] with the addition of flavour tagging, is presented. Flavour tagging significantly reduces the uncertainty of the measured value of $\phi_s$ while also allowing a measurement of one of the strong phases. Previous measurements of these quantities have been reported by the DØ, CDF and LHCb collaborations [7–9]. The analysis presented here uses 4.9 fb$^{-1}$ of LHC $pp$ data at $\sqrt{s} = 7$ TeV collected by the ATLAS detector in 2011.

II. ATLAS DETECTOR AND MONTE CARLO SIMULATION

The ATLAS experiment [10] is a multipurpose particle physics detector with a forward-backward symmetric cylindrical geometry and near 4$\pi$ coverage in solid angle. The inner tracking detector (ID) consists of a silicon pixel detector, a silicon microstrip detector and a transition radiation tracker. The ID is surrounded by a thin superconducting solenoid providing a 2 T axial magnetic field, and by a high granularity liquid-argon sampling electromagnetic calorimeter. A steel/scintillator tile calorimeter provides hadronic coverage in the central rapidity range. The end-cap and forward regions are instrumented with Liquid-argon calorimeters for both electromagnetic and hadronic measurements. The muon spectrometer (MS) surrounds the calorimeters and consists of three large superconducting toroids with eight coils each, a system of tracking chambers, and detectors for triggering.

The muon and tracking systems are of particular importance in the reconstruction of $B$ meson candidates. Only data where both systems were operating correctly and where the LHC beams were declared to be stable are used. A muon identified using a combination of MS and ID track parameters is referred to as combined. A

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I. INTRODUCTION

New phenomena beyond the predictions of the Standard Model (SM) may alter CP violation in $B$-decays. A channel that is expected to be sensitive to new physics contributions is the decay $B_s^0 \rightarrow J/\psi\phi$. CP violation in the $B_s^0 \rightarrow J/\psi\phi$ decay occurs due to interference between direct decays and decays with $B_s^0 - \bar{B}_s^0$ mixing. The oscillation frequency of $B_s^0$ meson mixing is characterized by the mass difference $\Delta m_s$ of the heavy ($B_{LH}$) and light ($B_{LL}$) mass eigenstates. The CP violating phase $\phi_s$ is defined as the weak phase difference between the $B_s^0 - \bar{B}_s^0$ mixing amplitude and the $b \rightarrow c\bar{s}d$ decay amplitude. In the absence of CP violation, the $B_{LH}$ state would correspond to the CP odd state and the $B_{LL}$ to the CP even state. In the SM the phase $\phi_s$ is small and can be related to CKM quark mixing matrix elements via the relation $\phi_s \simeq -2\beta_s$, with $\beta_s = \arg[-(V_{ts}V_{sb}^*)/(V_{cs}V_{cb}^*)]$; a value of $\phi_s \simeq -2\beta_s = -0.037 \pm 0.002 \text{ rad}$ [1] is predicted in the SM. Many new physics models predict large $\phi_s$ values whilst satisfying all existing constraints, including the precisely measured value of $\Delta m_s$ [2] [3].

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In this paper, an update to the previous measurement [6] with the addition of flavour tagging, is presented. Flavour tagging significantly reduces the uncertainty of the measured value of $\phi_s$ while also allowing a measurement of one of the strong phases. Previous measurements of these quantities have been reported by the DØ, CDF and LHCb collaborations [7–9]. The analysis presented here uses 4.9 fb$^{-1}$ of LHC $pp$ data at $\sqrt{s} = 7$ TeV collected by the ATLAS detector in 2011.

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The muon and tracking systems are of particular importance in the reconstruction of $B$ meson candidates. Only data where both systems were operating correctly and where the LHC beams were declared to be stable are used. A muon identified using a combination of MS and ID track parameters is referred to as combined. A
muon formed by track segments which are not associated with an MS track, but which are matched to ID tracks extrapolated to the MS is referred to as segment tagged.

The data were collected during a period of rising instantaneous luminosity, and the trigger conditions varied over this time. The triggers used to select events for this analysis are based on identification of a $J/\psi \rightarrow \mu^+\mu^-$ decay, with either a 4 GeV transverse momentum ($p_T$) threshold for each muon or an asymmetric configuration.

Monte Carlo (MC) simulation is used to study the detector response, estimate backgrounds and model systematic effects. For this study, 12 million MC-simulated $B^0 \rightarrow J/\psi\phi$ events were generated using PYTHIA 6 [12] tuned with recent ATLAS data [13]. No $p_T$ cuts were applied at the generator level. Detector responses for these events were simulated using the ATLAS simulation package based on GEANT4 [14] [15]. No $p_T$ cuts were applied to the conditions during data taking was included. In order to take into account varying trigger configurations during data-taking, the MC events were weighted to have the same trigger composition as the collected collision data. Additional samples of the background decay $B^0 \rightarrow J/\psi K^0_s$ as well as the more general $bb \rightarrow J/\psi X$ and $pp \rightarrow J/\psi X$ backgrounds were also simulated using PYTHIA.

