Branching Fractions and CP Asymmetries in $B^0 \to K^+ K^- K_S^0$ and $B^+ \to K^+ K_S^0 K_S^0$

We measure the branching fractions and $CP$ asymmetries in the decays $B^0 \to K^+ K^- K_S^0$ and $B^+ \to K^+ K^0_S K_S^0$ using a sample of approximately $122 \times 10^6$ $B\bar{B}$ pairs collected by the BABAR detector. From a time-dependent analysis of the $K^+ K^- K_S^0$ sample that excludes $\phi K_S^0$, the values of the $CP$-violation...
In the standard model (SM) of particle physics, the decays $B^0 \rightarrow K^+ K^- K^0_S$ and $B^+ \rightarrow K^+ K^0_S K^0_S$ [1] are dominated by $b \rightarrow s \bar{s}s$ gluonic penguin diagrams [2]. CP violation in such decays arises from the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing mechanism [3]. Neglecting CKM-suppressed contributions, the expectation for the CP-asymmetry parameters in $B^0 \rightarrow K^+ K^- K^0_S$ decays is the same as in $B^0 \rightarrow J/\psi K^0_S$ decays, where CP violation has been observed [4,5]. The decay rates for $B^+ \rightarrow K^+ K^0_S K^0_S$ and $B^- \rightarrow K^- K^0_S K^0_S$ are expected to be equal. However, contributions from physics beyond the SM could invalidate these predictions [6]. Since $b \rightarrow s \bar{s}s$ decays involve one-loop transitions, they are especially sensitive to additional contributions. Present results in decays of neutral B mesons are inconclusive due to large statistical errors. Belle measures the CP asymmetry parameter in $\phi K^0_S$ decays of $\sin 2\beta = -0.96 \pm 0.50^{+0.11}_{-0.11}$ [7] which is 3.5 standard deviations from the SM expectation of $\sin 2\beta = 0.73 \pm 0.056$ [4,5]. A BABAR measurement of $\sin 2\beta = 0.47 \pm 0.34^{+0.08}_{-0.06}$ [8] is consistent with the SM and disagrees with Belle by 2.3 standard deviations. 

A more accurate CP measurement can be made using all the decays to $K^+ K^- K^0_S$ that do not contain a $\phi$ meson. This sample is several times larger than the sample of $\phi K^0_S$ [9,10]. As Belle noted [10], the CP content of the final state can be extracted using an isospin analysis. In decays that exclude $\phi K^0_S$, Belle measures $\sin 2\beta = 0.51 \pm 0.26 \pm 0.05^{+0.18}_{-0.18}$ [7], consistent with the SM expectation. In this Letter we present measurements of CP asymmetry and CP content in $K^+ K^- K^0_S$ decays, and the first measurement of the charge asymmetry rate in $B^+ \rightarrow K^+ K^0_S K^0_S$ decays.

This analysis is based on about 122 $\times 10^6$ $B \bar{B}$ pairs collected with the BABAR detector [11] at the PEP-II asymmetric-energy $e^+e^-$ storage rings at SLAC, operating on the Y(4S) resonance. We reconstruct B mesons from $K^0_S \rightarrow \pi^+ \pi^-$ and $K^\pm$ candidates. Charged kaons are distinguished from pions and protons using energy-loss ($dE/dx$) information in the tracking system and from the Cherenkov angle and number of photons measured by the detector of internally reflected Cherenkov light. We accept $K^0_S \rightarrow \pi^+ \pi^-$ candidates that have a two-pion invariant mass within 12 MeV/$c^2$ of the nominal $K^0_S$ mass [12], a decay length greater than 3 standard deviations, and a cosine of the angle between the line connecting the $B$ and $K^0_S$ decay vertices and the $K^0_S$ momentum greater than 0.999. The three daughters in the $B$ decay are fitted constraining their paths to a common vertex, and the $K^0_S$ mass to the nominal value.

