Search for $D^0 \overline{D}^0$ Mixing and a Measurement of the Doubly Cabibbo-Suppressed Decay Rate in $D^0 \to K\pi$ Decays


(The BABAR Collaboration)
Within the standard model, the level of $D^0-\bar{D}^0$ mixing is predicted to be below the sensitivity of current experiments [1]. For this reason $D^0-\bar{D}^0$ mixing is a good place to look for signals of new physics beyond the standard model [2]. Because new physics may not conserve $CP$, it is important to consider $CP$ violation when measuring $D^0-\bar{D}^0$ mixing.
mixing. Observation of \( CP \) violation in \( D^0 \overline{D}^0 \) mixing would be an unambiguous sign of new physics [1,3].

Mixing can be characterized by the two parameters \( x \equiv \Delta m / \Gamma \) and \( y \equiv \Delta \Gamma / 2 \Gamma \), where \( \Delta m = m_1 - m_2 \) and \( \Delta \Gamma = \Gamma_1 - \Gamma_2 \) is the difference in mass (width) between the two mass eigenstates and \( \Gamma \) is the average width.

The dominant two-body decay of the \( D^0 \) is the right-sign (RS) Cabibbo-favored (CF) decay \( D^0 \to K^+ \pi^- \). Evidence for mixing and \( CP \) violation, if present, will appear in the wrong-sign (WS) decay \( D^0 \to K^- \pi^+ \). Charge conjugates are implied unless otherwise stated. Two amplitudes contribute to the production of this final state: the tree-level amplitude for doubly Cabibbo-suppressed (DCS) decay of the \( D^0 \), and an amplitude for mixing followed by CF decay of the \( \overline{D}^0 \). Assuming that \( x, y \ll 1 \) and \( CP \) is conserved, and with the convention \( \Delta \Gamma = \Gamma (CP = +1) - \Gamma (CP = -1) \), the time-dependent, WS decay rate \( T_{WS}(t) \) for \( D^0 \to K^+ \pi^- \) can be approximately related [4] to the RS decay rate \( T_{RS}(t) \) by

\[
T_{WS}(t) = T_{RS}(t) \left( R_D + \sqrt{R_D} y \cos \varphi \frac{x^2 + y^2}{4} \right),
\]

In Eq. (1), \( t \) is the proper time of the \( D^0 \) decay measured in units of the \( D^0 \) lifetime \( \tau_{D^0} \), \( T_{RS}(t) \propto e^{-t/\tau_{D^0}} \), \( R_D \) is the time-integrated rate of the direct DCS decay \( D^0 \to K^+ \pi^- \) relative to the RS decay, and \( x', y' \) are related to \( x, y \) by

\[
\begin{align*}
x' &= x \cos \delta_{K\pi} + y \sin \delta_{K\pi} \\
y' &= -x \sin \delta_{K\pi} + y \cos \delta_{K\pi}
\end{align*}
\]

where \( \delta_{K\pi} \) is the relative strong phase between the CF and DCS amplitudes. Physics beyond the standard model may include additional phases that are not \( CP \) conserving. Such terms can be absorbed into a phase \( \varphi \), described below. The time-integrated WS decay rate is

\[
R_{WS} = R_D + \sqrt{R_D} y' \frac{x'^2 + y'^2}{2}.
\]

Previous experiments have searched for CP violation using wrong-sign hadronic [4–6] and semileptonic [7] \( D^0 \) decays, or have searched for width differences between \( CP = +1 \) and \( CP = -1 \) states directly [8–10]. Since \( x' \) appears only quadratically in Eq. (1), its sign cannot be determined in an analysis based on the WS decay alone.

