Observation of the Decay $B^\pm \rightarrow \pi^\pm \pi^0$, Study of $B^\pm \rightarrow K^\pm \pi^0$, and Search for $B^0 \rightarrow \pi^0 \pi^0$

of final states plays an important role in the understanding of CP violation in the B system. In the standard model, CP violation arises from a single complex phase in the Cabibbo-Kobayashi-Maskawa quark-mixing matrix $V_{ij}$ [1]. Measurements of the time-dependent CP-violating asymmetry in the $B^0 \rightarrow \pi^+ \pi^-$ decay mode by the BABAR and Belle collaborations [2] provide information...
CsI (Tl) crystals. Tracks are identified as pions or kaons.

1.5 T superconducting solenoidal magnet. Photon (neutral tracker and a 40-layer drift chamber (DCH) inside a 

measured with a 5-layer double-sided silicon vertex 
detail in Ref. [8]. Charged particle (track) momenta are 
asymmetric-energy beams at PEP-II and is described in 

BABAR collected with the 

fiducial volume, originate from the interaction point, 

 backgrounds from false 

candidates, the angle 

candidates, the angle 

or 

events where an 

each quark randomly combine to mimic a B decay. 

Both backgrounds are separated from the signal using 

the kinematic constraints of B mesons produced at the 

Y(4S). The first kinematic variable is the beam-energy 

substituted mass 

where 

is the total center-of-mass (c.m.) energy. 

is the four momentum of the initial 

system and 

is the B momentum, both measured in the laboratory frame. The second variable is 

where 

is the B candidate energy in the c.m. frame. The pion mass is assigned to all 

candidates for the 

calculation.

The 

background to 

is reduced by using only candidates with 

GeV. Remaining 

background is further suppressed by removing candidates in which the additional 

is identified. The track that gives a 

invariant mass and 

combination most consistent with the 

and 

mass is selected. 

Requirements on the resulting 

invariant mass and on the 

combination remove roughly 

of the remaining 

background, with 

efficiency for 

. Only 

of 

decays, and a negligible fraction of non-resonant 

decays, remain after all cuts. For 

the 

background is suppressed by selecting candidates with 

GeV.

The jetlike 

background is suppressed by requiring that the angle 

between the sphericity 

axes of the 

candidate and of the remaining tracks and neutral clusters in the event, in the c.m. frame, satisfies 

in Table I. The error in the estimated efficiency is dominated by the 5% systematic uncertainty in the single 

reconstruction efficiency.

The number of signal 

candidates is determined in an extended unbinned maximum likelihood fit. The probability 

for a signal or background hypothesis is the product of probability density functions (PDFs) for the variables 

given the set of parameters 

. The likelihood function is given by a product over all 

events and the 

signal and background hypotheses:

\[
L = \exp \left( - \sum_{i=1}^{M} n_i \right) \prod_{j=1}^{N} \left( \sum_{i=1}^{M} N_i P_j (\tilde{x}_j; \tilde{\alpha}_i) \right). \tag{1}
\]
TABLE I. The results for both $B^\pm \to h^\pm \pi^0$ and $B^0 \to \pi^0 \pi^0$ are summarized. The number of $B$ candidates $N$, total detection efficiencies $\epsilon$, fitted signal yields $N_s$, significances $S$, charge-averaged branching fractions $\mathcal{B}$, asymmetries $\mathcal{A}$, and 90% C.L. asymmetry limits are shown. Errors are statistical and systematic, respectively, with the exception of $\epsilon$ whose error is purely systematic. The upper limit for the $B^0 \to \pi^0 \pi^0$ branching fraction corresponds to the 90% C.L., and the central value is shown in parentheses.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$N$</th>
<th>$\epsilon$ (%)</th>
<th>$N_s$</th>
<th>$S(\sigma)$</th>
<th>$\mathcal{B}$ (10$^{-6}$)</th>
<th>$\mathcal{A}$</th>
<th>$\mathcal{A}$ (90% C.L.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+ \pi^0$</td>
<td>21752</td>
<td>26.1 $\pm$ 1.7</td>
<td>125$^{+21}_{-21}$</td>
<td>10</td>
<td>7.7</td>
<td>$5.5^{+1.0}_{-1.0} \pm 0.6$</td>
<td>$0.03^{+0.15}_{-0.17} \pm 0.02$</td>
</tr>
<tr>
<td>$K^\pm \pi^0$</td>
<td>28.0 $\pm$ 2.0</td>
<td>239$^{+21}_{-21}$</td>
<td>6</td>
<td>17.4</td>
<td>$12.8^{+1.2}_{-1.1} \pm 1.0$</td>
<td>$0.09 \pm 0.09 \pm 0.01$</td>
<td>$[-0.24, 0.06]$</td>
</tr>
<tr>
<td>$\pi^0 \pi^0$</td>
<td>3020</td>
<td>16.5 $\pm$ 1.7</td>
<td>23$^{+10}_{-10}$</td>
<td>8</td>
<td>2.5</td>
<td>$&lt;3.6 (1.6 -0.6 -0.3)$</td>
<td></td>
</tr>
</tbody>
</table>

