Branching fraction measurements of $B \to \eta K$ decays

We study the decays $B^+ \to \eta K^+$ and $B^0 \to \eta K^0$, where the $\eta$ is reconstructed in the $K^0_S K^+ K^-$ or $K^+ K^- \pi^0$ decay modes. Results are based on a sample of 86 million $B\bar{B}$ pairs collected with the BABAR detector at the SLAC $e^+e^- B$ Factory. We measure the product of branching fractions $B(B^+ \to \eta K^+) \times B(\eta \to K\bar{K}\pi) = (7.40 \pm 0.50 \pm 0.70) \times 10^{-5}$ and $B(B^0 \to \eta K^0) \times B(\eta \to K\bar{K}\pi) = (6.48 \pm 0.85 \pm 0.71) \times 10^{-5}$, where the first error is statistical and the second is systematic. In addition, we search for $B \to \eta K$ events with $\eta \to 2(K^+ K^-)$ and $\eta \to \phi \phi$ and determine the $\eta_-$ decay branching fraction ratios $B(\eta_- \to 2(K^+ K^-))/B(\eta_\pi \to K\bar{K}\pi) = (2.3 \pm 0.7 \pm 0.6) \times 10^{-2}$ and $B(\eta_\pi \to \phi \phi)/B(\eta_\pi \to K\bar{K}\pi) = (5.5 \pm 1.4 \pm 0.5) \times 10^{-2}$.

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The decay $B \to \eta K$ is used to measure $\sin 2\beta$ [1,2], but is interesting dynamically as well. The ratio of its decay rate to that of $B \to J/\psi K$ reflects the underlying strong dynamics and can be used to check models of heavy quark systems.

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The $b \rightarrow c \bar{c}s$ weak decay should respect isospin invariance, an expectation that can be checked and then used to combine results for higher precision. It is therefore interesting to measure accurately the branching fractions for $B^0 \rightarrow \eta_s K^0$ and $B^+ \rightarrow \eta_s K^+$ [8].

We use data collected with the BABAR detector at the PEP-II energy-asymmetric $e^+ e^-$ storage rings. The data sample contains $86.1 \times 10^6 B \bar{B}$ pairs, corresponding to an integrated luminosity of $79.4 \text{ fb}^{-1}$ taken at a center-of-mass energy equivalent to the mass of the $Y(4S)$ resonance. An additional $9.6 \text{ fb}^{-1}$ of data, collected $40 \text{ MeV}$ below the resonance, is used to study the background from light quark and $c \bar{c}$ production.

A detailed description of the BABAR detector can be found elsewhere [9]; only detector components relevant to this analysis are mentioned here. Charged-particle trajectories are measured by a five-layer double-sided silicon vertex tracker (SVT) and a 40-layer drift chamber (DCH), operating in the field of a 1.5-T solenoid. Charged particles are identified by combining measurements of ionization energy loss (dE/dx) in the DCH and SVT with angular information from a detector of internally reflected Cherenkov light (DIRC). Photons are identified as isolated electromagnetic showers in a CsI(Tl) electromagnetic calorimeter.

In this analysis, $\eta_s$ mesons are reconstructed in the $K^0_s K^\pm \pi^\mp$, $K^+ K^—\pi^0$, $2(K^+ K^-)$ and $\phi \phi$ decay modes. Candidates for $K^0_s$ are identified through the decay $K^0_s \rightarrow \pi^+ \pi^-$, $\phi$ candidates through $\phi \rightarrow K^+ K^—\pi^0$ and $\pi^0$ candidates through $\pi^0 \rightarrow \gamma \gamma$. Note that $\eta_s$ decays to $2(K^+ K^-)$ include both nonresonant and resonant ($\phi \phi$, $\phi K^+ K^-$) components, so we expect a partial overlap of the $2(K^+ K^-)$ and $\phi \phi$ samples.

