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Observation of the decay $B^0 \to \rho^+ \rho^-$ and measurement of the branching fraction and polarization

Charmless $B$-meson decays provide an opportunity to measure the angles of the unitary triangles constructed from the elements of the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix [1]. There has been interest in the study of $B \rightarrow \pi \pi$ and $\rho \pi$ decays, where the time-dependent CP-violating asymmetries are related to the CKM angle $\alpha = \arg(-V_{td}V_{tb}^*/V_{ud}V_{ub}^*)$, and interference between tree and loop (penguin) amplitudes could give rise to direct CP violation. The decay $B^0 \rightarrow \rho^+ \rho^-$ is another promising mode for CP-violation studies and has the advantage of a larger expected decay rate and smaller uncertainty in penguin contributions. The measurements of the amplitudes in $B$ decays to two vector particles provide additional tests of theoretical calculations [2–4].
The decay $B^0 \rightarrow \rho^+ \rho^-$ is expected to proceed through the tree-level $b \rightarrow u$ transition and through CKM-suppressed $b \rightarrow d$ penguin transitions, as illustrated in Fig. 1 [4,5]. The extraction of $\alpha$ from measurements made with this decay requires an understanding of the contributing amplitudes. It also requires proper accounting for CP-even ($S$- and $D$-wave) and CP-odd ($P$-wave) components in the decay amplitude. The recent limit on the $B^0 \rightarrow \rho^+ \rho^0$ decay rate [6] and the measurements of the $B^+ \rightarrow \rho^+ \rho^0$ branching fraction [6,7] place experimental limits on the contribution of penguin amplitudes. Measurements of the longitudinal polarization, defined as the ratio between the longitudinal and total energy and angular resolution for detection of photons with energy in the range 20 MeV to 4 GeV. The energy and angular resolutions are 3% and 4 mrad, respectively, for a 1 GeV photon.

Hadronic events are selected based on track multiplicity and event topology. We fully reconstruct $B^0 \rightarrow \rho^+ \rho^-$ candidates from the decay products of the $\rho^+ \rightarrow \pi^+ \pi^0$ and $\pi^0 \rightarrow \gamma \gamma$ decays. Charged-track candidates are required to originate from the interaction point, have at least 12 DCH hits and have a minimum transverse momentum of 0.1 GeV. Charged-pion tracks are distinguished from kaon and proton tracks with a likelihood ratio that includes $dE/dx$ information from the SVT and DCH, and, for momenta above 0.7 GeV, the Cherenkov angle and number of photons measured by the DIRC. Charged pions are distinguished from electrons primarily on the basis of their EMC shower energy and spatial profile.

We reconstruct $\pi^0$ mesons from pairs of photons. Photon candidates are required to have a minimum energy of 30 MeV, have a shower shape consistent with the photon hypothesis, and not be matched to a track. The typical experimental resolution for the measured $\pi^0$ mass is 7 MeV. We require $\pi^0$ candidates to have an invariant mass within 15 MeV of the true $\pi^0$ mass. The invariant mass of the $\rho^\pm$ candidate ($m_{\pi^0 \pi^\pm}$) is required to be in the range 0.52 to 1.02 GeV. The helicity angles $\theta_1$ and $\theta_2$ of $\rho^+ \rho^-$ are defined as the angles between the $\pi^0$ direction and the direction opposite the $B$ in each $\rho$ rest frame as shown in Fig. 2. The helicity angles are restricted to the region $-0.75 \leq \cos \theta_{1,2} \leq 0.95$ to suppress combinatorial background and reduce acceptance uncertainties due to low-momentum pion reconstruction.

The $B$ meson candidates are identified from two nearly independent kinematic observables [8], the beam energy-substituted mass $m_{ES} = [(s/2 + p_T^B)^2/E_i^B - p_B^2]^{1/2}$ and the energy difference $\Delta E = (E_i E_B - p_T^B s/2)^2/\sqrt{s}$, where $(E_i, p_i)$ is the $e^+ e^-$ initial state four-momentum, and $(E_B, p_B)$ is the four-momentum of the reconstructed $B$ candidate, all defined in the laboratory frame. For signal events, the $m_{ES}$ distribution peaks at the $B$ mass and the $\Delta E$ distribution peaks near zero. Our selection requires $m_{ES} > 5.2$ GeV and $|\Delta E| < 0.2$ GeV, while the signal resolution is roughly 3 MeV and 50 MeV, respectively. The sideband regions are defined as $5.2$ GeV $< m_{ES} < 5.27$ GeV or $0.1$ GeV $< |\Delta E| < 0.2$ GeV.

