Study of the Decay $B^0(\overline{B}^0) \to \rho^+ \rho^-$, and Constraints on the Cabibbo-Kobayashi-Maskawa Angle $\alpha$

Using a data sample of $89 \times 10^6 \ Y(4S) \rightarrow \ B \bar{B}$ decays collected with the $\text{BABAR}$ detector at the PEP-II asymmetric $B$ Factory at SLAC, we measure the $B^0(\bar{B}^0) \rightarrow \rho^+ \rho^-$ branching fraction.
The recently observed [1] decay $B^0(\overline{B}^0) \rightarrow \rho^+\rho^-$ proceeds mainly through the $b \rightarrow uudd$ tree diagram. Interference between direct decay and decay after $B^0,\overline{B}^0$ mixing results in a time-dependent decay-rate asymmetry between $B^0$ and $\overline{B}^0$ that is sensitive to the Cabibbo-Kobayashi-Maskawa (CKM) [2] angle $\alpha \equiv \text{arg}[-V_{ub}V_{ub}^*/V_{ud}V_{ud}^*]$. The presence of loop (penguin) contributions introduces additional phases that can shift the experimentally measurable parameter $\alpha_{\text{eff}}$ away from the value of $\alpha$. In the presence of penguin contributions $\alpha_{\text{eff}} = \alpha + \delta\alpha_{\text{penguin}}$. A constraint on $\alpha$ tests the standard model description of CP violation. Recent measurements of the $B^+ \rightarrow \rho^+\rho^0$ branching fraction and upper limit for $B^0 \rightarrow \rho^0\rho^0$ [3] indicate small penguin contributions in $B \rightarrow \rho\rho$, as has been found in some calculations [4]. Here we present a time-dependent analysis of $B^0(\overline{B}^0) \rightarrow \rho^+\rho^-$. The CP analysis of $B$ decays to $\rho^+\rho^-$ is complicated by the presence of three helicity states ($h \equiv 0, \pm 1$). The $h = 0$ state corresponds to longitudinal polarization and is CP even, while neither the $h = +1$ nor the $h = -1$ state is an eigenstate of CP. The longitudinal polarization fraction $f_1$ is defined as the fraction of the helicity zero state in the decay. The angular distribution is

$$ \frac{d^2\Gamma}{d\cos\theta_1 d\cos\theta_2} = \frac{9}{4} f_1 \cos^2\theta_1 \cos^2\theta_2 + \frac{1}{4} (1 - f_1) \sin^2\theta_1 \sin^2\theta_2, $$

where $\theta_i$, $i = 1, 2$, is defined for each $\rho$ meson as the angle between the $\pi^0$ momentum in the $\rho$ rest frame and the flight direction of the $B^0$ in this frame. We have integrated over the angle between the $\rho$-decay planes. A full angular analysis of the decays is needed in order to separate the definite CP contributions; if, however, a single CP channel dominates the decay, this is not necessary [5]. The longitudinal polarization dominates this decay [1,6].

This measurement is based on 89 $\times 10^6$ $Y(4S) \rightarrow B\overline{B}$ decays collected with the BA BAR [7] detector at the PEP-II asymmetric B factory at SLAC. We reconstruct $B^0(\overline{B}^0) \rightarrow \rho^+\rho^-$ candidates ($B_{\text{rec}}$) from combinations of two charged tracks and two $\pi^0$ candidates. We require that both tracks have particle identification information inconsistent with the electron, kaon, and proton hypotheses. The $\pi^0$ candidates are formed from pairs of photons that have measured energies greater than 50 MeV. The reconstructed $\pi^0$ mass must satisfy $0.10 < m_{\gamma\gamma} < 0.16$ GeV/$c^2$. The mass of the $\rho$ candidates, $m_{\rho^+\rho^-}$, must satisfy $|m_{\rho^+\rho^-} - 0.770$ GeV/$c^2| < 0.375$ GeV/$c^2$.

When multiple $B$ candidates can be formed, we select the one that minimizes the sum of the deviations of the reconstructed $\pi^0$ masses from the true $\pi^0$ mass. Combinatorial backgrounds dominate near $|\cos\theta_1| = 1$, while backgrounds from $B$ decays, such as $B^0 \rightarrow \rho^+\pi^-$, with an additional low energy $\pi^0$ from the rest of the event (ROE), tend to concentrate at negative values of $\cos\theta_1$. We reduce these backgrounds with the requirement $-0.8 < \cos\theta_1 < 0.98$.