To allow for \( CP \) violation, we apply Eq. (1) to \( D^0 \) and \( \overline{D}^0 \) separately. We determine \( \{ R_{WS}, x'^+, y'^+ \} \) for \( D^0 \) candidates and \( \{ R_{WS}, x'^-, y'^- \} \) for \( \overline{D}^0 \) candidates. The separate \( D^0 \) and \( \overline{D}^0 \) results can be combined to form the quantities

\[
A_D = \frac{R_D^+ - R_D^-}{R_D^+ + R_D^-}, \quad A_M = \frac{R_M^+ - R_M^-}{R_M^+ + R_M^-},
\]

where \( R_M = (x'^2 + y'^2)/2 \). \( A_D \) and \( A_M \) are related to \( CP \) violation in the DCS decay and mixing amplitudes, respectively. \( CP \) violation in the interference of DCS decay and mixing is parametrized by the phase \( \varphi \):

\[
x'^+ = \frac{1 \pm A_M}{\sqrt{1 + A_M^2}} (x' \cos \varphi \pm y' \sin \varphi),
\]

\[
y'^- = \frac{1 \pm A_M}{\sqrt{1 + A_M^2}} (y' \cos \varphi \pm x' \sin \varphi),
\]

An offset of \( \varphi \) of \( \pm \pi \) is equivalent to interchanging the labels of the two physical \( D^0 \) states. To avoid this labeling ambiguity, we use the convention that \( |\varphi| < \pi/2 \).

We select a very clean sample of RS and WS decays from a 57.1 fb\(^{-1} \) dataset collected with the BABAR detector [11] at the PEP-II \( e^+ e^- \) storage ring. We fit for parameters describing mixing and DCS amplitudes from the WS decay-time distribution. To avoid potential bias, we finalized our data selection criteria and the procedures for fitting and extracting the statistical limits without examining the mixing results.

We select \( D^0 \) candidates from reconstructed \( D^{*+} \to D^0 \pi^+ \) decays; this provides a clean sample of \( D^0 \) decays, and the charge of the pion (the “tagging pion”) identifies the production flavor of the neutral \( D \). We retain each RS and WS \( D^0 \) candidate whose invariant mass \( m_{K\pi} \) is within 60 MeV/c\(^2 \) of the \( D^0 \) mass. We require the mass difference \( \delta m \) between the \( D^{*+} \) and the \( D^0 \) candidate to be less than \( m_1 + 25 \) MeV/c\(^2 \). Only \( D^{*+} \) candidates with center-of-mass momenta above 2.6 GeV/c are retained, thereby rejecting \( D^{*+} \) candidates from \( B \) decays.

We determine the \( D^0 \) vertex by requiring that the \( D^0 \) decay tracks originate from a common point with a probability \( p(x^2) > 1 \% \), and then determine the \( D^{*+} \) vertex by extrapolating the \( D^0 \) flight path back to the beam-beam interaction region. We constrain the trajectory of the tagging pion to originate from the \( D^{*+} \) vertex, and calculate the \( D^0 \) proper decay-time \( t \) from its flight length. The typical resolution is 0.2 ps.

We determine the mixing parameters by unbinned, extended maximum-likelihood fits to the RS and WS samples simultaneously. We consider four separate fit cases: (i) a general case allowing for possible \( CP \) violation (by treating WS \( D^0 \) and \( \overline{D}^0 \) candidates separately), fitting for \( \{ R_{WS}, x'^+, y'^+ \} \) for \( D^0 \) candidates and \( \{ R_{WS}, x'^-, y'^- \} \) for \( \overline{D}^0 \) candidates; (ii) a case assuming \( CP \) conservation, not differentiating between \( D^0 \) and \( \overline{D}^0 \) candidates, fitting for \( R_{WS}, x'^2, y'^2 \); (iii) a case assuming no mixing, but allowing \( CP \) violation in the DCS amplitudes, fitting for \( \{ R_D, A_D \} \); and (iv) a case assuming both \( CP \) conservation and no mixing, fitting for \( R_D \) only.

We assign each candidate to one of four categories based on its origin as \( D^0 \) or \( \overline{D}^0 \) and its decay as RS or WS. For each category we construct probability density functions (PDFs) that model signal and background components. The independent variables in the PDFs are \( m_{K\pi}, \delta m, \) the \( D^0 \) proper time \( t \), and its error \( \sigma_t \).