To reject the dominant continuum background (from $e^+ e^- \rightarrow q \bar{q}$ events, $q = u, d, s, c$), we require $|\cos \theta_E| < 0.8$, where $\theta_E$ is the angle between the thrust axis of the $B$ can-

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**FIG. 1.** Diagrams describing the decay $B^0 \rightarrow \rho^+ \rho^-$: (a) dominant tree diagram; (b) gluonic penguin diagram.

**FIG. 2.** Definition of helicity angles $\theta_1$, $\theta_2$, and $\phi$, for the decay $B^0 \rightarrow \rho^+ \rho^-$. The $\pi^+ \pi^0$ final states are shown in the $\rho^\pm$ rest frames.
didate and the thrust axis of the rest of the tracks and photon
candidates in the event, calculated in the c.m. frame. The
distribution of $|\cos \theta_1|$ is sharply peaked near 1.0 for jetlike
events originating from $q\bar{q}$ pairs and nearly uniform for the
isotropic decays of the $B$ meson. A Fisher discriminant ($F$)
combines 11 observables: the polar angle of the $B$
combinatorial background. The likelihood for each $B^0
\rightarrow \rho^+\rho^-$ candidate $i$ is defined as

$$L_i = \sum_{j=1}^{3} n_j P_j(\vec{x}_i; \vec{\beta}), \quad (1)$$

where each of the $P_j(\vec{x}_i; \vec{\beta})$ is the probability density
function (PDF) for seven observables $\vec{x}_i$ ($m_{ES}$, $\Delta E$, $F$, $m_{\pi^+\pi^0}$, $m_{\pi^-\pi^0}$, $\theta_1$, $\theta_2$) and is described by the PDF parameters $\vec{\beta}$. The
event yields $n_j$ for each category $j$ are free parameters in the
fit. We allow for multiple candidates in a given event by
assigning to each selected candidate a weight of $1/N_i$, where
$N_i$ is the number of candidates in that event. The average
count of candidates per event is 1.27. MC simulation shows
that this procedure does not introduce bias while provid-
ing a statistical improvement over the random
choice of a candidate in a given event. The extended likeli-
hood for a sample of $N_{\text{cand}}$ candidates is

$$L = \exp \left( -\sum_{j=1}^{3} n_j \prod_{i=1}^{N_{\text{cand}}} \exp \left( \frac{\ln L_i}{N_i} \right) \right), \quad (2)$$

The correlations among the input observables $\vec{x}_i$ are found
to be small for both the background ($<5\%$) and signal
($<10\%$), except for angular correlations in the signal. The
$P_j(\vec{x}_i; \vec{\beta})$, for a given candidate $i$, is the product of PDFs for
each of the observables and a joint PDF for the helicity
angles, which accounts for the angular correlations in the
signal and for detector acceptance effects. We integrate over
the angle $\phi$ between the two decay planes shown in Fig. 2,
leaving a PDF that depends only on $\theta_1$, $\theta_2$, and the
unknown longitudinal polarization $f_L$. The differential decay
rate [2] is

$$\frac{1}{\Gamma} \frac{d^2\Gamma}{d\cos \theta_1 d\cos \theta_2} = \frac{9}{4} \left( 1 - f_L \right) \frac{1}{4} \sin^2 \theta_1 \sin^2 \theta_2$$

$$+ f_L \cos^2 \theta_1 \cos^2 \theta_2. \quad (3)$$

The PDF parameters $\vec{\beta}$, except for $f_L$, are extracted from
MC simulation and on-resonance $m_{ES}$ and $\Delta E$ sidebands,
and are fixed in the fit. The resolutions are adjusted by com-
paring data and simulation in calibration channels with simi-
lar kinematics and topology, such as $B \rightarrow D^0 \pi^0$, $D \pi^+$ with
$D \rightarrow K^+ \pi^- (\pi^0), K^0 \pi^- (\pi^0), K^+ \pi^- \pi^-, K^0 \pi^- \pi^-$. To
describe the signal distributions, we use Gaussian functions for
the parametrization of the PDFs for $m_{ES}$ and $\Delta E$, and a
relativistic $P$-wave Breit-Wigner distribution for the $\rho^+$ reso-
nance masses. The angular acceptance effects are parametri-
ized with empirical polynomial functions for each helicity
angle and are included in the joint helicity-angle PDF as a
factor multiplying the ideal distribution in Eq. (3).

