Observation of the Decay $B^\pm \to \pi^\pm \pi^0$, Study of $B^\pm \to K^\pm \pi^0$, and Search for $B^0 \to \pi^0 \pi^0$
We present results for the branching fractions and charge asymmetries in $B^z \to h^z \pi^0$ (where $h^z = \pi^z, K^z$) and a search for the decay $B^0 \to \pi^0 \pi^0$ using a sample of approximately $88 \times 10^9 \ B\bar{B}$ pairs collected by the BABAR detector at the PEP-II asymmetric-energy $B$ Factory at SLAC. We measure $\mathcal{B}(B^z \to \pi^z \pi^0) = (5.5^{+1.0}_{-0.9} \pm 0.6) \times 10^{-6}$, where the first error is statistical and the second is systematic. The $B^z \to \pi^z \pi^0$ signal has a significance of $7.7\sigma$ including systematic uncertainties. We simultaneously measure the $K^z \pi^0$ branching fraction to be $\mathcal{B}(K^z \to K^0 \pi^0) = (12.8^{+1.1}_{-1.0} \pm 1.0) \times 10^{-6}$. The charge asymmetries are $A_{K^z \pi^0} = -0.03^{+0.11}_{-0.10} \pm 0.02$ and $A_{K^z \pi^0} = -0.09 \pm 0.09 \pm 0.01$. We place a 90% confidence-level upper limit on the branching fraction $\mathcal{B}(B^0 \to \pi^0 \pi^0)$ of $3.6 \times 10^{-6}$.
on the angle \( \alpha \equiv \arg[-V_{uds}V_{us}^*/V_{uds}V_{us}^*] \) of the unitarity triangle. However, in contrast to the theoretically clean determination of the angle \( \beta \) in \( B^0 \) decays to charmonium final states [3,4], the extraction of \( \alpha \) in \( B^0 \to \pi^+\pi^- \) is complicated by the interference of tree and penguin amplitudes with different weak phases. The shift between \( \alpha_{eff} \), derived from the measured \( B^0 \to \pi^+\pi^- \) asymmetry, and \( \alpha \) may be evaluated or constrained using measurements of the isospin-related decays \( B^0(\overline{B}^0) \to \pi^0\pi^0 \) and \( B^0 \to \pi^+\pi^- \) [5].

The CP-violating charge asymmetry for \( B^\pm \) modes, defined as \( A_{CP} = (|A|^2 - |A|^2)/(|A|^2 + |A|^2) \), where \( A(A) \) is the \( B^+ (B^-) \) decay amplitude, will deviate from zero if the tree and penguin amplitudes each have different weak and strong phases. In the standard model the decay \( B^\pm \to \pi^\pm\pi^0 \) has only a tree amplitude, so no charge asymmetry is expected. Both the rate and asymmetry of the decay \( B^\pm \to K^\pm\pi^0 \) may constrain the value of the unitarity triangle angle \( \gamma \). In particular, the ratio of \( B(B^\pm \to K^\pm\pi^0) \) and \( B(B^\pm \to K^0\pi^\pm) \) provides a lower bound for \( \gamma \) [6]. The decay \( B^\pm \to K^\pm\pi^0 \) can also exhibit a significant charge asymmetry; different models for hadronic \( B \) decays predict a range of values [7].

In this paper, we report on an observation of the decays \( B^\pm \to \pi^\pm\pi^0 \) and \( B^\pm \to K^\pm\pi^0 \), a measurement of their CP-violating charge asymmetries, and a search for the decay \( B^0 \to \pi^0\pi^0 \), using \((87.9\pm1.0)\times10^6\) \( BB \) pairs collected with the BABAR detector.

BABAR is a solenoidal detector optimized for the asymmetric-energy beams at PEP-II and is described in detail in Ref. [8]. Charged particle (track) momenta are measured with a 5-layer double-sided silicon vertex tracker and a 40-layer drift chamber (DCH) inside a 1.5 T superconducting solenoidal magnet. Photons (neutral cluster) positions and energies are measured with an electromagnetic calorimeter (EMC) consisting of 6580 CsI (Tl) crystals. Tracks are identified as pions or kaons by the Cherenkov angle \( \theta_c \) measured with a detector of internally reflected Cherenkov light (DIRC).