We require that charged tracks, other than those used to reconstruct $K^0_s \rightarrow \pi^+ \pi^-$ candidates, have a minimum transverse momentum of 0.1 GeV/$c$, and that they originate from the interaction point to within 10 cm along the beam direction and 1.5 cm in the transverse plane. The “fast” kaon candidate at the two-body $B^+ \rightarrow D^+ \pi^-$ decay vertex is required to have at least 12 hits in the drift chamber and to have a momentum in the $Y(4S)$ rest frame larger than $1.5 \text{ GeV}/c$.

The cuts used to select $K^+$, $K^0_s$, $\phi$ and $\pi^0$ candidates from $\eta_s$ decays are described below. They are optimized to maximize the statistical sensitivity of the signal, defined as $S/\sqrt{S+B}$, with $S$ and $B$ being the estimated numbers of signal and combinatorial-background events.

All charged-kaon candidates are required to have momentum greater than 250 MeV/$c$ and a polar angle between 0.35 and $2.54 \text{ rad}$ with respect to the detector axis, to restrict them to a fiducial region where the particle identification performance can be determined with small uncertainty. Kaon identification is based on a neural network (NN) algorithm [10] that combines information from the DCH, the SVT, and the DIRC. Particle identification criteria are crucial for background suppression, especially for $\eta_s \rightarrow 2(K^+ K^-)$ decays. In this channel, three of the four kaons must pass a tight cut on the NN output variable. Less restrictive requirements on the NN signature are used for identifying the fourth kaon from $\eta_s \rightarrow 2(K^+ K^-)$ candidates, the charged kaons in the other $\eta_s$ decay modes, and the “fast” kaon from $B^+ \rightarrow \phi$ decays. The kaon-identification efficiency depends on the momentum and polar angle of the track, as well as on the chosen NN cut. For the tightest selection above, the average kaon efficiency exceeds 85%; the corresponding pion-rejection efficiency is about 98%.

We reconstruct $K^0_s$ candidates from pairs of oppositely charged tracks fitted to a common vertex. We require the $K^0_s$ candidate from the $B (\eta_s)$ decay to have a reconstructed mass within $13 (16) \text{ MeV}/c^2$ of the $K^0_s$ mass [11]. Furthermore, the cosine of the opening angle between the flight direction and the momentum vector of the $K^0_s$ candidate is required to be larger than 0.9995 (0.9930), and the flight distance from the $B$ vertex larger than four times its error.

We reconstruct $\phi$ candidates from pairs of oppositely charged kaons with an invariant mass within $14 \text{ MeV}/c^2$ of the $\phi$ mass [11].

We use pairs of photons to reconstruct $\pi^0 \rightarrow \gamma \gamma$ candidates, requiring a minimum energy of 120 MeV for one photon and $80 \text{ MeV}$ for the other. The reconstructed $\gamma \gamma$ mass is required to lie within $18 \text{ MeV}/c^2$ of the $\pi^0$ mass [11].

We reconstruct $\eta_s$ candidates by fitting the appropriate combination of charged tracks, $K^0_s$, $\phi$, or $\pi^0$ candidates to a common vertex. Neutral or charged $B$ candidates are formed from reconstructed $\eta_s$ and $K^0_s$ or $K^+$ candidates. In reconstructing the $B$ decay chain, the measured momentum vector of each intermediate particle is determined by refitting the momenta of its daughters, constraining the mass to the nominal mass of the particle, and requiring that the decay products originate from a common point. In the case of the $\eta_s$, only the geometrical vertex constraint is applied because of the large intrinsic width of the resonance. Charmion candidates are accepted if they have an invariant mass between 2.7 and 3.3 GeV/$c^2$. Note that this procedure also reconstructs $J/\psi$ decays, which are used to measure the mass resolution and for other cross-checks.