For the background PDFs, we use polynomials or, in the
case of $m_{ES}$, an empirical phase-space function [11]. In the
background PDF we incorporate a small linear correlation
between the curvature $\xi$ of the phase-space function and the
value of $F$. The background parametrizations for the $\rho^+$ can-
didate masses also include a resonant component to account
for $\rho^+$ production. The background helicity-angle distribu-
tion is obtained from contributions from combinatorial
background from real $\rho^+$ mesons, both described by
polynomials. For both signal and background, the PDF for $F$
is represented by a Gaussian distribution with different
widths above and below the peak.

PDF parameters for the background from other $B$ decays
are determined from MC simulation. The contribution from
charmless $B$ decays with similar topology (cross-feed modes)
such as $B \rightarrow \rho \pi$, $\rho^0 \rho^+$, $\rho K^\pm$, $a_1 \pi$, and $a_1 \rho$ is estimated
with MC modeling and is fixed in the fit. Each branching
fraction for the cross-feed modes is estimated to be in the
range $(1-3) \times 10^{-5}$. The branching fractions for these and
many other modes are taken from the most recent measure-
ments [6,7,12] or extrapolated from other results with a
flavor-SU(3)-symmetry approximation.

The selected $B^0 \rightarrow \rho^+ \rho^-$ events fall into three categories.
MC simulation of events with longitudinal polarization
shows that roughly $30\%$ of the events contain only misrecon-
structed candidates. Approximately $20\%$ of the events con-
tain both correctly and incorrectly reconstructed candidates.
The remainder contain only correct candidates. Misrecon-
struction occurs when at least one candidate photon in a $\pi^0$
candidate or one charged track in a $\rho$ candidate belongs to
the decay products of the other $B$. The distributions that
show peaks for correctly reconstructed events have substan-
tial tails, with large uncertainties in MC simulation, when
misreconstructed events are included. These tails would re-
duce the power of the distributions to discriminate between
the background and the collection of correctly and incor-
correctly reconstructed events. We choose, therefore, to repre-
sent only the correctly reconstructed candidates in the signal
PDF. Misreconstructed candidates are predominantly accom-
TABLE I. Summary of the fit results; \( n_{\text{sig}} \) is the fitted number of signal events, \( S \) is the significance, \( f_L \) is the longitudinal polarization, \( \varepsilon \) denotes the reconstruction efficiency, and \( B \) is the branching fraction of the \( B^0 \to \rho^+ \rho^- \) decays. The first uncertainty is statistical and the second systematic. The efficiency (\( \varepsilon \)) and significance (\( S \)) include systematic uncertainties, and the significance without systematics is given in parentheses.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Measured value</th>
</tr>
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<tbody>
<tr>
<td>( n_{\text{sig}} )</td>
<td>( 88_{-21}^{+21} \pm 9 )</td>
</tr>
<tr>
<td>( S )</td>
<td>( 5.1 \sigma ) (5.5 ( \sigma ))</td>
</tr>
<tr>
<td>( f_L )</td>
<td>( 0.98^{+0.02}_{-0.08} \pm 0.03 )</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>( 3.9^{+0.9}_{-0.6} % )</td>
</tr>
<tr>
<td>( B )</td>
<td>( (25_{-6}^{+4}) \times 10^{-6} )</td>
</tr>
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</table>

modulated by the combinatorial background PDF. Fitting to determine the number of correctly reconstructed candidates has an efficiency less than 100% since some fraction of the events have both correctly and incorrectly reconstructed candidates. Monte Carlo simulation finds this efficiency to be 87%.

In this analysis, we do not include a fit component for other \( B \) decays with the same final-state particles selected within the \( \rho \) resonance mass window, such as nonresonant decays \( B^0 \to \pi^+ \pi^- \pi^0 \pi^0 \) and \( B^0 \to \rho^+ \rho^- \pi^0 \). The contribution of these decays to the fit results is significantly suppressed by the selection requirements on the masses and by the mass and helicity-angle information in the fit; they are examined in the context of mass and helicity-angle distributions, as discussed below.

