Measurement of the Inclusive Charmless Semileptonic Branching Ratio of $B$ Mesons and Determination of $|V_{ub}|$

We report a measurement of the inclusive charmless semileptonic branching fraction of $B$ mesons in a sample of $89 \times 10^6 \, \bar{B}B$ events recorded with the $BaBar$ detector at the $Y(4S)$ resonance. Events are selected by fully reconstructing the decay of one $B$ meson and identifying a charged lepton from the decay of the other $B$ meson. The number of signal events is extracted from the mass distribution of the hadronic system accompanying the lepton and is used to determine the ratio of branching fractions $B(\bar{B} \rightarrow X_{\nu} \ell \bar{\nu})/B(\bar{B} \rightarrow X \ell \bar{\nu}) = [2.06 \pm 0.25 \text{(stat)} \pm 0.23 \text{(syst)} \pm 0.36 \text{(theo)}] \times 10^{-2}$. Using the measured branching fraction for inclusive semileptonic $B$ decays, we find $B(\bar{B} \rightarrow X_{\nu} \ell \bar{\nu}) = [2.24 \pm 0.27 \text{(stat)} \pm 0.26 \text{(syst)} \pm 0.39 \text{(theo)}] \times 10^{-3}$ and derive the Cabibbo-Kobayashi-Maskawa matrix element $|V_{ub}| = [4.62 \pm 0.28 \text{(stat)} \pm 0.27 \text{(syst)} \pm 0.48 \text{(theo)}] \times 10^{-3}$. 

DOI: 10.1103/PhysRevLett.92.071802 PACS numbers: 13.20.He, 12.15.Hh
The element $|V_{ub}|$ of the Cabibbo-Kobayashi-Maskawa quark-mixing matrix [1] plays a critical role in testing the consistency of the standard model description of CP violation. In this Letter, we present a determination of $|V_{ub}|$ from a measurement of inclusive charmless semileptonic decays $\bar{B} \to X_c \ell \bar{\nu}$ [2]. The analysis uses $Y(4S) \to B \bar{B}$ events in which one of the $B$ meson decays hadronically and is fully reconstructed ($B_{\text{reco}}$) and the semileptonic decay of the recoiling $\bar{B}$ meson is identified by the presence of an electron or muon. While this approach results in a low overall event selection efficiency, it allows for the determination of the momentum, charge, and flavor of the $B$ mesons. We use the invariant mass $m_X$ of the hadronic system to separate $\bar{B} \to X_c \ell \bar{\nu}$ decays from the dominant $\bar{B} \to X_c \ell \bar{\nu}$ background, which clusters above the $D$ meson mass [3]. We achieve a higher signal purity and acceptance than previous analyses [4] and obtain smaller theoretical uncertainties. By measuring the fraction of charmless semileptonic decays $R_u = \mathcal{B}(\bar{B} \to X_c \ell \bar{\nu})/\mathcal{B}(\bar{B} \to X \ell \bar{\nu})$, we minimize experimental uncertainties.

The measurement presented here is based on a sample of $89 \times 10^6 B \bar{B}$ pairs collected near the $Y(4S)$ resonance by the BABAR detector [5] at the PEP-II asymmetric-energy $e^+ e^-$ storage ring operating at SLAC.

We use Monte Carlo (MC) simulations of the BABAR detector based on GEANT [6] to optimize selection criteria and to determine signal efficiencies and background distributions. Charmless semileptonic $\bar{B} \to X_c \ell \bar{\nu}$ decays are simulated as a combination [see Fig. 1(a)] of resonant three-body decays ($X_u = \pi, \eta, \rho, \omega, \ldots$) [7] and decays to nonresonant hadronic final states $X_u$ [8], for which the hadronization is performed by string fragmentation as implemented in the program JETSET [9]. The motion of the $b$ quark inside the $B$ meson is implemented with the shape function parametrization given in Ref. [8]. The simulation of the $\bar{B} \to X_c \ell \bar{\nu}$ background uses an HQET parametrization of form factors for $\bar{B} \to D^* \ell \bar{\nu}$ [10], and models for $\bar{B} \to D \pi \ell \bar{\nu}, D^* \pi \ell \bar{\nu}$ [11], and $\bar{B} \to D \ell \bar{\nu}, D^{**} \ell \bar{\nu}$ [7].

