$J/\psi$ production via initial state radiation in $e^+e^-\rightarrow \mu^+\mu^-\gamma$ at an $e^+e^-$ center-of-mass energy near 10.6 GeV
The possibility of using $e^+e^-$ annihilation with initial state radiation (ISR), $e^+e^-\rightarrow\text{hadrons}+\gamma$, to measure the $e^+e^-$ cross sections into hadrons over a wide range of center-of-mass (c.m.) energies in a single experiment has been discussed in the literature [1]. In this paper, we have implemented this idea by studying the process $e^+e^-\rightarrow\mu^+\mu^-\gamma$ for $\mu^+\mu^-$ masses in the range from 2.8 to 3.4 GeV/$c^2$. We measure the cross section $\sigma(e^+e^-\rightarrow\mu^+\mu^-\gamma)$ and derive the product of electronic and total widths: $\Gamma(J/\psi\rightarrow e^+e^-) = 5.61 \pm 0.20$ keV and $\Gamma = 94.7 \pm 4.4$ keV.

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width times the branching fraction $\Gamma(J/\psi \rightarrow e^+e^-)B(J/\psi \rightarrow \mu^+\mu^-)$. Using the world averages for $B(J/\psi \rightarrow \mu^+\mu^-)$ and $B(J/\psi \rightarrow e^+e^-)$ [2], we then derive the electronic and total widths of the $J/\psi$ meson. The data used in this analysis were collected with the BABAR detector [3] at the SLAC PEP-II asymmetric $e^+e^-$ storage ring [4].

The Born cross section for the process $e^+e^- \rightarrow \mu^+\mu^-\gamma$ in the $J/\psi$ mass region has contributions from three Feynman diagrams, as illustrated in Fig. 1. The first and second of these diagrams describe the pure QED processes corresponding to initial state radiation and final state radiation (FSR), respectively, while the third diagram corresponds to the $J/\psi$ production. The function $W(s,x)$ is the Born cross section for $J/\psi$ production and QED amplitudes after convolution with the detector resolution function.

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The Born cross section for $J/\psi$ production [Fig. 1c] is given by

$$\frac{d\sigma^{\text{Born}}_{J/\psi}(s,x)}{dx} = W(s,x)\sigma_0(s(1-x)),$$

where $\sqrt{s}$ is the $e^+e^-$ invariant mass, $x=2E_\gamma/\sqrt{s}$, $E_\gamma$ is the photon energy in the c.m., and $\sigma_0$ is the Born cross section for $e^+e^- \rightarrow J/\psi \rightarrow \mu^+\mu^-$. The function

$$W(s,x) = \frac{2\alpha}{\pi x} \left(2 \ln \frac{\sqrt{s}}{m_e} - 1 \right) \left(1 - x + \frac{x^2}{2}\right)$$

describes the probability of ISR photon emission. Here $\alpha$ is the fine structure constant and $m_e$ is the electron mass. ISR photons are emitted predominantly at small angles relative to the electron direction. About 10% of the photons have c.m. polar angles in the range $30^\circ < \theta < 150^\circ$ and can be detected in BABAR.

As a first approximation, the Born cross section for $e^+e^- \rightarrow J/\psi \rightarrow \mu^+\mu^-$ is given by the Breit-Wigner formula

$$\sigma_{J/\psi}(s) = \frac{12\pi\Gamma_{ee}B_{\mu\mu}}{m^2} \frac{m^2\Gamma^2}{(s-m^2)^2+m^2\Gamma^2},$$

where $m$ and $\Gamma$ are the $J/\psi$ mass and total width, respectively, $\Gamma_{ee}$ and $B_{\mu\mu}$ are the branching fractions $B(J/\psi \rightarrow e^+e^-)$ and $B(J/\psi \rightarrow \mu^+\mu^-)$. For a narrow resonance, such as the $J/\psi$, we can replace the Breit-Wigner with a $\delta$ function $\pi m\Gamma \delta(s-m^2)$ and integrate over photon energy to find

$$\sigma_{J/\psi}^{\text{Born}}(s) = \frac{12\pi^2\Gamma_{ee}B_{\mu\mu}m}{m\cdot s} W(s,x_0), \quad x_0 = 1 - \frac{m^2}{s},$$

where $\Gamma_{ee} = \Gamma \cdot B_{ee}$. These formulas do not account for interference between the $e^+e^- \rightarrow J/\psi \rightarrow \mu^+\mu^-$ and the nonresonant (QED) $e^+e^- \rightarrow \mu^+\mu^-$ amplitudes.