In the characterization of the $B$ candidates we use two kinematic variables. The energy difference $\Delta E = E_{\bar{B}} - \sqrt{s}/2$ is reconstructed from the energy of the $B$ candidate $E_{\bar{B}}$ and the total energy $\sqrt{s}$ in the $e^+e^-$ center-of-mass (c.m.) frame. The $\Delta E$ resolution for signal events is 18 MeV. We also use the beam-energy-substituted mass $m_{BS} = \sqrt{(s/2 + \vec{p}_t \cdot \vec{p}_{\bar{B}})^2/E_{\bar{B}}^2 - \vec{p}_{\bar{B}}^2}$, where $(\vec{p}_t, E_t)$ is the four-momentum of the initial $e^+e^-$ system and $\vec{p}_{\bar{B}}$ is the momentum of the $B$ candidate, both measured in the laboratory frame. The $m_{BS}$ resolution for signal events is 2.6 MeV/$c^2$. We retain candidates with $|\Delta E| < 200$ MeV and 5.2 < $m_{BS}$ < 5.3 GeV/$c^2$.

The background is dominated by random combinations of tracks created in $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$) continuum events. We suppress this background by utilizing the difference in the topology in the c.m. frame between jetlike $q\bar{q}$ events and spherical signal events. The topology is described using angle $\theta_t$ between the thrust axis of the $B$ candidate and the thrust axis of the charged and neutral particles in the rest of the event (ROE) [11]. Other quantities that characterize the event topology are two sums over the ROE: $\sum |\vec{p}_{i^*}|$ and $\sum |\vec{p}_{i^*}| \cos^2 \theta_i$, where $\theta_i$ is the angle between the momentum $\vec{p}_{i^*}$ and the thrust axis of the $B$ candidate. Additional separation is achieved using the angle $\theta_{B'}$ between the $B'$-momentum direction and the beam axis. After requiring $|\cos \theta_{B'}| < 0.9$, these four event shape variables are combined into a Fisher discriminant $\mathcal{F}$ [13].

The remaining background originates from $B$ decays where a neutral or charged pion is missed during reconstruction (peaking $B$ background). We use Monte Carlo (MC) events to model the signal and the peaking background, and data sidebands to model continuum background.

We suppress background from $B$ decays that proceed through a $b \rightarrow c$ transition leading to the $K^+ K^- K^0_S$ ($K^+ K^0_S K^0_S$) final state by applying invariant mass cuts to remove $D^0 \rightarrow K K$, $D^+ \rightarrow K^+ K^0_S$, $J/\psi \rightarrow K K$, and $X_{c,0} \rightarrow K K (K^0_S K^0_S)$ decays. Finally, $B$ decays into final states with pions are eliminated by requiring the pion misidentification rate to be less than 2%.
The time-dependent CP asymmetry is obtained by measuring the proper time difference $\Delta t$ between a fully reconstructed neutral B meson ($B_{CP}$) decaying into $K^+ K^- K_S^0$, and the partially reconstructed recoil B meson ($B_{tag}$). Decay products of the recoil side are used to determine the $B_{tag}$ meson’s flavor (flavor tag) and to classify the event into five mutually exclusive tagging categories [4]. If the fraction of events in category $c$ is $\epsilon_c$ and the mistag probability is $w_c$, the overall quality of the tagging, $\sum \epsilon_c (1 - 2w_c)^2$, is $(28.0 \pm 0.4)$%.

The time difference $\Delta t$ is extracted from the measurement of the separation $\Delta z$ between the $B_{CP}$ and $B_{tag}$ vertices, along the boost axis ($z$) of the $B\bar{B}$ system. The vertex position of the $B_{CP}$ meson is reconstructed primarily from kaon tracks, and its MC-estimated resolution ranges between 40 and 80 $\mu$m, depending on the opening angle and direction of the kaon pair. The final $\Delta t$ resolution is dominated by the uncertainty on the $B_{tag}$ vertex which allows the $\Delta t$ ($\Delta z$) precision with rms of 1.1 ps (180 $\mu$m). We retain events that have $|\Delta t| < 20$ ps and whose estimated uncertainty $\sigma_{\Delta t}$ is less than 2.5 ps. The $\Delta t$ resolution function is parameterized as a sum of two Gaussian distributions whose widths are given by a scale factor times the event-by-event uncertainty $\sigma_{\Delta t}$. A third Gaussian distribution, with a fixed large width, accounts for a small fraction of outlying events [4].