To reconstruct a large sample of $B$ mesons, hadronic decays $B_{\text{reco}} \to D Y^+, \bar{D}^* Y^+$ are selected. Here, the system $Y^+$ consists of hadrons with a total charge of $+1$, composed of $n_1 \pi^+ n_2 K^\pm n_3 K^{*0} n_4 \pi^0$, where $n_1 + n_2 = 5$, $n_3 \leq 2$, and $n_4 \leq 2$. We reconstruct $D^+ \to D^{0} \pi^+$, $\bar{D}^{*0} \to \bar{D}^0 \pi^0$, $\bar{D}^{0} \gamma$, $D^- \to K^- \pi^- \pi^-$, $K^+ \pi^- \pi^0$, $K^0_S \pi^- \pi^0$, $K^- \pi^- \pi^0$, $K^0_S \pi^- \pi^0$, and $\bar{D}^{0} \to K^+ \pi^0$, $K^+ \pi^- \pi^0$, $K^+ \pi^- \pi^0$, $K^+_S \pi^- \pi^0$. The kinematic consistency of $B_{\text{reco}}$ candidates is checked with two variables, the beam energy-substituted mass $m_{ES} = s^{1/2} - \vec{p}_{\ell}^2$ and the energy difference $\Delta E = E_B - \sqrt{s}/2$. Here $\sqrt{s}$ is the total energy in the $Y(4S)$ center of mass frame, and $\vec{p}_B$ and $E_B$ denote the momentum and energy of the $B_{\text{reco}}$ candidate in the same frame. We require $\Delta E = 0$ within 3 standard deviations as measured for each mode.

For each of the reconstructed $B$ decay modes, the purity $P$ is estimated as the signal fraction in events with $m_{ES} > 5.27 \text{ GeV}/c^2$. The number of signal events is derived from a fit to the $m_{ES}$ distribution that uses an empirical description [12] of the combinatorial background, together with a signal [13] peaked at the $B$ meson mass [Fig. 2(a)]. We use 311 modes for which $P$ exceeds a decay mode dependent threshold in the range of 8% to 24%. In events with more than one reconstructed $B$ decay, we select the decay mode with the highest purity. We reconstruct one $B$ candidate in 0.3% (0.5%) of the $B^0 \bar{B}^0$ ($B^+ B^-$) events.

Semileptonic decays $\bar{B} \to X \ell \bar{\nu}$ of the $B$ recoiling against the $B_{\text{reco}}$ candidate are identified by an electron or muon with a minimum momentum of $p^* > 1 \text{ GeV}/c$ in the $\bar{B}$ rest frame. After this requirement, the purity of the event sample is 67%. For charged $B_{\text{reco}}$ candidates, we require the charge of the lepton to be consistent with a primary semileptonic $B$ decay. For neutral $B_{\text{reco}}$ candidates, both charge-flavor combinations are retained and the known average $B^0 \bar{B}^0$ mixing rate is used to determine the primary lepton yield. Electrons are identified [14] with 92% average efficiency and a hadron misidentification rate ranging between 0.05% and 0.1%. Muons are identified [5] with an efficiency ranging between 60% ($p^* > 1 \text{ GeV}/c$) and 75% ($p^* > 2 \text{ GeV}/c$) and hadron misidentification rate between 1% and 3%.

To reconstruct a large sample of $B$ mesons, hadronic decays $B_{\text{reco}} \to D Y^+, \bar{D}^* Y^+$ are selected. Here, the system $Y^+$ consists of hadrons with a total charge of $+1$, composed of $n_1 \pi^+ n_2 K^\pm n_3 K^{*0} n_4 \pi^0$, where $n_1 + n_2 = 5$, $n_3 \leq 2$, and $n_4 \leq 2$. We reconstruct $D^+ \to D^{0} \pi^+$, $\bar{D}^{*0} \to \bar{D}^0 \pi^0$, $\bar{D}^{0} \gamma$, $D^- \to K^- \pi^- \pi^-$, $K^+ \pi^- \pi^0$, $K^0_S \pi^- \pi^0$, $K^- \pi^- \pi^0$, $K^0_S \pi^- \pi^0$, and $\bar{D}^{0} \to K^+ \pi^0$, $K^+ \pi^- \pi^0$, $K^+ \pi^- \pi^0$, $K^+_S \pi^- \pi^0$. The kinematic consistency of $B_{\text{reco}}$ candidates is checked with two variables, the beam energy-substituted mass $m_{ES} = s^{1/2} - \vec{p}_{\ell}^2$ and the energy difference $\Delta E = E_B - \sqrt{s}/2$. Here $\sqrt{s}$ is the total energy in the $Y(4S)$ center of mass frame, and $\vec{p}_B$ and $E_B$ denote the momentum and energy of the $B_{\text{reco}}$ candidate in the same frame. We require $\Delta E = 0$ within 3 standard deviations as measured for each mode.

