Measurement of the $B \to J/\psi K^*(892)$ Decay Amplitudes


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We present a measurement of the decay amplitudes in $B \rightarrow J/\psi K^-$ (892) channels using 20.7 fb$^{-1}$ of data collected at the $\Upsilon(4S)$ resonance with the BABAR detector at PEP-II. We measure a $P$-wave fraction $R_\perp = (16.0 \pm 3.2 \pm 1.4)\%$ and a longitudinal polarization fraction $(59.7 \pm 2.8 \pm 2.4)\%$. The measurement of a relative phase that is neither 0 nor $\pi$, $\phi_\parallel = 2.50 \pm 0.20 \pm 0.08$ radians, favors a departure from the factorization hypothesis. Although the decay $B \rightarrow J/\psi K \pi$ proceeds mainly via $K^*(892)$, there is also evidence for $K^*(1430)$ and $K \pi$ $S$-wave contributions.

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The decay $B^0 \to J/\psi K^{*0}$ with $K^{*0} \to K^0 \pi^0$ allows a measurement of the CP violation parameter $\sin 2\beta$ that is theoretically as clean as for $B^0 \to J/\psi K^0$ [1]. However, due to the presence of even $(L = 0, 2)$ and odd $(L = 1)$ orbital angular momenta in the $J/\psi K^*$ system, there can be CP-even and CP-odd contributions to the decay rate. If the information contained in the decay angles is ignored, the measured time-dependent CP asymmetry is reduced by the dilution factor $D_{CP} = 1 - 2R_\perp$, where $R_\perp$ is the fraction of the $P$-wave. If the angular information is used, the CP components can be separated [2].

The angular analysis also provides a test of the factorization hypothesis, the validity of which is in question for color-suppressed modes [3,4]. In this scheme, the weak decay is described by a product of $J/\psi$ and $B \to K^*$ hadronic currents, and final state interactions are neglected. If factorization holds, the decay amplitudes should have relative phase 0 or $\pi$.

The decay $B \to J/\psi K^*(892)$ is described by three amplitudes. In the transversity basis [2,5] used by CLEO [6] and CDF [7], the amplitudes $A_0$, $A_0$, and $A_1$ have CP eigenvalues $+1$, $+1$, and $-1$, respectively. $A_0$ corresponds to longitudinal polarization, and $A_\parallel$ and $A_\perp$, respectively, to parallel and perpendicular transverse polarizations of the vector mesons; $R_\perp$ is defined as $|A_\perp|^2$. For a $\Delta t = 0$ transition, all $K^* \to K\pi$ channels involve the same amplitudes, and so the data for different decay modes can be combined.

The transversity frame is defined in the $J/\psi$ rest frame. The $K^*$ direction defines the negative $x$ axis. The $K\pi$ decay plane defines the $(x, y)$ plane, with $y$ such that $p_y(K) > 0$. The $z$ axis is the normal to this plane, and the coordinate system is right handed. The transversity angles $\theta_{1\ell}$ and $\phi_{1\ell}$ are defined as the polar and azimuthal angles of the positive lepton from the $J/\psi$ decay; $\theta_{K^*}$ is the $K^*$ helicity angle defined as the angle between the $K$ direction and the direction opposite the $J/\psi$ in the $K^*$ rest frame. The normalized angular distribution $g(\cos \theta_{1\ell}, \cos \theta_{K^*}, \phi_{1\ell})$ is

$$g = \frac{d^3\Gamma}{d \cos \theta_{1\ell} d \cos \theta_{K^*} d \phi_{1\ell}} = f_1|A_0|^2 + f_2|A_\parallel|^2 + f_3|A_\perp|^2 + f_4 \text{Im}(A_\parallel^* A_\perp) + f_5 \text{Re}(A_0^* A_\parallel) + f_6 \text{Im}(A_0^* A_\perp),$$