The event yields \( n_j \) and polarization \( f_L \) are obtained by minimizing the quantity \( \chi^2 = -2 \ln L \). The dependence of \( \chi^2 \) on a fit parameter \( n_j \) or \( f_L \) is obtained with the other fit parameters floating. Their values are constrained to the physical range \( n_j \geq 0 \) and \( 0 \leq f_L \leq 1 \). Statistical uncertainties correspond to a unit increase in \( \chi^2 \). The statistical significance of the signal is defined as the square root of the change in \( \chi^2 \) when the number of signal events is constrained to zero in the likelihood fit.

The results of our maximum-likelihood fits are summarized in Table I. The statistical significance of the \( B^0 \to \rho^+ \rho^- \) signal is 5.5 \( \sigma \). We find that the \( \rho \) mesons in \( B^0 \to \rho^+ \rho^- \) decays are almost fully longitudinally polarized. To compute the branching fraction, equal production rates for \( B^0 B^0 \) and \( B^+ B^- \) are assumed. To check the stability of our results we refit, removing each observable from the fit in turn, and find consistent results. The measured uncertainties in the number of fitted events and the polarization, the statistical significance, and the fit \( \chi^2 \) value are well reproduced with generated MC samples.

The projections of the fit input observables are shown in Fig. 3. The projections are made after a requirement on the signal-to-background probability ratio \( \mathcal{P}_{\text{sig}}(x_i; \beta)/ \mathcal{P}_{\text{bkg}}(x_i; \beta) \), where \( \mathcal{P}_{\text{sig}} \) and \( \mathcal{P}_{\text{bkg}} \) are the signal and the dominant continuum background PDFs defined in Eq. (1), but with the PDF for the plotted observable excluded. The points with error bars show the data with (40–60\%) of signal retained, while the lines show the corresponding PDF projections.

To check the sensitivity of our results to the presence of nonresonant \( B^0 \to \pi^+ \pi^- \pi^0 \pi^0 \) and \( B^0 \to \rho^+ \rho^- \pi^0 \) decays, we explicitly include a fit component for them, assuming a phase-space decay model. The selection requirements alone suppress the \( B \to 4 \pi \) \( (B \to \rho \pi \pi \pi) \) efficiency by two (one) orders of magnitude relative to \( B^0 \to \rho^+ \rho^- \). The fit results with a nonresonant component indicate a potential \( B \to \rho \pi \pi \pi \) contribution of \( (10 \pm 10)\% \) (statistical uncertainty only) of our nominal \( B^0 \to \rho^+ \rho^- \) event yield in Table I; interference effects between the resonant and nonresonant components were ignored in this fit. The hypothesis that all the signal is nonresonant \( B \to 4 \pi \) \( (B \to \rho \pi \pi \pi) \) is excluded with 5.1 \( \sigma \) (4.4 \( \sigma \)) statistical significance. These results are consistent with our assumption that the nonresonant contribution is negligible.

The systematic uncertainty in the fitted number of signal events \( (n_{\text{sig}}) \) originates from the uncertainty in the cross-feed \( B \)-decay modeling, which was studied with MC generated samples and estimated to be half of the variation with cross-feed set to zero (3\% uncertainty in \( n_{\text{sig}} \)). Systematic uncertainties in the fit originate from assumptions about the background and signal PDF parameters. Uncertainties in the PDF parameters arise from the limited number of events in the background sideband data and signal control samples. We vary them within their respective uncertainties, and derive the associated systematic uncertainty on the event yield (9\%). The signal remains statistically significant with these variations (5.1 \( \sigma \) including systematics).
The systematic uncertainties in the efficiency ($e$) are due to track finding (2% for two tracks), particle identification (2% for two tracks), and $\pi^0$ reconstruction (13% for two $\pi^0$s). The fit efficiency is less than 100% because of misconstructed signal events. This has an additional systematic uncertainty due to uncertainties in the modeling of misconstructed events. We account for this with a systematic uncertainty on the efficiency of 7%, which is half of the inefficiency; the fit efficiency cannot exceed 100% and the frequency of multiple candidate selection is estimated in the $B$ decay control samples. The reconstruction efficiency depends on the decay polarization. We calculate the efficiencies using the measured polarization and assign a systematic uncertainty ($+17\%$) corresponding to the total polarization measurement uncertainty. Smaller systematic uncertainties arise from event-selection criteria, MC statistics, and the number of produced $B$ mesons.

For the polarization measurement ($f_L$), we include systematic uncertainties from detector MC variations that account for uncertainties in the detector acceptance, estimated with MC, and background parametrizations. This results in a total absolute uncertainty of 0.025. The biases from the resolution in helicity-angle measurement and dilution due to the presence of the misconstructed combinations are studied with MC simulation and give a systematic uncertainty of 0.02.