For each of the reconstructed $B$ decay modes, the purity $P$ is estimated as the signal fraction in events with $m_{ES} > 5.27 \text{ GeV}/c^2$. The number of signal events is derived from a fit to the $m_{ES}$ distribution that uses an empirical description [12] of the combinatorial background, together with a signal [13] peaked at the $B$ meson mass [Fig. 2(a)]. We use 311 modes for which $P$ exceeds a decay mode dependent threshold in the range of 8% to 24%. In events with more than one reconstructed $B$ decay, we select the decay mode with the highest purity. We reconstruct one $B$ candidate in 0.3% (0.5%) of the $B^0 \bar{B}^0$ ($B^+ B^-$) events.

Semileptonic decays $\bar{B} \to X \ell \bar{\nu}$ of the $B$ recoiling against the $B_{\text{reco}}$ candidate are identified by an electron or muon with a minimum momentum of $p^* > 1 \text{ GeV}/c$ in the $\bar{B}$ rest frame. After this requirement, the purity of the event sample is 67%. For charged $B_{\text{reco}}$ candidates, we require the charge of the lepton to be consistent with a primary semileptonic $B$ decay. For neutral $B_{\text{reco}}$ candidates, both charge-flavor combinations are retained and the known average $B^0 \bar{B}^0$ mixing rate is used to determine the primary lepton yield. Electrons are identified [14] with 92% average efficiency and a hadron misidentification rate ranging between 0.05% and 0.1%. Muons are identified [5] with an efficiency ranging between 60% ($p^* > 1 \text{ GeV}/c$) and 75% ($p^* > 2 \text{ GeV}/c$) and hadron misidentification rate between 1% and 3%.
The hadronic system $X$ in the decay $\bar{B} \to X \ell \nu$ is reconstructed from charged tracks and energy depositions in the calorimeter that are not associated with the $B_{\text{reco}}$ candidate or the identified lepton. Care is taken to eliminate fake charged tracks, as well as low-energy beam-generated photons and energy depositions in the calorimeter from charged and neutral hadrons. The neutrino four-momentum $p_\nu$ is estimated from the missing momentum four-vector $p_{\text{miss}} = p_{Y(4S)} - p_{B_{\text{reco}}'} - p_\gamma - p_{\ell}$, where all momenta are measured in the laboratory frame and $p_{Y(4S)}$ refers to the $Y(4S)$ meson momentum.

To select $\bar{B} \to X_s \ell \nu$ candidates we require exactly one charged lepton with $p^\ell > 1$ GeV/c, charge conservation ($Q_X + Q_\ell + Q_\nu = 0$), and a missing mass consistent with zero ($m^2_{\text{miss}} < 0.5$ GeV$^2$/c$^4$). These criteria suppress the dominant $\bar{B} \to X_c \ell \nu$ decays, many of which contain additional neutrinos or an undetected $K^0_{\text{sl}}$ meson. The determination of the mass of the hadronic system is improved by a kinematic fit that imposes four-momentum conservation, the equality of the masses of the two $B$ mesons, and forces $p_\nu^2 = 0$. The resulting $m_\ell$ resolution is 350 MeV/c$^2$ on average. We suppress the $B^0 \to D^{*+} \ell^+ \nu$ background by reconstructing only the $\pi^+$ (from the $D^{*+} \to D^0 \pi^+$ decay) and the lepton: since the momentum of the $\pi^+$ is almost collinear with the $D^{*+}$ momentum in the laboratory frame, we can approximate the energy of the $D^{*+}$ as $E_{D^{*+}} = m_{D^{*+}}E_{\pi^+}/145$ MeV/c$^2$ and eliminate events with $(p_B - p_{D^{*+}} - p_\ell)^2 > 3$ GeV$^2$/c$^4$. We veto events with charged or neutral kaons (reconstructed as $K^0_{\text{sl}} \to \pi^+ \pi^-$) in the recoil $B$ to reduce the background from $\bar{B} \to X_c \ell \nu$ decays. The impact of the event selection on the $m_\ell$ distribution is illustrated in Fig. 1(b). If all charged particles of the $X$ system are reconstructed, the selection efficiency is $>50\%$, but lost particles lower the efficiency significantly. Therefore, resonant states (e.g., the $\rho$ meson) decaying into few particles are selected with higher efficiency.

