Limits on the Decay-Rate Difference of Neutral B Mesons and on $CP$, $T$, and $CPT$ Violation in $B^0\bar{B}^0$ Oscillations


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Using events in which one of two neutral $B$ mesons from the decay of an $Y(4S)$ meson is fully reconstructed, we determine parameters governing decay ($\Delta \Gamma_0 / \Gamma_0$), $CP$, and $T$ violation ($|q/p|$), and $CP$ and $CPT$ violation ($Re \, Z$, $Im \, Z$). The results, obtained from an analysis of $88 \times 10^6$ $Y(4S)$ decays recorded by BABAR, are $sign(Re \, \Lambda_{CP}) \Delta \Gamma_0 / \Gamma_0 = -0.008 \pm 0.037 (stat) \pm 0.018 syst [0.084, 0.068], |q/p| = 1.029 \pm 0.013 (stat) \pm 0.011 (syst) [1.001, 1.057], (Re \, \Lambda_{CP}) / |\Lambda_{CP}| Re \, Z = 0.014 \pm 0.035 (stat) \pm 0.034 (syst) \times [-0.072, 0.101], Im \, Z = 0.038 \pm 0.029 (stat) \pm 0.025 (syst) [-0.028, 0.104]$. The values inside the square
In this Letter, we provide a direct limit on the total decay-rate difference $\Delta \Gamma_d$ between the $B_d$ mass eigenstates and set limits on $CP$, $T$, and $CPT$ violation inherent in the mixing of neutral $B$ mesons. In the standard model $CPT$ violation is forbidden, and the other effects are expected to be nonzero but small, but new physics could provide enhancements [1–4]. We test these predictions by analyzing the time dependences of decays of the $Y(4S)$ resonance in which one neutral $B$ meson ($B_{\text{rec}}$) is fully reconstructed and the flavor of the other $B$ ($B_{\text{tag}}$) is identified as being either $B^0$ or $\bar{B}^0$. The $B_{\text{rec}}$ sample is composed of flavor- and $CP$-eigenstate subsamples, $B_{\text{flav}}$ and $B_{CP}$. We reconstruct the flavor eigenstates [5] $B_{\text{flav}} = D^{(*)-}\pi^+(p^+, a_1^+) + J/\psi K_{S}^{0}(\rightarrow K^+ \pi^-)$ and the $CP$ eigenstates $B_{CP} = J/\psi K_{S}^{0}, \psi(2S)K_{S}^{0}, \chi_{c0}(2S), \chi_{c0}^{*}(2S)$, and $J/\psi K_{S}^{*}$. The flavor of the $B$ that is not completely reconstructed is “tagged” on the basis of the charges of leptons and kaons, as well as other indicators [6]. The data come from $88 \times 10^6$ $Y(4S) \rightarrow B\bar{B}$ decays collected with the $\bar{B} a r$ detector [7] at the PEP-II asymmetric-energy $B$ Factory at SLAC.

The light and heavy $B_d$ mass eigenstates $B_{L,H}$ are superpositions of $B^0$ and $\bar{B}^0$. This mixing is a consequence of transitions between $B^0$ and $\bar{B}^0$ through intermediate states. Flavor oscillations between $B^0$ and $\bar{B}^0$ occur with a frequency $\Delta m_{d} \equiv m_{H} - m_{L}$. A state that is initially $B^0 (\bar{B}^0)$ will develop a $\bar{B}^0 (B^0)$ component over time, whose amplitude is proportional to a complex factor denoted $q/p (p/q)$ [8]. Since $|q/p| \approx 1$ in the standard model, this factor is usually assumed to be a pure phase.