III. RECONSTRUCTION AND CANDIDATE SELECTION

Events passing the trigger and the data quality selections described in section [17] are required to pass the following additional criteria: the event must contain at least one reconstructed primary vertex, built from at least four ID tracks, and at least one pair of oppositely charged muon candidates that are reconstructed using information from the MS and the ID [16]. Both combined and segment tagged muons are used. In this analysis the muon track parameters are taken from the ID measurement alone, since the precision of the measured track parameters for muons in the $p_T$ range of interest for this analysis is dominated by the ID track reconstruction. The pairs of muon tracks are refitted to a common vertex and accepted for further consideration if the fit resolution for these three subsets. When both muons have $|\eta| < 1.05$, the dimuon invariant mass must fall in the range (2.959–3.229) GeV to be accepted as a $J/\psi$ candidate. When one muon has $1.05 < |\eta| < 2.5$ and the other muon $|\eta| < 1.05$, the corresponding signal region is (2.913–3.273) GeV. For the third subset, where both muons have $1.05 < |\eta| < 2.5$, the signal region is (2.852–3.332) GeV. In each case the signal region is defined so as to retain 99.8% of the $J/\psi$ candidates identified in the fits.

The candidates for $\phi \rightarrow K^+K^-$ are reconstructed from all pairs of oppositely charged particles with $p_T > 0.5$ GeV and $|\eta| < 2.5$ that are not identified as muons. Candidates for $B_s^0 \rightarrow J/\psi(\mu^+\mu^-)\phi(K^+K^-)$ are sought by fitting the tracks for each combination of $J/\psi \rightarrow \mu^+\mu^-$ and $\phi \rightarrow K^+K^-$ to a common vertex. Each of the four tracks is required to have at least one hit in the pixel detector and at least four hits in the silicon microstrip detector. The fit is further constrained by fixing the invariant mass calculated from the two muon tracks to the $J/\psi$ mass [17]. These quadruplets of tracks are accepted for further analysis if the vertex fit has a $\chi^2$/d.o.f. $< 3$, the fitted $p_T$ of each track from $\phi \rightarrow K^+K^-$ is greater than 1 GeV and the invariant mass of the track pairs (under the assumption that they are kaons) falls within the interval 1.0085 GeV $< m(K^+K^-) < 1.0305$ GeV. If there is more than one accepted candidate in the event, the candidate with the lowest $\chi^2$/d.o.f. is selected. In total 131513 $B^0_s$ candidates are collected within a mass range of 5.15 $< m(B^0_s) < 5.65$ GeV.

For each $B^0_s$ meson candidate the proper decay time $t$ is estimated by the expression:

$$t = \frac{L_{xy} M_B}{p_{TB}}$$

where $p_{TB}$ is the reconstructed transverse momentum of the $B^0_s$ meson candidate and $M_B$ denotes the world average mass value [17] of the $B^0_s$ meson. The transverse decay length, $L_{xy}$, is the displacement in the transverse plane of the $B^0_s$ meson decay vertex with respect to the primary vertex, projected onto the direction of the $B^0_s$ transverse momentum. The position of the primary vertex used to calculate this quantity is refitted following the removal of the tracks used to reconstruct the $B^0_s$ meson candidate.

For the selected events the average number of pileup interactions is 5.6, necessitating a choice of the best candidate for the primary vertex at which the $B^0_s$ meson is produced. The variable used is the three-dimensional impact parameter $d_0$, which is calculated as the distance between the line extrapolated from the reconstructed $B^0_s$ meson vertex in the direction of the $B^0_s$ momentum, and each primary vertex candidate. The chosen primary vertex is the one with the smallest $d_0$. Using MC simulation it is shown that the fraction of $B^0_s$ candidates which are assigned the wrong primary vertex is less than 1% and that the corresponding effect on the final results is negligible. No $B^0_s$ meson decay time cut is applied in the analysis.
IV. FLAVOUR TAGGING

The determination of the initial flavour of neutral $B$-mesons can be inferred using information from the $B$-meson that is typically produced from the other $b$-quark in the event [13]. This is referred to as the opposite-side tagging (OST).

To study and calibrate the OST methods, events containing the decays of $B^± \rightarrow J/ψK^±$ can be used, where flavour of the $B$-meson at production is provided by the kaon charge. Events from the entire 2011 run period satisfying the same data quality selections as described in section 2 are used.

A. $B^± \rightarrow J/ψK^±$ event selection

To be selected for use in the calibration analysis, events must satisfy a trigger condition requiring two oppositely charged muons within an invariant mass range around the nominal $J/ψ$ mass. Candidate $B^± \rightarrow J/ψK^±$ decays are identified using two oppositely charged combined muons forming a good vertex using information supplied by the inner detector. Each muon is required to have a transverse momentum of at least 4 GeV and pseudo-rapidity within $|η| < 2.5$. The invariant mass of the dimuon candidate is required to satisfy $2.8 < m(μ^+μ^-) < 3.4$ GeV. To form the $B$ candidate an additional track with the charged kaon mass hypothesis, $p_T > 1$ GeV and $|η| < 2.5$ is combined with the dimuon candidate and a vertex fit is performed with the mass of the di-muon pair constrained to the known value of the $J/ψ$ mass. To reduce the prompt component of the combinatorial background, the requirement $L_{xy} > 0.1$ mm is applied to the $B$ candidate. The choice of primary vertex is determined using the same procedure as done for the $B^0$ candidates.