We determine $R_u$ from $N_u$, the observed number of $\bar{B} \to X_u \ell \nu$ candidates with $m_\ell < 1.55$ GeV/c$^2$, and $N_{\text{sl}}$, the number of events with at least one charged lepton:

$$R_u = \frac{B(\bar{B} \to X_u \ell \nu)}{B(\bar{B} \to X \ell \nu)} = \frac{N_u}{N_{\text{sl}}} \times \frac{e_{\text{sl}}^u/e_{\text{reco}}^{\text{sl}}}{e_{\ell}^u/e_{\text{reco}}^{\ell}}.$$ 

Here $e_{\text{sl}}^{u} = (34.2 \pm 0.6_{\text{stat}})\%$ is the efficiency for selecting $\bar{B} \to X_u \ell \nu$ decays once a $\bar{B} \to X \ell \nu$ candidate has been identified, $e_{\text{reco}}^{u} = (73.3 \pm 0.9_{\text{stat}})\%$ is the fraction of signal events with $m_\ell < 1.55$ GeV/c$^2$, $e_{\ell}^u/e_{\text{reco}}^{\ell} = 0.887 \pm 0.008_{\text{stat}}$ corrects for the difference in the efficiency of the lepton momentum cut for $\bar{B} \to X \ell \nu$ and $\bar{B} \to X_u \ell \nu$ decays, and $e_{\text{reco}}^{u}/e_{\text{reco}}^{\text{sl}} = 1.00 \pm 0.03_{\text{stat}}$ accounts for a possible efficiency difference in the $B_{\text{reco}}$ reconstruction in events with $\bar{B} \to X \ell \nu$ and $\bar{B} \to X_u \ell \nu$ decays.

We derive $N_{\text{sl}}$ from a fit to the $m_{\text{reco}}$ distribution shown in Fig. 2(a). The residual background in $N_{\text{sl}}$ from misidentified leptons and semileptonic charm decays amounts to $(6.8 \pm 0.1_{\text{stat}})\%$ and is subtracted. We extract $N_u$ from the $m_\ell$ distribution by a minimum $\chi^2$ fit to the sum of three contributions: the signal, the background $N_c$ from $\bar{B} \to X_c \ell \nu$, and a background of $<1\%$ from other sources (misidentified leptons, secondary $\tau$, and charm decays). In each bin of the $m_\ell$ distribution, the combinatorial $B_{\text{reco}}$ background for $m_{\text{reco}} > 5.27$ is subtracted on the basis of a fit to the $m_{\text{reco}}$ distribution [Fig. 2(b)]. Figure 3(a) shows the fitted $m_\ell$ distribution. To minimize the model dependence, the first bin covers the region up to $m_{\text{reco}}^\ell = 1.55$ GeV/c$^2$. The fit reproduces the data well with $\chi^2$/dof $= 7.6/6$. Figure 3(b) shows the $m_\ell$ distribution after background subtraction with finer binning. Table I summarizes the results of fits with different sizes of the first $m_\ell$ bin, for electrons and muons, for neutral and charged $B_{\text{reco}}$ candidates, and for different ranges of the $B_{\text{reco}}$ purity $P$. The results are all consistent within the uncorrelated errors of signal and background samples.