Candidate \( \pi^0 \) mesons are reconstructed as pairs of photons, spatially separated in the EMC, with an invariant mass within \( 3\sigma \) of the \( \pi^0 \) mass. The resolution is approximately 8 MeV/c^2 for high momentum \( \pi^0 \)s. Photon candidates are required to be consistent with the expected lateral shower shape, not be matched to a track, and have a minimum energy of 30 MeV. To reduce the background from false \( \pi^0 \) candidates, the angle \( \theta_\gamma \) between the photon momentum vector in the \( \pi^0 \) rest frame and the \( \pi^0 \) momentum vector in the laboratory frame is required to satisfy \( |\cos\theta_\gamma| < 0.95 \).

Candidate tracks are required to be within the tracking fiducial volume, originate from the interaction point, consist of at least 12 DCH hits, and be associated with at least six Cherenkov photons in the DIRC.

\( B \) meson candidates are reconstructed by combining a \( \pi^0 \) with a pion or kaon \((h^\pm)\) or by combining two \( \pi^0 \) mesons. Backgrounds arise from two sources: \( B \to \rho\pi \) decays in which one pion is emitted nearly at rest in the \( B \) frame so that the remaining decay products are kinematically consistent with a \( B^\pm \to \pi^\pm\pi^0 \) or \( B^0 \to \pi^0\pi^0 \) decay, and \( e^+e^- \to q\overline{q} \) \((q = u, d, s, c)\) events where an \( h^\pm \) or \( \pi^0 \) from each quark randomly combine to mimic a B decay.

Both backgrounds are separated from the signal using the kinematic constraints of \( B \) mesons produced at the \( \Upsilon(4S) \). The first kinematic variable is the beam-energy substituted mass \( m_{ES} = \sqrt{(s/2 + \mathbf{p}_B^2)/E_i^2 - \mathbf{p}_B^2} \), where \( \sqrt{s} \) is the total center-of-mass (c.m.) energy. \( E_i, \mathbf{p}_B \) is the four momentum of the initial \( e^+e^- \) system and \( \mathbf{p}_B \) is the B momentum, both measured in the laboratory frame. The second variable is \( \Delta E = E_B - \sqrt{s}/2 \), where \( E_B \) is the \( B \) candidate energy in the c.m. frame. The pion mass is assigned to all \( h^\pm \) candidates for the \( \Delta E \) calculation.

The \( B^\pm \to \rho^\pm\pi^0 \) background to \( B^0 \to \pi^0\pi^0 \) is reduced by using only candidates with \( |\Delta E| < 0.2 \) GeV. Remaining \( B^\pm \to \rho^\pm\pi^0 \) background is further suppressed by removing candidates in which the additional \( \pi^\pm \) is identified. The track that gives a \( \pi^+\pi^- \) invariant mass and \( m_{ES} \) of the \( \pi^+\pi^-\pi^0 \) combination most consistent with the \( \rho \) and \( B \) mass is selected. Requirements on the resulting \( \pi^+\pi^- \) invariant mass and on the \( \Delta E \) of the \( \pi^+\pi^-\pi^0 \) combination remove roughly 50% of the remaining \( B^\pm \to \rho^\pm\pi^0 \) background, with 93% efficiency for \( B^0 \to \pi^0\pi^0 \). Only \( (0.40 \pm 0.04)\% \) of \( B^\pm \to \rho^\pm\pi^0 \) decays, and a negligible fraction of nonresonant \( B^\pm \to \pi^+\pi^-\pi^0 \) decays, remain after all cuts. For \( B^\pm \to h^\pm\pi^0 \) the \( B \to \rho\pi \) background is suppressed by selecting candidates with \( -0.11 < \Delta E < 0.15 \) GeV.

The jetlike \( g\overline{q} \) background is suppressed by requiring that the angle \( \theta_\gamma \) between the sphericity [9] axes of the \( B \) candidate and of the remaining tracks and neutral clusters in the event, in the c.m. frame, satisfies \( |\cos\theta_\gamma| < 0.8 \) (0.7) for \( B^\pm \to h^\pm\pi^0 \) \((B^0 \to \pi^0\pi^0)\). Also, we require \( m_{ES} > 5.2 \) GeV/c^2. The number of \( B^\pm \to h^\pm\pi^0 \) and \( B^0 \to \pi^0\pi^0 \) candidates satisfying these requirements and the estimated efficiencies, obtained from simulated data, are shown in Table I. The error in the estimated efficiency is dominated by the 5% systematic uncertainty in the single \( \pi^0 \) reconstruction efficiency.