Observation of the $B^0 \rightarrow \rho^+ \rho^-$ decay completes a first set of measurements of the isospin-related $B \rightarrow \rho \rho$ modes [6,7]. The measured branching fraction is consistent with recent predicted values in the range ($18-35) \times 10^{-6}$ [4] and the dominant longitudinal polarization implies a suppression of the transverse amplitude, which is expected to be suppressed by a factor of $m_{\rho}/m_{B}$ [4]. The rates of the $B^0 \rightarrow \rho^+ \rho^-$ and $B^+ \rightarrow \rho^0 \rho^+$ decays appear to be larger than the corresponding rates of $B \rightarrow \pi \pi$ decays [12]. At the same time, the recent measurement of the $B^+ \rightarrow \rho^0 \ell^+ \ell^-$ branching fraction [6] does not show significant enhancement with respect to $B \rightarrow \pi K$ decays [12], both of which are expected to be dominated by $b \rightarrow s$ penguin diagrams. We can use flavor SU(3) to relate $b \rightarrow s$ and $b \rightarrow d$ penguin diagrams analogous to Fig. 1(b) [13]; the measured branching fractions indicate that the relative penguin contributions in the $B \rightarrow \rho \rho$ decays are smaller than in the $B \rightarrow \pi \pi$ case.

We make a more quantitative estimate of penguin contributions in $B \rightarrow \rho \rho$ decays using our previous measurements of $B^0 \rightarrow \rho^+ \rho^-$ and $B^+ \rightarrow \rho^+ \rho^0$ branching fractions and polarization [6]. Since the tree contribution to the $B^0 \rightarrow \rho^0 \rho^0$ decay is color-suppressed, the decay rate is sensitive to the penguin diagram analogous to Fig. 1(b). Using the earlier $BABAR$ measurements [6], we obtain a 90% confidence level (C.L.) upper limit on the ratio of the longitudinal amplitudes $A_L$ in the $B \rightarrow \rho \rho$ decays:

$$\frac{|A_L(B^0 \rightarrow \rho^0 \rho^0)|^2 + |A_L(B^+ \rightarrow \rho^0 \rho^+)|^2}{|A_L(B^0 \rightarrow \rho^0 \rho^+)|^2 + |A_L(B^+ \rightarrow \rho^0 \rho^-)|^2} \leq \frac{B(B^0 \rightarrow \rho^0 \rho^0) \times f_L(B^0 \rightarrow \rho^0 \rho^0)}{B(B^+ \rightarrow \rho^0 \rho^+) \times f_L(B^+ \rightarrow \rho^0 \rho^+)} < 0.10.$$ (4)

In the above calculation we conservatively assume that the $B^0 \rightarrow \rho^0 \rho^0$ decay polarization is fully longitudinal ($f_L = 1$) and use the average branching fraction measurements for the $B$ and $\bar{B}$ decays. From Eq. (4) we can deduce the uncertainty from penguin contributions for future measurements of $\alpha$ based on the time-dependence of longitudinally-polarized $B^0 \rightarrow \rho^+ \rho^-$ decays using isospin relations analogous to those discussed in the context of $B \rightarrow \pi \pi$ [14]. In the event that for the $\rho^0 \rho^0$ final state we have only the upper bound Eq. (4), the induced uncertainty in $\alpha$ is $19\%$ at 90% C.L. neglecting the nonresonant and $I=1$ isospin contributions as discussed in Ref. [15].

In summary, we have observed $B^0 \rightarrow \rho^+ \rho^-$, measured its branching fraction $B = (25 \pm 5_{-6}^{+8}) \times 10^{-6}$, and determined the longitudinal polarization fraction $f_L = 0.98_{-0.08}^{+0.02} \pm 0.03$. Our quantitative estimates of penguin contributions in $B^0 \rightarrow \rho^+ \rho^-$ decays and the dominance of the CP-even longitudinal polarization make this decay a promising channel for the measurement of the CKM angle $\alpha$.

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[5] We use the name of one member of a charge conjugate pair to refer to both unless explicitly stated otherwise.

[11] With $x = \frac{2m_{ES}}{\sqrt{s}}$ and $\xi$ a parameter to be fit, $f(x) \propto x \sqrt{1-x^2} \exp\left(-\xi(1-x^2)\right)$; see ARGUS Collaboration, H. Albrecht et al., Phys. Lett. B 241, 278 (1990).


