PDF hosted at the Radboud Repository of the Radboud University Nijmegen

The following full text is a publisher's version.

For additional information about this publication click this link.
http://hdl.handle.net/2066/129080

Please be advised that this information was generated on 2019-03-22 and may be subject to change.
This purpose of this paper is to prove that for every Grothendieck topos $\mathcal{E}$ there exist a space $X$ and a covering $\varphi : X \to \mathcal{E}$ which induces an isomorphism in cohomology

$$H^n(\mathcal{E}, A) \to H^n(X, \varphi^* A) \quad (n \geq 0)$$

for any abelian group $A$ in $\mathcal{E}$. Moreover for $n = 1$ this is also true for nonabelian $A$. This implies, by a result of Artin and Mazur, that $\varphi$ induces an isomorphism of etale homotopy groups.

1. Construction of the cover. Let $\mathcal{E}$ be a Grothendieck topos, and let $G$ be an object of $\mathcal{E}$. $\text{En}(G)$ is the space (in this paper 'space' means space in the sense of [JT], chapter IV, unless explicitly said otherwise) of infinite-to-one partial enumerations of $G$; in other words, $\text{En}(G)$ is characterized by the property that for any map $f: \mathcal{T} \to \mathcal{E}$ of toposes, the points of the induced space $f^*(\text{En}(G))$ in $\mathcal{T}$ correspond to diagrams $\mathbb{N} \leftarrow U \rightarrow f^*G$ in $\mathcal{T}$ with the property that for any $n \in \mathbb{N}, U - \{0, \ldots, n\} \to f^*G$ is still epi. We write $\mathcal{E}[^{\text{En}}(G)]$ for the category of sheaves in $\mathcal{E}$ on the space $\text{En}(G)$, and $\varphi : \mathcal{E}[^{\text{En}}(G)] \to \mathcal{E}$ for the corresponding geometric morphism. The properties of the space $\text{En}(G)$ and the map $\varphi$ were extensively discussed in [JM]. For the present purpose, we recall the following basic facts. First of all, for a suitable object $G$ of $\mathcal{E}$, $\mathcal{E}[^{\text{En}}(G)]$ is equivalent to the topos $\text{Sh}(X_\varphi)$ of sheaves on a space $X_\varphi$ in Sets, so that $\varphi$ corresponds to a cover

$$(1) \quad \varphi : \text{Sh}(X_\varphi) \to \mathcal{E}.$$
This geometric morphism is connected and locally connected; in particular, $\varphi^*: \mathcal{E} \to \text{Sh}(X_\phi)$ has a left adjoint $\varphi$, such that for any $E \in \mathcal{E}$ and $S \in \text{Sh}(X_\phi)$,

$$\varphi_!(\varphi^*(E) \times S) \cong E \times \varphi_!(S).$$

For any $G$ in $\mathcal{E}$, there exists a surjective geometric morphism $p: \mathcal{B} \to \mathcal{E}$ where $\mathcal{B}$ is the category of sheaves on a complete Boolean algebra (Barr’s theorem, [B]), such that $p^*G$ is countable (cf. [JT]). $\mathcal{B}$ is a model of set theory, and the induced space $\text{En}(p^*G) \cong p^*(\text{En}(G))$ in $\mathcal{B}$ has enough points, i.e. is an ordinary topological space, which can be described as follows: the points of $\text{En}(p^*(G))$ are functions $\alpha: U \to p^*G$ with $U \subset \mathbb{N}$ and $\alpha^{-1}(g)$ infinite for all $g \in p^*(G)$; the basic open sets are the sets of the form $V_u = \{\alpha | \forall i \in \text{domain}(u): i \in U \text{ and } \alpha(i) = u(i)\}$, where $u$ ranges over all functions $u: K \to p^*G$ defined on a finite set $K \subset \mathbb{N}$. It is not difficult to prove that each basic open set $V_u$ (in particular, the space itself, $V_\phi$) is contractible ([JM]).

2. **Relative Čech cohomology.** In this section, let $Y$ be a space in a topos $\mathcal{E}$. One can define the relative Čech cohomology groups of $Y$ with coefficients in an abelian group object in $\mathcal{E}[Y]$, i.e. a sheaf (or in fact, just a presheaf) of abelian groups on $Y$ in $\mathcal{E}$,

$$\tilde{H}^p_\mathcal{E}(Y, A).$$

These cohomology groups are group objects in $\mathcal{E}$. Their construction is completely parallel to the usual construction of the Čech cohomology groups of a topological space; indeed, the latter construction immediately translates to the context of a space in a topos $\mathcal{E}$, by viewing $\mathcal{E}$ as a universe for (constructive) set theory (cf. [BJ]).