For $B^\pm \to h^\pm \pi^0$ the probability coefficients are $N_i = \frac{1}{2}(1 - q_j \mathcal{A}_j)n_i$, where $q_j$ is the charge of the track $h$, and the fit parameters $n_i$ and $\mathcal{A}_j$ are the number of events and asymmetry for the four $\pi^+ \pi^0$ and $K^+ \pi^0$ signal and background components. For $B^0 \to \pi^0 \pi^0$ the coefficients are $N_i = n_i$ where the three $n_i$ are the number of signal candidates, $B^\pm \to \rho^0 \pi^0$ background, and $q\bar{q}$ background. Monte Carlo simulations are used to verify that the likelihood fits are unbiased.

The variables $\tilde{x}_j$ used for $B^\pm \to h^\pm \pi^0$ are $m_{ES}$, $\Delta E$, the Cherenkov angle $\theta_c$ of the $h^\pm$ track, and a Fisher discriminant $\mathcal{F}$. The Fisher discriminant is given by an optimized linear combination of $\sum_i p_i$ and $\sum_i p_i |\cos \theta_i|^2$, where $p_i$ is the momentum and $\theta_i$ is the angle with respect to the thrust axis of the $B$ candidate, both in the c.m. frame, for all tracks and neutral clusters not used to reconstruct the $B$ meson.

The PDFs for $m_{ES}$, $\Delta E$, $\theta_c$, and $\mathcal{F}$ for the background are determined using data, while the PDFs for signal are found from a combination of simulated events and data. The $m_{ES}$ distribution for background is modeled as a threshold function [10], whose shape parameter is a free parameter of the fit. The $\Delta E$ distribution for background is modeled as a quadratic function whose parameters are determined from the $m_{ES}$ sideband in the data. The $m_{ES}$ and $\Delta E$ distributions for the signal are modeled as Gaussian distributions with a low-side power-law tail whose parameters are found with simulated events. The $\Delta E$ resolution is approximately 42 MeV based on simulated events and $B^\pm \to D^0 \rho^\pm (\rho^- \to \pi^- \pi^0)$ events with an energetic $\pi^0$. To allow for EMC energy scale variations, the mean of the $\Delta E$ PDF is a free parameter of the fit. To account for the use of the pion mass hypothesis, the mean of $\Delta E$ is shifted for the $K^\pm \pi^0$ PDFs. The $\mathcal{F}$ distribution is modeled as a bifurcated Gaussian and a double Gaussian for signal and background, respectively, whose parameters are determined from the signal from simulation and for the background from $m_{ES}$ sidebands. The difference of the measured and expected values of $\theta_c$ for the pion or kaon hypothesis, divided by the uncertainty on $\theta_c$, is modeled as a double Gaussian function. A control sample of kaon and pion tracks, from the decay $D^{*+} \to D^0 \pi^+$, $D^0 \to K^- \pi^+$, is used to parametrize $\sigma_{\theta_c}$ as a function of the track polar angle.