We use a Fisher discriminant [12] to suppress $e^+ e^- \rightarrow q \bar{q}$ background processes. Our Fisher discriminant is a linear combination of 18 variables, the most important of which are the normalized second Fox-Wolfram moment and the angle between the thrust axis of the $B$ candidate and that of the rest of the event. Also contributing are the energy flow in nine $10^\circ$ polar angle intervals coaxial around the $\eta_s$ direction in the center-of-mass frame [13], the polar angles of the $B$ candidate and of the overall thrust axis, and other event-shape variables that distinguish between $B \bar{B}$ events and continuum background. The discriminant is tuned on simulated signal events and on off-resonance data to achieve maximum separation between signal and continuum background. Fisher coefficients are determined individually for each $\eta_s$ decay mode, and threshold values are set as part of the cut-optimization procedure described earlier.

We select $B$ candidates using two nearly independent kinematic variables: $m_{\text{ES}}$, the beam-energy-substituted mass, and $\Delta E$, the difference between the energy of the $B$ and the beam energy in the center-of-mass frame [9]. The $m_{\text{ES}}$ resolution is $2.6 \text{ MeV}/c^2$, dominated by the beam-energy spread. The $\Delta E$ resolution varies from 15 to $28 \text{ MeV}$, depending on
the $\eta_c$ decay mode. Signal events are expected to have $m_{ES}$ close to the $B$ mass and $\Delta E$ close to zero. Our selection requires $5.2 < m_{ES} < 5.3$ GeV/$c^2$. After all the cuts listed above, 10–25% of the selected events, depending on the $\eta_c$ decay channel, contain more than one $B \rightarrow \eta_c K$ candidate in a $\Delta E$ window $\pm 250$ MeV wide; we then retain only the candidate with the smallest value of $|\Delta E|$. We have verified with simulated events that this procedure selects the correct candidate in 90–98% of the cases, and that it does not bias the measurement. Finally, we require candidates to lie within an optimized interval of $\Delta E$ that varies from $\pm 35$ to $\pm 70$ MeV, depending on the decay mode.

Events surviving the full selection chain originate from four different sources: $B \rightarrow \eta_c K$ decays, i.e., the signal; $B \rightarrow J/\psi K$ decays, with the $J/\psi$ decaying into the same final state as the $\eta_c$; a combinatorial background, arising from random track combinations in continuum and in $BB$ final states; and a background component from other $B$ decays to the same final state particles as the $B \rightarrow \eta_c K$ decay mode under consideration. The last background component can contribute events that cluster at the signal peak in $m_{ES}$ and $\Delta E$ and is therefore termed “peaking background.”

Examples of peaking background for $B \rightarrow \eta_c K (\eta_c \rightarrow K_S^0 K^- \pi^+)$ are $B^+ \rightarrow K^+ K^- K^+ (K^0 \rightarrow K_S^0 \pi^-)$ or $B^0 \rightarrow K^+ K^- K^0 (K^0 \rightarrow K_S^0 \pi^+)$ in the particular case of the decay $B^+ \rightarrow \eta_c K^+$ (and $\eta_c \rightarrow K_S^0 K^- \pi^+$), another important source of peaking background comes from $B^+ \rightarrow D^0 K^0_S K^+ (D^0 \rightarrow K^- \pi^+)$. For this decay mode, therefore, candidates with a $K^+ \pi^-$ invariant mass within 15 MeV ($3\sigma$) of the $D^0$ mass are explicitly vetoed. Other processes, such as nonresonant $B$ decays to the selected final state, whose branching fractions are not well known, can also contribute. The mass $m_X$ of the system recoiling against the fast kaon is used to separate $B \rightarrow \eta_c K$ and $B \rightarrow J/\psi K$ events, which peak at the mass of the corresponding charmonium system, from the peaking (in $m_{ES}$ and $\Delta E$) background, which is expected to exhibit a linear dependence on $m_X$. This assumption is verified with large samples of simulated $BB$ events. These studies also show that inclusive $B$ decays into $\eta_c$, and potential cross-feed among different $\eta_c$ decay modes are negligible after the event selection.