In order to study the distributions corresponding to the signal processes with the background component removed, a sideband subtraction method is defined. Events are separated into five equal regions of $B$ candidate rapidity from $0 - 2.5$ and three mass regions. The mass regions are defined as a signal region around the fitted peak signal mass position $μ ± 2σ$ and the sidebands are $[μ - 5σ, μ - 3σ]$ and $[μ + 3σ, μ + 5σ]$, where $μ$ and $σ$ are the mean and width of the Gaussian function describing the $B$ signal mass, for each rapidity region. Individual binned extended maximum likelihood fits to the invariant mass distribution are performed in each region of rapidity.

The background is modelled by an exponential to describe combinatorial background and a hyperbolic tangent function to parameterize the low-mass contribution from incorrectly or partially reconstructed $B$ decays. A Gaussian function is used to model the $B^± \rightarrow J/ψπ^±$ contribution. The contributions of non-combinatorial backgrounds are found to have a negligible effect in the tagging procedure. Figure 1 shows the invariant mass distribution of $B$ candidates for all rapidity regions overlaid with the fit result for the combined data.

B. Tagging methods

Several methods are available to infer the flavour of the opposite-side $b$-quark, with varying efficiencies and discriminating powers. The measured charge of a muon from the semileptonic decay of the $B$ meson provides strong separation power; however, the $b \rightarrow μ$ transitions are diluted through neutral $B$ meson oscillations, as well as by cascade decays $b \rightarrow c \rightarrow μ$ which can alter the sign of the muon relative to the one from direct semileptonic decays $b \rightarrow μ$. The separation power of tagging muons can be enhanced by considering a weighted sum of the charge of the tracks in a cone around the muon. If no muon is present, a weighted sum of the charge of tracks associated with the opposite-side $B$ meson decay will provide some separation. The tagging methods are described in detail below.

For muon-based tagging, an additional muon is required in the event, with $p_T > 2.5$ GeV, $|η| < 2.5$ and with $|Δz| < 5$ mm from the primary vertex. Muons are classified according to their reconstruction class, combined or segment tagged, and subsequently treated as distinct tagging methods. In the case of multiple muons, the muon with highest transverse momentum is selected.

A muon cone charge variable is constructed, defined as

$$Q_μ = \frac{\sum_{i}N \text{ tracks } q_i \cdot (p_T^i)^κ}{\sum_{i}N \text{ tracks } (p_T^i)^κ},$$

where $κ$ is a parameter that can be tuned to optimize the separation between signal and background.
with the highest value of the b-tag weight is used.

A jet charge is defined as

\[ Q_{\text{jet}} = \sum_{i}^{N} \frac{q^{i} \cdot (p_{T}^{i})^{\kappa}}{\sum_{i}^{N} (p_{T}^{i})^{\kappa}}, \]

where \( \kappa = 1.1 \), and the sum is over the tracks associated with the jet, using the method described in ref. [21].

Figure 3 shows the distribution of charges for opposite-side jet charge from \( B^\pm \) signal candidate events.

The efficiency \( \epsilon \) of an individual tagger is defined as the ratio of the number of tagged events to the total number of candidates. A probability that a specific event has a signal decay containing a \( b \)-quark given the value of the discriminating variable \( P(B|Q) \) is constructed from the calibration samples for each of the \( B^+ \) and \( B^- \) samples, defining \( P(Q|B^+) \) and \( P(Q|B^-) \) respectively. The probability to tag a signal event as containing a \( b \)-quark is therefore \( P(B|Q) = P(Q|B^+)/\left( P(Q|B^+) + P(Q|B^-) \right) \) and \( P(B|Q) = 1 - P(B|Q) \). The tagging power is defined as \( \epsilon D^2 = \sum_{i} \epsilon_i \cdot (2P_i(B|Q_i) - 1)^2 \), where the sum is over the bins of the probability distribution as a function of the charge variable and \( \epsilon_i \) is the number of tagged events in each bin divided by the total number of candidates. An effective dilution \( D \) is calculated from the tagging power and the efficiency.

The combination of the tagging methods is applied according to the hierarchy of performance, based on the dilution of the tagging method. The single best performing tagging measurement is taken, according to the order: combined muon cone charge, segment tagged muon cone charge, jet charge. If it is not possible to provide a tagging response for the event, then a probability of 0.5 is assigned. A summary of the tagging performance is given in table 1.
TABLE I. Summary of tagging performance for the different tagging methods described in the text. Uncertainties shown are statistical only. The efficiency and tagging power are each determined by summing over the individual bins of the charge distribution. The effective dilution is obtained from the measured efficiency and tagging power. The uncertainties are determined by combining the appropriate uncertainties on the individual bins of each charge distribution.