where

$$f_1 = (9/32\pi) \times 2 \cos^2 \theta_{K^*} (1 - \sin^2 \theta_{1\ell} \cos^2 \phi_{1\ell}),$$
$$f_2 = (9/32\pi) \times \sin^2 \theta_{K^*} (1 - \sin^2 \theta_{1\ell} \sin^2 \phi_{1\ell}),$$
$$f_3 = (9/32\pi) \times \sin^2 \theta_{K^*} \sin^2 \theta_{1\ell},$$
$$f_4 = (9/32\pi) \times \sin^2 \theta_{K^*} \sin 2\theta_{1\ell} \sin \phi_{1\ell} \cdot \zeta,$$
$$f_5 = - (9/32\pi) \times (1/\sqrt{2}) \sin 2\theta_{K^*} \sin^2 \theta_{1\ell} \sin 2\phi_{1\ell},$$
$$f_6 = (9/32\pi) \times (1/\sqrt{2}) \sin 2\theta_{K^*} \sin 2\phi_{1\ell} \cos \phi_{1\ell} \cdot \zeta.$$
than 0.2 rad. A photon is defined as a neutral cluster of energy greater than 30 MeV in the EMC that agrees in lateral shower shape with an electromagnetic shower. A \( \pi^0 \) candidate consists of a pair of photons with invariant mass in the interval 106–153 MeV/c\(^2\). The \( J/\psi, K_S^0, \) and \( \pi^0 \) are constrained to the corresponding nominal masses [10]. \( K^+ \) candidates must have \( K\pi \) invariant mass within 100 MeV/c\(^2\) of the nominal \( K^*(892) \) mass [10].

\( B \) mesons are formed from \( J/\psi \) and \( K^+ \) candidates. For \( B \to J/\psi(K\pi^0)^* \), \( \cos\theta_K \) is required to be smaller than 0.667. This reduces the cross feed (CF) from \( J/\psi(K\pi^\pm)^* \) modes, where the \( \pi^\pm \) is lost, and the self-cross feed (SCF) due to a wrongly reconstructed \( \pi^0 \). The SCF is the most important background source since it tends to peak in the signal region.

The signal region is defined using two variables. The first is the difference \( \Delta E = E_B^\text{obs} - E_B^\text{beam} \) between the candidate \( B \) energy and the beam energy, in the \( Y(4S) \) rest frame. The second is the beam-energy substituted mass \( m_{ES} = (E_{\exp}^2 - \vec{p}_B^2)^{1/2} \) where, in the laboratory frame, \( E_{\exp} = (s/2 + \vec{p}_B \cdot \vec{p}_i)/E_i \) is the \( B \) candidate expected energy, \( \vec{p}_B \) is its measured momentum, and \( E_i \) is the \( e^+ e^- \) initial-state four-momentum. \( \sqrt{s} \) is the center of mass energy. For the signal region, \( \Delta E \) is required to be between \(-70\) and \(+50\) MeV for channels involving a \( \pi^0 \), and within \( \pm30 \) MeV otherwise. If several \( B \) candidates are found in an event, the one having the smallest \( |\Delta E| \) is retained. The corresponding \( m_{ES} \) distributions are shown in Fig. 1.

With the signal region defined by \( m_{ES} > 5.27 \text{ GeV/c}^2 \) and the above \( \Delta E \) ranges, the \( B \) reconstruction efficiencies are 9.9\%, 23.9\%, 17.2\%, and 13.8\% for the \( K_S^0 \pi^0, K^+ \pi^-, K_S^0 \pi^+, \) and \( K^+ \pi^0 \) modes, respectively, with corresponding total yields of 43, 547, 135, and 216 events. The CF (SCF) contamination levels, obtained from a full simulation of the BABAR detector, are 9.9(15.8)\%, 1.2(2.4)\%, 2.4(3.0)\%, and 8.1(15.7)\% of the pure signal, respectively.

The fit maximizes an unbinned likelihood that uses a probability density function (pdf) that depends on angular and \( m_{ES} \) information. From the observed \( m_{ES} \) value, a signal probability is computed with a Gaussian \( G(m_{ES}) \) to describe the signal and a phase-space background function [11] \( F(m_{ES}) \).