The most general time dependence allowed for the decays of the two neutral $B$ mesons coming from an $Y(4S)$ is [6]

$$dN/d\Delta t = e^{-\Gamma_{d}\Delta t}\left[\frac{1}{2}(a_+^2 + a_-^2)\cos\left(\frac{\Delta \Gamma_{d}\Delta t}{2}\right) + \frac{1}{2}(a_+^2 - a_-^2)\cos(\Delta m_{d}\Delta t)
- \Re(a_+^*a_-)\sin\left(\frac{\Delta \Gamma_{d}\Delta t}{2}\right) + \Im(a_+^*a_-)\sin(\Delta m_{d}\Delta t)\right],$$

where $\Delta t \equiv t_{\text{rec}} - t_{\text{tag}}$ is the signed difference in proper decay times, $\Gamma_{d}$ is the mean decay rate of the two neutral mass eigenstates, and $\Delta \Gamma_{d} \equiv \Gamma_{H} - \Gamma_{L}$ is their decay-rate difference. The values of the complex parameters $a_+$ differ for the various combinations of flavor and $CP$ eigenstates into which the $B$ mesons decay [6].

In the simplest picture, where $\Delta \Gamma_{d} = 0$, and $CP$, $T$, and $CPT$ violation in mixing are neglected, if the fully reconstructed state is a flavor eigenstate the time distributions $dN/d\Delta t$ with perfect tagging are proportional to $e^{-\Gamma_{d}\Delta t}[1 \pm \cos(\Delta m_{d}\Delta t)]$. In practice, the tagging is imperfect and its performance is measured directly from the data. Imperfect tagging reduces the coefficient of $\cos(\Delta m_{d}\Delta t)$ by a factor of $1 - 2w$ called the dilution, where $w$ is the probability of tagging incorrectly.

$B$ decays to a $CP$ eigenstate $f_{CP}$ are conveniently parametrized by $\lambda_{CP} \equiv \langle q/p|\mathcal{A}_{CP}/\mathcal{A}_{CP}\rangle$, where $\mathcal{A}_{CP}$ is the amplitude for $B^0 \rightarrow f_{CP}$ ($\bar{B}^0 \rightarrow f_{CP}$). $CP$ violation is parametrized by $\lambda_{CP} \neq \eta_{CP}$ where $\eta_{CP} = \pm 1$ is the final state’s $CP$ eigenvalue. The $CP$ violation observed in decays like $B \rightarrow J/\psi K_{S}^{0}$ [9,10] involves interference between decays with and without net oscillation, and leads to $\lambda_{CP} \neq 0$. Other possible sources of $CP$ violation are $|q/p| \neq 1$ and $|\mathcal{A}_{CP}/\mathcal{A}_{CP}| \neq 1$. We include a test of the former possibility here.

The time distributions $dN/d\Delta t$ for the $B_{CP}$ samples, in the simplest picture (defined above) and with perfect tagging, are proportional to

$$e^{-\Gamma_{d}\Delta t}\left[1 + |\lambda_{CP}|^2 \pm (1 - |\lambda_{CP}|^2)\cos(\Delta m_{d}\Delta t) \mp 2 \Im \lambda_{CP}\sin(\Delta m_{d}\Delta t)\right].$$

In the standard model we have $\lambda_{CP} = -e^{-2i\beta}$ for $J/\psi K_{S}^{0}$ with the approximation $\Delta \Gamma_{d} = 0$, where $\beta = \arg\left[\frac{V_{cb}V_{cb}^*}{V_{ub}V_{ub}^*}\right]$ is one of the angles of the triangle [11] that represents the unitarity of the quark mixing matrix $V_{ij}$. Since $|\lambda_{CP}| = 1$, the $\cos(\Delta m_{d}\Delta t)$ term is absent. Again, wrongly tagged events reduce the amplitude of the oscillatory terms.

To measure $\Delta \Gamma_{d}$, or $CP$, $T$, or $CPT$ violation in mixing alone we need to find small deviations from these simple patterns. Other effects that can mimic the behavior we seek must be included in the analysis. Among these are asymmetries in the response of the detector to $B^0$ and $\bar{B}^0$ decays [6] and interference between dominant and suppressed decay amplitudes to flavor eigenstates, both those that are fully reconstructed and those that contribute to tagging [6,12].