<table>
<thead>
<tr>
<th>Tagger</th>
<th>Efficiency [%]</th>
<th>Dilution [%]</th>
<th>Tagging Power [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combined $\mu$</td>
<td>3.37 ± 0.04</td>
<td>50.6 ± 0.5</td>
<td>0.86 ± 0.04</td>
</tr>
<tr>
<td>Segment Tagged $\mu$</td>
<td>1.08 ± 0.02</td>
<td>36.7 ± 0.7</td>
<td>0.15 ± 0.02</td>
</tr>
<tr>
<td>Jet</td>
<td>27.7 ± 0.1</td>
<td>12.68 ± 0.06</td>
<td>0.45 ± 0.03</td>
</tr>
<tr>
<td>Total</td>
<td>32.1 ± 0.1</td>
<td>21.3 ± 0.08</td>
<td>1.45 ± 0.05</td>
</tr>
</tbody>
</table>

V. MAXIMUM LIKELIHOOD FIT

An unbinned maximum likelihood fit is performed on the selected events to extract the parameters of the $B_s^0 \rightarrow J/\psi (\mu^+ \mu^-) \phi (K^+ K^-)$ decay. The fit uses information about the reconstructed mass $m$, its uncertainty $\sigma_m$, the measured proper decay time $t$, and its uncertainty $\sigma_t$, the tag probability, and the transversity angles $\Omega$ of each $B_s^0 \rightarrow J/\psi \phi$ decay candidate. There are three transversity angles $\Omega = (\theta_T, \psi_T, \phi_T)$ and these are defined in section IV.A.

The likelihood function is defined as a combination of the signal and background probability density functions as follows:

$$\ln L = \sum_{i=1}^{N} \{ w_i \cdot \ln(f_s \cdot F_s(m_i, t_i, \Omega_i, P(B|Q))) + f_s \cdot F_{B^0} \cdot F_{B^0}(m_i, t_i, \Omega_i, P(B|Q)) + (1 - f_s \cdot (1 + f_{B^0} \cdot F_{B^0}(m_i, t_i, \Omega_i, P(B|Q))) \}$$  \hspace{1cm} (1)

where $N$ is the number of selected candidates, $w_i$ is a weighting factor to account for the trigger efficiency, $f_s$ is the fraction of signal candidates, $f_{B^0}$ is the fraction of $B^0$ ($B^0 \rightarrow J/\psi K_{S}^0$ and $B^0 \rightarrow J/\psi K^+ \pi^-$) mesons mis-identified as $B_s^0$ candidates calculated relative to the number of signal events; this parameter is fixed in the likelihood fit. The mass $m_i$, the proper decay time $t_i$ and the decay angles $\Omega_i$ are the values measured from the data for each event $i$. $F_s$, $F_{B^0}$ and $F_{Bkg}$ are the probability density functions (PDF) modelling the signal, the specific $B^0$ background and the other background distributions, respectively. A detailed description of the signal PDF terms in equation (1) is given in section IV.A. The two background functions are, with the exception of new terms dependent on $P(B|Q)$ which are explained in section IV.B unchanged from the previous analysis [6]. They are each described by the product of eight terms which describe the distribution of each measured parameter. With the exception of the lifetime and its uncertainty the background parameters are assumed uncorrelated.

A. Signal PDF

The PDF describing the signal events, $F_s$, has the form of a product of PDFs for each quantity measured from the data:

$$F_s(m_i, t_i, \Omega_i, P(B|Q)) = P_s(m_i, \sigma_m) \cdot P_t(\sigma_t) \cdot P_s(\Omega_i, \sigma_{\Omega_i}) \cdot P_s(P(B|Q)) \cdot A(\Omega_i, \sigma_{\Omega_i}) \cdot P_s(\phi_T) \cdot P_s(\psi_T) \cdot P_s(\theta_T).$$

The terms $P_s(m_i, \sigma_m)$, $P_t(\sigma_t)$, $P_s(\Omega_i, \sigma_{\Omega_i})$ and $A(\Omega_i, \sigma_{\Omega_i})$ are explained in the current section. The tagging probability term $P_s(P(B|Q))$ is described in section IV.B. The remaining probability terms $P_s(\sigma_m)$, $P_{s}(\sigma_{\Omega_i})$ and $P_s(\phi_T)$ are described by Gamma functions. They are unchanged from the previous analysis and explained in detail in ref. [6]. Ignoring detector effects, the joint distribution for the decay time $t$ and the transversity angles $\Omega$ for the $B_s^0 \rightarrow J/\psi (\mu^+ \mu^-) \phi (K^+ K^-)$ decay is given by the differential decay rate [22]:

$$\frac{d\Gamma}{dt \ d\Omega} = \sum_{k=1}^{10} O^{(k)}(t) g^{(k)}(\theta_T, \psi_T, \phi_T),$$

where $O^{(k)}(t)$ are the time-dependent amplitudes and $g^{(k)}(\theta_T, \psi_T, \phi_T)$ are the angular functions, given in table [4]. The formulae for the time-dependent amplitudes have the same structure for $B_s^0$ and $B_{s}^0$ but with a sign reversal in the terms containing $\Delta m_s$. The addition of flavour tagging to the analysis means that these terms no longer cancel, so there are more terms in the fit that contain $\phi_s$. In addition to this, the strong phase variable $\delta_{LT}$ becomes accessible and one of the symmetries in the untagged fit is removed. $A_{\perp}(t)$ describes a CP odd final-state configuration while both $A_{0}(t)$ and $A_{\parallel}(t)$ correspond to $CP$ even final-state configurations. $A_{\perp}(t)$ describes the contribution of the CP odd non-resonant $B_s^0 \rightarrow J/\psi K^+ K^- S$-wave state as well as the $B_s^0 \rightarrow J/\psi f_0$ decays. The corresponding amplitudes are given in the last four lines of table [4] ($k = 7–10$) and follow the convention used in the previous analysis [23] The likelihood is independent of the $K^+ K^-$ mass distribution.
The equations are normalized, such that the squares of the amplitudes sum to unity; three of the four amplitudes are fit parameters and \(|A_\perp(0)|^2\) is determined according to this constraint.