More explicitly, let $S \in \mathcal{E}$ and let $\mathcal{U}: S \to \mathcal{O}(Y)$ be an open cover of $Y$ indexed by $S$. Let $A$ be a (pre)sheaf of abelian groups on $Y$ in $\mathcal{E}$; so $A$ is given by a map $A \to \mathcal{O}(Y)$ in $\mathcal{E}$ equipped with the structure of a (pre)sheaf. Let

$$\mathcal{U}_p: S_p = S \times \cdots \times S \xrightarrow{\text{aggl}} \mathcal{O}(Y)^{p+1} \xrightarrow{\wedge} \mathcal{O}(Y)$$

be the map in $\mathcal{E}$ obtained from $\mathcal{U}$ by intersection in $Y$, and let
(3) \[ C^p(\mathcal{U}, A) = \prod_{S_p} (A \times S_p \to S_p) \]

where \( \Pi_{S_p} : \mathcal{E}/S_p \to \mathcal{E} \) is the right adjoint of the functor \( S_p^* : \mathcal{E} \to \mathcal{E}/S_p \) (cf. [J], p. 36). The \( C^p(\mathcal{U}, A) \), \( p \geq 0 \), give a cochain complex \( C^p(\mathcal{U}, A) \to C^{p+1}(\mathcal{U}, A) \to \cdots \) in the usual way, with the differential defined via alternating sums. The cohomology groups of this complex are denoted by \( H^p(\mathcal{U}, A) \). One may now take the colimit of these groups over the \textit{internal} diagram in \( \mathcal{E} \) of all open covers \( \mathcal{O}(Y) \) (so this involves internal covers of \( Y \) in \( \mathcal{E}/E \) for arbitrary \( E! \)), and obtain the relative Čech cohomology groups

(4) \[ \check{H}^p(\mathcal{E}, A) = \lim_{\to \mathcal{U}} H^p(\mathcal{U}, A) \quad (p \geq 0). \]

Straightforward modifications of the standard argument show that these cohomology groups have the usual properties. For instance, if we write \( \varphi : \mathcal{E}[Y] \to \mathcal{E} \) for the canonical geometric morphism and \( e_E : \mathcal{E}/E \to \mathcal{E} \) for the geometric morphism given by \( e_E^* = E^* = (X \mapsto X \times E \to E) \), then for any open cover \( \mathcal{U} \) of \( e_E^*(Y) \) in \( \mathcal{E}/E \),

(5) \[ H^p(\mathcal{U}, e_E^*(A)) \equiv e_E^* \varphi A, \]

where \( E \) is any object of \( \mathcal{E} \); hence

(6) \[ H^p(\mathcal{E}, A) \equiv \varphi A. \]

And for an injective object \( I \) of the category \( \text{Ab} \mathcal{E}[Y] \) of abelian sheaves on \( Y \) in \( \mathcal{E} \),

(7) \[ H^p(\mathcal{U}, e_E^*I) = 0 \quad (n > 0) \]

for any \( E \) in \( \mathcal{E} \) and any open cover \( \mathcal{U} \) of \( e_E^*(Y) \) in \( \mathcal{E}/E \), so

(8) \[ \check{H}^p(\mathcal{E}, I) = 0 \quad (n > 0) \]

3. A relative Cartan-Leray spectral sequence. As before, let \( Y \) be a space in a topos \( \mathcal{E} \), and let \( \varphi : \mathcal{E}[Y] \to \mathcal{E} \) be the corresponding
geometric morphism. \( \mathcal{E}[Y] \) is a subtopos of the topos \( \mathcal{E}^{O(Y)} \) of presheaves on \( O(Y) \) in \( \mathcal{E} \), and we write \( i: \mathcal{E}[Y] \hookrightarrow \mathcal{E}^{O(Y)} \) for the inclusion. The following is a relative version of SGA4, exp V, p. 24.