Parameters describing the tagging performance and the $\Delta t$ resolution function are extracted from approximately 30,000 $B^0$ decays into $D^{(*)-} X^+$ ($X^+ = \pi^+, \rho^+, a_1^+$) flavor eigenstates ($B_{flav}$ sample).

The decay rate $f_+(f_-)$ when the flavor of the tagging meson is a $B^0$ ($\bar{B}^0$) is given by

$$f_\pm (\Delta t) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4 \tau_{B^0}^2} \left[ 1 \pm S \sin(\Delta m_d \Delta t) \right] \frac{C \cos(\Delta m_d \Delta t)}{1 - 2w_c},$$

where $\tau_{B^0}$ is the mean $B^0$ lifetime and $\Delta m_d$ is the $B^0$-$\bar{B}^0$ oscillation frequency. The parameters $C$ and $S$ describe the magnitude of CP violation in the decay and the interference between decay and mixing, respectively. In the SM, we expect $C = 0$ because there can be no direct CP violation when there is only one decay mechanism. If we exclude $\phi K_S^0$ events by applying a $K^+ K^-$ invariant mass cut of 15 MeV/$c^2$ around the nominal $\phi$ mass [12], and assume that the remaining $B_{CP}$ candidates are CP even, as our analysis below indicates, we expect $S = -\sin 2\beta = -0.731 \pm 0.056$ [4,5].

Direct CP violation in $B^+ \rightarrow K^+ K_S^0 K_S^0$ decays is measured as an asymmetry in the decay rates

$$A_{CP} = \frac{\Gamma_{K^- K_S^0 K_S^0} - \Gamma_{K^+ K_S^0 K_S^0}}{\Gamma_{K^- K_S^0 K_S^0} + \Gamma_{K^+ K_S^0 K_S^0}}.$$  

The SM expectation for $A_{CP}$ is zero.

Branching fractions and CP asymmetries are extracted in unbinned extended maximum likelihood fits to the different samples. The likelihood function $L$, with event yields $N_i$ and probability density functions (PDFs) $P_{i,j}$, is

$$L = \exp \left( -\sum_i N_i \right) \prod_j \left[ \sum_i N_i P_{i,j} \right],$$

where $j$ runs over events and $i$ over event yields. We have a total of 6144 events in the $K^+ K^- K_S^0$ mode, and 13 864 (12 862) in the $K^+ K^- K_S^0$ mode with $\phi K_S^0$ included (excluded).

In the measurement of the branching fractions $B$, the total PDF is formed as $P(m_{ES}) P(\Delta E) P(\mathcal{E})$. Event yields for signal, continuum, and peaking $B$ background are varied in the fit. In the extraction of the charge asymmetry $A_{CP}$ in $K^+ K_S^0 K_S^0$ decays, the yields are split by the charge, which brings the total number of varied parameters to six. To extract the branching fractions, we assign a weight for each event to belong to the signal decay, $W_i = (\sum V_{i,j} P_{i,j})/\sum N_i P_{i,j}$, where $V_{i,j}$ is the signal row of the covariance matrix obtained from the fit [14]. The branching fraction is calculated as $B = \frac{1}{N_{ES}} \sum W_i e_j$, where $N_{ES}$ is the total number of $B\bar{B}$ pairs. Since the efficiency $e_j$ varies across the phase space, $e_j$ is computed in small phase-space bins using simulated events. The method is cross-checked with a simple counting analysis. Distributions of $m_{ES}$ are shown in Fig. 1, and the fit results are given in Table I.