Continuum $e^+e^- \rightarrow q\overline{q}$ ($q = u, d, s, c$) events are the dominant background. To discriminate signal from continuum we use a neural network ($\mathcal{N}$) to combine six variables: the event-shape variables used in the Fisher discriminant of Ref. [8]; the cosine of the angle between the direction of the $B$ and the collision axis ($\zeta$) in the center-of-mass (c.m.) frame; the cosine of the angle between the $B$ thrust axis and the $z$ axis; the cosine of the angle between the $B$ thrust axis and the thrust axis of the ROE; the decay angle of the $\pi^0$ (defined in analogy to the $\rho$ decay angle, $\theta_1$); and the sum of transverse momenta in the ROE relative to the $z$ axis.

Signal events are identified kinematically using two variables: the difference $\Delta E$ between the c.m. energy of the $B$ candidate and $\sqrt{s}/2$, and the beam-energy-substituted mass $m_{\text{ES}} = \sqrt{(s/2 + p_\text{B})^2/E^2 - p_\text{B}^2}$, where $\sqrt{s}$ is the total c.m. energy. The $B$ momentum $p_\text{B}$ and four-momentum of the initial state ($E_\text{B}, p_\text{B}$) are defined in the laboratory frame. We accept candidates that satisfy $5.21 < m_{\text{ES}} < 5.29$ GeV/c$^2$ and $-0.12 < \Delta E < 0.15$ GeV. The asymmetric $\Delta E$ window suppresses background from higher-multiplicity $B$ decays.

To study the time-dependent asymmetry one needs to measure the proper time difference, $\Delta t$, between the two $B$ decays in the event, and to determine the flavor tag of the other $B$ meson. The time difference between the decays of the two neutral $B$ mesons in the event ($B_{\text{rec}}, B_{\text{tag}}$) is calculated from the measured separation $\Delta z$ between the $B_{\text{rec}}$ and $B_{\text{tag}}$ decay vertices [9,10]. We determine the $B_{\text{rec}}$ vertex from the two charged-pion tracks in its decay. The $B_{\text{tag}}$ decay vertex is obtained by fitting the other tracks in the event, with constraints from the $B_{\text{rec}}$ momentum and the beam-spot location. The rms resolution on $\Delta t$ is 1.1 ps. We use only events for which the proper time difference between the $B_{\text{rec}}$ and $B_{\text{tag}}$ decays satisfies $|\Delta t| < 20$ ps and the error on $\Delta t$, $\sigma(\Delta t)$, is less than 2.5 ps. The flavor of the $B_{\text{tag}}$ meson is determined with a multivariate technique [8] that has a total effective tagging efficiency of $(28.4 \pm 0.7)\%$. The events are as-
signed to five mutually exclusive tagging categories Lepton, Kaon 1, Kaon 2, Inclusive, and Untagged, listed in order of decreasing reliability of the tag.

Signal candidates may pass the selection even if one or more of the pions assigned to the $\rho^+\rho^-$ state is from the other $B$ in the event. These self-cross-feed (SCF) candidates comprise 39% (16%) of the accepted signal for $f_L = 1$ ($f_L = 0$).

The efficiency of the selection is 7.7% (14.9%) for the longitudinally (transversely) polarized signal as determined with Monte Carlo simulations (MCS) [11]. The signal efficiency taking into account the measured polarization is 7.7%. We select 24,288 events, which are dominated by combinatoric backgrounds: roughly 86% from $q\bar{q}$ and 13% from $B\bar{B}$. We distinguish the following candidate types: (i) correctly reconstructed signal, for $B^0(\bar{B}^0) \rightarrow \rho^+\rho^-$ decays where the correct particles are combined to form the $B_{\text{rec}}$ candidate; (ii) SCF signal; (iii) charm $B^\pm$ background ($b \rightarrow c$); (iv) charm $B^0$ background ($b \rightarrow c$); (v) charmless $B$ backgrounds; and (vi) continuum $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$) background. We consider both types (i) and (ii) as signal. The charmless decays $B^\pm \rightarrow \rho^\pm\pi^0$, $B^0 \rightarrow \rho^0\rho^0$, $B^0 \rightarrow \pi^0\pi^0$, and $B^{\pm,0} \rightarrow \pi^0\pi^0$, $\rho^0\rho^0$, or $\phi\phi$ are expected to contribute to the final sample. For these decays we assume the following branching fractions: $11.0 \pm 2.7$ [12], $26.4^{+6.1}_{-6.4}$ [13], $30 \pm 15$, and $20 \pm 20$, in units of $10^{-3}$, corresponding to $17.4 \pm 4, 30 \pm 15$, and $26 \pm 26$ events in the data, respectively. The latter two are estimated from the measured branching fractions of related decays. We expect an additional 283 ± 283 candidates of charmless $B$ decays with more than four mesons in the final state; since most branching fractions for such modes have not been measured yet, we generate them using the JETSET simulation [14]. We expect 1,700 (1,016) charged (neutral) $B$ decays to final states containing charm mesons. The rest of the background is composed of continuum. Each of these backgrounds is included as an individual component in the fit, where the continuum yield is allowed to vary in the fit.

