Search for $B^+ \rightarrow [K^+ \pi^\pm]_D K^+$ and Upper Limit on the $b \rightarrow u$ Amplitude in $B^+ \rightarrow D K^+$

We search for $B^- \to [K^+ \pi^-]_D K^-$ decays, where $[K^+ \pi^-]_D$ indicates that the $K^+\pi^-$ pair originates from the decay of a $D^0$ or $\bar{D}^0$. Results are based on $120 \times 10^6$ $Y(4S) \to BB$ decays collected with the BABAR detector at SLAC. We set an upper limit on the ratio $R_{K^+} = \frac{\Gamma(B^- \to [K^+ \pi^-]_D K^-) + \Gamma(B^- \to [K^+ \pi^-]_D K^-)}{\Gamma(B^- \to D^0 K^-)} < 0.026$ (90% C.L.). This constrains the amplitude ratio $r_B = |A(B^- \to \bar{D}^0 K^-)/A(B^- \to D^0 K^-)| < 0.22$ (90% C.L.), consistent with expectations. The small value
Following the discovery of CP violation in B-meson decays and the measurement of the angle $\beta$ of the unitarity triangle [1] associated with the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix, focus has turned towards the measurements of the other angles $\alpha$ and $\gamma$. The angle $\gamma = \arg(-V_{ub}^{*}V_{ud}/V_{cb}^{*}V_{cd})$, where $V_{ij}$ are CKM matrix elements; in the Wolfenstein convention [2], $\gamma = \arg(V_{ub}^{*})$.

Several proposed methods for measuring $\gamma$ exploit the interference between $B^{-} \rightarrow D_{0}^{0}K^{-}$ and $B^{-} \rightarrow D_{0}^{0}K^{-}$ (Fig. 1) which occurs when the $D_{0}^{0}$ and the $\bar{D}_{0}^{0}$ decay to common final states, as first suggested in Ref. [3].

Following the proposal in Ref. [4], we search for $B^{-} \rightarrow \bar{D}_{0}^{0}K^{-}$ followed by $\bar{D}_{0}^{0} \rightarrow K^{-}\pi^{-}\rho$, as well as the charge conjugate sequence, where the symbol $D_{0}^{0}$ indicates either a $D_{0}^{0}$ or a $\bar{D}_{0}^{0}$. Here the favored $B$ decay followed by the doubly CKM-suppressed $D$ decay interferes with the suppressed $B$ decay followed by the CKM-favored $D$ decay. We use the notation $B^{-} \rightarrow [h_{1}^{0}h_{2}^{0}]_{D}h_{3}^{-}$ (with each $h_{i}^{0} = \pi$ or $K$) for the decay chain $B^{-} \rightarrow D_{0}^{0}h_{1}^{-}$, $D_{0}^{0} \rightarrow h_{2}^{0}h_{3}^{-}$. We also refer to $h_{3}^{-}$ as the bachelor $\pi$ or $K$. Then, ignoring $D$ mixing,

$$R_{K_{\pi}}^{\pi} = \frac{\Gamma([K^{\pm}\pi^{\mp}]_{D}K^{\pm})}{\Gamma([K^{\pm}\pi^{\mp}]_{D}K^{\pm})} = r_{B}^{2} + r_{D}^{2} + 2r_{B}r_{D}\cos(\pm \gamma + \delta),$$

where

$$r_{B} = \frac{|A(B^{-} \rightarrow \bar{D}_{0}^{0}K^{-})|}{|A(B^{-} \rightarrow D_{0}^{0}K^{-})|}, \quad \delta = \delta_{B} + \delta_{D},$$

$$r_{D} = \frac{|A(D_{0}^{0} \rightarrow K^{+}\pi^{-})|}{|A(D_{0}^{0} \rightarrow K^{-}\pi^{+})|} = 0.060 \pm 0.003$$

[5], and $\delta_{B}$ and $\delta_{D}$ are strong phase differences between the two $B$ and $D$ decay amplitudes, respectively. The expression for $R_{K_{\pi}}^{\pi}$ neglects the tiny contribution to the $[K^{\pm}\pi^{\mp}]_{D}K^{\pm}$ mode from the color-suppressed $B$ decay followed by the doubly CKM-suppressed $D$ decay.