The angles \((\theta_T, \psi_T, \phi_T)\), are defined in the rest frames of the final-state particles. The \(x\)-axis is determined by the direction of the \(\phi\) meson in the \(J/\psi\) rest frame, and the \(K^+K^-\) system defines the \(x-y\) plane, where \(p_\psi(K^+) > 0\). The three angles are defined as follows:

- \(\theta_T\), the angle between \(\vec{p}(\mu^+)\) and the normal to the \(x-y\) plane, in the \(J/\psi\) meson rest frame
- \(\phi_T\), the angle between the \(x\)-axis and \(\vec{p}_\mu(\mu^+)\), the projection of the \(\mu^+\) momentum in the \(x-y\) plane, in the \(J/\psi\) meson rest frame
- \(\psi_T\), the angle between \(\vec{p}(K^+)\) and \(-\vec{p}(J/\psi)\) in the \(\phi\) meson rest frame

The angular sculpting of the detector and kinematic cuts on the angular distributions are included in the likelihood function through \(A(\Omega, p_T)\). This is calculated using a 4D binned acceptance method, applying an event-by-event efficiency according to the transversity angles \((\theta_T, \psi_T, \phi_T)\) and the \(p_T\) of the candidate. The \(p_T\) binning is necessary, because the angular sculpting is influenced by the \(p_T\) of the \(B_0^\pm\). The acceptance was calculated from the \(B_0^\pm \rightarrow J/\psi K^+ K^- (f_0)\) and has a related strong phase \(\delta_5\). The \(\pm\) and \(\mp\) terms denote two cases: the upper sign describes the decay of a meson that was initially a \(B_0^+\), while the lower sign describes the decays of a meson that was initially \(B_0^-\).

The signal PDF, \(P_s(\Omega, t, P(BQ), \sigma_t)\), needs to take into account lifetime resolution, so each time element in table II is smeared with a Gaussian function. This smearing is done numerically on an event-by-event basis where the width of the Gaussian function is the proper decay time uncertainty, measured for each event, multiplied by a scale factor to account for any mis-measurements.

### Table II

<table>
<thead>
<tr>
<th>(k)</th>
<th>(\mathcal{O}^{(k)}(t))</th>
<th>(g^{(k)}(\theta_T, \psi_T, \phi_T))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\frac{1}{2}</td>
<td>A_0(0)</td>
</tr>
<tr>
<td>2</td>
<td>(\frac{1}{2}</td>
<td>A_1(0)</td>
</tr>
<tr>
<td>3</td>
<td>(\frac{1}{2}</td>
<td>A_\perp(0)</td>
</tr>
<tr>
<td>4</td>
<td>(\frac{1}{2}</td>
<td>A_0(0)</td>
</tr>
<tr>
<td>5</td>
<td>(</td>
<td>A_\parallel(0)</td>
</tr>
<tr>
<td>6</td>
<td>(</td>
<td>A_0(0)</td>
</tr>
<tr>
<td>7</td>
<td>(</td>
<td>A_\perp(0)</td>
</tr>
<tr>
<td>8</td>
<td>(</td>
<td>A_0(0)</td>
</tr>
<tr>
<td>9</td>
<td>(</td>
<td>A_\perp(0)</td>
</tr>
<tr>
<td>10</td>
<td>(</td>
<td>A_0(0)</td>
</tr>
</tbody>
</table>

The tag probability for each \(B_0^\pm\) candidate is determined from a weighted sum of charged-particle tracks.

### B. Using tag information in the fit

The tag PDF, \(P_t(m)\), is modeled using a single Gaussian function smeared with an event-by-event mass resolution. The PDF is normalized over the range \(5.15 < m(B_0^\pm) < 5.65\) GeV.
in a cone, as described in section IV. The tag probability is obtained from this tag charge using the calibrations measured in the $B^{\pm} \rightarrow J/\psi K^{\pm}$ data. For the case where there is only one track, the cone charge can only be ±1. This leads to a tag probability distribution with continuous and discrete parts (spikes), which are estimated separately. The distributions of tag probabilities for the signal and background are also different and since the background cannot be factorized out, extra PDF terms are included to account for this difference.

To describe the continuous parts, the sidebands are parameterized first. Sidebands are selected according to $B^0_s$ mass, i.e. $m(B^0_s) < 5.317$ GeV or $m(B^0_s) > 5.417$ GeV. In the fit the same function as for the sidebands is used to describe events in the signal region: background parameters are fixed to the values obtained in sidebands while signal parameters are free in this step. The ratio of background to signal (obtained from simultaneous mass–lifetime fit) is fixed as well. The function describing tagging using combined muons has the form of a fourth-order Chebychev polynomial. A third-order polynomial is used for the segment tagged muons’ tagging algorithm. A fourth-order Chebychev polynomial is also applied for the jet charge tagging algorithm. In all three cases unbinned maximum likelihood fits are used. Results of fits projected on histograms are shown in figure 4.