Each candidate is described with the eight $B_{\text{rec}}$ kinematic variables $m_{\text{ES}}$ and $\Delta E$, the $m_{\pi^\pm\pi^0}$ and $\cos\theta$, values of the two $\rho$ mesons, $\Delta t$, and $N$. For each different candidate type considered, we construct a probability density function (PDF) that is the product of PDFs in each of these variables, assuming that they are uncorrelated. The total PDF is used in the fit to data.

The parameters of the PDFs for continuum-background $m_{\text{ES}}$, $\Delta E$, $\cos\theta$, and $N$ are allowed to vary in the final fit to the data. The distribution of the continuum as a function of $m_{\pi^\pm\pi^0}$ is described by a nonparametric PDF [15] derived from $m_{\text{ES}}$ and $\Delta E$ data sidebands. For all other types these distributions are extracted from high-statistics MCS. The $\cos\theta$ distributions for the background are described by a nonparametric PDF derived from the MCS, as the detector acceptance and selection criteria modify the known vector-meson decay distribution. The signal distribution is given by Eq. (1) multiplied by an acceptance function determined from signal MCS. We take into account known differences between data and the MCS.

The signal $\Delta t$ distribution is described by an exponential ($B$ lifetime) multiplied by a $CP$ violating term, convoluted with three Gaussians ($\sim 90\%$ core, $\sim 9\%$ tail, $\sim 1\%$ outliers) and takes into account $\sigma(\Delta t)$ from the vertex fit. The resolution is parametrized using a large sample of fully reconstructed hadronic $B$ decays [9]. The nominal $\Delta t$ distribution for the $B$ backgrounds is a non-parametric representation of the MCS; in the study of systematic errors, we replace this model with the one used for signal. The continuum background is described by the sum of three Gaussian distributions whose parameters are determined by fitting the data.

The signal decay-rate distribution $f_+(f_-)$ for $B_{\text{tag}} = B^0(\bar{B}^0)$ is given by

$$f_{\pm}(\Delta t) = \frac{e^{-|\Delta t|/\tau}}{4\tau} \left[ 1 \pm S \sin(\Delta m_d\Delta t) \mp C \cos(\Delta m_d\Delta t) \right],$$

where $\tau$ is the mean $B^0$ lifetime, $\Delta m_d$ is the $B^0$-$\bar{B}^0$ mixing frequency, and $S = S_L$ or $S_T$ and $C = C_L$ or $C_T$ are the $\bar{CP}$ asymmetry parameters for the longitudinal and transversely polarized signal. The fitting function takes into account mistag dilution and is convoluted with the $\Delta t$ resolution function described above. We set $S_T$ and $C_T$ to zero since the transverse polarization in the fit is small.

We perform an unbinned extended maximum likelihood fit that assumes the event types mentioned previously. The results of the fit are $246 \pm 29$ signal events with $f_L = 0.99 \pm 0.03$, $S_L = -0.42 \pm 0.42$, and $C_L = -0.17 \pm 0.27$. There is a bias on the yield coming from the neglect of correlations in the fit (six events) and $B$-background modeling (16 events). The former is estimated using MCS and the latter is dominated by the change in signal yield when the $a_1\rho$ component is allowed to vary in a fit to the data. The signal yield remains stable when allowing the yield of other background types to vary. The corrected signal yield is $224 \pm 29$ events. Figure 1 shows distributions of $m_{\text{ES}}$, $\cos\theta$, and $m_{\pi^\pm\pi^0}$ for Lepton and Kaon 1 tagged events, enhanced in signal content by cuts on the signal-to-background likelihood ratios of the discriminating variables not projected. The additional cuts retain $O(15\%)$ of the signal events in the analysis sample. For $m_{\text{ES}}$ and $\Delta E$, we show a projection of the data for all tag categories; in these plots, we retain $O(60\%)$ of the signal events in the analysis sample. Figure 2 shows the raw $\Delta t$ distribution for $B^0$ and $\bar{B}^0$ tagged events. The time-dependent decay-rate asymmetry $A = [R(\Delta t) - \bar{R}(\Delta t)]/[R(\Delta t) + \bar{R}(\Delta t)]$ is also shown, where $R(\bar{R})$ is the decay rate for $B^0(\bar{B}^0)$ tagged events.