In the time-dependent CP fit, $K^+ K^- K_S^0$ events that exclude $\phi K_S^0$ decays are fit simultaneously with the $B_{flav}$ sample. The PDFs are formed as $P(m_{ES}) P(\Delta E) P(\mathcal{E}) P(\Delta t; \sigma_{\Delta t})$ for $B_{CP}$ events and $P(m_{ES}) P(\mathcal{E}) P(\Delta t; \sigma_{\Delta t})$ in the $B_{flav}$ sample. The $\Delta t$ resolution and tagging parameters are allowed to be different for each tagging category $c$. Fit parameters that are common to both samples are the signal fractions in tagging categories $\epsilon_c$, the average mistag fraction $w_c$, the difference between $B^0$ and $\bar{B}^0$ mistag rates $\Delta w_c$, and the $\Delta t$ resolution functions for signal and background events. We also vary the $K^+ K^- K_S^0$ signal yield and background yields in $m_{ES}$ bins.
tag categories, the CP parameters, and the parameters describing the $\Delta t$ shape of the background. The total number of floated parameters is 38. The largest correlation between $S$ or $C$ with any linear combination of other parameters is 6.6%.

Results of the time-dependent CP asymmetry measurement in $K^+K^-K_S^0$ are given in Table I. Figure 2 shows the $\Delta t$ distributions of events with $B^0$ and $\bar{B}^0$ tags, with projections from the likelihood fit superimposed. The fit procedure is verified with the $K^+K_S^0K_S^0$ sample (Table I), where one expects zero asymmetry, and the $J/\psi K_S^0$ sample where the results are consistent with our previous measurement [4].

We evaluate the fraction $f_{even}$ of CP-even final states in $B^0 \rightarrow K^+K^-K_S^0$ decays by comparing $K^+K^-K^0$ and $K^+K_S^0K_S^0$ decay rates: $f_{even} = \frac{2\Gamma(B^0 \rightarrow K^+K^-K^0)}{\Gamma(B^0 \rightarrow K^+K^-K_S^0)}$ [10]. The results listed in Table I are in agreement with Belle's measurements of 0.86 $\pm$ 0.15 $\pm$ 0.05 and 1.04 $\pm$ 0.19 $\pm$ 0.06 for the total sample and the CP sample that excludes $\phi K_S^0$ events, respectively [10]. We estimate the fraction of remaining $\phi K_S^0$ events in the CP sample, using a non-interfering Breit-Wigner for the $\phi$ shape and measured branching fractions, to be (1.1 $\pm$ 0.4)%.

As a consistency check, we examine the distribution of the cosine of the helicity angle $\theta_H$, which is defined as the angle between the $K^+$ and $B^0$ directions in the $K^+K^-$ center-of-mass frame [15]. The distribution in several $K^+K^-$ invariant mass bins of the CP sample is approximately uniform which is consistent with $S$-wave decays. However, the existence of interference effects due to CP-odd amplitudes cannot be completely ruled out with the present statistics.

If we account for a small CP-odd fraction in the CP sample, we can extract the SM parameter $\sin 2\beta$. In a fit with $C = 0$ we get $\sin 2\beta = -S/(2f_{even} - 1) = 0.57 \pm 0.26 \pm 0.04^{+0.17}_{-0.08}$, where the last error is due to uncertainty on the CP content.

Systematic uncertainties in the branching-fraction measurements are given in Table II. We include contributions from the signal reconstruction efficiency and from the modeling of the efficiency variation over the phase space. Other errors come from the fit bias, the counting of $B\bar{B}$ pairs, and the misidentification of kaons. We assume equal production rates of $B^0\bar{B}^0$ and $B^+B^-$. The systematic uncertainty on $A_{CP}$ due to charge asymmetry in track finding and identification is 0.02.

The systematic errors on the time-dependent CP-asymmetry parameters are given in Table III. The errors account for the fit bias, the presence of double CKM-suppressed decays (DCSD) in $B_{sig}$ [16], uncertainty in the beam spot and detector alignment, and the asymmetry in the tagging efficiency for signal and background events. Other smaller effects come from $\Delta t$ resolution, PDF parametrization of yield variables, and uncertainty on the $B^0$ lifetime and mixing frequency. In the fit we use $\tau_{B^0} = 1.537 \pm 0.015$ ps and $\Delta m_1 = 0.502 \pm 0.007$ ps$^{-1}$ [12].