The spikes have their origin in tagging objects formed from a single track, providing a tag charge of exactly +1 or −1. When a background candidate is formed from a random combination of a $J/\psi$ and a pair of tracks, the positive and negative charges are equally probable. However, some of the background events are formed of partially reconstructed $B$ hadrons, and in these cases tag charges of +1 or −1 are not equally probable. For signal events the tag charges are obviously not symmetric. The fractions $f_{+1}$ and $f_{-1}$ of events tagged with charges of +1 and −1 are derived separately for signal and background. The remaining $(1 - f_{+1} - f_{-1})$ is the fraction of events in the continuous region. The fractions $f_{+1}$ and $f_{-1}$ are determined using the same $B^0_s$ mass sidebands and signal regions as in case of continuous parts. Table III summarizes the obtained relative probabilities between tag charges +1 and −1 for signal and background events and for all tag methods.

Similarly, the sideband subtraction method is also used to determine, for signal and background events, the relative fraction of each tagging method. The results are summarized in table IV.

If the tag-probability PDFs were ignored in the likelihood fit, equivalent to assuming identical signal and background behaviour, the impact on the fit result would be small, affecting the results by less than 10% of the statistical uncertainty.
TABLE III. Table summarizing the obtained relative probabilities between tag charges +1 and −1 for signal and background events for the different tagging methods. Only statistical errors are quoted. The asymmetry in the signal combined-muon tagging method has no impact on the results as it affects only 1% of the signal events (in addition to the negligible effect of the tag-probability distributions themselves).

<table>
<thead>
<tr>
<th>Tag method</th>
<th>Signal</th>
<th>Background</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( f_{+1} )</td>
<td>( f_{-1} )</td>
</tr>
<tr>
<td>Combined ( \mu )</td>
<td>0.106 ± 0.019</td>
<td>0.187 ± 0.022</td>
</tr>
<tr>
<td>Segment tag ( \mu )</td>
<td>0.152 ± 0.043</td>
<td>0.153 ± 0.043</td>
</tr>
<tr>
<td>Jet charge</td>
<td>0.167 ± 0.010</td>
<td>0.164 ± 0.010</td>
</tr>
</tbody>
</table>

TABLE IV. Table summarizing the relative population of the tagging methods in the background and signal events. Only statistical errors are quoted.

<table>
<thead>
<tr>
<th>Tag method</th>
<th>Signal</th>
<th>Background</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( f_{+1} )</td>
<td>( f_{-1} )</td>
</tr>
<tr>
<td>Combined ( \mu )</td>
<td>0.0372 ± 0.0023</td>
<td>0.0272 ± 0.0005</td>
</tr>
<tr>
<td>Segment tag ( \mu )</td>
<td>0.277 ± 0.007</td>
<td>0.254 ± 0.002</td>
</tr>
</tbody>
</table>

VI. RESULTS

The full simultaneous maximum likelihood fit contains 25 free parameters. These include the nine physics parameters: \( \Delta \Gamma_s \), \( \phi_s \), \( \Gamma_s \), \( |A_0(0)|^2 \), \( |A_\parallel(0)|^2 \), \( |A_\perp(0)|^2 \), \( |A_S|\) and \( \delta_S \). The other parameters in the likelihood function are the \( B_s \) signal fraction \( f_s \), the parameters describing the \( J/\psi\phi \) mass distribution, the parameters describing the \( B_s \) meson decay time plus angular distributions of background events, the parameters used to describe the estimated decay time uncertainty distributions for signal and background events, and scale factors between the estimated decay time and mass uncertainties and their true uncertainties.

The number of signal \( B_s \) meson candidates extracted from the fits is 22670 ± 150. The results and correlations for the measured physics parameters of the simultaneous unbinned maximum likelihood fit are given in tables V and VI. Fit projections of the mass, proper decay time and angles are given in figures 5 and 6 respectively.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Statistical uncertainty</th>
<th>Systematic uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_s )[rad]</td>
<td>0.12</td>
<td>0.25</td>
<td>0.05</td>
</tr>
<tr>
<td>( \Delta \Gamma_s[\text{ps}^{-1}] )</td>
<td>0.053</td>
<td>0.021</td>
<td>0.010</td>
</tr>
<tr>
<td>( \Gamma_s[\text{ps}^{-1}] )</td>
<td>0.677</td>
<td>0.007</td>
<td>0.004</td>
</tr>
<tr>
<td>(</td>
<td>A_0(0)</td>
<td>^2 )</td>
<td>0.220</td>
</tr>
<tr>
<td>(</td>
<td>A_\parallel(0)</td>
<td>^2 )</td>
<td>0.529</td>
</tr>
<tr>
<td>(</td>
<td>A_\perp(0)</td>
<td>^2 )</td>
<td>0.024</td>
</tr>
<tr>
<td>( \delta_\perp )</td>
<td>3.89</td>
<td>0.47</td>
<td>0.11</td>
</tr>
<tr>
<td>( \delta_\parallel )</td>
<td>3.34</td>
<td>[3.04, 3.23]</td>
<td>0.09</td>
</tr>
<tr>
<td>( \delta_\perp - \delta_S )</td>
<td>[3.02, 3.25]</td>
<td>0.04</td>
<td></td>
</tr>
</tbody>
</table>

TABLE V. Fitted values for the physical parameters with their statistical and systematic uncertainties. For the parameters \( \delta_\parallel \) and \( \delta_\perp - \delta_S \) a 68% confidence level interval is given. The reason for this is described in section 8.
FIG. 5. (Top) Mass fit projection for the $B_s^0 \to J/\psi \phi$. The red line shows the total fit, the dashed green line shows the signal component while the dotted blue line shows the contribution from $B_s^0 \to J/\psi K^{*0}$ events. (Bottom) Proper decay time fit projection for the $B_s^0 \to J/\psi \phi$. The red line shows the total fit while the green dashed line shows the total signal. The light and heavy components of the signal are shown in green as a dotted and a dash-dotted line, respectively. The total background is shown as a blue dashed line with a grey dotted line showing the prompt $J/\psi$ background. The pull distributions at the bottom show the difference between data and fit value normalized to the data statistical uncertainty.