Since $r_{B}$ is expected to be of the same order as $r_{D}$, CP violation could manifest itself as a large difference between $R_{K_{\pi}}^{\pi}$ and $R_{K_{\pi}}^{\pi}$. Measurements of $R_{K_{\pi}}^{\pi}$ are not sufficient to extract $\gamma$, since these two quantities are functions of three unknowns: $\gamma$, $r_{B}$, and $\delta$. However, they can be combined with measurements for other $D_{0}^{0}$ modes to extract $\gamma$ in a theoretically clean way [4].

The value of $r_{B}$ determines, in part, the level of interference between the diagrams of Fig. 1. In most techniques for measuring $\gamma$, high values of $r_{B}$ lead to better sensitivity. Since $R_{K_{\pi}}^{\pi}$ depend quadratically on $r_{B}$, measurements of $R_{K_{\pi}}^{\pi}$ can constrain $r_{B}$. In the standard model, $r_{B} = |V_{ub}V_{us}^{*}/V_{cb}V_{cd}|, F_{c} = 0.4F_{cs}$, and $F_{cs} < 1$ accounts for the additional suppression, beyond that due to CKM factors, of $B^{-} \rightarrow \bar{D}_{0}^{0}K^{-}$ relative to $B^{-} \rightarrow D_{0}^{0}K^{-}$. Naively, $F_{cs} = 1/4$, which is the probability for the color of the quarks from the virtual $W$ in $B^{-} \rightarrow \bar{D}_{0}^{0}K^{-}$ to match that of the other two quarks; see Fig. 1. Early estimates gave $F_{cs} = 0.22$ [6], leading to $r_{B} = 0.09$; however, recent measurements [7] of color-suppressed $b \rightarrow c$ decays $[B \rightarrow D_{0}^{0}h^{0}; h^{0} = \pi^{0}, \rho^{0}, \omega, \eta, \eta']$ suggest that $F_{cs}$, and therefore $r_{B}$, could be larger, e.g., $r_{B} = 0.2$ [8]. A study by the Belle Collaboration of $B^{-} \rightarrow D_{0}^{0}K^{\mp}\pi^{\pm}$, $D_{0}^{0} \rightarrow K_{S}\pi^{+}\pi^{-}$, favors a large value of $r_{B}$: $r_{B} = 0.26^{+0.11}_{-0.15}$ [9].

Our results are based on $120 \times 10^{8}$ $\Upsilon(4S) \rightarrow B\bar{B}$ decays, corresponding to an integrated luminosity of 109 fb$^{-1}$, collected between 1999 and 2003 with the BABAR detector [10] at the PEP-II B Factory at SLAC. A 12 fb$^{-1}$ off-resonance data sample, with a c.m. energy 40 MeV below the $\Upsilon(4S)$ resonance, is used to study continuum events, $e^{+}e^{-} \rightarrow q\bar{q}$ ($q = u, d, s$, or $c$).

The event selection was developed from studies of simulated $B\bar{B}$ and continuum events, and off-resonance data. A large on-resonance data sample of $B^{-} \rightarrow D_{0}^{0}\pi^{-}$, $D_{0}^{0} \rightarrow K^{-}\pi^{+}\pi^{-}$ events was used to validate several aspects of the simulation and analysis procedure. We refer to this mode and its charge conjugate as $B \rightarrow D\pi$.