![FIG. 2. (a),(b) The $\Delta t$ distributions of $B^0$- and $\bar{B}^0$-tagged $K^+K^-K_S^0$ events. The solid lines refer to the fit for all events; the dashed lines correspond to the background. (c) The raw asymmetry, where the solid line is obtained from the fit and the dotted line is the SM expectation for the measured CP content. The signal-to-background ratio is enhanced with a cut on the event probability.](image-url)

<table>
<thead>
<tr>
<th>Source</th>
<th>$K^+K^-K_S^0$</th>
<th>$K^+K_S^0K_S^0$</th>
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<tr>
<td>Efficiency</td>
<td>5.6</td>
<td>8.6</td>
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<td>PDF parametrization</td>
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<td>2.5</td>
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<tr>
<td>Other</td>
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<td>1.6</td>
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<tr>
<td>Total</td>
<td>6.9</td>
<td>9.6</td>
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TABLE I. Summary of branching-fraction ($B$), time-dependent ($S$, $C$), and direct CP-asymmetry ($A_{CP}$) results. $N_{sig}$ and $\varepsilon$ are the signal yield and the average total efficiency in the branching-fraction fit; $f_{even}$ is the CP-even fraction of the final states. The 90% confidence-level interval for $A_{CP}$ is $[-0.23, 0.15]$.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\varepsilon$ (%)</th>
<th>$N_{sig}$</th>
<th>$B$ (10$^{-6}$)</th>
<th>$f_{even}$</th>
<th>$S$</th>
<th>$C$</th>
<th>$A_{CP}$</th>
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<tbody>
<tr>
<td>$K^+K^-K_S^0$ (CP$^+$)</td>
<td>8.58</td>
<td>201 $\pm$ 16</td>
<td>20.2 $\pm$ 1.9</td>
<td>1.4</td>
<td>0.98 $\pm$ 0.15</td>
<td>$\pm$ 0.04</td>
<td>$-0.56 \pm 0.25 \pm 0.04$</td>
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<tr>
<td>$K^+K^-K_S^0$ (all)</td>
<td>8.78</td>
<td>249 $\pm$ 20</td>
<td>23.8 $\pm$ 2.0</td>
<td>1.6</td>
<td>0.83 $\pm$ 0.12</td>
<td>$\pm$ 0.03</td>
<td>$\cdots$</td>
</tr>
<tr>
<td>$K^+K_S^0K_S^0$</td>
<td>9.7</td>
<td>122 $\pm$ 14</td>
<td>10.7 $\pm$ 1.2</td>
<td>1.0</td>
<td>$\cdots$</td>
<td>$\cdots$</td>
<td>$-0.16 \pm 0.35$</td>
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</table>

$^a$CP excludes $\phi K_S^0$ events.
In summary, we have measured branching fractions for charmless decays of $B$ mesons into the three-body final states $B^0 \rightarrow K^+ K^0 S$ and $B^+ \rightarrow K^+ K^0 S K^0_S$. Using two independent approaches, we find that the $K^+ K^0 S$ final state is dominated by a $CP$-even component. The results agree with previous measurements [9,10]. In the first measurement of the charge asymmetry in $B^0 \rightarrow K^+ K^0 S K^0_S$ decays, we find no evidence for direct $CP$ violation. We measure a time-dependent $CP$ asymmetry in $B^0 \rightarrow K^+ K^- K^0_S$ decays at the $1.9\sigma$ level. The obtained $\sin^2 \theta$ is consistent with the SM expectation and previous measurements in decays into the $K^+ K^- K^0_S$ final state [7,8], but differs from Belle’s measurement in $\phi K^0_S$ decays [7] by 2.7 standard deviations.

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TABLE III. Systematic uncertainties in $CP$ parameters.

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<td>Fit bias</td>
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<tr>
<td>Total</td>
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<td>0.10</td>
</tr>
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</table>

*Present address: Department of Physics, University of Warwick, Coventry, United Kingdom.
†Also with Università della Basilicata, Potenza, Italy.
‡Also with IFIC, Instituto de Física Corpuscular, CSIC-Universidad de Valencia, Valencia, Spain.
§Deceased.

[1] Charge-conjugate states are included unless explicitly stated otherwise.