FIG. 6. Fit projections for transversity angles. (Top): $\phi_T$, (middle): $\cos \theta_T$, (bottom): $\cos \psi_T$. In all three plots, the red line shows the total fit, the dashed green line shows the signal component and the dotted blue line shows the background contribution.
VII. SYSTEMATIC UNCERTAINITIES

Systematic uncertainties are assigned by considering several effects that are not accounted for in the likelihood fit. These are described below.

- **Inner detector alignment**: Residual misalignments of the inner detector affects the impact parameter distribution with respect to the primary vertex. The effect of the residual mis-alignment is estimated using simulated events with and without distorted geometry. For this, the impact parameter distribution with respect to the primary vertex is measured with data as a function of $\eta$ and $\phi$ with the maximum deviation from zero being less than $10 \, \mu m$. The measurement is used to distort the geometry for simulated events in order to reproduce the impact parameter distribution measured as a function of $\eta$ and $\phi$. The difference between the measurement using simulated events with and without the distorted geometry is used as the systematic uncertainty.

- **Trigger efficiency**: It is observed that the muon trigger biases the transverse impact parameter of muons toward smaller values. To correct for this bias the events are re-weighted according to
  \[ w = e^{-|t|/\tau_{\text{sing}} + \epsilon} / e^{-|t|/\tau_{\text{sing}}}, \]
  where $\tau_{\text{sing}}$ is a single $B^0_s$ lifetime measured before the correction, using an unbinned mass-lifetime maximum likelihood fit. The value of the parameter $\epsilon$ and its uncertainty are described in \[5\]. The systematic uncertainty is calculated by varying the value of $\epsilon$ by its uncertainty and rerunning the fit.

- **$B^0$ contribution**: Contaminations from $B^0 \rightarrow J/\psi K^{0*}$ and $B^0 \rightarrow J/\psi K\pi$ events mis-reconstructed as $B^0_s \rightarrow J/\psi\phi$ are accounted for in the default fit. The fractions of $B^0 \rightarrow J/\psi K^{0*}$ and $B^0 \rightarrow J/\psi K\pi$ events in the default fit are $(6.5 \pm / -2.4)\%$ and $(4.5 \pm / -2.8)\%$ respectively. They were determined in MC simulation and using branching fractions from ref. [17]. To estimate the systematic uncertainty arising from the precision of the fraction estimates, the data are fitted with these fractions increased and decreased by $1\sigma$. The largest shifts in the fitted values from the default case are taken as the systematic uncertainty for each parameter of interest.

- **Tagging**: For the uncertainties in the fit parameters due to uncertainty in the tagging, the statistical and systematic components are separated. The statistical uncertainty is due to the sample size of $B^\pm \rightarrow J/\psi K^\pm$ decays available and is included in the overall statistical error. The systematic uncertainty arises from the precision of the tagging calibration and is estimated by varying the model parameterizing the probability distribution, $P(B|Q)$, as a function of tag charge. The default model is a linear function. For the combined-muon cone-charge tag and the segment tagged muons the alternative fit function is a third-order polynomial. For the jet-charge tag with no muons, a third- and a fifth-order polynomial are used. The $B^0_s$ fit was repeated using the alternative models and the largest difference was assigned as the systematic uncertainty.

- **Angular acceptance method**: The angular acceptance is calculated from a binned fit to Monte Carlo data. A separate set of Monte Carlo signal events were generated and fully simulated. Background was generated using pseudo-experiments as described below. There is sufficient data to perform 166 fits. The systematic uncertainty is calculated using the bias of the pull distribution multiplied by the statistical uncertainty of each parameter. In order to estimate the size of the systematic uncertainty introduced from the choice of binning, different acceptance functions are calculated using different bin widths and central values. These effects are found to be negligible.

- **Signal and background mass model, resolution model, background lifetime and background angles model**: In order to estimate the size of systematic uncertainties caused by the assumptions made in the fit model, variations of the model are tested in pseudo-experiments. A set of 2400 pseudo-experiments is generated for each variation considered, and fitted with the default model. The systematic error quoted for each effect is the difference between the mean shift of the fitted value of each parameter from its input value for the pseudo-experiments with the systematic alteration included. The variations are: two different scale factors are used to generate the signal mass; the background mass is generated from an exponential function; two different scale factors are used to generate the lifetime uncertainty; the background lifetimes are generated by sampling data from the mass sidebands; pseudo-experiments are generated with background angles taken from histograms from sideband data and are fitted with the default fit model to assess the systematic uncertainty to the parameterization of the background angles in the fit.

- **Default fit model**: The systematic uncertainty of the default fit model is calculated using the bias of the pull distribution of 2400 pseudo-experiments, multiplied by the statistical uncertainty of each parameter.