Kaon and pion candidates in $B^{-} \rightarrow \{K\pi\}_{D}K_{\pm}$ must satisfy $K$ or $\pi$ identification criteria that are typically 90% efficient, depending on momentum and polar angle. Misidentification rates are at the few percent level. The invariant mass of the $K\pi$ pair must be within 18.8 MeV (2.5$\sigma$) of the mean reconstructed $D_{0}^{0}$ mass. The remaining background from other $B^{-} \rightarrow [h_{1}^{0}h_{2}^{0}]_{D}h_{3}^{0}$ modes is eliminated by removing events where any $h_{i}^{0}h_{j}^{0}$ pair, with any particle-type assignment except for the signal hypothesis for the $h_{1}^{0}h_{2}^{0}$ pair, is consistent with $D_{0}^{0}$ decay. We also reject $B$ candidates where the $D_{0}^{0}$ paired with a $\pi^{0}$ or $\pi^{0}$ in the event is consistent with $D^{+} \rightarrow D\pi$ decay.
After these requirements, backgrounds are mostly from continuum, mainly $e^+e^+ \to c\bar{c}$, with $\bar{c} \to D^0 \to K^+\pi^-$ and $c \to D \to K^-$. These are reduced with a neural network based on nine quantities that distinguish continuum and $B\bar{B}$ events: (i) A Fisher discriminant based on the quantities $L_0 = \sum_i p_i$ and $L_2 = \sum_i p_i \cos^2 \theta_i$ calculated in the c.m. frame. Here, $p_i$ is the momentum and $\theta_i$ is the angle with respect to the thrust axis of the $B$ candidate of tracks and clusters not used to reconstruct the $B$. (ii) $|\cos \theta|$, where $\theta$ is the angle in the c.m. frame between the thrust axes of the $B$ and the detected remainder of the event. (iii) $\cos \theta_B$, where $\theta_B$ is the polar angle of the $B$ in the c.m. frame. (iv) $\cos \theta_K^0$, where $\theta_K^0$ is the decay angle in $D^0 \to K\pi$, i.e., the angle between the direction of the $K$ and the line of flight of the $D^0$ in the $D^0$ rest frame. (v) $\cos \theta_B^0$, where $\theta_B^0$ is the decay angle in $B \to \bar{D}^0K$. (vi) The difference $\Delta Q$ between the sum of the charges of tracks in the $\bar{D}^0$ hemisphere and the sum of the charges of the tracks in the opposite hemisphere excluding the tracks used in the reconstructed $B$. For signal, $\langle \Delta Q \rangle = 0$, while for the $c\bar{c}$ background $\langle \Delta Q \rangle = \frac{2}{3} \times Q_B$, where $Q_B$ is the $B$ candidate charge. The $\Delta Q$ rms is 2.4. (vii) $Q_B \cdot Q_K$, where $Q_K$ is the sum of the charges of all kaons not in the reconstructed $B$. Many signal events have $Q_B \cdot Q_K \leq -1$, while most continuum events have no kaons outside of the reconstructed $B$, and hence $Q_K = 0$. (viii) The distance of the closest approach between the bachelor track and the trajectory of the $\bar{D}^0$. This is consistent with zero for signal events, but can be larger in $c\bar{c}$ events. (ix) The existence of a lepton ($e$ or $\mu$) and the invariant mass ($m_{KL}$) of the lepton and the bachelor $K$. Continuum events have fewer leptons than signal events. Moreover, most leptons in $c\bar{c}$ events are from $D \to K\ell\nu$, where $K$ is the bachelor kaon, so that $m_{KL} < m_D$.

The neural net is trained with simulated continuum and signal events. We find agreement between the distributions of all nine variables in simulation and in control samples of off-resonance data and of $B \to D\pi$. The neural net requirement is 66% efficient for signal, and rejects 96% of the continuum background. An additional requirement, $\cos \theta_K^0 > -0.75$, rejects 50% of the remaining $B\bar{B}$ backgrounds and is 93% efficient for signal.

A $B$ candidate is characterized by the energy-substituted mass $m_{ES} = \sqrt{\frac{1}{2} + \frac{p_0}{2} - \frac{p_B^2}{2}} / \sqrt{E_0^2 - p_B^2}$ and the energy difference $\Delta E = E^* + \frac{1}{2} \sqrt{5}/E_0$, where $E$ and $p$ are energy and momentum, the asterisk denotes the c.m. frame, the subscripts 0 and $B$ refer to the $Y(4S)$ and $B$ candidates, respectively, and $s$ is the square of the c.m. energy. For signal events $m_{ES} = m_B$ within the resolution of about 2.5 MeV, where $m_B$ is the known $B$ mass.

We require $\Delta E$ to be within 47.8 MeV (2.5$\sigma$) of the mean value of $-4.1$ MeV found in the $B \to D\pi$ control sample. The yield of signal events is extracted from a fit to the $m_{ES}$ distribution of events satisfying all of the requirements discussed above.

Our selection includes contributions from backgrounds with $m_{ES}$ distributions peaked near $m_B$ (peaking backgrounds). We distinguish those with a real $D^0 \to K^+\pi^-$ and those without, e.g., $B^+ \to h^+h^-h^-$. The latter are estimated from events with $K^+\pi^-$ mass in a sideband of the $D^0$. The former are from $B^+ \to D^0\pi^+$, followed by the CKM-suppressed decay $D^0 \to K^+\pi^-$, with the bachelor $\pi$ misidentified as a $K$. These are estimated as $N^0_{\text{peak}} = r_D^0 N_{D\pi}$, where $N_{D\pi}$ is the number of observed $B \to D\pi$ events with the $\pi$ misidentified as a $K$. The technique used to measure $N_{D\pi}$ is described below. Studies of simulated $B\bar{B}$ events indicate that other peaking background contributions are negligible.