**Lemma 1.** For any abelian group \( A \) in \( \mathcal{E}[Y] \), there exists a spectral sequence

\[
E_2^{p,q} = \tilde{H}_{\mathcal{E}}^p(Y, R^q i_*(A)) \Rightarrow R^{p+q} \varphi_*(A).
\]

**Proof.** Let \( 0 \to A \to I' \) be an injective resolution of \( A \) in \( \text{Ab} \mathcal{E}[Y] \). For an open cover \( \mathcal{U} \) of \( Y \) in \( \mathcal{E} \), one has a double complex of abelian groups \( C^{p,q}(\mathcal{U}) = C^p(\mathcal{U}, I') \) (cf. (3)). By (5) and (7) above, the cohomology of the total complex is \( H^p H^q(C**(\mathcal{U}))) = R^p \varphi_*(A) \), so we obtain a spectral sequence

\[
E_2^{p,q}(\mathcal{U}) = H^p H^q(C**(\mathcal{U}))) = H_{\mathcal{E}}^p(\mathcal{U}, R^q i_! A) \Rightarrow R^{p+q} \varphi_*(A)
\]

in the standard way ([G]). The same applies to open covers of \( e^*_E(Y) \) in \( \mathcal{E}/E \) for any object \( E \) of \( \mathcal{E} \), so by taking the internal colimit in \( \mathcal{E} \) over all open covers of \( Y \), we obtain a spectral sequence as stated in the lemma.

Now let \( B \subset O(Y) \) be a basis for \( Y \) in \( \mathcal{E} \) which is closed under binary meets. Call \( B \) \( A \)-acyclic if for every morphism \( B: E \to B \) in \( \mathcal{E} \),

\[
\tilde{H}_{\mathcal{E}/E}^q(B, A|B) = 0. \quad (q > 0)
\]

In (10), \( B \) stands for the open subspace of \( e^*_E(Y) \) determined by the given morphism \( B: E \to B \subset O(Y) \), and \( A|B \in \text{Ab}(\mathcal{E}/E)[B] \) is the sheaf induced by \( A \).

\[
(\mathcal{E}/E)[B] \hookrightarrow (\mathcal{E}/E)[e^*_E Y] \longrightarrow \mathcal{E}[Y] \]

(11)

\[
(\mathcal{E}/E) \quad \overset{e^*_E}{\longrightarrow} \quad \mathcal{E}
\]

**Lemma 2.** If \( B \) is an \( A \)-acyclic basis for \( Y \) as above, then \( \tilde{H}_{\mathcal{E}}^p(Y, A) \equiv R^p \varphi_! A \), for all \( p \geq 0 \).
Proof. We show by induction on \( n \) that \( E_{\mathcal{E}}^{q,n} = 0 \) for all \( p \) and all \( q \) with \( 0 < q < n \), in the spectral sequence of Lemma 1. Suppose this holds for \( n \). Then (cf. [CE], p. 328) \( \tilde{H}_{\mathcal{E}}^n(Y, A) = R^i\varphi_*A \) for \( i < n \), and there is an exact sequence \( 0 \to \tilde{H}_{\mathcal{E}}^n(Y, A) \to R^i\varphi_*A \to E_{\mathcal{E}}^{0,n} \to \tilde{H}_{\mathcal{E}}^{n+1}(Y, A) \to R^n\varphi_*A \). But \( E_{\mathcal{E}}^{0,n} = \tilde{H}_{\mathcal{E}}^n(Y, R^n\varphi_*A) \to \varphi_*i^nR^n\varphi_*(A) = \varphi_*(0) = 0 \) \( n > 0 \), so \( H_{\mathcal{E}}^n(Y, A) \equiv R^n\varphi_*A \). Applying this argument not to \( Y \), but to any open subspace \( B \) (for any morphism \( B: E \to B \), cf the diagram (11)), our assumption on \( B \) gives that \( R^n\varphi_*(A) \mid B = 0 \), where \( \varphi_*(A) \mid B \) denotes the restriction functor \( \mathcal{E}^{\mathcal{O}(X)} \to \mathcal{E}^{\mathcal{O}(B)} \). Thus if in the spectral sequence (9) above, \( \mathcal{U} \) is a cover consisting of basic opens from \( B \), then \( E_{\mathcal{E}}^{q,n}(\mathcal{U}) = H_{\mathcal{E}}^q(\mathcal{U}, 0) = 0 \) for all \( p \). Since such covers consisting of basic opens are cofinal in the internal system of all covers, it follows by passing to the colimit that \( E_{\mathcal{E}}^{q,n} = 0 \) \( (p, q) \) in the spectral sequence of Lemma 1. So the inductive statement in the beginning of the proof holds for \( n + 1 \), and Lemma 2 is proved.