The number of signal \( B \) candidates is determined in an extended unbinned maximum likelihood fit. The probability \( P_i(\mathbf{x}_j; \mathbf{\alpha}_i) \) for a signal or background hypothesis is the product of probability density functions (PDFs) for the variables \( \mathbf{x}_j \) given the set of parameters \( \mathbf{\alpha}_i \). The likelihood function is given by a product over all \( N \) events and the \( M \) signal and background hypotheses:

\[
\mathcal{L} = \exp \left( -\sum_{i=1}^{M} n_i \prod_{j=1}^{N} \frac{P_i(\mathbf{x}_j; \mathbf{\alpha}_i)}{\sum_{i=1}^{M} P_i(\mathbf{x}_j; \mathbf{\alpha}_i)} \right). \tag{1}
\]
TABLE I. The results for both $B^+ \to h^+ \pi^0$ and $B^0 \to \pi^0 \pi^0$ are summarized. The number of $B$ candidates $N$, total detection efficiencies $\epsilon$, fitted signal yields $N_s$, significances $S$, charge-averaged branching fractions $\mathcal{B}$, asymmetries $\mathcal{A}$, and 90% C.L. asymmetry limits are shown. Errors are statistical and systematic, respectively, with the exception of $\epsilon$ whose error is purely systematic. The upper limit for the $B^0 \to \pi^0 \pi^0$ branching fraction corresponds to the 90% C.L., and the central value is shown in parentheses.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$N$</th>
<th>$\epsilon$ (%)</th>
<th>$N_s$</th>
<th>$S(\sigma)$</th>
<th>$\mathcal{B} \times 10^{-6}$</th>
<th>$\mathcal{A}$</th>
<th>$\mathcal{A}$ (90% C.L.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+ \pi^0$</td>
<td>21752</td>
<td>26.1 $\pm$ 1.7</td>
<td>125$^{+22}_{-21} \pm$ 10</td>
<td>7.7</td>
<td>5.5$^{+1.1}_{-0.9} \pm$ 0.6</td>
<td>$-0.03^{+0.15}_{-0.17}$</td>
<td>$[-0.32, 0.27]$</td>
</tr>
<tr>
<td>$K^+ \pi^0$</td>
<td>280.0 $\pm$ 2.0</td>
<td>239$^{+21}_{-22} \pm$ 6</td>
<td>17.4</td>
<td></td>
<td>12.8$^{+1.2}_{-1.1} \pm$ 1.0</td>
<td>$-0.09 \pm 0.09 \pm 0.01$</td>
<td>$[-0.24, 0.06]$</td>
</tr>
<tr>
<td>$\pi^0 \pi^0$</td>
<td>3020</td>
<td>16.5 $\pm$ 1.7</td>
<td>23$^{+9}_{-8} \pm$ 4.8</td>
<td>2.5</td>
<td>&lt;3.6 /0.06 /0.03</td>
<td></td>
<td></td>
</tr>
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</table>

For $B^+ \to h^+ \pi^0$ the probability coefficients are $N_i = \frac{1}{2} (1 - q_j \mathcal{A}_i) n_i$, where $q_j$ is the charge of the track $h$, and the fit parameters $n_i$ and $\mathcal{A}_i$ are the number of events and asymmetry for the four $\pi^+ \pi^0$ and $K^+ \pi^0$ signal and background components. For $B^0 \to \pi^0 \pi^0$ the coefficients are $N_i = n_i$ where the three $n_i$ are the number of signal candidates, $B^+ \to \rho^+ \pi^0$ background, and $q\bar{q}$ background. Monte Carlo simulations are used to verify that the likelihood fits are unbiased.

The variables $\vec{x}_i$ used for $B^+ \to h^+ \pi^0$ are $m_{ES}; \Delta E$, the Cherenkov angle $\theta_c$ of the $h^+$ track, and a Fisher discriminant $\mathcal{F}$. The Fisher discriminant is given by an optimized linear combination of $\sum_i p_i$ and $\sum_i |p_i | \cos \theta_c|^2$, where $p_i$ is the momentum and $\theta_c$ is the angle with respect to the thrust axis of the $B$ candidate, both in the c.m. frame, for all tracks and neutral clusters not used to reconstruct the $B$ meson.

