Measurement of the $B^0$ and $B^+$ meson lifetimes with fully reconstructed hadronic final states

The BaBar Collaboration

The $B^0$ and $B^+$ meson lifetimes have been measured in $e^+e^-$ annihilation data collected in 1999 and 2000 with the BABAR detector at center-of-mass energies near the $\Upsilon(4S)$ resonance. Events are selected in which one $B$ meson is fully reconstructed in a hadronic final state while the second $B$ meson is reconstructed inclusively. A combined fit to the $B^0$ and the $B^+$ decay time difference distributions yields $\tau_{B^0} = 1.546 \pm 0.032$ (stat) $\pm 0.022$ (syst) ps, $\tau_{B^+} = 1.673 \pm 0.032$ (stat) $\pm 0.023$ (syst) ps and $\tau_{B^+}/\tau_{B^0} = 1.082 \pm 0.026$ (stat) $\pm 0.012$ (syst).

PACS numbers: 13.25.Hw, 12.39.Hg

The spectator quark model predicts that the two charge states of a meson with one heavy quark $Q$ ($Q\pi$ and $Q\bar{q}$) have the same lifetime. Deviations from this simple picture are expected to be proportional to $1/m_Q^2$. Therefore, any lifetime differences are anticipated to be much smaller for bottom than for charm mesons. Various models [1, 2] predict the ratio of the $B^+$ and $B^0$ meson lifetimes to differ by up to 10% from unity. At present, this ratio is measured to be $\tau_{B^+}/\tau_{B^0} = 1.062 \pm 0.029$ [3], with the most precise values obtained by experiments operating near the $Z$ and at hadron colliders.

The lifetime measurements described here are based on a sample of approximately 23 million $B\bar{B}$ pairs recorded near the $\Upsilon(4S)$ resonance with the BABAR detector at the Stanford Linear Accelerator Center. The PEP-II asymmetric-energy $e^+e^-$ collider produces $B^+$-$B^-$ and $B^0\bar{B}^0$ pairs moving along the beam axis ($z$ direction) with a nominal Lorentz boost of $\beta\gamma = 0.56$. Hence, on average, the two $B$ decay vertices are separated by $\langle|\Delta z|\rangle = \beta\gamma \Gamma_{B^{\mp}} \tau \approx 270 \mu m$, where $\tau$ is either the $B^0$ or $B^+$ lifetime, and $\Gamma_{B^{\mp}}$ is the Lorentz factor of the $B$ mesons in the $\Upsilon(4S)$ rest frame. This separation allows $B$ lifetimes to be measured at the $\Upsilon(4S)$, with good statistical precision and systematic error sources different from those in previously published results.

In this analysis, one of the $B$ mesons in an event, denoted $B_{\text{rec}}$, is fully reconstructed in a variety of two-body charm and charmonium final states. The decay point of the other $B$ in the event, $B_{\text{opp}}$, is reconstructed inclusively. The decay probability distribution is given by

$$g(\Delta t|\tau) = \frac{1}{N} \frac{dN}{d(\Delta t)} = \frac{1}{2\tau} e^{-|\Delta t|/\tau},$$

where $\Delta t = t_{\text{rec}} - t_{\text{opp}}$ is the (signed) difference of the proper decay times of the $B$ mesons. The time interval $\Delta t$ between the two $B$ decays is determined from $\Delta z$, including an event-by-event correction for the direction of the $B$ mesons with respect to the $z$ direction in the $\Upsilon(4S)$ frame. The challenge of the measurement is to disentangle the resolution in $\Delta z$, 190 $\mu m$ on average, from the effects of the $B$ lifetime, since both contribute to the width of the $\Delta t$ distribution. In the absence of background, the measured $\Delta t$ distribution is described by the probability density function (PDF)

$$G(\Delta t, \sigma | \tau, \tilde{a}) = \int_{-\infty}^{+\infty} g(\Delta t'|\tau) R(\Delta t - \Delta t', \sigma | \tilde{a}) d(\Delta t'),$$

where $R$ is the $\Delta t$ resolution function with parameters $\tilde{a}$, and $\sigma$ is the event-by-event error on $\Delta t$ calculated from the vertex fits. An unbinned maximum likelihood fit is used to extract the $B^0$ and $B^+$ lifetimes from the $\Delta t$ distributions for $B^0\bar{B}^0$ and $B^+B^-$ events.