The time dependence of the $B_{CP}$ sample includes a $\sin(\Delta \Gamma_{d}\Delta t/2)$ term that is effectively linear in $\Delta \Gamma_{d}$, while the flavor sample has an effective second-order sensitivity to $\Delta \Gamma_{d}$ through a $\cos(\Delta \Gamma_{d}\Delta t/2)$ term. Untagged data are included in this analysis and improve our sensitivity to $\Delta \Gamma_{d}$ since the contributions of $\Delta \Gamma_{d}$-dependent terms do not depend on whether $B_{\text{tag}}$ is a $B^0$ or $\bar{B}^0$. With our sample sizes and small measured value
and complements earlier limits on the dominant difference assume $j_{181801-5} \propto j_{181801-5}$. g gives $j_{003}$ $j_{003} \propto j_{003}$ proportional to $j_{003}$. We find that, working to first order in the small quantities $\Delta m_{\chi}/m_{\chi}$ and $\Delta m_{\chi}/m_{\chi}$, the cumulative effect of each ensemble does not modify the expected decay-rate distributions. We combine all the data for the CP eigenstates taking into account $\Delta m_{\chi}/m_{\chi} = 1$ (but vary this ratio as a system).

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model, while the imaginary parts and magnitudes of these effective parameters are treated as independent variables. For all sets of nonleptonic flavor eigenstates analyzed, the magnitude of each \(|\Delta|\) is fixed to 0.02 (up to a factor \(|q/p|\) or \(|p/q|\)) but \(\text{Im} \lambda \mid \lambda\) is left unconstrained. The decay model uses 26 more parameters to model the effects of experimental \(\Delta t\) resolution \((10), B^0/\bar{B}^0\) tagging capability \((11),\) and reconstruction and tagging efficiencies \((5)\). An additional 22 parameters model the levels and \(\Delta t\) dependence of backgrounds. A total of 58 free parameters are determined with a simultaneous unbinned maximum-likelihood fit to the \(\Delta t\) distributions of \(CP\) and flavor-eigenstate samples \([6]\).

Table I summarizes the results of fits allowing \((z=0)\) CPT violation in \(B^0\bar{B}^0\) oscillations. The largest statistical correlations involving the parameters of interest are between \(|q/p|\) and parameters modeling \(B^0\bar{B}^0\) asymmetries in reconstruction efficiency and mistag probabilities, and between \(\text{Im} z\) and the DCS contributions to \(B_{\text{tag}}\) decay amplitudes. The fitted values of \(\Delta m_d\) and \(\text{Im} \lambda_{CP}/|\lambda_{CP}|\) are consistent with recent \(B\) Factory measurements \([9,10,13,15]\). When \(z\) is fixed, the value of \(\text{Im} \lambda_{CP}/|\lambda_{CP}|\) decreases by 0.011, equal to 15% of the statistical uncertainty on \(\text{Im} \lambda_{CP}/|\lambda_{CP}|\) which is consistent with the correlations observed in the fit with \(z\) free, while the value of uncertainty in \(\Delta m_d\) are unchanged. No statistically significant \(B^0/\bar{B}^0\) differences in reconstruction and tagging efficiencies are observed.

We have used data and Monte Carlo samples to validate our analysis technique. Tests with large, parameterized Monte Carlo samples demonstrate that the observed statistical uncertainties and correlations are consistent with expectations. Analyses of Monte Carlo samples generated with a detailed detector simulation verify that the analysis procedure is unbiased. Fits to data subsamples selected by tagging category, running period, and \(B_{\text{rec}}\) decay mode give consistent results. Changes to the algorithms used to estimate \(\Delta t\) and \(\sigma_{\Delta t}\) or to their allowed ranges also have no statistically significant effect. Fits to samples of charged \(B\) decays, in which no oscillations are present, give the expected results.

We identify four general sources of systematic uncertainty with the contributions shown in Table II for the fit in which \(z\) is free \([6]\). The first is possible bias in the event selection and fit method: we see no evidence of such bias when analyzing Monte Carlo samples and assign the statistical uncertainty of these checks as a systematic uncertainty on the final results. The second is the \(\Delta t\) measurement. The choice of parametrization of the resolution function dominates this uncertainty, but assumptions about the beam spot and detector alignment contribute as well. Assumptions about the properties of signal \(\psi(4S) \rightarrow B_{\text{rec}}B_{\text{tag}}\) decays include the values of the lifetime, \(|\mathcal{A}_{CP}/\mathcal{A}_{CP}|\), and DCS parameters, and are the third source of systematic uncertainty. Uncertainties in the size and \(\Delta t\) distributions of background (BG) events incorrectly identified as \(\psi(4S) \rightarrow B_{\text{rec}}B_{\text{tag}}\) make small contributions to the systematic uncertainties.