Remark. Let \( Y \) be a space in \( \mathcal{E} \), as above. Recall (see [JT]) that an open \( U \subset Y \) is called surjective if it holds in \( \mathcal{E} \) that every cover of \( U \) is inhabited. If \( A \) is a sheaf on \( Y \) and \( \{ U_\alpha : \alpha \in \mathcal{A} \} \) is a family of opens, then \( \Pi\{ A(U_\alpha) \mid \alpha \in \mathcal{A} \} \equiv \Pi\{ A(U_\alpha) \mid \alpha \in \mathcal{A} \}, U_\alpha \) surjective \( \) (where \( \Pi \) is the internal product \( \mathcal{E}/\mathcal{A} \to \mathcal{E} \), as in Section 2). This is analogous to the fact that for \( \mathcal{E} = \text{Sets} \), \( A(U) = \{^*\} \) if the empty set covers \( U \). Therefore in Lemma 2 it is enough to assume that \( B \) is closed under surjective binary meets (i.e. \( B \wedge B' \in B \) whenever \( B \) and \( B' \in B \) and \( B \wedge B' \) is surjective), since by this isomorphism, surjective intersections are the only ones that need to be considered in the complexes \( C_{\mathcal{E}}^{p,q}(\mathcal{U}) \).

4. The main theorem. Let \( \mathcal{E} \) be a Grothendieck topos, and let \( X_\mathcal{E} \) be the space constructed in Section 1. In the following theorem, \( H^q(X_\mathcal{E}, -) \) denotes the sheaf cohomology of \( X_\mathcal{E} \).

Theorem. The geometric morphism \( \varphi : \text{Sh}(X_\mathcal{E}) \to \mathcal{E} \) has the property that for any abelian group \( A \) in \( \mathcal{E} \), \( R^q\varphi_*(\varphi^*A) = 0 \) for \( q > 0 \) \( (\)for \( q = 0, R^q\varphi_*(\varphi^*A) \equiv A \); consequently, \( \varphi \) induces an isomorphism

\[
H^q(\mathcal{E}, A) \to H^q(X_\mathcal{E}, \varphi^*A)
\]

for each \( q \geq 0 \).
Proof. The second statement follows from the first by the Leray spectral sequence (SGA4, exp V, p. 35). The first statement is a special case (by construction of $X_\phi$) of the general fact that for any object $G$ in $\mathcal{C}$, the corresponding geometric morphism $\varphi: \mathcal{C}[\text{En}(G)] \to \mathcal{C}$ induces isomorphisms $H^q(\mathcal{C}, A) \to H^q(\mathcal{C}[E(G)], \varphi^*A)$, for any abelian group $A$ in $\mathcal{C}$ and any $q \geq 0$. Let $\mathcal{B}$ be the basis consisting of opens of the form $V_u$ ($u$ a finite partial function from $\mathbb{N}$ to $G$, cf. [JM]). $\text{En}(G) = V_\varphi \in \mathcal{B}$, and $V_u \land V_w$ is surjective iff $u$ and $w$ are compatible finite functions, and in that case $V_u \land V_w = V_{u \lor w}$, so $\mathcal{B}$ is closed under surjective finite meets (cf. the remark in Section 3).

We will show that for any injective object $I$ of $\text{Ab}(\mathcal{C})$ and any $q > 0$

\[(12)\quad R^q\varphi_*(\varphi^*I) = 0.\]

This is enough, because $\varphi$ is connected, i.e. $\varphi_*\varphi^* \cong \text{id}$, and (12) says that $\varphi^*$ maps injectives to $\varphi_*$-acyclic objects, so there is a spectral sequence ([G]) for the composition $\varphi^* \circ \varphi_*$, $E_2^{p,q} = (R^p\varphi_*)(R^q\varphi^*)A \Rightarrow R^{p+q}(\varphi_*\varphi^*)A$; $\varphi^*$ is exact and $\varphi_*\varphi^* \cong \text{id}$, so $E_2^{p,q} = 0$ for $q > 0$ and $E_2^{p,0} = R^p(\text{id})(A) = 0$ for $p > 0$. Thus $R^p\varphi_*\varphi^*(A) = 0$ for $p > 0$.