The pdf \( g_{\text{obs}} = g(\bar{\omega}_j \cdot \epsilon(\bar{\omega}_j))/\epsilon \) is used to describe signal events; \( \bar{\omega}_j \) represents the angular variables \( \cos\theta_B, \cos\theta_K, \phi_B \) for event \( j \), and \( \epsilon(\bar{\omega}_j) \) is the efficiency at \( \bar{\omega}_j \). Rewriting Eq. (1) as \( g = \sum_{i=1}^6 f_i A_i \), where the \( A_i \) (\( i = 1, \ldots, 6 \)) represent \( |A_0|^2, |A_1|^2, |A_2|^2, \) \( \text{Im}(A_0 A_1), \text{Re}(A_0 A_1), \) and \( \text{Im}(A_0 A_1) \), the mean value is \( \epsilon = \int \epsilon \cdot d\bar{\omega} = \sum_{i=1}^6 \epsilon_i \), where the \( \epsilon_i = \int f_i \epsilon \cdot d\bar{\omega} \) are constants. The signal part of the log likelihood, \( \ln L_{\text{signal}} = \sum_{i=1}^{N_{\text{obs}}} \ln[g_{\text{obs}}(\bar{\omega}_j)] \), where \( N_{\text{obs}} \) is the number of observed events, becomes \( \ln L_{\text{signal}} = \sum_{j=1}^{N_{\text{obs}}} \ln[g(\bar{\omega}_j)] + \sum_{j=1}^{N_{\text{obs}}} \ln[\epsilon(\bar{\omega}_j)] - N_{\text{obs}} \ln(\sum_{i=1}^6 \epsilon_i) \). Since the \( \epsilon(\bar{\omega}_j) \) are constants, the second term can be discarded. Only the coefficients \( \epsilon_i \) are required, and detailed representation of the acceptance is unnecessary [12].

The coefficients \( \xi_i \) are evaluated with a Monte Carlo simulation. Separate sets of \( \xi_i \) are used for each channel, and for \( \xi = \epsilon, \mu \). The values of \( \xi_i \) (\( i = 1, 2, 3 \)) are close to that of \( \epsilon \); \( \xi_1 \) is always smallest, especially in channels involving a \( \pi^0 \), because of the requirement on \( \cos\theta_K \).

The values of \( \xi_i \) (\( i = 4, 5, 6 \)), which are related to the interference terms, are compatible with zero.

The angular dependence of combinatorial background events, \( g_{\text{bkg}} \), is described by a pdf similar to that in Eq. (1) with amplitudes \( B_i, i = 0, \|, \bot, \) and corresponding terms \( B_i (i = 1, \ldots, 6) \).

The angular distribution of the (S)CF background is amplitude dependent. We correct for the effect of this background by evaluating the modified values \( \xi_i \) of the \( \xi_i \) by