Different sources dominate the systematic uncertainty for each parameter. Most systematic uncertainties are determined with data and will decrease with additional statistics. The largest single source of uncertainty is the contribution of the DCS parameters to \((\Re \lambda_{CP}/|\lambda_{CP}|) \Re z\), and it is estimated by varying the DCS phase parameters over their full allowed range, and \(|\mathcal{A}_{Bf}/\mathcal{A}_{Bf}|\) and \(|\mathcal{A}_{\bar{B}f}/\mathcal{A}_{\bar{B}f}|\) over the range 0–0.04. Systematic uncertainties on \(\text{sgn}(\Re \lambda_{CP})\Delta \Gamma_d^{-1}\) and \(|q/p|\) for the analysis assuming \(z=0\) were evaluated similarly as \(\pm 0.018\) and \(\pm 0.011\), respectively.

Using the world-average value of \(\Delta m_d\) \([8]\), we derive the value \(\text{sgn}(\Re \lambda_{CP})\Delta \Gamma_d^{-1}/\Delta m_d = -0.011 \pm 0.049\) (stat) \(\pm 0.024\) (syst), corresponding to the range \([-0.112, 0.091]\) at the 90% confidence level, from the fit results with \(z\) free. The limit on \(CP\) and \(T\) violation in oscillations is independent of and consistent with our previous measurement based on an analysis of inclusive dilepton events \([20]\). Using Eq. (3) and taking the world-average \(B_q\) mass \([8]\,\)

**Table II. Summary of systematic uncertainties (z free).**

| Source | \(\text{sgn}(\Re \lambda_{CP})\times \Delta \Gamma_d^{-1}/\Gamma_d| q/p \) | \(\Re \lambda_{CP}/|\lambda_{CP}|\times \Re z\) | Im z |
|--------|-----------------|-----------------|-------|
| Analysis method | 0.006 | 0.007 | 0.005 | 0.016 |
| \(\Delta t\) Resolution | 0.013 | 0.003 | 0.008 | 0.016 |
| Signal properties | 0.010 | 0.008 | 0.033 | 0.009 |
| BG properties | 0.005 | 0.003 | 0.007 | 0.004 |
| Total | 0.018 | 0.011 | 0.034 | 0.025 |

![FIG. 1. Favored regions at 68% confidence level in the \((|q/p| - 1, |z|)\) plane determined by this analysis and by the BABAR measurement of the dilepton asymmetry \([20]\). Labels reflect the requirements that both \(CP\) and \(T\) be violated if \(|q/p| \neq 1\) and that both \(CP\) and \(CPT\) be violated if \(|z| \neq 0\). The dilepton measurement constrains \(|q/p|\) without assumptions on the value of \(|z|\). The standard model expectation of \(|q/p| - 1 = (2.5 - 6.5) \times 10^{-4}\) is obtained from Ref. \([2]\).](181801-6)
we derive $|\Delta m_d|/m_{B_d} < 1.0 \times 10^{-14}$ and $-0.156 < \delta \Gamma_d/
abla \Gamma_d < 0.042$ at the 90% confidence level. Figure 1 shows the results of the fit with $\mathbf{z}$ free in the $(|q/p| - 1, |z|)$ plane, compared to the previous $\text{BABAR}$ measurement of $|q/p|$, and to standard model expectations.

Conventional analyses of oscillations and $\text{CP}$ violation in the $B_d$ system neglect possible contributions from several sources that are expected to be small in the standard model. This analysis includes these effects and finds results consistent with standard model expectations. While the standard model predictions for $|q/p|$, $\mathbf{z}$ are well below our current sensitivity, higher-precision measurements may still bring surprises.

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[5] Charge conjugation is implied throughout this Letter, unless explicitly stated otherwise.
[6] $\text{BABAR}$ Collaboration, B. Aubert et al., hep-ex/0403002 (to be published).