The variables $\tilde{x}_j$ used for $B^0 \to \pi^0 \pi^0$ are $m_{ES}$, $\Delta E$, and another Fisher discriminant $\mathcal{F}_T$. The $\mathcal{F}_T$ combines $\mathcal{F}$ with information from the $B$ tagging algorithm described in Ref. [3]. The tagging algorithm uniquely classifies events according to their lepton, kaon, and slow pion (from $D^{*+} \to D^0 \pi^0_{slow}$) content, using all tracks in the event. Nine event classes, in decreasing order of their significance, on the background rejection, contain the following: a high momentum electron and a kaon, a high momentum muon and a kaon, a high momentum electron, a high momentum muon, a kaon and a slow pion, a well-identified kaon, a slow pion, any kaon, or none of the above. These event classes are assigned an index, which is a new discriminating variable, and is combined with $\mathcal{F}$ into a second Fisher discriminant $\mathcal{F}_T$, optimized using simulated events.

The $m_{ES}$ distribution for $q\bar{q}$ background is parameterized by the same threshold function used in the $B^\pm \to h^\pm \pi^0$ analysis, where the shape parameter is determined from the data with $|\cos \theta_{\tilde{q}}| > 0.9$. The $\Delta E$ distribution for the $q\bar{q}$ background is modeled as a quadratic polynomial with parameters found from on-resonance data in the $m_{ES}$ sidebands and off-resonance data. The $m_{ES}$ and $\Delta E$ variables in both $B^0 \to \pi^+ \pi^0$ and $B^0 \to \rho^+ \pi^0$ are correlated, so a two dimensional PDF derived from a smoothed simulated distribution is used. The $\Delta E$ resolution is approximately 80 MeV. The $\mathcal{F}_T$ distribution for $q\bar{q}$, $B^\pm \to \rho^\pm \pi^0$, and $B^0 \to \pi^0 \pi^0$ is modeled as the sum of three Gaussians. For $q\bar{q}$ the parameters are found using both $m_{ES}$ sideband and off-resonance data. For $B^0 \to \pi^0 \pi^0$ and $B^0 \to \rho^+ \pi^0$ the parameters are found using a sample of fully reconstructed $B^0 \to D^{(*)} n \pi$ ($n = 1, 2, 3$) events.

The decay $B^0 \to \rho^+ \pi^0$ has not been observed; Ref. [11] sets an upper limit of $|B(B^0 \to \rho^+ \pi^0)| < 4.3 \times 10^{-5}$ at 90% C.L. based on a measured central value of $B(B^0 \to \rho^+ \pi^0) = 2.4 \times 10^{-5}$. Therefore we fix the number of $B^0 \to \rho^+ \pi^0$ events in the fit to $n_{\rho \pi^0} = 8.4$, based on this central value, and evaluate the systematic uncertainty of allowing $n_{\rho \pi^0}$ to vary from 4.2 to 15 events.

The results of the maximum likelihood fits are summarized in Table I. Distributions of some of the variables used in the fits are shown in Figs. 1 and 2 for $B^\pm \to h^\pm \pi^0$ and $B^0 \to \pi^0 \pi^0$, respectively. The data shown are for
events that have passed a probability ratio cut optimized to enhance the signal to background fraction. The likelihood function for $B^0 \to \pi^0 \pi^0$ is shown in Fig. 2(d). The statistical errors on the number of events are given by the change in signal yield $n_i$ that corresponds to an increase in $-2 \ln L$ of one unit. The dominant systematic uncertainty in the likelihood fit is estimated by varying the PDF parameters by their statistical errors or by comparing the result with an alternate parametrization.

For $B^\pm \to \pi^\pm \pi^0$, the dominant systematic uncertainty is due to the $\mathcal{F}$ PDF for signal ($\pm 6.2$ events) and background ($\pm 7.6$ events) PDFs, while for $B^\pm \to K^\pm \pi^0$ it is due to the $m_{ES}$ PDF for signal ($\pm 0.8$ events). Systematic uncertainties on the CP asymmetries are evaluated from PDF parameter variations, which mostly cancel in the asymmetry ratio, and from the upper limit on intrinsic charge bias in the detector (1.0%) [12].