The cross section for $e^+e^- \rightarrow \mu^+\mu^-$ including QED and resonant $J/\psi$ production amplitudes and their interference can be written as [5]

$$\sigma(s) = \frac{4\pi\alpha^2}{3s} \left[1 - Q \frac{m^2 \Gamma^2}{m^2 - s - im\Gamma}\right]^2,$$

where $Q = 3 \sqrt{B_{ee}B_{\mu\mu}/\alpha}$. The interference term changes sign at the $J/\psi$ mass. Therefore, it does not change the integrated cross section of Eq. (4) significantly, but does change the shape of the mass distribution. Because the $J/\psi$ cross section is so much greater than the QED cross section at resonance ($Q^2 \approx 600$), the power-law behavior of the Breit-Wigner tails produces observable interference even 1000 widths from resonance. The expected di-muon mass spectrum, convolved with the detector resolution, is shown in Fig. 2. The interference is clearly seen, despite the experimental resolution.
14.5 MeV/c², being more than 100 times the J/ψ natural linewidth. The maximum relative difference between the spectra calculated with and without interference is about 7%. The interference also leads to a 1.3 MeV/c² shift between the maximum of the resonance peak and the actual J/ψ mass.

The shape of the expected mass spectrum is very sensitive to the tails of the Breit-Wigner approximation used in Eqs. (3) and (5), where its validity far from resonance is questionable. To estimate the sensitivity of our analysis to the details of the shape assumptions, we will take the full difference between fits that do, and do not, use interference as a measure of the systematic uncertainty.

The width of the J/ψ has been measured directly in p ¯p annihilation with the result 99±12±6 keV [6]. In e⁺e⁻ annihilation, measuring the area under the resonance curve for e⁺e⁻→J/ψ→μ⁺μ⁻ gives the product λψBμμ as seen in Eq. (4). Combining this with the leptonic branching ratio yields the total width. The BES Collaboration made a comprehensive collection of measurements at the J/ψ from which they determined Γ=84.4±8.9 keV [7]. This superceded results obtained from original measurements made of the area under the excitation curve in 1975. More recently, the BES Collaboration has measured the leptonic branching ratio with a 1.5% uncertainty using J/ψ’s from the decay ψ(2S)→J/ψγππ [8]. It is this result that we combine with our measurement of ΓψBμμ to obtain the highest precision result to date for the total width of the J/ψ.

Charged particle tracking for the BABAR detector is provided by a five-layer silicon vertex tracker (SVT) and a 40-layer drift chamber (DCH), operating in a 1.5 T axial magnetic field. The transverse momentum resolution is 0.47% at 1 GeV/c. Energies of photons and electrons are measured by a CsI(Tl) electromagnetic calorimeter (EMC) with resolution of 3% at 1 GeV. Charged particle identification is provided by ionization measurements in the SVT and DCH, and by an internally reflecting ring-imaging Cherenkov detector (DIRC). Muons are identified in the solenoid’s instrumented flux return (IFR), which consists of iron plates interleaved with resistive plate chambers. The data sample used for this analysis corresponds to an integrated luminosity of 88.4 fb⁻¹ recorded in the vicinity of the Y(4S) resonance.

The initial selection of μ⁺μ⁻γ candidates requires that all particles are detected inside a fiducial volume and that the event kinematics are consistent with the hypothesis e⁺e⁻→μ⁺μ⁻γ. Photons must have polar angles in the range 0.35<θ<2.4 radians and must have a c.m. energy above 3 GeV. Muon candidates must have polar angles in the range 0.35<θ<2.4 radians and transverse momenta above 0.1 GeV/c, and must originate from the interaction point. Energy and momentum balance is provided by the conditions |E_{total}−E_{beams}|<1.5 GeV and ΔΨ<0.07. Here E_{total} is the summed energy of the muon candidates and the photon, ΔΨ is the angle between the photon and the direction of the di-muon missing momentum, p_{miss}=p_μ⁺+p_μ⁻−p_μ⁺−p_μ⁻. We reduce backgrounds using a one-constraint fit to the hypothesis that the recoil mass against the di-muon is zero. Requiring χ²<20 rejects 90% of the multihadron ISR contamination (general e⁺e⁻→qqγγ reactions) and about 10% of signal events.