The BABAR detector is described in detail elsewhere [3]. Charged particle trajectories are measured by a combination of a silicon vertex tracker (SVT) and a drift chamber (DCH) in a 1.5-T solenoidal field. For 1 GeV/c tracks, the impact parameter resolutions in $z$ and in the transverse plane are 65 $\mu m$ and 55 $\mu m$, respectively. Photons and electrons are detected in the CsI(Tl) electromagnetic calorimeter (EMC). A ring imaging Cherenkov detector, the DIRC, is used for charged hadron identification. The DCH and SVT also provide ionization measurements, $dE/dx$, for particle identification. The instrumented flux return (IFR) is segmented and contains resistive plate chambers to identify muons. Electron candidates are required to have a ratio of EMC energy to track momentum, an EMC cluster shape, DCH $dE/dx$ and DIRC Cherenkov angle consistent with the electron hypothesis. Muon candidates are required to have an energy deposit in the EMC consistent with the muon hypothesis, IFR hits located consistently on the extrapolated DCH track, and an IFR penetration in interaction lengths consistent with the muon hypothesis.

$B^0$ and $B^+$ mesons are reconstructed in a sample of multihadron events in the modes $B^0 \rightarrow D^{(*)}+\pi^+$, $D^{(*)}+\rho^+$, $D^{(*)}+\pi^0$, $J/\psi K^{*0}$ and $B^+ \rightarrow D^{(*)}\pi^+$, $J/\psi K^+$, $\psi(2S)K^+$. Multihadron events must have a minimum of three reconstructed charged tracks, a total charged and neutral energy greater than 4.5 GeV, and an event vertex within 0.5 cm of the beam spot center in $xy$ and within 6 cm in $z$. The event vertex is determined from all charged tracks that have an impact parameter with respect to the beam spot center smaller than 1 cm in $xy$ and 3 cm in $z$.

For $\pi^0$ candidates, pairs of photons in the EMC, each with more than 30 MeV of energy, are selected if their
invariant mass is within 20 MeV/$c^2$ of the $\pi^0$ mass and their total energy exceeds 200 MeV (100 MeV for the soft $\pi^0$ in $D^*$ decays). A mass constraint is applied to selected candidates for use in the subsequent reconstruction chain.

$K^0_d \rightarrow \pi^+\pi^-$ candidates are required to have an invariant mass between 462 and 534 MeV/$c^2$. A geometrical vertex fit with $\chi^2$ probability above 0.1% is required, and the transverse flight distance from the event vertex must be greater than 0.2 cm.

$D^0$ candidates are reconstructed in the decay channels $K^+\pi^-$, $K^+\pi^-\pi^0$, $K^+\pi^-\pi^-$ and $K^0\pi^-\pi^-$ and $D^-$ candidates in the decay channels $K^+\pi^-\pi^-$ and $K^0\pi^-\pi^-$. Kaons from $D^-$ decays and charged daughters from $\bar{D}^0 \rightarrow K^+\pi^-$ are required to have momenta greater than 200 MeV/$c$. All other charged $\bar{D}$ daughters are required to have momenta greater than 150 MeV/$c$. For $\bar{D}^0 \rightarrow K^+\pi^-\pi^0$, we only reconstruct the dominant resonant mode, $\bar{D}^0 \rightarrow K^+\rho^-$, followed by $\rho^- \rightarrow \pi^-\pi^0$. The $\pi^-\pi^0$ mass is required to lie within 150 MeV/$c^2$ of the $\rho$ mass and the angle between the $\pi^-$ and $\bar{D}^0$ in the $\rho$ rest frame, $\theta_{\rho\pi^-}$, must satisfy $|\cos \theta_{\rho\pi^-}| > 0.4$. All $\bar{D}^0$ and $D^-$ candidates are required to have a momentum greater than 1.3 GeV/$c$ in the $\Upsilon(4S)$ frame, an invariant mass within 3$\sigma$ of the nominal value and a geometrical vertex fit with a $\chi^2$ probability greater than 0.1%. A mass constraint is applied to selected $\bar{D}$ candidates.

Charged and neutral $\bar{D}^0$ candidates are formed by combining a $\bar{D}^0$ with a $\pi^-$ or $\pi^0$. The momentum of the pion in the $\Upsilon(4S)$ frame is required to be less than 450 MeV/$c$. The soft $\pi^-$ is constrained to originate from the beam spot when the $D^-$ vertex is fit. After the mass constraint to the $\bar{D}^0$ daughter, candidates within 2$\sigma$ of the nominal mass for $D^-$, or within 4$\sigma$ of the nominal mass, are selected.