The number of signal events is determined from an unbinned maximum-likelihood fit to the joint $m_{ES}$ and $m_X$ distribution. Four hypotheses are considered to build the 2D likelihood function: $B \rightarrow \eta_c K$, modeled by the product of a Gaussian resolution function in $m_{ES}$ and of a nonrelativistic Breit-Wigner function convoluted with a Gaussian resolution function in $m_X$; $B \rightarrow J/\psi K$ component, given by the product of Gaussian resolution functions in $m_{ES}$ and $m_X$; combinatorial background, modeled by an “ARGUS” end-point function in $m_{ES}$ [14], and a linear function in $m_X$; and peaking background, described by a function linear in $m_X$ and Gaussian in $m_{ES}$. The widths of the $m_X$ and $m_{ES}$ resolution functions, and the mean value of the $m_{ES}$ distribution are free parameters common to the $\eta_c$ and $J/\psi$ probability density functions (p.d.f.). The latter two parameters also determine the $m_{ES}$ dependence of the peaking-background p.d.f., reflecting the evidence that this background is dominated by $B$ decays to the same final states as the signal. We set the $\eta_c$ and $J/\psi$ masses to their world-average values [11], the end point of the combinatorial background function to 5.29 GeV/$c^2$, and the $\eta_c$ width to the value recently measured by BABAR [15]. All other parameters and the number of events in the different components are determined by the fit, which is performed separately for each decay channel.

For $B \rightarrow \eta_c K$ modes with $\eta_c \rightarrow K\bar{K}\pi$, candidates are weighted to take into account small efficiency variations across the $\eta_c$ Dalitz plot. The weighting procedure compensates for any resonant structure in $\eta_c$ three-body decays unaccounted for by the simulated phase-space distribution, which is uniform over the Dalitz plot. Since all weights are close to one, they do not affect the shape of the different components and have only a marginal influence (0.6–4%) on the fitted event yield. Samples of simulated events are used to verify that the likelihood fit is unbiased.

The measured $\eta_c$ signal yields are reported in Table I. We observe a significant signal in all modes with the exception of $B^0$ with $\eta_c$ decaying into $2(K^+ K^-$) and $\phi \phi$. The $m_{ES}$ distributions of $B^+ \rightarrow \eta_c K^+$ candidates are shown in Fig. 1. In the largest sample ($\eta_c \rightarrow K_S^0 K^- \pi^+$) we can determine the $\eta_c$ width $\Gamma(\eta_c)$ from a simultaneous fit to neutral and charged $B$ data, shown in Fig. 2(a). We find $\Gamma(\eta_c) = 39.7 \pm 6.6$ MeV/$c^2$, where the error is statistical only, consistent with the BABAR measurement, $\Gamma(\eta_c) = 34.3 \pm 2.3 \pm 0.9$ MeV/$c^2$ [15]. The $m_X$ distributions for the other $\eta_c$ decay modes are shown in Fig. 2(b,c,d).

The systematic uncertainty associated with the fitted signal yield includes three components: the uncertainty in the fixed parameters, the uncertainty associated with the Dalitz weighting procedure, and the uncertainty associated with the p.d.f. models. The first component is evaluated by varying each fixed parameter, one at a time, by one standard deviation and repeating the fit. This component is dominated by the uncertainty on $\Gamma(\eta_c)$ (0–3% fractional uncertainty in $B$, depending on the mode). For the second component, the fit is

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### Table I. Number of $B \rightarrow \eta_c K$ events and statistical significance $S$