The large background from e⁺e⁻→e⁺e⁻→e⁺e⁻→J/ψγ is suppressed by requiring that the charged track momenta be greater than 0.5 GeV/c and that the corresponding energies detected in the calorimeter be small: E_{EMCM}<0.4 GeV for each track. The average energy deposition of muons in the calorimeter is about 0.2 GeV, while electrons typically deposit more than 90% of their energy in the calorimeter. Additional suppression of this background is achieved by requiring large angular separation (in the c.m.) between the charged tracks and the photon (cos⁡θ<0.5). This also reduces the level of FSR (μ⁺→e⁺γ) events in the final sample by a factor of two. The invariant mass distribution of approximately 70000 di-muon pairs in our final sample is shown in Fig. 3. About 7800 events are in the J/ψ peak.

The remaining electron contamination in our final sample is estimated using a subsample of events enriched with electrons. To select this subsample we require that neither muon be identified in the IFR. Muon identification requires that a track penetrate at least 2 nuclear interaction lengths (λ) of IFR material, and that the difference between the measured and expected muon ranges be less than 2λ. This algorithm is 90% efficient for true muons and misidentifies about 10% of real electrons as muons. We then require that the DCH based dE/dx measurements for the two tracks be consistent with the di-electron hypothesis and inconsistent with the di-muon hypothesis. This eliminates 95% of di-muons and retains 85% of di-electrons. From the number of electron events in the selected subsample we estimate the fraction of electron events in the full sample to be (0.1±0.1)%.

ISR events with hadronic final states are another source of background, both on resonance and off. For e⁺e⁻→J/ψγ→π⁺π⁻γ and e⁺e⁻→J/ψγ→K⁺K⁻γ, the cross sections are proportional to the ratios of branching fractions B(J/ψ→π⁺π⁻)/B(J/ψ→μ⁺μ⁻)≈2.5×10⁻³ and B(J/ψ→K⁺K⁻)/B(J/ψ→μ⁺μ⁻)≈4×10⁻⁴. To first approximation, the off-resonance ratios, σ(e⁺e⁻→π⁺π⁻)/σ(e⁺e⁻→μ⁺μ⁻) and σ(e⁺e⁻→K⁺K⁻)/σ(e⁺e⁻→μ⁺μ⁻) are similar to those on resonance. As off-resonance production proceeds via virtual photon intermediate states while on-resonance production proceeds via both virtual photon intermediate states and hadronic intermediate states [9], the on-

![FIG. 3. The mass spectrum for observed events. The curve is the result of the fit described in the text.](image-url)
resonance ratios are overestimates of the off-resonance ratios. Thus, we consider the on-resonance background rate as an upper limit for both.

The suppression of kaon and pion reactions was studied using samples of $e^+e^- \rightarrow \omega \gamma \rightarrow 3 \pi \gamma$ and $e^+e^- \rightarrow \phi \gamma \rightarrow K^+K^- \gamma$ events. About two thirds of these events are rejected by the calorimeter energy deposition requirement. Under the di-muon hypothesis, the peak of the $J/\psi \rightarrow K^+K^-$ distribution transforms into a broad distribution with $M_{\mu\mu} < 2.95 \text{ GeV}/c^2$. The only background peaking under the $J/\psi \rightarrow \mu^+\mu^-$ signal is that due to $J/\psi \rightarrow \pi^+\pi^-\gamma$; its contribution is estimated to be $0.09 \pm 0.03\%$. The only other decay into two charged hadrons, $J/\psi \rightarrow p\bar{p}$, produces events with $M_{\mu\mu} < 2.4 \text{ GeV}/c^2$, and thus contributes no background in the di-muon mass range studied here. The total nonresonant background from $e^+e^- \rightarrow e^+e^- \gamma$, $\pi^+\pi^-\gamma$, $K^+K^-\gamma$ processes is estimated to be $0.3 \pm 0.2\%$.