Because of the small number of events, we combine the $B^+$ and $B^-$ samples. We define the quantity

$$R_{K^\pi} = \frac{\Gamma(B^- \to [K^+\pi^-]_D K^-) + \Gamma(B^+ \to [K^-\pi^+]_D K^+)}{\Gamma(B^- \to [K^+\pi^-]_D K^-) + \Gamma(B^+ \to [K^-\pi^+]_D K^+)},$$

$$R_{K^{\pm}} = \frac{R_{K^+} + R_{K^-}}{2} = r_B + r_D + 2r_B r_D \cos \gamma \cos \delta,$$

assuming no CP violation in $[K^+\pi^+]_D K^-$. We determine $R_{K^\pi}$ as $c N_{\text{sig}}/N_{DK}$, where $N_{\text{sig}}$ is the number of $B^\pm \to [K^+\pi^-]_D K^-\pi^0$ signal events and $N_{DK}$ is the number of $B^\pm \to [K^+\pi^-]_D K^-\pi^0$ events, a mode that we denote by $B \to DK$. Most systematic uncertainties cancel in the ratio. The factor $c = 0.93 \pm 0.04$, determined from simulation, accounts for a difference in the event selection efficiency between the signal mode and $B \to DK$. This difference is mostly due to a correlation between the efficiencies of the $\cos \theta_K^0$ requirement and the $\bar{D}^0$ veto constructed using the bachelor track and the oppositely charged track in the $[K\pi]$ pair. This correlation depends on the relative sign of the kaon and the bachelor track, and is different in the two modes.

The value of $R_{K^\pi}$ is obtained from a simultaneous unbinned maximum likelihood fit to four $m_{ES}$ and three $\Delta E$ distributions. These distributions are used to extract the parameters needed to calculate $R_{K^\pi}$ (e.g., $N_{\text{sig}}$) or to constrain the shapes of other distributions. The likelihood is expressed directly in terms of $R_{K^\pi}$.

The $m_{ES}$ distribution for signal candidates is fit to the sum of a threshold background function and a Gaussian centered at $m_B$. The number of events in the Gaussian is $N_{\text{sig}} + N_{\text{peak}}^{D^0} + N_{\text{peak}}^{hhhh}$, where $N_{\text{peak}}^{D^0}$ and $N_{\text{peak}}^{hhhh}$ are the number of peaking background events with and without a real $\bar{D}^0$, respectively. The Gaussian parameters are constrained by the fit to the $m_{ES}$ distribution of $B \to DK$ events. The shape of the threshold function is constrained by fitting the $m_{ES}$ distribution of candidates in a sideband of $\Delta E (-125 < \Delta E < 200$ MeV, excluding the signal region). The $m_{ES}$ distribution for events passing all signal requirements, but with $K^+\pi^-$ mass in the sideband of the $\bar{D}^0$ is fit in the same manner. We estimate $N_{\text{peak}}^{hhhh}$ from the Gaussian yield of this last fit, accounting for the different sizes of the signal and sideband $\bar{D}^0$ mass ranges. The $m_{ES}$
distributions for signal and $D^0$ sideband candidates are shown in Figs. 2(a) and 2(b).

The $m_{ES}$ distribution for $B \to DK$ candidates with $|\Delta E + 4.1 \text{ MeV}| < 47.8 \text{ MeV}$ [see Fig. 2(c)] is also fit to a Gaussian and a threshold function. The number of events in the Gaussian is $N_{DK} + N_{D\pi}$, where, as previously defined, $N_{DK}$ is the number of $B \to DK$ events and $N_{D\pi}$ is the number of $B \to D\pi$ events with the bachelor $\pi$ misidentified as a $K$. The ratio $N_{DK}/N_{D\pi}$ is obtained by fitting the $\Delta E$ distribution for $B \to DK$ candidate events with $m_{ES} > 5.27$ GeV [see Fig. 2(d)]. This is modeled as the sum of a combinatoric background function, a double Gaussian for the $B \to D\pi$ background, and a Gaussian for the $B \to DK$ signal. The parameters of the Gaussians in the $\Delta E$ fit are constrained from fits to the $\Delta E$ distributions of well-identified $B \to D\pi$ events with the bachelor $\pi$ assumed to be a $\pi$ or a $K$.