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\eta_c$ yield</th>
<th>$S$</th>
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<tbody>
<tr>
<td>$B^+ \rightarrow \eta_c K^+$</td>
<td>306.4 $\pm$ 24.4 $\pm$ 14.0</td>
<td>20.5</td>
</tr>
<tr>
<td>$\eta_c \rightarrow K_S^0 K^- \pi^+$</td>
<td>136.8 $\pm$ 17.5 $\pm$ 9.3</td>
<td>11.7</td>
</tr>
<tr>
<td>$\eta_c \rightarrow K^+ K^- \pi^0$</td>
<td>26.2 $\pm$ 8.4 $\pm$ 4.5</td>
<td>4.5</td>
</tr>
<tr>
<td>$\eta_c \rightarrow \phi \phi$</td>
<td>19.1 $\pm$ 4.9 $\pm$ 0.6</td>
<td>6.6</td>
</tr>
<tr>
<td>$B^0 \rightarrow \eta_c K^0$</td>
<td>79.4 $\pm$ 12.7 $\pm$ 4.3</td>
<td>9.7</td>
</tr>
<tr>
<td>$\eta_c \rightarrow K_S^0 K^- \pi^+$</td>
<td>40.9 $\pm$ 9.5 $\pm$ 2.7</td>
<td>6.2</td>
</tr>
<tr>
<td>$\eta_c \rightarrow K^+ K^- \pi^0$</td>
<td>3.9 $\pm$ 3.7 $\pm$ 1.6</td>
<td>1.4</td>
</tr>
<tr>
<td>$\eta_c \rightarrow \phi \phi$</td>
<td>3.0 $\pm$ 1.7 $\pm$ 0.1</td>
<td>3.6</td>
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repeated without applying the Dalitz-correction procedure; half of the difference on the $\eta_c$ signal yield (0–2% in $B$) is conservatively assigned as the corresponding systematic uncertainty. The last component is dominated by the uncertainty in the peaking background model. This error is evaluated by varying the assumed $m_X$ dependence from a first- to a second-order polynomial; it typically amounts to 4%, and exceeds 10% only for the $\eta_c \rightarrow 2(K^+K^-)$ modes. The error associated with the $m_{ES}$ resolution function model (0–5%) is estimated by using, instead of a single Gaussian function fitted to the data, double-Gaussian resolution functions fitted to each simulated signal sample.

Efficiencies are computed with simulated signal events that are reconstructed and selected using the same procedure as for the data, including the yield-extraction fit. We apply small corrections, determined from data, to the efficiency calculation to account for the overestimation of the tracking and particle-identification performance, and of the $\pi^0$ and $K^0_S$ reconstruction efficiencies. A systematic uncertainty is assigned to each correction to account for the limited size and purity of the control sample used in computing that correction. For example, the track finding efficiency corrections are determined from multihadron events in the data [16]. For the fast kaon identification, we correct the simulation using a pure sample of $D^{*+} \rightarrow \pi^+ D^0$ decays with $D^0 \rightarrow K^- \pi^+$. We include in the particle-identification systematic uncertainty contributions associated with the sample size, the background subtraction, and the different kinematics of this decay chain compared to the two-body $B^+ \rightarrow \eta_c K^+$ decay. Similarly, corrections affecting the $\pi^0$ reconstruction are calibrated using real and simulated $e^+ e^- \rightarrow \tau^+ \tau^-$ and multihadron samples.

In order to quantify the ability of the simulation to model the kinematic and event-shape variables used in the event selection, we compare our signal simulation, after all corrections, to appropriate control samples with similar kinematics or final-state topology. The small residual differences in the efficiencies at the cut value are assigned as systematic uncertainties affecting the selection procedure.

Finally, we assign a systematic uncertainty to the yield-extraction fit by evaluating the influence of mixing background events with simulated signal events. Values for the efficiencies, the corrections, and the corresponding systematic uncertainties are reported in Table II.

The results on the products of the branching fractions for each mode are listed in Table III. We use the world-average values for the $K_{S}^{0} \rightarrow \pi^{+} \pi^{-}$, $\pi^{0} \rightarrow \gamma \gamma$ and $\phi \rightarrow K^+K^-$ branching fractions [11] and include their uncertainties in the systematic error. The systematic error also comprises the uncertainties from the determination of the number of $B\overline{B}$ pairs (1.1%), from the likelihood fit, and from the signal efficiency. We assume that the branching fraction of the $Y(4S)$ into $B\overline{B}$ is 100%, with an equal admixture of charged and neutral $B$ final states. We do not include any additional uncertainty due to these assumptions. Possible interference effects between the $B \rightarrow \eta_c K$ signal and the peaking background are neglected.