Within a category, the likelihood is a sum of PDFs, one for each signal or background component, weighted by
the number of events for that component. Each component’s PDF factorizes into a portion describing the behavior of each independent variable convoluted with a corresponding resolution function. The parameters describing the mass resolutions and shapes and the lifetime resolution are shared between PDFs. These are determined primarily by the large RS sample. We limit the fit to the fiducial range $|t| < 4$ ps and $\sigma_t < 0.4$ ps.

We characterize the WS background by three components: true $D^0$ decays that are combined with unassociated pions to form $D^{\pi^+}$ candidates; combinatorial background where one or both of the tracks in the $D^0$ candidate do not originate from a $D^0$ decay; and background where the kaon and the pion in the $D^0$ decay have both been misidentified, thus converting a RS decay into an apparent WS decay (double misidentification). Kaons (pions) are identified with an average efficiency of 84% (85%); the average misidentification rate is 3% (2%). Fitting the double misidentification background is particularly important due to the large size of the RS sample; its level as obtained from the fit agrees well with predictions based on our particle identification performance.

We normalize $D^0$ and $\bar{D}^0$ WS candidates separately, resulting in two signal and six background WS components. We assume $CP$ conservation in the RS data; it has one signal and three background components.

We perform the fit in steps. Parameters corresponding to the $m_{K\pi}$ and $\delta m$ distributions and the number of candidates in each category are determined first. Then these parameters are fixed while fitting the WS proper time distribution. The shapes of the distributions in $m_{K\pi}$ and $\delta m$ allow the fit to differentiate between the various signal and background components. Figure 1 shows projections from the WS sample overlaid on the fit result.

We fit the RS decay-time distribution using a model that combines the RS signal decay-time distribution $\Gamma_{RS}(t)$ in Eq. (1) and the expected decay-time distributions of each background component, convolving each with a common decay-time resolution model that uses the decay-time error for each candidate and a scaling factor determined in the fit. For the WS signal component, we use the same resolution model but with a lifetime distribution including the mixing parameters as given by $\Gamma_{WS}(t)$ in Eq. (1) or its $CP$-violating counterparts. For the unassociated pion and double misidentification backgrounds, we also use the $\Gamma_{RS}(t)$ lifetime distribution because they are true $D^0$ decays. The combinatorial background is assigned a zero-lifetime distribution and a signal-type resolution model based on studies of mass sidebands and Monte Carlo (MC) samples.

Table I summarizes the fit results for the four cases. Figure 2 shows the decay-time distribution of the WS sample for the signal and a background region. We select a signal (background) region with 73% signal (50% combinatorial background) candidates based on the reconstructed values of $m_{K\pi}$ and $\delta m$. The selected signal region contains 64% of all signal events according to the fit. We observe about 120 000 RS (430 WS) signal decays.

Our fit permits $x^2$ to take unphysical negative values. We use a frequentist approach utilizing toy MC experiments to interpret nonphysical results and to construct 95% confidence-level (C.L.) contours in $(x^2, y^2)$. In each toy experiment, we generate a WS dataset (the part sensitive to mixing) for a given $(x^2, y^2)$ with the same number of $D^0$ and $\bar{D}^0$ events as observed in the data, but with a decay-time distribution appropriate for the chosen point. Fit parameters for the $m_{K\pi}$ and $\delta m$ distributions and other parameters not sensitive to mixing are fixed at their fitted values from the data. The $\sigma_t$ distribution and background fractions from the data fit are used as well. We fit each toy MC dataset, obtaining values for the mixing parameters and the corresponding log-likelihood surface. We construct contours such that for any point $\hat{\alpha} = (x^2, y^2)$ on the contour 95% of the experiments at