The background from ISR production of higher multiplicity multihadron events is estimated from Monte Carlo simulation. We estimate the background from multihadron $J/\psi$ decays to be less than 0.05% using simulated $e^+e^- \rightarrow J/\psi \gamma$, $J/\psi \rightarrow 3 \pi$ events and $J/\psi$ charged particle multiplicity data [8]. We also note that such events populate the mass region below 3 GeV/c^2 when misidentified as signal events. We use the JETSET [10] event generator to simulate the hadronic part of the $e^+e^- \rightarrow q\bar{q}\gamma$, $q=u,d,s$ cross section. We find the background due to such events to be less than 0.3%. As these background rates are not the dominant sources of systematic uncertainty in our final results, we have not tried to determine them with greater precision.

We use a binned maximum likelihood fit to describe the mass spectrum of Fig. 3. The mass range used, 2.8–3.4 GeV/c^2, is divided into 150 bins of width 4 MeV/c^2. The probability density function (PDF) for the $J/\psi$ signal is modeled as the convolution of a $J/\psi$ Breit-Wigner line shape and the resolution function shown in Fig. 4. This is derived from detector simulation in conjunction with an $e^+e^- \rightarrow \mu^+\mu^-\gamma$ event generator based on the differential cross sections of Ref. [11]. Soft photon radiation is generated with the use of the structure function method of Ref. [12] and the PHOTOS package [13] for electron and muon bremsstrahlung, respectively. Muon bremsstrahlung leads to the low mass tail observed in the spectrum of Fig. 4. To account for possible resolution differences between simulation and data, the resolution function shown in Fig. 4 is convolved with an additional Gaussian smearing function of width $\sigma_G$. Both $\sigma_G$ and the observed $J/\psi$ peak position, $M_{J/\psi}$, are parameters in our fit. A Monte Carlo calculation shows that the shape of the nonresonant cross section can be described well by a linear function. To account for possible deviations from this hypothesis (e.g., due to detector response) a second-order polynomial is used to fit the experimental spectrum. The full PDF is written as

$$f(m_i) = \frac{N_0}{C(m_i)} \left[ R \cdot H(m_i; M_{J/\psi}, \sigma_G) + 1 + a(m_i - M_{J/\psi}) + b(m_i - M_{J/\psi})^2 \right].$$

where $m_i$ is the central value of the $i$th bin of the data histogram, $N_0$ is the number of events, $h$ is the level of the nonresonant mass distribution at $m = M_{J/\psi}$, $\Delta m = 4 \text{ MeV}/c^2$ is the bin width, $H$ is the PDF for the $J/\psi$ signal with detector resolution, and $a$ and $b$ are the background polynomial coefficients. To account for the interference between resonant and nonresonant amplitudes described in Eq. (5), the PDF is divided by the correction function $C(m_i)$, which is the ratio of the di-muon mass spectra calculated with and without interference as shown in Fig. 2. Because the shape of this function depends on the $J/\psi$ parameters (mass, full width), an iterative procedure is used to calculate it. The ratio

$$R = \frac{N_{J/\psi}}{dN/dm} \cdot \frac{\Delta m}{\sigma_{J/\psi}}$$

is the main fit parameter. Here $N_{J/\psi}$ is the number of observed $J/\psi$ decays. After substituting cross sections for numbers of events, this ratio can be rewritten

$$R = \frac{\frac{d\sigma_{\text{Born}}}{d\sigma_{\text{ISR}}}}{\frac{d\sigma_{\text{Born}}}{d\sigma_{\text{ISR}}}} \frac{1}{K} = \frac{d\sigma_{\text{Total}}/dm}{d\sigma_{\text{ISR}}/dm},$$

Detector acceptances and radiative corrections to the initial particles are the same for nonresonant and $J/\psi$ contributions to ISR production of $\mu^+\mu^-\gamma$ and cancel in the ratio. The total nonresonant cross section includes FSR contributions, which we parametrize in terms of $K$, the ratio of the visible nonresonant and ISR-only (FSR switched off) cross sections. Using simulated events, we determine $K = 1.11 \pm 0.01$ (statistical error only) for our selection criteria.

FIG. 4. The distribution of the reconstructed mass minus the generated mass in Monte Carlo events.
The result of the fit is shown in Fig. 3. We find $R = 18.94 \pm 0.44$ with $\chi^2$ per degree of freedom $\chi^2/\nu = 122/144$. This fitted value of $R$ must be multiplied by 1.002 to correct for nonresonant and resonant contributions from $e^+ e^- \rightarrow e^+ e^- \gamma$, $\pi^+ \pi^- \gamma$, $K^+ K^- \gamma$.