The PDFs for $m_{ES}; \Delta E$, $\theta_c$, and $\mathcal{F}$ for the background are determined using data, while the PDFs for signal are found from a combination of simulated events and data. The $m_{ES}$ distribution for background is modeled as a threshold function [10], whose shape parameter is a free parameter of the fit. The $\Delta E$ distribution for background is modeled as a quadratic function whose parameters are determined from the $m_{ES}$ sideband in the data. The $m_{ES}$ and $\Delta E$ distributions for the signal are modeled as Gaussian distributions with a low-side power-law tail whose parameters are found with simulated events. The $\Delta E$ resolution is approximately 42 MeV based on simulated events and $B^+ \to D^0 \rho^+ (\rho^+ \to \pi^0 \pi^0)$ events with an energetic $\pi^0$. To allow for EMC energy scale variations, the mean of the $\Delta E$ PDF is a free parameter of the fit. To account for the use of the pion mass hypothesis, the mean of $\Delta E$ is shifted for the $K^+ \pi^0$ PDFs. The $\mathcal{F}$ distribution is modeled as a bifurcated Gaussian and a double Gaussian for signal and background, respectively, whose parameters are determined for the signal from simulation and for the background from $m_{ES}$ sidebands. The difference of the measured and expected values of $\theta_c$ for the pion or kaon hypothesis, divided by the uncertainty on $\theta_c$, is modeled as a double Gaussian function. A control sample of kaon and pion tracks, from the decay $D^{*+} \to D^0 \pi^+$, $D^0 \to K^- \pi^+$, is used to parametrize $\sigma_{\theta_c}$ as a function of the track polar angle.

The variables $\vec{x}_i$ used for $B^0 \to \pi^0 \pi^0$ are $m_{ES}; \Delta E$, and another Fisher discriminant $\mathcal{F}_T$. The $\mathcal{F}_T$ combines $\mathcal{F}$ with information from the $B$ tagging algorithm described in Ref. [3]. The tagging algorithm uniquely classifies events according to their lepton, kaon, and slow pion (from $D^{*+} \to D^0 \pi_{slow}$) content, using all tracks in the event. Nine event classes, in decreasing order of their background rejection, contain the following: a high momentum electron and a kaon, a high momentum muon and a kaon, a high momentum electron, a high momentum muon, a kaon and a slow pion, a well-identified kaon, a slow pion, any kaon, or none of the above. These event classes are assigned an index, which is a new discriminating variable, and is combined with $\mathcal{F}$ into a second Fisher discriminant $\mathcal{F}_T$, optimized using simulated events.

The $m_{ES}$ distribution for $q\bar{q}$ background is parameterized by the same threshold function used in the $B^+ \to h^+ \pi^0$ analysis, where the shape parameter is determined from the data with $| \cos \theta_c | > 0.9$. The $\Delta E$ distribution for the $q\bar{q}$ background is modeled as a quadratic polynomial with parameters found from on-resonance data in the $m_{ES}$ sidebands and off-resonance data. The $m_{ES}$ and $\Delta E$ variables in both $B^0 \to \pi^0 \pi^0$ and $B^0 \to \rho^+ \pi^0$ are correlated, so a two dimensional PDF derived from a smoothed simulated distribution is used. The $\Delta E$ resolution is approximately 80 MeV. The $\mathcal{F}_T$ distribution for $q\bar{q}$, $B^+ \to \rho^+ \pi^0$, and $B^0 \to \pi^0 \pi^0$ is modeled as the sum of three Gaussians. For $q\bar{q}$ the parameters are found using both $m_{ES}$ sideband and off-resonance data. For $B^0 \to \pi^0 \pi^0$ and $B^0 \to \rho^+ \pi^0$ the parameters are found using a sample of fully reconstructed $B^0 \to D(s) n \pi (n = 1, 2, 3)$ events.

The decay $B^0 \to \rho^+ \pi^0$ has not been observed; Ref. [11] sets an upper limit of $\mathcal{B}(B^0 \to \rho^+ \pi^0) < 4.3 \times 10^{-5}$ at 90% C.L. based on a measured central value of $\mathcal{B}(B^0 \to \rho^+ \pi^0) = 2.4 \times 10^{-5}$. Therefore we fix the number of $B^0 \to \rho^+ \pi^0$ events in the fit to $n_{\rho^+\pi^0} = 8.4$, based on this central value, and evaluate the systematic uncertainty of allowing $n_{\rho^+\pi^0}$ to vary from 4.2 to 15 events.