The decay amplitudes for $\eta_c \rightarrow K^+K^-\pi^0$ and $\eta_c \rightarrow K^0K^-\pi^+$ are related by isospin symmetry. The expected ratio of branching fractions, using the appropriate Clebsch-Gordon coefficients, is $0.25$. Our measurements are consis-
tent with this value for both the $B^+$ (0.27±0.04±0.03) and the $B^0$ (0.26±0.07±0.03) sample. We therefore combine our results for these modes to obtain $\mathcal{B}(B^+ \to \eta K^0) \times \mathcal{B}(\eta \to K \bar{K} \pi) = (7.40\pm 0.50 \pm 0.70) \times 10^{-5}$ and $\mathcal{B}(B^0 \to \eta K^0) \times \mathcal{B}(\eta \to K \bar{K} \pi) = (6.48\pm 0.85 \pm 0.71) \times 10^{-5}$. In the combination we separate correlated and uncorrelated uncertainties to weight the individual results and obtain the total systematic error. Using the world average for the $\eta \to K \bar{K} \pi$ branching fraction (0.055±0.017 [11]) we derive

$$\mathcal{B}(B^+ \to \eta K^0) = (1.34\pm 0.09 \pm 0.13 \pm 0.41) \times 10^{-3},$$

$$\mathcal{B}(B^0 \to \eta K^0) = (1.18\pm 0.16 \pm 0.13 \pm 0.37) \times 10^{-3},$$

where the first error is statistical, the second systematic, and the third due to the uncertainty on the $\eta \to K \bar{K} \pi$ branching fraction. We also compute the ratio of neutral over charged $B$ decays $\mathcal{B}(B^+ \to \eta K^0)/\mathcal{B}(B^+ \to \eta K^+)$ = 0.87±0.13±0.07, and, multiplying by the mean lifetime ratio $\tau_{B^+}/\tau_{B^0}$ = 1.085±0.017 [11], we derive the ratio of partial widths

$$\Gamma(B^0 \to \eta K^0)/\Gamma(B^+ \to \eta K^+) = 0.94\pm 0.14\pm 0.08.$$

To determine $R_K = \Gamma(B \to \eta K)/\Gamma(B \to J/\psi K)$, we use the BABAR measurements [16] of the branching fractions, $\mathcal{B}(B^+ \to J/\psi K^+) = (10.1\pm 0.3 \pm 0.5) \times 10^{-4}$ and $\mathcal{B}(B^0 \to J/\psi K^0) = (8.5\pm 0.5 \pm 0.6) \times 10^{-4}$. We obtain

$$R_K(B^+) = 1.33\pm 0.10\pm 0.12\pm 0.41,$$

where the first error is statistical, the second systematic, and the third due to $\eta \to K \bar{K} \pi$ branching fraction. Our results agree with most predictions for $R_K$, which range from 0.9 to 2.3 [3–7].

The measured values of $\mathcal{B}(\eta \to 2(K^+ K^-))$ and $\mathcal{B}(\eta \to \phi \phi)$ have higher uncertainties and therefore these modes are not used for averages. We can express our $\eta \to 2(K^+ K^-)$ and $\eta \to \phi \phi$ results in terms of ratios to the best-measured branching fractions of $\eta \to K \bar{K} \pi$, thereby canceling all fully-correlated systematic uncertainties. We average results on charged $B$ decays and neutral $B$ decays, taking into account correlations in the systematic uncertainties, to obtain $\mathcal{B}(\eta \to 2(K^+ K^-))/\mathcal{B}(\eta \to K \bar{K} \pi) = (2.3\pm 0.7 \pm 0.6) \times 10^{-2}$ and $\mathcal{B}(\eta \to \phi \phi)/\mathcal{B}(\eta \to K \bar{K} \pi) = (5.5\pm 1.4 \pm 0.5) \times 10^{-2}$. These results can be translated into $\eta$ branching fractions:

$$\mathcal{B}(\eta \to 2(K^+ K^-)) = (1.3\pm 0.4 \pm 0.3 \pm 0.4) \times 10^{-3},$$

$$\mathcal{B}(\eta \to \phi \phi) = (3.0\pm 0.8 \pm 0.3 \pm 0.9) \times 10^{-3},$$

where the third error is due to the uncertainty of $\mathcal{B}(\eta \to K \bar{K} \pi)$. Note that about half of the $\eta \to 2(K^+ K^-)$ events are due to $\eta \to \phi \phi$, $\phi \to K^+ K^-$ decays. Our measured branching fractions for $\eta \to 2(K^+ K^-)$ and $\eta \to \phi \phi$ are consistent with recent results from Belle and BES [17,18] and are smaller than those of earlier experiments [11] by a factor twenty and two, respectively.