<table>
<thead>
<tr>
<th>Quantity</th>
<th>( K_S^0 \pi^0 )</th>
<th>( K^+ \pi^- )</th>
<th>( K_S^0 \pi^+ )</th>
<th>( K^+ \pi^0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>A_0</td>
<td>^2 )</td>
<td>0.65 ± 0.13</td>
<td>0.60 ± 0.04</td>
</tr>
<tr>
<td>(</td>
<td>A_1</td>
<td>^2 )</td>
<td>0.07 ± 0.11</td>
<td>0.17 ± 0.05</td>
</tr>
<tr>
<td>(</td>
<td>A_2</td>
<td>^2 )</td>
<td>0.28 ± 0.14</td>
<td>0.23 ± 0.05</td>
</tr>
<tr>
<td>( \phi_\perp ) (rad)</td>
<td>( \cdots )</td>
<td>(-0.1 \pm 0.2 )</td>
<td>( 0.0 \pm 0.3 )</td>
<td>(-0.4 \pm 0.4 )</td>
</tr>
<tr>
<td>( \phi_\parallel ) (rad)</td>
<td>2.1 ± 0.7</td>
<td>2.5 ± 0.3</td>
<td>2.8 ± 0.4</td>
<td>2.6 ± 0.5</td>
</tr>
</tbody>
</table>
TABLE II. Fitted parameter values for the combined data samples. The first uncertainty is statistical, and the second is systematic. Note that \((\phi_0, \phi_j) \rightarrow (\pi - \phi_0, -\phi_j)\) is also a solution.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>(</td>
<td>A_0</td>
</tr>
<tr>
<td>(</td>
<td>A_1</td>
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<tr>
<td>(</td>
<td>A_2</td>
</tr>
<tr>
<td>(\phi_0) (rad)</td>
<td>-0.17 ± 0.16 ± 0.07</td>
</tr>
<tr>
<td>(\phi_j) (rad)</td>
<td>2.50 ± 0.20 ± 0.08</td>
</tr>
</tbody>
</table>

including the (S)CF events, in the \(m_{ES}\) signal region, in addition to the signal [12]. In contrast to the \(\xi_i\), the \(\xi_i\) depend on the amplitudes used in the simulation, but the maximum effect on the fitted amplitudes is found to be on the order of \(10^{-3}\). The complete log likelihood is

\[
\ln L = \sum_{i=1}^{N_{\text{obs}}} \ln \left[ x G(m_{ES}) g(\tilde{\omega}_i) \right] \\
+ (1 - x) F(m_{ES}) g_B(\tilde{\omega}_i) \\
- N_{\text{obs}} \ln \left( \sum_{i=1}^{N_{\text{obs}}} \tilde{\xi}_i x A_i + (1 - x) B_i \right) - \mathcal{N},
\]

where \(x\) is the fraction of signal integrated over the \(m_{ES}\) range 5.2–5.3 GeV/c². The normalization of \(g\) and \(g_B\) is relaxed in an extended likelihood approach [13], with convergence to the required condition \(a^2 = |A_i|^2 + |A_j|^2 + |A_k|^2 = 1\) imposed through the additional term \(\mathcal{N} = N_{\text{obs}} a^2\) while \(|B_0|^2 + |B_1|^2 + |B_2|^2 = a^2\) holds by construction. The fit parameters are the mean and width of \(G(m_{ES})\); the shape parameter of \(F(m_{ES})\); the fraction \(x\); the signal amplitudes and phases \(|A_0|^2, |A_1|^2, |A_2|^2, \phi_0, \phi_1\); and the corresponding background amplitudes and phases.

The agreement among the results for the individual decay channels is shown in Table I, while the fit result for the combined sample is summarized in Table II.

A partial representation of the fit is given by the one-dimensional projections of the angular distribution in Fig. 2. As a check of the fit quality, fits were performed to Monte Carlo samples with the angular distribution and number of events observed in the data. The maximum likelihood in the data is 1.35 standard deviations below the mean obtained from the Monte Carlo fits. The probability of obtaining a lower likelihood is 8.8%.

Systematic uncertainties are detailed in Table III. Limited simulation statistics (32 000 events per mode) give rise to a systematic uncertainty in the acceptance and (S)CF corrections (first row). Monte Carlo simulation has been used to estimate uncertainties due to the assumed form for the \(m_{ES}\) angular distributions of the background (second row). In particular, this accounts for any possible absorption of the (S)CF background by the \(F(m_{ES})\) function. The differences between simulated tracking and PID efficiencies and measurements obtained with control samples in the data lead to systematic uncertainties (third row) through their impact on acceptance corrections.