We find $R_{K\pi} = (4 \pm 12) \times 10^{-3}$, consistent with zero. The number of signal, normalization, and peaking background events are $N_{sig} = 1.1 \pm 3.0$, $N_{DK} = 261 \pm 22$, $N_{D\pi}^{\text{peak}} = 72 \pm 0.07$, and $N_{D\pi}^{\text{hh}} = 0.4 \pm 1.1$. The uncertainties are mostly statistical. From this likelihood, we set a Bayesian limit $R_{K\pi} < 0.026$ at the 90% confidence level (C.L.), assuming a constant prior probability for $R_{K\pi} > 0$ (see Fig. 3).

In Fig. 4 we show the dependence of $R_{K\pi}$ on $r_B$, together with our limit. This is shown allowing a $\pm 1\sigma$ variation on $r_D$, for the full range $0^\circ - 180^\circ$ for $\gamma$ and $\delta$, as well as with the restriction $48^\circ < \gamma < 73^\circ$ suggested by global CKM fits [11]. The least restrictive limit on $r_B$ is computed assuming maximal destructive interference: $\gamma = 0^\circ$, $\delta = 180^\circ$ or $\gamma = 180^\circ$, $\delta = 0^\circ$. This limit is $r_B < 0.22$ at 90% C.L.

In summary, we find no evidence for $B^\pm \to [K^\mp \pi^\pm]_{D}K^\pm$. We set a 90% C.L. limit on the ratio $R_{K\pi}$ of rates for this mode and the favored mode $B^\pm \to [K^- \pi^+]_{D}K^-$. Our limit is $R_{K\pi} < 0.026$ at 90% C.L. With the most conservative assumption on the values of $\gamma$ and of the strong phases in the $B$ and $D$ decays, this results in a limit on the ratio of the magnitudes of the $B^- \to D^0K^-$ and $B^- \to D^0K^-$ amplitudes $r_B < 0.22$ at 90% C.L. Our analysis suggests that $r_B$ is smaller than the value reported by the Belle Collaboration, $r_B = 0.26^{+0.11}_{-0.15}$ [9], but given the uncertainties the two results are not in disagreement. A small value of $r_B$ will make it diffi-

![FIG. 2 (color online). $m_{ES}$ distributions for (a) signal $[K^- \pi^+]_{D}K^-$ candidates, (b) candidates from the $D^0$ sideband, and (c) $B \to DK$ candidates. The $D^0$ sideband selection uses a $K^- \pi^-$ invariant mass range 2.72 times larger than the signal selection. (d) $\Delta E$ distribution for $B \to DK$ candidates; the peak centered at $\approx 0.05$ GeV is from $B \to D\pi$. The superimposed curves are described in the text. In (c), the dashed Gaussian centered at $m_B$ represents the $B \to D\pi$ contribution estimated from (d).](image-url)

![FIG. 3 (color online). Likelihood as a function of $R_{K\pi}$. The integral for $0 < R_{K\pi} < 0.026$ is 90% of the integral for $R_{K\pi} > 0$.](image-url)

![FIG. 4 (color online). Expectations for $R_{K\pi}$ and $N_{sig}$ vs $r_B$. Shaded area: allowed region for any value of $\delta$, with a $\pm 1\sigma$ variation on $r_D$, and $48^\circ < \gamma < 73^\circ$. Hatched area: additional allowed region with no constraint on $\gamma$. The horizontal line represents the 90% C.L. limit $R_{K\pi} < 0.026$. The dashed lines are drawn at $r_D = 0.196$ and $r_D = 0.224$. They represent the 90% C.L. upper limits on $r_B$ with and without the constraint on $\gamma$.](image-url)
cult to measure $\gamma$ with other methods [3, 12] based on $B \to \bar{D} K$.

We are grateful for the excellent luminosity and machine conditions provided by our PEP-II colleagues, and for the substantial dedicated effort from the computing organizations that support BABAR. The collaborating institutions thank SLAC for its support and kind hospitality. This work is supported by DOE and NSF (USA), NSERC (Canada), IHEP (China), CEA and CNRS-IN2P3 (France), BMBF and DFG (Germany), INFN (Italy), FOM (The Netherlands), NFR (Norway), MIST (Russia), and PPARC (United Kingdom). Individuals have received support from the A.P. Sloan Foundation, Research Corporation, and Alexander von Humboldt Foundation.

*Also with Università della Basilicata, Potenza, Italy.
†Also with IFIC, Instituto de Física Corpuscular, CSIC-Universidad de Valencia, Valencia, Spain.
‡Deceased.