We have performed extensive studies to determine the systematic uncertainties on $R_u$. To establish that the background from $\bar{B} \to X_c \ell \nu$ events is adequately simulated we use previously excluded events with charged or neutral kaons as a control sample. The fraction of events removed by the application of selection criteria is very well described by the MC simulation for both the signal and the control samples. The relative systematic error due to uncertainties in the detection of photons is estimated to be $4.7\%$ by varying the corrections applied to the MC simulation to match the data control samples. An error of $1.0\%$ due to the simulation of showers generated by $K^0_{\text{sl}}$ interactions is estimated by removing the $K^0_{\text{sl}}$ energy depositions in the MC simulation. An error of $1.0\%$ is due to the uncertainty in the track-finding efficiency. The errors due to identification of electrons, muons, and kaons are estimated to be $1.0\%$, $1.0\%$, and $2.3\%$, respectively, by varying identification efficiencies by $\pm 2\%$, $\pm 3\%$, and $\pm 2\%$ for $e^\pm$, $\mu^\pm$, and $K^\pm$, and the misidentification rates by $\pm 15\%$ for all particle types (see Ref. [14]).

The uncertainty due to the $B_{\text{reco}}$ combinatorial background subtraction is $3.8\%$. It is estimated by changing the empirical $m_{\text{reco}}$ signal function to a Gaussian distribution and by...
varying the parameters within 1 standard deviation of the default values. The limited statistics of the simulated event samples adds an uncertainty of 4.5%. The choice of bins for \( m_X > 1.55 \text{GeV}/c^2 \) impacts the fit result at a level of 1.2%.

The uncertainties in the background modeling due to branching fraction measurements for \( \bar{B} \to D\bar{c} \), \( D^*\bar{c} \bar{\nu} \), ..., and for inclusive and exclusive \( D \) meson decays [15] contribute 4.4%. The error due to the hadronization in the \( \bar{B} \to X_s \ell \bar{\nu} \) final state is estimated to be 3.0% by measuring \( R_u \) as a function of the charged and neutral particle multiplicities and performing the fit with only the nonresonant part of the signal model. We assign an additional 2.8% error to account for the uncertainties in the inclusive and exclusive branching fractions for charmless semileptonic \( B \) decays [15], plus 3.7% for the veto on strange particles. Here, we assume a 100% uncertainty in the \( s\bar{s} \) contents for the resonant and 30% for the nonresonant component [16].

The efficiencies \( e^{u}_{m} \) and \( e^{u}_{mX} \) are sensitive to the choice of the shape function parameters [8], which we assume to be directly related to the HQET parameters \( \Lambda \) and \( \lambda_1 \). We assess the uncertainties by varying within their errors \( \Lambda = 0.48 \pm 0.12 \text{GeV} \) and \( \lambda_1 = -0.30 \pm 0.11 \text{GeV}^2 \) values obtained from the results in Ref. [17] by removing terms proportional to \( 1/m_b^3 \) and \( \alpha_s^2 \) from the relation between the measured observables and \( \Lambda \) and \( \lambda_1 \). We have verified that significantly larger variations of these parameters are inconsistent with our measured \( m_X \) distribution. Taking into account the correlation of \(-0.8\) between \( \Lambda \) and \( \lambda_1 \), we arrive at a theoretical error of 17.5%.

In summary, we have \( R_u = (2.06 \pm 0.25 \pm 0.23 \pm 0.36) \times 10^{-2} \), where the errors are statistical, systematic (experimental plus signal and background modeling), and theoretical, respectively. Taking into account common errors we compute the double ratio \( \frac{\mathcal{B}(\bar{B} \to X_s \ell \bar{\nu})/\mathcal{B}(\bar{B} \to X_s \ell \bar{\nu})}{\mathcal{B}(\bar{B} \to X_s \ell \bar{\nu})/\mathcal{B}(\bar{B} \to X_s \ell \bar{\nu})} = 0.72 \pm 0.18_{\text{stat}} \pm 0.19_{\text{syst}} \), consistent with theoretical expectations.

Combining \( R_u \) with the measured inclusive semileptonic branching fraction \( \mathcal{B}(\bar{B} \to X_s \ell \bar{\nu}) = (10.87 \pm 0.18_{\text{stat}} \pm 0.30_{\text{syst}})\% \) [14], we obtain

\[
\mathcal{B}(\bar{B} \to X_s \ell \bar{\nu}) = (2.24 \pm 0.27 \pm 0.26 \pm 0.39) \times 10^{-3}.
\]

We combine this result with the average \( B \) lifetime of \( \tau_B = 1.608 \pm 0.012 \text{ps} \) [15,18] and obtain [19]

\[
|V_{ub}| = (4.62 \pm 0.28 \pm 0.27 \pm 0.40 \pm 0.26) \times 10^{-3}.
\]

The first error is statistical, the second systematic, the third gives the theoretical uncertainty in the signal efficiency, and the fourth is the uncertainty in the extraction of \( |V_{ub}| \) from the total decay rate. No error is assigned to the assumption of parton-hadron duality.