The \(K\pi\) S-wave systematic uncertainty (fourth row) is obtained as follows. Although the \(K\pi\) mass distribution for \(B \rightarrow J/\psi K\pi\) is dominated by the \(K^*(892)\) [Fig. 3(a)], a significant number of candidates are at higher mass with
a clear peak at \(\sim 1.4\) GeV/\(c^2\). The states in this region that couple strongly to \(K\pi\) are the \(K_0^*(1430)\) and the \(K_2^*(1430)\) [10]. Since it has width \(\sim 300\) MeV/\(c^2\), the \(K_0^*(1430)\) alone would yield significantly more events above and below the peak than are observed. The \(K_2^*(1430)\) alone describes the high mass region but, when combined with the \(K^*(892)\) tail, yields too few events in the \(1.1-1.3\) GeV/\(c^2\) range [Fig. 3(b)]. This suggests a significant \(S\)-wave contribution, in which case the recoil \(J/\psi\) has a helicity angle distribution \(\sim \sin^2(\theta_{J/\psi})\). The observed behavior [Fig. 3(c)] agrees with this conjecture. This, together with the absence of the \(S\)-wave above \(1.5\) GeV/\(c^2\), is consistent with the mass dependence of the \(S\)-wave \(K\pi\) scattering amplitude [14]. If the \(K\pi\) \(S\)-wave in \(B\) decay behaves like this, a coherent \(S\)-wave amplitude should also be present in the \(K^*(892)\) region; \(S-P\) interference should occur, which, if ignored, can affect the \(P\)-wave amplitudes extracted from the data.

The effect of the \(S\)-wave in the \(K^*(892)\) region has been estimated by including a scalar term in the total amplitude. This yields a more complicated angular distribution \(g_S\), with ten \(f_i\) functions. A fit of \(g_S\) to the data in the \(1.1-1.3\) GeV/\(c^2\) region yields an \(S\)-wave fraction of \((62 \pm 9)\%\), in agreement with the failure of a \(P\)- and \(D\)-wave fit to describe the mass spectrum. By repeating the analysis using \(g_S\), we find the \(S\)-wave contribution in the \(K^*(892)\) region to be \((1.2 \pm 0.7)\%\). The differences in the \(P\)-wave results with and without \(S\)-wave are taken as estimates of systematic uncertainty (Table III, fourth row) since, with the present statistics, the presence of the \(S\)-wave in the \(K^*(892)\) region cannot be confirmed.

Table IV compares our results to those of CLEO [6] and CDF [7]. They are consistent, but the present measurement is significantly more precise. Longitudinal polarization is seen to dominate and the \(P\)-wave intensity is small. If \(\sin 2\beta\) were measured in the \(B \rightarrow J/\psi K_S^0\pi^0\) channel from the decay-time information only, the value of the dilution from the present measurement, \(D_\perp = 0.68 \pm 0.07\), would contribute a \(10\%\) uncertainty.

Finally, we find that \(|\phi_\|\) differs significantly from \(\pi\). This agrees with the CDF measurement, and indicates a departure from the factorization of the hadronic currents.

In addition, there is evidence that \(S\)- and \(D\)-wave amplitude contributions are necessary for a description of the \(K\pi\) mass spectrum from \(B \rightarrow J/\psi K\pi\) decay.

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**TABLE IV.** Comparison with other experiments. Statistical and systematic uncertainties are added in quadrature.

|       | \(|A_\parallel|^2\) | \(|A_\perp|^2\) | \(\phi_\perp\) | \(\phi_\|\) |
|-------|-----------------|----------------|-------------|-------------|
| CLEO  | 0.52 \(\pm 0.08\) | 0.16 \(\pm 0.09\) | \(-0.11 \pm 0.46\) | \(3.00 \pm 0.37\) |
| CDF   | 0.59 \(\pm 0.06\) | 0.13 \(\pm 0.11\) | \(-0.56 \pm 0.54\) | \(2.16 \pm 0.47\) |
| BABAR | 0.60 \(\pm 0.04\) | 0.16 \(\pm 0.03\) | \(-0.17 \pm 0.17\) | \(2.50 \pm 0.22\) |

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[8] Throughout the paper, the use of charge conjugate modes is implied wherever relevant.