For $B^0 \to \pi^0 \pi^0$, systematic uncertainties from the PDFs are due to the $\mathcal{F}_T$ PDF for $q\bar{q}$ background ($\pm 1.7$ events), the $m_{ES}$ PDF for $q\bar{q}$ background ($\pm 1.0$ events), and the $\Delta E$ PDF for $q\bar{q}$ background ($\pm 0.2$ events). Additional systematic uncertainties for $B^0 \to \pi^0 \pi^0$ arise from uncertainty in the EMC energy scale ($\pm 0.5$ events), the $B^\pm \to \rho^\pm \pi^0$ rejection cut ($\pm 1.3$ events), and uncertainty in the assumed $\mathcal{B}^\pm \to \rho^\pm \pi^0$ branching fraction ($\pm 1.0$ events). The significance of the event yield, also listed in Table I, is evaluated from the maximum likelihood fit with the signal yield fixed to zero. The upper limit for $B^0 \to \pi^0 \pi^0$ is evaluated by finding $n_{\pi^0 \pi^0}$ where $\int_0^{n_{\pi^0 \pi^0}} L(n) dn / \int_0^\infty L(n) dn = 0.9$. For both signal and upper limits, systematic uncertainties are included with a worst case assumption for efficiencies and PDF variations.

We observe $\mathcal{B}(B^\pm \to \pi^\pm \pi^0) = (5.5^{+1.0}_{-1.0} \pm 0.6) \times 10^{-6}$, with a statistical significance of $7.7 \sigma$ from zero. This result is consistent with several prior measurements reporting evidence for this decay [13–15]. We measure $\mathcal{B}(B^\pm \to K^\pm \pi^0) = (12.8^{+1.2}_{-1.1} \pm 1.0) \times 10^{-6}$. The charge asymmetries are $\mathcal{A}_{\pi^+ \pi^-} = -0.03^{+0.18}_{-0.07} \pm 0.02$ and $\mathcal{A}_{K^+ \pi^-} = -0.09 \pm 0.09 \pm 0.01$, and no evidence of direct CP violation is observed. Our limit $\mathcal{B}(B^+ \to \rho^+ \pi^-) < 3.6 \times 10^{-6}$ improves upon prior results [14,16].
\[ \mathcal{B}(B^0 \to \pi^0 \pi^0)/\mathcal{B}(B^\pm \to \pi^\pm \pi^0) < 0.61 \text{ at a 90\% confidence level. Assuming isospin relations for } B \to \pi \pi [5], \]

this corresponds to an upper limit of \( |\alpha_{eff} - \alpha| < 51^\circ \).

We are grateful for the excellent luminosity and machine conditions provided by our PEP-II colleagues, and for the substantial dedicated effort from the computing organizations that support BABAR. The collaborating institutions wish to thank SLAC for its support and kind hospitality. This work is supported by DOE and NSF (U.S.A.), NSERC (Canada), IHEP (China), CEA and CNRS-IN2P3 (France), BMBF and DFG (Germany), INFN (Italy), FOM (The Netherlands), NFR (Norway), MIST (Russia), and PPARC (United Kingdom). Individuals have received support from the A. P. Sloan Foundation, the Research Corporation, and the Alexander von Humboldt Foundation.

*Also with Università di Perugia, Perugia, Italy.
† Also with Università della Basilicata, Potenza, Italy.
‡ Also with IFIC, Instituto de Física Corpuscular, CSIC-Universidad de Valencia, Valencia, Spain.
§ Deceased.

[10] The threshold function used for the \( m_{ES} \) PDF is \( (m_{ES}/m_0)\sqrt{1 - (m_{ES}/m_0)^2} \exp[-\xi(1 - (m_{ES}/m_0)^2)] \), where \( m_0 \) is the \( m_{ES} \) end point, and \( \xi \) the shape parameter. See ARGUS Collaboration, H. Albrecht et al., Z. Phys. C 48, 543 (1990).