To prove (12), let $I$ be an injective in $\text{Ab}(\mathcal{C})$, and let $\mathcal{U}$ be an open cover of $\text{En}(G)$ by basic opens, say $\mathcal{U} : S \to \mathcal{B} \subset \mathcal{O}(Y)$ as in Section 2. Let us consider the nerve $N(\mathcal{U})$ of $\mathcal{U}$. This is the simplicial complex in $\mathcal{E}$ defined as follows: $S = (S_p, p \geq 0)$ is a simplicial complex in $\mathcal{E}$, with as face $d_i : S_p \to S_{p-1}$ the projection $S^{p+1} \to S^p$ which deletes the $i$-th coordinate. The morphism $\mathcal{U}_p : S_p \to \mathcal{B} \subset \mathcal{O}(Y)$ can be viewed as an $S_p$-indexed sum of subobjects of the terminal object 1 of $\mathcal{E}[Y]$, and we write $\Sigma_{S_p} \mathcal{U}_p$ for their internal sum. Then

$$N_p(\mathcal{U}) = \varphi_*(\Sigma_{S_p} \mathcal{U}_p),$$

and the faces and degeneracies of $S$ give $N,(\mathcal{U})$ the structure of a simplicial complex over $\mathcal{E}$. Moreover,

\[(13)\quad C^p(\mathcal{U}, \varphi^*I) \cong I^{N_p(\mathcal{U})}.\]

(cf. (2)), where the differentials on the left correspond to the differentials obtained on the right by alternating sums from the cofaces of the co-
simplicial object $I N(\mathcal{U})$. We claim that $C^p(\mathcal{U}, \varphi^* I)$ is an acyclic complex. Since $I$ is injective, it suffices to prove that $\text{Free}(N\mathcal{U})$ is an acyclic chain complex in $\text{Ab}(\mathcal{E})$, where $\text{Free}(\cdot)$ denotes the free abelian group functor. To this end, let $p : \mathcal{B} \to \mathcal{E}$ be a Boolean extension as at the end of Section 1, and consider the pullback square

$$
\begin{array}{ccc}
\mathcal{B}[\text{En}(p^* G)] & \to & \mathcal{E}[\text{En}(G)] \\
\downarrow \psi & & \downarrow \varphi \\
\mathcal{B} & \to & \mathcal{E}.
\end{array}
$$

Since $\varphi$ is locally connected so is $\psi$, and the Beck-Chevalley condition holds, i.e.

$$
p^* \varphi_! \cong \psi_! p^*.
$$

Consequently, if we write $\mathcal{U}'$ for the cover of $\text{En}(p^* G)$ induced by $\mathcal{U}$ via pullback along $p$, we have $p^*(\text{Free}(N\mathcal{U})) \cong \text{Free}(N\mathcal{U}')$. But $\mathcal{B}$ is a model for set theory (with the axiom of choice), so we are now in a position to apply results from classical topology: the cover $\mathcal{U}'$ of $\text{En}(p^* G)$ is a cover by basic opens, and $\text{En}(p^* G)$ as well as each of its basic open subspaces are contractible, so the nerve $N(\mathcal{U}')$ of this cover is a contractible simplicial set, and $\text{Free}(N\mathcal{U}')$ is an acyclic chain complex. Since $p^*(\text{Free} N\mathcal{U}) = \text{Free}(N\mathcal{U}')$ and $p^*$ is faithful, it follows that $\text{Free}(N\mathcal{U})$ is acyclic, as was to be shown.

Now apply this argument not just to $\text{En}(G)$, but to any basic open $B \subseteq \text{e}_G^p(\text{En}(G))$ and any $E \in \mathcal{E}$ (cf (11), where $Y = \text{En}(G)$ now). Then we conclude that $\mathcal{B}$ is an $I$-acyclic basis. (12) now follows by Lemma 2, since the whole space $\text{En}(G)$ is a member of $\mathcal{B}$. This completes the proof of the theorem.