The results of the maximum likelihood fits are summarized in Table I. Distributions of some of the variables used in the fits are shown in Figs. 1 and 2 for $B^+ \to h^+ \pi^0$ and $B^0 \to \pi^0 \pi^0$, respectively. The data shown are for
events that have passed a probability ratio cut optimized to enhance the signal to background fraction. The likelihood function for \( B^0 \rightarrow \pi^0 \pi^0 \) is shown in Fig. 2(d). The statistical errors on the number of events are given by the change in signal yield \( n_i \) that corresponds to an increase in \(-2 \ln L\) of one unit. The systematic uncertainty in the likelihood fit is estimated by varying the PDF parameters by their statistical errors or by comparing the result with an alternate parametrization.

For \( B^z \rightarrow \pi^z \pi^0 \), the dominant systematic uncertainty is due to the \( \mathcal{F} \) PDF for signal (\( \pm 6.2 \) events) and background (\( \pm 7.6 \) events) PDFs, while for \( B^z \rightarrow K^z \pi^0 \) it is due to the \( m_{ES} \) PDF for signal (\( \pm 2.7\) events). Systematic uncertainties on the CP asymmetries are evaluated from PDF parameter variations, which mostly cancel in the asymmetry ratio, and from the upper limit on intrinsic charge bias in the detector (1.0%) [12].

For \( B^0 \rightarrow \pi^0 \pi^0 \), systematic uncertainties from the PDFs are due to the \( \mathcal{F}_T \) PDF for \( q\bar{q} \) background (\( \pm 1.7\) events), the \( m_{ES} \) PDF for \( q\bar{q} \) background (\( \pm 1.7\) events), and the \( \Delta E \) PDF for \( q\bar{q} \) background (\( \pm 0.2\) events). Additional systematic uncertainties for \( B^0 \rightarrow \pi^0 \pi^0 \) arise from uncertainty in the EMC energy scale (\( \pm 0.1\) events), the \( B^z \rightarrow \rho^z \pi^0 \) rejection cut (\( \pm 1.3 \) events), and uncertainty in the assumed \( B^z \rightarrow \rho^z \pi^0 \) branching fraction (\( \pm 1.0\) events). The significance of the event yield, also listed in Table I, is evaluated from the square root of the change in \(-2 \ln L\) with the signal yield fixed to zero. The upper limit for \( B^0 \rightarrow \pi^0 \pi^0 \) is evaluated by finding \( n_{\pi^0 \pi^0} \) where \( \int_0^{n_{\pi^0 \pi^0}} \mathcal{L}(n)dn/\int_0^{\infty} \mathcal{L}(n)dn = 0.9 \). For both significance and upper limits, systematic uncertainties are included with a worst case assumption for efficiencies and PDF variations.

We observe \( B(B^z \rightarrow \pi^z \pi^0) = (5.5^{+1.0}_{-1.0} \pm 0.6) \times 10^{-6} \), with a statistical significance of 7.7\( \sigma \) from zero. This result is consistent with several prior measurements reporting evidence for this decay [13–15]. We measure \( B(B^z \rightarrow K^z \pi^0) = (12.8^{+1.2}_{-1.1} \pm 1.0) \times 10^{-6} \). The charge asymmetries are \( \mathcal{A}_{\pi^z \pi^0} = -0.03^{+0.18}_{-0.17} \pm 0.02 \) and \( \mathcal{A}_{K^z \pi^0} = -0.09 \pm 0.09 \pm 0.01 \); no evidence of direct CP violation is observed. Our limit \( B(B^z \rightarrow \rho^z \pi^0) < 3.6 \times 10^{-6} \) improves upon prior results [14,16]. Removing correlated systematic uncertainties from the luminosity and \( \pi^0 \) efficiency, we bound the ratio...
$\mathcal{B}(B^0 \to \pi^0 \pi^0)/\mathcal{B}(B^+ \to \pi^+ \pi^0) < 0.61$ at a 90% confidence level. Assuming isospin relations for $B \to \pi \pi$ [5], this corresponds to an upper limit of $|\alpha_{\text{eff}} - \alpha| < 51^\circ$.

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§ Deceased.

[10] The threshold function used for the $m_{\text{ES}}$ PDF is $(m_{\text{ES}}/m_0)\sqrt{1-(m_{\text{ES}}/m_0)^2}\exp[-\xi(1-(m_{\text{ES}}/m_0)^2)]$, where $m_0$ is the $m_{\text{ES}}$ end point, and $\xi$ the shape parameter. See ARGUS Collaboration, H. Albrecht et al., Z. Phys. C 48, 543 (1990).