The nominal fit does not account for nonresonant background. If we add a nonresonant component of
$B \rightarrow \rho \pi \pi^0$ events to the likelihood, the fitted signal yield changes by less than 11% (90% C.L.). Any possible $B \rightarrow 4\pi$ component would be significantly smaller. The dominant systematic uncertainties in the yield arise from the assumed $B$-background branching fractions (20 events) and the uncertainty on the fraction of SCF events (14 events). The uncertainty on the estimated fraction of misreconstructed events is extrapolated from a control sample of fully reconstructed $B^0 \rightarrow D^{*+}\rho^+$ decays. A 10% systematic error on the branching fraction comes from $\pi^0$ reconstruction. The dominant systematic error on $f_L$ is from the uncertainty in PDF parametrization ($\pm 0.03$). We vary $CP$ violation in the $B$ background within reasonable limits. This is the main systematic uncertainty on the $CP$ results: 0.08 (0.11) on $S_L$ ($C_L$). The systematic uncertainty on $S_L$ ($C_L$) from $B$-background branching fractions is 0.02 (0.03). Uncertainty in the vertex-detector alignment contributes an error of 0.06 (0.04) on $S_L$ ($C_L$). In half of the SCF events the misreconstructed signal contains at least one wrong track; the difference in resolution function for these events corresponds to an uncertainty of 0.03 (0.01) on $S_L$ ($C_L$). The uncertainty in the parametrization of the likelihood contributes an error of 0.05 (0.02) on $S_L$ ($C_L$).

We estimate the systematic error from ignoring interference with nonresonant modes and $a_1\pi$ to be 0.02 on $S_L$ and $C_L$, and 2.4% on the signal yield. The uncertainty from possible $CP$ violation in the doubly Cabibbo-suppressed decays on the tag side of the event [16] is assumed to be the same as for $B^0(B^0) \rightarrow \pi^+ \pi^-$: 0.012 (0.037) for $S_L$ ($C_L$). We also apply a correction to account for possible dilution from $B$-background (5%) and SCF (3%) events.

Our results are

$$B(B^0 \rightarrow \rho^+ \rho^-) = [33 \pm 4(\text{stat}) \pm 5(\text{syst})] \times 10^{-6},$$

$$f_L = 0.99 \pm 0.03(\text{stat})^{+0.04}_{-0.03}(\text{syst}),$$

$$C_L = -0.17 \pm 0.27(\text{stat}) \pm 0.14(\text{syst}),$$

$$S_L = -0.42 \pm 0.42(\text{stat}) \pm 0.14(\text{syst}).$$
The correlation coefficient between $S_L$ and $C_L$ is $-0.016$. We average this branching fraction with the less precise result from Ref. [1], taking into account correlations where appropriate [13], to obtain the final value of $(30 \pm 4 \pm 5) \times 10^{-6}$. This measurement supersedes the previous BABAR result presented in Ref. [1].

Using the Grossman-Quinn bound [1] with the recent results on $B \to \rho^+ \rho^- \rho^0 \rho^0$ from [3], we limit $|\alpha_{\text{eff}} - \alpha| < 13^\circ$ (68% C.L.). Ignoring possible nonresonant contributions, and $I = 1$ amplitudes [18], one can relate the $CP$ parameters $S_L$ and $C_L$ to $\alpha$, up to a four-fold ambiguity. If we select the solution closest to the CKM best fit central value of $\alpha = 95^\circ - 98^\circ$ [19], the measured $CP$ parameters of the longitudinal polarization correspond to $\alpha = 102.1^{+15.8}_{-10.1} \, ^{(\text{stat})}^{+11.2}_{-7.2} \, ^{(\text{syst})} \pm 13(\text{penguin})^\circ$. Figure 3 shows the confidence level as a function of $\alpha_{\text{eff}} = \arcsin(S_L/\sqrt{1 - C_L^2})/2$ for this result, (dotted line) taking into account the systematic uncertainties and (solid line) also including the penguin contribution. We exclude values of $\alpha$ between $19^\circ$ and $71^\circ$ (90% C.L.).

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