<table>
<thead>
<tr>
<th>Fit case</th>
<th>Parameter</th>
<th>$D^0$</th>
<th>$\bar{D}^0$</th>
<th>$D^0 + \bar{D}^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mixing allowed</td>
<td>$R_{WS}^{(x)}$</td>
<td>3.9</td>
<td>3.2</td>
<td>3.6</td>
</tr>
<tr>
<td></td>
<td>$x_{\pi}^{(x)}$</td>
<td>$-0.79$</td>
<td>$-0.17$</td>
<td>$-0.32$</td>
</tr>
<tr>
<td></td>
<td>$x_{\pi}^{(y)}$</td>
<td>17</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>No mixing</td>
<td>$R_{WS}^{(x)}$</td>
<td>3.9</td>
<td>3.2</td>
<td>3.6</td>
</tr>
</tbody>
</table>
generated at that point will have a log-likelihood difference $\Delta \ln \mathcal{L}(\tilde{a}_e) = \ln \mathcal{L}_{\text{max}} - \ln \mathcal{L}(\tilde{a}_e)$ less than the corresponding value $\Delta \ln \mathcal{L}_{\text{data}}(\tilde{a}_e)$ evaluated for the data. $\mathcal{L}_{\text{max}}$ is the maximum-likelihood obtained from any fit [12].

Where we assume $CP$ conservation, we apply this method to the combined $D^0$ and $\bar{D}^0$ WS samples. The resulting contour is shown by the dotted line in Fig. 3. The 95% C.L. for $R_D$ and for $R_M$ are obtained by finding their extreme values on the 95% C.L. contour.

To consider $CP$ violation, we divide the WS sample into candidates produced as a $D^0$ or as a $\bar{D}^0$ and calculate separate contours for $(x^{+}, y^{+})$ and $(x^{-}, y^{-})$, each corresponding to a C.L. of $1 - \sqrt{0.05} = 77.6\%$. Each point on the $D^0$ contour is combined with each point on the $\bar{D}^0$ contour using Eqs. (3)–(5) to produce two potential solutions of $(x', y')$ for each relative sign of $x^+$ and $x^-$. The outer envelope of these points is presented as the 95% C.L. contour in the $(x'^2, y')$ plane (see Fig. 3). The peculiar shape of the contour arises from the two solutions for each point. This contour is more stringent than the $CP$-conserving case in some cases, which is acceptable since the definition of coverage is slightly different. No value for $x'^2$ exists if either $x'^+$ or $x'^- < 0$.

We summarize results including uncertainties in Table II. We obtain limits on the mixing parameters by projecting the contours onto the corresponding coordinate axes. Since the no-mixing solution is well within the 95% C.L. contour, we cannot place limits on $\Lambda_M$ and $\varphi$.

To estimate systematic uncertainties, we evaluate contributions from uncertainties in the PDF parametrization, detector effects, and event selection criteria. The small systematic effects of fixing the $m_{K\pi}$ and $\delta m$ parameters and the number of events in each category in the final fit is evaluated by varying these parameters within statistical uncertainties while accounting for correlations.

For detector effects such as alignment errors or charge asymmetries, we measure their effect on the RS sample. Assuming that RS decay is exponential and has no direct $CP$ violation, this method is very sensitive. The systematic error due to the size of the MC sample is insignificant since all distributions are obtained from the data.

Each systematic check yields a small shift in the mixing parameters. We use MC experiments to determine the significance of each shift using the same method employed for the 95% C.L. statistical contour. We scale the statistical contour with respect to the central fitted point by $\sqrt{1 + \sum m_i^2}$, where $m_i$ is the relative significance of each check. For the general case, we carry out this procedure for the $D^0$ and $\bar{D}^0$ contours separately before combination. In all fits, the largest effect for $x'^2$ and $y'$ is the $D^{+}$ momentum selection cut, with $m_i^2 = 0.04$; all others are at least 3 times smaller. For $R_D$, the largest effect is the decay-time range. We show systematic error contours in Fig. 3 as a dashed line in the $CP$-conserving case and as a dash-dotted line in the general case.

FIG. 2. The proper time distribution for the WS candidates in (a) the signal region (73% signal purity) and (b) a background region (50% combinatorial background). See Fig. 1 for component definitions.
We have set improved limits on $D^0$-$\bar{D}^0$ mixing and $CP$ violation in WS decays of $D^0$ mesons. These are compatible with previous results [4–6] and with no mixing and no $CP$ violation, agreeing with standard model predictions.

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\[ R_D = [0.357 \pm 0.020(\text{stat}) \pm 0.027(\text{syst})] \%
\]

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