Candidates for leptonic decays of charmonium mesons must have at least one decay product positively identified as an electron or a muon. If it traverses the calorimeter, the second muon must be consistent with a minimum ionizing particle. $J/\psi$ candidates are required to lie in the invariant mass interval 2.95 (3.06) to 3.14 GeV/$c^2$ for the $e^+e^-$ ($\mu^+\mu^-$) channel. The $e^+e^-$ ($\mu^+\mu^-$) invariant mass of $\psi(2S)$ candidates must be between 3.44 (3.64) and 3.74 GeV/$c^2$. A mass constraint is applied to selected candidates. $\psi(2S) \rightarrow J/\psi \pi^+\pi^-$ candidates are selected if the $\pi^+\pi^-$ mass is between 0.4 and 0.6 GeV/$c^2$ and the $\psi(2S)$ mass is within 15 MeV/$c^2$ of the nominal value. All $\psi(2S)$ candidates must have momenta between 1.0 and 1.6 GeV/$c$ in the $\Upsilon(4S)$ rest frame.

$B$ candidates are formed by combining a $\bar{D}^{(*)}$, $J/\psi$ or $\psi(2S)$ candidate with a $\pi^+$, $\rho^+$, $a_1^+$ ($a_1^- \rightarrow \pi^+\pi^-\pi^+$), $K^{*0}$ ($K^{*0} \rightarrow K^+\pi^-$) or $K^+$ candidate that has a momentum larger than 500 MeV/$c$ in the $\Upsilon(4S)$ frame. For $B^0 \rightarrow D^{(*)}\rho^+$, the $\rho^+$ from the $\rho^+$ decay must have an energy greater than 300 MeV. For $B^0 \rightarrow D^{(*)}a_1^+$, the $a_1^+$ must have an invariant mass between 1.0 and 1.6 GeV/$c^2$, and the $\chi^2$ probability of a vertex fit of the $a_1^+$ candidate is required to be greater than 0.1%. Positive identification of kaons is required for modes with higher background, such as $B^{+} \rightarrow D^{0}\pi^+$ with $D^{0} \rightarrow K^+\pi^+\pi^-\pi^-$. Continuum $e^+e^- \rightarrow q\bar{q}$ background is rejected by requiring the normalized second Fox-Wolfram moment for the event to be less than 0.5. Further suppression is achieved by a mode-dependent restriction on the angle between the $B_{\text{rec}}$ and $B_{\text{opp}}$ thrust axes in the $\Upsilon(4S)$ frame.

$B^0$ and $B^{+}$ candidates are identified on the basis of the difference $\Delta E$ between the reconstructed energy and the beam energy $\sqrt{s}/2$ in the $\Upsilon(4S)$ frame, and the beam-energy substituted mass $m_{\text{ES}}$ calculated from $\sqrt{s}/2$ and the reconstructed momentum of the candidate. $B$ candidates are selected with $m_{\text{ES}} > 5.2$ GeV/$c^2$ and $|\Delta E| < 3\sigma_{\Delta E}$, where $\sigma_{\Delta E}$ (10 to 30 MeV) is the measured resolution for each decay mode.

The decay position of the $B_{\text{rec}}$ candidate is determined by requiring convergence of a vertex fit, where in addition the masses of the $D$ mesons are constrained to their nominal values. Precisions between 60 and 100 $\mu$m $rms$ for the $B_{\text{rec}}$ decay position in $z$ and in the transverse plane are achieved, depending on the decay mode.

The vertex of the $B_{\text{opp}}$ is determined from all tracks in the event after removing those associated with the $B_{\text{rec}}$ candidate. Tracks from photon conversion candidates are rejected. Daughter tracks from $K^0_s$ or $\Lambda$ candidates are replaced by the neutral parents. An additional constraint is imposed on the $B_{\text{opp}}$ vertex using the $B_{\text{rec}}$ vertex and three-momentum, the beam spot position, and the average $\Upsilon(4S)$ momentum. To reduce the bias in the forward $z$ direction from charm decay tracks, the track with the largest contribution to the vertex $\chi^2$, if above 6, is removed and the fit iterated until no track fails this requirement. Events are required to have at least 2 tracks remaining in the $B_{\text{opp}}$ vertex, an error on $\Delta z$ smaller than 400 $\mu$m and $|\Delta z| < 3000$ $\mu$m. The precision achieved on $\Delta z$, 190 $\mu$m $rms$ on average, is dominated by the resolution on the $B_{\text{opp}}$ vertex. A remaining bias of $\sim 35$ $\mu$m due to charm decays on the $B_{\text{opp}}$ side is observed. We require $|\Delta t| < 18$ ps and find $6064 \pm 70$ $B^0$ and $6336 \pm 63$ $B^+$ signal events in a $\pm 2.5\sigma$ ($\sigma = 2.7$ MeV/$c^2$ and 2.6 MeV/$c^2$, respectively) window around the $m_{\text{ES}}$ peak above a small background ($\simeq 10\%$). The $m_{\text{ES}}$ distributions for the final samples are shown in Fig. 5 along with the results of a fit with a Gaussian distribution for the signal and an ARGUS background function.