The nonresonant cross section extracted from this measurement is close to the value expected from simulation. Their ratio is $0.968 \pm 0.016$. The quoted uncertainty includes a 0.4% statistical error, a 1% statistical error from simulation, and a 1.2% uncertainty in luminosity. We have not studied the systematic uncertainties on the efficiency for the nonresonant process in detail, as most of these cancel in $R$. We estimate the uncertainty due to data–Monte Carlo differences by studying the stability of $R \cdot K$ for different selection criteria. We vary the photon and muon angular selection criteria and the muon momentum requirement over a wide range of values. While the value of $R$ varies from 1.08 to 1.19, the maximum deviation from our reference mean value $R \cdot K = 21.03$ is only 1.3%. Although this variation might be a statistical fluctuation (at least in part), we treat it as a systematic uncertainty associated with the value of $K$.

As described earlier, we use Monte Carlo simulations of specific ISR and other processes to estimate the level of nonresonant background to be less than 0.4%. We also use the data themselves to estimate this quantity. We compare the fit results for data selected with the standard selection criteria and for data selected with additional muon identification for one of the charged particles. This reduces pion (kaon) contamination by a factor of 9 (3). From the difference in $R \cdot K$, we estimate that the level of nonresonant background does not exceed 0.5%.

The fit results do depend significantly on the model assumed for the $J/\psi$ line shape. The shape of the signal distribution varies with the selection of the maximum allowed value of the $\chi^2$ from the one-constraint fit. Requiring lower values tends to reject events with extra photons, thus reducing the fraction of events in the low mass tail of the $J/\psi$ peak. The fraction of $J/\psi$ events with mass less than $(M_{J/\psi} - 0.1)$ GeV/c$^2$ changes from 2.4% for no $\chi^2$ cut, to 0.4% for $\chi^2 < 5$. Refitting data with different requirements on the value of $\chi^2$ does not change the result for $R \cdot K$ significantly.

The maximum deviation of $R \cdot K$ from our reference mean value, 1.4%, is taken as a systematic uncertainty.

We also consider an additional contribution to the line-shape uncertainty by refitting the data with a model that does not include interference between the nonresonant and $J/\psi$ production amplitudes. The quality of this fit is good: $\chi^2/\nu = 138/144$. As the data do not distinguish between the two models statistically, we take the difference in $R$, 0.3%, as the corresponding systematic uncertainty. The total systematic error for $K \cdot R$ is 2.2%, compared to the statistical error of 2.3%.

The Born cross section for the process $e^+ e^- \rightarrow J/\psi \gamma \rightarrow \mu^+ \mu^- \gamma$ can be evaluated from Eq. (8). The nonresonant Born cross section in this formula is calculated to be $d\sigma_{\text{ISB}}/dm \cdot 4$ MeV/c$^2 = 101.0 \text{ fb}$. Following the generally accepted practice [14] of including the vacuum polarization correction in the value of the electron width $\Gamma_{ee}$, we multiply the pure Born cross section by 1.042 ± 0.002. From $R \cdot K = 21.03 \pm 0.49 \pm 0.47$ we calculate the cross section $\sigma_{J/\psi} = 2124 \pm 49 \pm 47 \text{ fb}$ and the product of the $J/\psi$ parameters

$$\Gamma_{ee} B_{\mu\mu} = 0.3301 \pm 0.0077 \pm 0.0073 \text{ keV}.$$  

From the PDG values [2] for $B_{ee}$ and $B_{\mu\mu}$, which are dominated by those measured in $\psi(2S) \rightarrow J/\psi \pi^+ \pi^-$ decays by the BES Collaboration [8], we derive the electronic and total widths of the $J/\psi$ meson,

$$\Gamma_{ee} = 5.61 \pm 0.20 \text{ keV}, \quad \Gamma = 94.7 \pm 4.4 \text{ keV},$$

using the correlated errors reported by BES. The statistical and systematic uncertainties are combined in quadrature. Our results agree with the previous world averages [2], $\Gamma_{ee} = 5.26 \pm 0.37 \text{ keV}$ and $\Gamma = 87 \pm 5 \text{ keV}$, but are more precise.

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