As a cross-check, we can extract the branching fraction of $J/\psi$ decaying into the $2(K^+ K^-)$ final state from the measured number of $J/\psi$ events in the appropriate $B^+$ and $B^0$ samples. Assuming the same efficiencies as for the $B \to \eta K \bar{K} \pi$ process, we obtain $\mathcal{B}(J/\psi \to 2(K^+ K^-)) = (1.0\pm 0.5) \times 10^{-3}$ ($B^+$) and $\mathcal{B}(J/\psi \to 2(K^+ K^-)) = (0.1\pm 0.2)$

<table>
<thead>
<tr>
<th>$\eta$ decay channel</th>
<th>$B^+ \to \eta K^+$</th>
<th>$B^0 \to \eta K^0$</th>
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<tbody>
<tr>
<td>$\eta \to K^0 K^- \pi^+$</td>
<td>48.6±3.9±4.9</td>
<td>42.6±6.8±5.2</td>
</tr>
<tr>
<td>$\eta \to K^- K^0 \pi^+$</td>
<td>12.9±1.7±1.6</td>
<td>11.1±2.6±1.3</td>
</tr>
<tr>
<td>$\eta \to 2(K^+ K^-)$</td>
<td>2.0±0.6±0.4</td>
<td>0.9±0.9±0.4</td>
</tr>
<tr>
<td>$\eta \to \phi \phi$</td>
<td>4.7±1.2±0.5</td>
<td>2.4±1.4±0.3</td>
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</table>
\times 10^{-3} \left( B^0 \right) \), where the error is statistical only. These results are consistent with the world average, \( \mathcal{B}(J/\psi \to 2(K^+K^-)) = (0.7 \pm 0.3) \times 10^{-3} \) [11].

In summary, we have studied \( B \to \eta_c K \) decays with \( \eta_c \) decaying into \( K_S^0K^+\pi^- \), \( K^+K^-\pi^0 \), \( 2(K^+K^-) \), and \( \phi\phi \). Using the first two decay channels, we have measured the branching fraction products \( \mathcal{B}(B^+ \to \eta_cK^+) \times \mathcal{B}(\eta_c \to K\bar{K}\pi) = (7.40 \pm 0.50 \pm 0.70) \times 10^{-5} \) and \( \mathcal{B}(B^0 \to \eta_cK^0) \times \mathcal{B}(\eta_c \to K\bar{K}\pi) = (6.48 \pm 0.85 \pm 0.71) \times 10^{-5} \), which improve the statistical precision of, and are in good agreement with, previous measurements [19,20]. We have also measured the branching-fraction ratios \( \mathcal{B}(\eta_c \to 2(K^+K^-))/\mathcal{B}(\eta_c \to K\bar{K}\pi) = (2.3 \pm 0.7 \pm 0.6) \times 10^{-2} \) and \( \mathcal{B}(\eta_c \to \phi\phi)/\mathcal{B}(\eta_c \to K\bar{K}\pi) = (5.5 \pm 1.4 \pm 0.5) \times 10^{-2} \), where \( \eta_c \to 2(K^+K^-) \) includes \( \eta_c \to \phi\phi \) events with \( \phi \to K^+K^- \). The inferred branching fractions of \( \eta_c \to 2(K^+K^-) \) and \( \eta_c \to \phi\phi \) are in good agreement with recent results and much smaller than suggested by earlier experiments.

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[8] Charge conjugate modes are implicity included throughout this paper.