As already noted, the modeling of the resolution function $R$ is a crucial element of the $B$ lifetime measurements. Studies with both Monte Carlo simulation and data show that the sum of a zero-mean Gaussian distribution and its convolution with a decay exponential provides a good trade-off between different sources of un-
The background $\Delta t$ distribution, $B$, for each $B$ species is modeled by the sum of a prompt component and a lifetime component convoluted with a resolution function of the form given in Eq. 3, but with a separate set of parameters. The fraction of non-prompt background, its effective lifetime and the background resolution parameters are determined separately for charged and neutral $B$ mesons. Signal and background outlier events have an assumed $\Delta t$ behavior $O$ given by a Gaussian distribution with zero mean and a fixed 10 ps width. The fractions of outliers in signal and background are determined separately in the lifetime fit.

Since the same resolution function is used for neutral and charged $B$ mesons, the fitting procedure maximizes the log-likelihood function $\ln \mathcal{L}$ formed from the sum of two terms, one for each $B$ meson species, with common parameters $\alpha$ for $R$:

$$
\ln \mathcal{L} = \sum_{i=0}^{1} \ln \left[ \mathcal{F}(\Delta t; \sigma_{0}, \rho_{0}|\tau_{B^{0}}; \tilde{\alpha}, \tilde{\beta}, f_{\text{out}}^{0}) \right] + \sum_{i=0}^{1} \ln \left[ \mathcal{F}(\Delta t; \sigma_{0}, \rho_{0}|\tau_{B^{+}}; \tilde{\alpha}, \tilde{\beta}, f_{\text{out}}^{+}) \right].
$$

The likelihood fit involves 19 free parameters. The parameter $\tau_{B^{0}}$ is replaced with $\tau_{B^{0}}/\tau_{B^{+}}$ to estimate the statistical error on the lifetime ratio $r$. The lifetime values were kept hidden until the event selection and the $\Delta t$ reconstruction method, as well as the fitting procedure, were finalized and the systematic errors were determined.

The fit results, after small corrections discussed below, are $\tau_{B^{0}} = 1.546 \pm 0.032$ ps, $\tau_{B^{+}} = 1.673 \pm 0.032$ ps and $\tau_{B^{0}}/\tau_{B^{+}} = 1.082 \pm 0.026$, where the errors are statistical only. The resolution parameters $\alpha$ (with $h = 0.69 \pm 0.07$, $s = 1.21 \pm 0.07$ and $\kappa = 1.04 \pm 0.24$) are consistent with those found in a Monte Carlo simulation that includes detector alignment effects. The fitted outlier fractions in the $B^{+}$ and $B^{0}$ signals are both $0.2^{+0.3}_{-0.2}$%. Figure 2 shows the results of the fit superimposed on the observed $\Delta t$ distributions for $B^{0}$ and $B^{+}$ events within 2.5 standard deviations of the $B$ mass in $m_{ES}$.

Table 3 summarizes the systematic uncertainties on the lifetime results. The full analysis chain, including event reconstruction and selection, has been tested with Monte Carlo simulation. The statistical precision on the consistency between the generated and fitted lifetimes is assigned as a systematic error. The resolution parameters $\alpha$ are determined from the data by the fit, contributing $\pm 0.017$ ps in quadrature to the statistical error of the individual lifetime results. Thus, a large part of the $\Delta t$ resolution uncertainty is included in the statistical error. Residual systematic uncertainties are attributed to limited flexibility of the resolution model. These contributions have been estimated by comparing results with different parametrizations. We correct our measurements for the small positive (negative) bias on the $B^{0}$ ($B^{+}$) lifetime due to differences in the $\Delta t$ resolution functions for
experiments with sets of fixed values for the outlier width and mean were simulated and subjected to the nominal lifetime fit. The largest observed bias is taken as systematic uncertainty. Additional systematic uncertainties are due to the SVT alignment. The z length scale was determined to better than 0.5% from secondary interactions in a beam pipe section of known length. Approximations in the calculation of $\Delta t$ from $\Delta z$ and the uncertainty on the boost lead to small systematic errors. The errors on the $m_{ES}$ fit parameters are used to determine the uncertainty on $p^{vis}$ and the corresponding systematic error. The main systematic uncertainties related to backgrounds arise from changes in the background composition as a function of $m_{ES}$. An additional contribution arises from a 1-2% $B^0$ contamination of the $B^+$ signal sample and vice versa. We use Monte Carlo simulation to correct for these background effects and assign the sum in quadrature of the corrections as systematic uncertainty.