The systematic uncertainties are provided in table VII. For each variable, the total systematic error is obtained by adding in quadrature the different contributions.
VIII. DISCUSSION

The PDF describing the $B_0^0 \to J/\psi \phi$ decay is invariant under the following simultaneous transformations:

$$\{ \phi_s, \Delta \Gamma_s, \delta_s, \phi_s \} \to \{ \pi - \phi_s, -\Delta \Gamma_s, \pi - \Delta \Gamma_s \}$$

$\Delta \Gamma_s$ has been determined to be positive $[24]$. Therefore there is a unique solution and only the case $\Delta \Gamma_s > 0$ is considered. Uncertainties on individual parameters were studied in detail in likelihood scans. Figure 7 shows the 1D likelihood scans for $\phi_s$ and $\Delta \Gamma_s$. Figure 8 shows the likelihood contours in the $\phi_s - \Delta \Gamma_s$ plane.

The behaviour of the amplitudes around their fitted values is Gaussian, however the strong phases are more complicated. Figure 9 shows the 1D likelihood scans for the three measured strong phases.

The likelihood behaviour of $\delta_s$ appears Gaussian and therefore it is reasonable to quote $\delta_s = 3.89 \pm 0.47 (\text{stat})$ rad. For $\Delta \Gamma_s - \delta_s$ the scan shows a minimum close to $\pi$; however, it is insensitive over the rest of the scan at the level of 2.1$\sigma$. Therefore the measured value of the difference $\delta_s - \Delta \Gamma_s$ is only given as 1$\sigma$ confidence interval $[3.02, 3.25]$ rad. For the strong phase $\delta_s$ the central fit value is close to $\pi$ ($3.14 \pm 0.10$) and the 1D likelihood scan shows normal Gaussian behaviour around this minimum. However, the systematic pull plot based on 2400 pseudo-experiments fits reveals a double-Gaussian shape with 68% of the results included in the interval $[2.92, 3.35]$ rad and so we quote the result in the form of a 68% C.L. interval $\delta_s \in [2.92, 3.35]$ rad (statistical only).
FIG. 7. 1D likelihood scans for $\phi_s$ (left) and $\Delta \Gamma_s$ (right).

FIG. 8. Likelihood contours in the $\phi_s - \Delta \Gamma_s$ plane. The blue line shows the 68% likelihood contour, the dashed pink line shows the 90% likelihood contour and the red dotted line shows the 95% likelihood contour (statistical errors only). The green band is the theoretical prediction of mixing-induced $CP$ violation.
FIG. 9. 1D likelihood scans for $\delta_{||}$ (top left), $\delta_{\perp}$ (top right) and $\delta_{\perp} - \delta_S$ (bottom).

IX. CONCLUSION

A measurement of time-dependent CP asymmetry parameters in $B^0_s \to J/\psi(\mu^+\mu^-)\phi(K^+K^-)$ decays from a 4.9 fb$^{-1}$ data sample of $pp$ collisions collected with the ATLAS detector during the 2011 $\sqrt{s} = 7$ TeV LHC run is presented. Several parameters describing the $B^0_s$ meson system are measured. These include the mean $B^0_s$ lifetime $1/\Gamma_s$, the decay width difference $\Delta \Gamma_s$ between the heavy and light mass eigenstates, and the transversity amplitudes $|A_0(0)|$ and $|A_\parallel(0)|$. Each of these is consistent with its respective world average. Likelihood contours in the $\phi_s - \Delta \Gamma_s$ plane are also provided. The fraction $|A_S(0)|^2$, the signal contribution from $B^0_s \to J/\psi K^+K^-$ and $B^0_s \to J/\psi f_0$ decays, is measured to be consistent with zero, at $0.024 \pm 0.014$ (stat.) $\pm 0.028$ (syst.).

The results are:

- $\phi_s = 0.12 \pm 0.25$ (stat.) $\pm 0.05$ (syst.) rad
- $\Delta \Gamma_s = 0.053 \pm 0.021$ (stat.) $\pm 0.010$ (syst.) ps$^{-1}$
- $\Gamma_s = 0.677 \pm 0.007$ (stat.) $\pm 0.004$ (syst.) ps$^{-1}$
- $|A_\parallel(0)|^2 = 0.220 \pm 0.008$ (stat.) $\pm 0.009$ (syst.)
- $|A_0(0)|^2 = 0.529 \pm 0.006$ (stat.) $\pm 0.012$ (syst.)
- $\delta_\perp = 3.89 \pm 0.47$ (stat.) $\pm 0.11$ (syst.) rad

The values are consistent with those obtained in our untagged analysis [6] and significantly reduce the overall uncertainty on $\phi_s$. These results are consistent with the values predicted in the Standard Model.
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[11] ATLAS uses a right-handed coordinate system with its origin at the nominal interaction point (IP) in the centre of the detector and the z-axis along the beam pipe. The x-axis points from the IP to the centre of the LHC ring, and the y-axis points upward. Cylindrical coordinates (r,φ) are used in the transverse plane, φ being the azimuthal angle around the beam pipe. The pseudorapidity is defined in terms of the polar angle θ as η = − ln tan(θ/2).
[19] ∆R^2 = ∆φ^2 + ∆η^2, where ∆φ and ∆η are the differences between the measured φ and η of the tracks respectively.