5. Torsors. Let $G$ be a group in a topos $\mathcal{E}$. A $G$-torsor in $\mathcal{E}$ (or principal $G$-bundle over $\mathcal{E}$) is an object $T$ of $\mathcal{E}$ equipped with an action $\mu : G \times T \to T$ of $G$ such that $T \to 1$ is epi and $(\mu, \pi_3) : G \times T \to T \times T$ is an isomorphism. Recall ([Gi]) that $H^i(\mathcal{E}, G)$ is the pointed set of isomorphism classes of $G$-torsors (this is a group if $G$ is abelian). For a space $X$ and a sheaf of groups $G$ on $X$, $H^i(X, G)$ stands for $H^i(\text{Sh}(X), G)$. 
THEOREM. Let $\mathcal{E}$ be a topos, and let $\varphi: \text{Sh}(X_{\mathcal{E}}) \to \mathcal{E}$ be the cover of Section 1. For any group $G$ in $\mathcal{E}$, $\varphi$ induces an isomorphism

$$H^1(\mathcal{E}, G) \cong H^1(\text{Sh}(X_{\mathcal{E}}), \varphi^*G)$$

Proof. The functor $\varphi^*: \mathcal{E} \to \text{Sh}(X_{\mathcal{E}})$ is fully faithful, so it restricts to a fully faithful functor from the category of $G$-torsors in $\mathcal{E}$ to that of $\varphi^*G$-torsors in $\text{Sh}(X_{\mathcal{E}})$. It thus suffices to show that this restriction of $\varphi^*$ is essentially surjective. By [JM], there is a class $P \subset (X_{\mathcal{E}})'$ of paths, such that $\mathcal{E}$ is equivalent to the full subcategory of $\text{Sh}(X_{\mathcal{E}})$ consisting of those sheaves on $X_{\mathcal{E}}$ which are constant along the paths in $P$. Let $T$ be a $\varphi^*G$-torsor in $\text{Sh}(X_{\mathcal{E}})$. Then $T$ is locally isomorphic to $\varphi^*(G)$, and $\varphi^*(G)$ is constant along all the paths in $P$. So $T$ is locally constant along the paths in $P$, and hence constant along those paths (since the interval $I$ is simply connected).

6. Étale homotopy. Let $\mathcal{E}$ be a locally connected topos, and let $p$ be a point of $\mathcal{E}$. Artin and Mazur ([AM]) define the étale homotopy groups $\pi_n(\mathcal{E}, p)$ ($n \geq 0$), and prove a Whitehead theorem for toposes: a geometric morphism $(\mathfrak{F}, q) \to (\mathcal{E}, p)$ of pointed locally connected toposes induces isomorphisms of étale homotopy groups iff it induces isomorphisms of cohomology groups with coefficients in a locally constant abelian group $A$ in $\mathcal{E}$, as well as an isomorphism of the fundamental progroups $\pi_1(\mathfrak{F}, q) \to \pi_1(\mathcal{E}, p)$. Our previous results give:

COROLLARY. For any locally connected pointed topos $(\mathcal{E}, p)$ there exists a pointed space $(X_{\mathcal{E}}, q)$ and a cover $\varphi: (\text{Sh}(X_{\mathcal{E}}), q) \to (\mathcal{E}, p)$ which induces isomorphisms in étale homotopy,

$$\pi_n(X_{\mathcal{E}}, q) \cong \pi_n(\mathcal{E}, p) \quad (n \geq 0)$$

Proof. First of all, we need to modify the construction of the space $X_{\mathcal{E}}$ slightly, in order to lift the point $p$: if we replace the set $\mathbb{N}$ of natural numbers by an arbitrary infinite set $S$ in the construction of Section 1 (and the space of infinite-to-one enumerations $\mathbb{N} \leftarrow U \rightarrow G$ by that of infinite-to-one partial maps $\Delta(S) \leftarrow U \rightarrow G$, where $\Delta S$ denotes the constant object of $\mathcal{E}$ corresponding to the set $S$), we obtain a cover (again called) $\varphi: X_{\mathcal{E}} \to \mathcal{E}$ with exactly the same properties as before. A straightforward classifying-topos argument shows that if we
choose the cardinality of $S$ sufficiently large (at least that of $p^*G$) then the given point $p$ can be lifted to a point $q$ of this (modified) space $X_\varepsilon$. $X_\varepsilon$ is locally connected since $\varepsilon$ is, and $\varphi$ is a locally connected map. Now the result of Section 5 shows that $\varphi$ induces an isomorphism in $\pi_1$ (since $H^1(\varepsilon, G) \cong \text{Hom}(\pi_1(\varepsilon, p), G)$, cf [AM], Section 10). The corollary follows by the Whitehead theorem just quoted and the theorem of Section 4.

UNIVERSITÉ DU QUÉBEC À MONTREAL, CANADA
UNIVERSITY OF CHICAGO

REFERENCES

[JM] A. Joyal and I. Moerdijk, Toposes as homotopy groupoids, (to appear in *Advances in Math.*).