In summary, the $B^0$ and $B^+$ meson lifetimes and their ratio have been determined to be:

$$\tau_{B^0} = 1.546 \pm 0.032 \text{ (stat)} \pm 0.022 \text{ (syst)} \text{ ps},$$
$$\tau_{B^+} = 1.673 \pm 0.032 \text{ (stat)} \pm 0.023 \text{ (syst)} \text{ ps},$$
$$\tau_{B^+}/\tau_{B^0} = 1.082 \pm 0.026 \text{ (stat)} \pm 0.012 \text{ (syst)}.$$

These are the most precise measurements to date, and they are consistent with the current world averages.

We are grateful for the extraordinary contributions of our PEP-II colleagues in achieving the excellent luminosity and machine conditions that have made this work possible. The collaborating institutions wish to thank SLAC for its support and the kind hospitality extended to them. This work is supported by DOE and NSF (USA), NSERC (Canada), IHEP (China), CEA and CNRS-IN2P3 (France), BMBF (Germany), INFN (Italy), NFR (Norway), MIST (Russia), and PPARC (United Kingdom). Individuals have received support from the Swiss NSF, A. P. Sloan Foundation, Research Corporation, and Alexander von Humboldt Foundation.

* Also with Università di Perugia, Perugia, Italy.
† Also with Università della Basilicata, Potenza, Italy.

[3] Throughout this paper, references to a hadron or to a decay reaction also imply their charge conjugate.

FIG. 2: $\Delta t$ distribution for the $B^0$ (top) and $B^+$ (bottom) events within 2.5$\sigma$ of the $B$ mass in $m_{ES}$. The results of the fit are superimposed on the data. The single-hatched areas are the background components $B$ and the cross-hatched areas represent the outlier contributions. The probability of obtaining a lower likelihood, evaluated with a Monte Carlo technique, is 7.3%.

| TABLE I: Summary of the systematic uncertainties. |
|---------------------|-----------------|-----------------|
| Effect               | $\delta(\tau_{B^0})$ (ps) | $\delta(\tau_{B^+})$ (ps) | $\delta(\tau_{B^+}/\tau_{B^0})$ |
| MC statistics        | 0.009            | 0.007            | 0.006                      |
| $R$ parametrization  | 0.008            | 0.004            | 0.003                      |
| same $R$ for $B^0$ and $B^+$ | 0.004 | 0.005 | 0.006 |
| Beam spot, $p_{B_{rec}}$ | 0.002       | 0.002            | 0.006                      |
| $\Delta t$ outliers | 0.011            | 0.011            | 0.005                      |
| SVT alignment        | 0.008            | 0.008            | 0.006                      |
| $z$ scale            | 0.008            | 0.008            | 0.006                      |
| $\Delta z$ to $\Delta t$ conversion | 0.006 | 0.006 | 0.006 |
| Signal probability   | 0.003            | 0.003            | 0.003                      |
| Background modeling  | 0.005            | 0.011            | 0.005                      |
| Total in quadrature  | 0.022            | 0.023            | 0.012                      |

$B^0$ and $B^+$ mesons arising from their decays to a different admixture of $D^-$ and $\overline{D}^0$ mesons and estimated with a high-statistics Monte Carlo sample. The size of the correction is assigned as a systematic error. A small systematic error results from uncertainties on the beam spot position and vertical size, and the $B_{rec}$ momentum vector, which are used to constrain the $B_{rec}$ vertex. To estimate the systematic error due to the assumptions on the shape of the $\Delta t$ outlier PDF, we first verified that the fitted lifetime results are stable when distributions wider than 10 ps or even flat are used in the fit. To investigate narrower shapes which are more signal-like, thousands of