This result is consistent with previous inclusive measurements [4], but is based on a sample with larger phase-space acceptance and higher purity. The results of exclusive measurements [20] tend to have a lower central value, but with a slightly larger error due to model-dependent form factor calculations. In the future, improved understanding of the signal composition and charm background will reduce the experimental errors, and this, together with independent measurements of \( b \to s \) transitions and semileptonic \( B \) decays, is expected to constrain the theoretical uncertainties.

We are grateful for the excellent luminosity and machine conditions provided by our PEP-II colleagues, and for the substantial dedicated effort from the computing organizations that support \( \text{BaBar} \). The collaborating institutions wish to thank SLAC for its support and kind hospitality. This work is supported by DOE and NSF (USA), NSERC (Canada), IHEP (China), CEA and CNRS-IN2P3 (France), BMBF and DFG (Germany), INFN (Italy), FOM (The Netherlands), NFR (Norway), MIST (Russia), and PPARC (United Kingdom). Individuals have received support from the A.P. Sloan Foundation, Research Corporation, and Alexander von Humboldt Foundation.

\*Also with Università di Perugia, Perugia, Italy.
\*Also with Università della Basilicata, Potenza, Italy.

TABLE I. Fit results for data subsamples.

<table>
<thead>
<tr>
<th>Sample</th>
<th>( N_d )</th>
<th>( N_u )</th>
<th>( N_s )</th>
<th>( R_u(%) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_X^u = 1.55 \text{GeV}/c^2 )</td>
<td>29982 ± 233</td>
<td>175 ± 21</td>
<td>90 ± 5</td>
<td>2.06 ± 0.25</td>
</tr>
<tr>
<td>( m_X^u = 1.40 \text{GeV}/c^2 )</td>
<td>29982 ± 233</td>
<td>143 ± 18</td>
<td>54 ± 3</td>
<td>1.89 ± 0.24</td>
</tr>
<tr>
<td>( m_X^u = 1.70 \text{GeV}/c^2 )</td>
<td>29982 ± 233</td>
<td>214 ± 26</td>
<td>145 ± 9</td>
<td>2.35 ± 0.28</td>
</tr>
<tr>
<td>neutral ( B_{\text{reco}} )</td>
<td>10862 ± 133</td>
<td>76 ± 15</td>
<td>22 ± 3</td>
<td>2.53 ± 0.50</td>
</tr>
<tr>
<td>charged ( B_{\text{reco}} )</td>
<td>19080 ± 191</td>
<td>100 ± 16</td>
<td>67 ± 4</td>
<td>1.82 ± 0.30</td>
</tr>
<tr>
<td>Electrons</td>
<td>17320 ± 173</td>
<td>101 ± 15</td>
<td>46 ± 3</td>
<td>2.27 ± 0.34</td>
</tr>
<tr>
<td>Muons</td>
<td>12622 ± 157</td>
<td>73 ± 15</td>
<td>41 ± 4</td>
<td>1.83 ± 0.37</td>
</tr>
<tr>
<td>( \mathcal{P} &gt; 80% )</td>
<td>4187 ± 68</td>
<td>20 ± 7</td>
<td>12 ± 1</td>
<td>1.68 ± 0.57</td>
</tr>
<tr>
<td>( 50% &lt; \mathcal{P} &lt; 80% )</td>
<td>12373 ± 141</td>
<td>68 ± 13</td>
<td>41 ± 3</td>
<td>1.94 ± 0.37</td>
</tr>
<tr>
<td>( \mathcal{P} &lt; 50% )</td>
<td>13144 ± 170</td>
<td>86 ± 15</td>
<td>34 ± 3</td>
<td>2.31 ± 0.41</td>
</tr>
</tbody>
</table>
‡Also with IFIC, Instituto de Física Corpuscular, CSIC-Universidad de Valencia, Valencia, Spain.
§Deceased.


[2] Charge conjugation is implied throughout the Letter.


[18] The impact of the uncertainty of the fraction of neutral and charged B mesons is negligible.
