Limits on the Decay-Rate Difference of Neutral $B$ Mesons and on $CP$, $T$, and $CPT$ Violation in $B^0\bar{B}^0$ Oscillations

Using events in which one of two neutral $B$ mesons from the decay of an $\Upsilon(4S)$ meson is fully reconstructed, we determine parameters governing decay ($\Delta \Gamma_d/\Gamma_d$), $CP$, and $T$ violation ($|q/p|$), and $CP$ and $CPT$ violation ($Re \, z$, $Im \, z$). The results, obtained from an analysis of $88 \times 10^9 \ \Upsilon(4S)$ decays recorded by $B\bar{B}$AR, are $sgn(Re \, \lambda_{CP})\Delta \Gamma_d/\Gamma_d = -0.008 \pm 0.037(stat) \pm 0.018(syst)[-0.084,0.068]$, $|q/p| = 1.029 \pm 0.013(stat) \pm 0.011(syst)[1.001,1.057]$, $(Re \, \lambda_{CP})/|\lambda_{CP}|Re \, z = 0.014 \pm 0.035(stat) \pm 0.034(syst) \times [-0.072,0.101]$, $Im \, z = 0.038 \pm 0.029(stat) \pm 0.025(syst)[-0.028,0.104]$. The values inside the square
In this Letter, we provide a direct limit on the total decay-rate difference $\Delta \Gamma_d$ between the $B_d$ mass eigenstates and set limits on $CP$, $T$, and $CPT$ violation inherent in the mixing of neutral $B$ mesons. In the standard model $CPT$ violation is forbidden, and the other effects are expected to be nonzero but small, but new physics could provide enhancements [1–4]. We test these predictions by analyzing the time dependence of decays of the $Y(4S)$ resonance in which one neutral $B$ meson ($B_{\text{rec}}$) is fully reconstructed and the flavor of the other $B$ ($B_{\text{tag}}$) is identified as being either $B^0$ or $\bar{B}^0$. The $B_{\text{rec}}$ sample is composed of flavor- and CP-eigenstate subsamples, $B_{\text{flav}}$ and $B_{\text{CP}}$. We reconstruct the flavor eigenstates [5] $B_{\text{flav}} = D(\pi^-)\pi^+(p^+, a_1^{+})$ and $J/\psi K_S^{0}(\rightarrow K^-\pi^-)$ and the CP eigenstates $B_{\text{CP}} = J/\psi K_L^0$, $\psi(2S)K_L^0$, $\chi_cJ/\psi K_S^0$, and $J/\psi K_S^0$. The flavor of the $B$ that is not completely reconstructed is “tagged” on the basis of the charges of leptons and kaons, as well as other indicators [6]. The data come from $88 \times 10^6$ $Y(4S) \rightarrow B\bar{B}$ decays collected with the Babar detector [7] at the PEP-II asymmetric-energy $B$ Factory at SLAC.

The light and heavy $B_d$ mass eigenstates $B_{L,H}$ are superpositions of $B^0$ and $\bar{B}^0$. This mixing is a consequence of transitions between $B^0$ and $\bar{B}^0$ through intermediate states. Flavor oscillations between $B^0$ and $\bar{B}^0$ occur with a frequency $\Delta m_d = m_H - m_L$. A state that is initially $B^0(\bar{B}^0)$ will develop a $\bar{B}^0(B^0)$ component over time, whose amplitude is proportional to a complex factor denoted $q/p$ ($p/q$) [8]. Since $|q/p| \approx 1$ in the standard model, this factor is usually assumed to be a pure phase.

The most general time dependence allowed for the decays of the two neutral $B$ mesons coming from an $Y(4S)$ is [6]

$$\frac{dN}{dt} = e^{-\Gamma_d|\Delta t|} \left[ \frac{|a_+|^2 + |a_-|^2}{2} \cos\left(\frac{\Delta \Gamma_d |\Delta t|}{2}\right) + \frac{|a_+|^2 - |a_-|^2}{2} \cos(\Delta m_d |\Delta t|) - \text{Re}(a_+^* a_-) \sin\left(\frac{\Delta \Gamma_d |\Delta t|}{2}\right) + \text{Im}(a_+^* a_-) \sin(\Delta m_d |\Delta t|) \right],$$

where $\Delta t = t_{\text{rec}} - t_{\text{tag}}$ is the signed difference in proper decay times, $\Gamma_d$ is the mean decay rate of the two neutral mass eigenstates, and $\Delta \Gamma_d = \Gamma_H - \Gamma_L$ is their decay-rate difference. The values of the complex parameters $a_+$ differ for the various combinations of flavor and CP eigenstates into which the $B$ mesons decay [6].

In the simplest picture, where $\Delta \Gamma_d = 0$, and $CP$, $T$, and $CPT$ violation in mixing are neglected, if the fully reconstructed state is a flavor eigenstate the time distributions $dN/d\Delta t$ with perfect tagging are proportional to $e^{-\Gamma_d|\Delta t|}[1 \pm \cos(\Delta m_d |\Delta t|)]$. In practice, the tagging is imperfect and its performance is measured directly from the data. Imperfect tagging reduces the coefficient of $\cos(\Delta m_d |\Delta t|)$ by a factor of $1 - 2w$ called the dilution, where $w$ is the probability of tagging incorrectly.

$B$ decays to a CP eigenstate $f_{CP}$ are conveniently parameterized by $\lambda_{CP} = (q/p) \frac{\mathcal{A}_{CP}}{\mathcal{A}_{CP}}$, where $\mathcal{A}_{CP}$ (\overline{\mathcal{A}}_{CP}) is the amplitude for $B^0 \rightarrow f_{CP}$ ($\bar{B}^0 \rightarrow f_{CP}$). $CP$ violation is characterized by $\lambda_{CP} \neq \eta_{CP}$ where $\eta_{CP} = \pm 1$ is the final state’s CP eigenvalue. The CP violation observed in decays like $B \rightarrow J/\psi K_S^0$ [9,10] involves interference between decays with and without net oscillation, and leads to $|\mathcal{A}_{CP}| \neq 0$. Other possible sources of CP violation are $|q/p| \neq 1$ and $|\overline{\mathcal{A}}_{CP}/\mathcal{A}_{CP}| \neq 1$. We include a test of the former possibility here.

The time distributions $dN/d|\Delta t|$ for the $B_{CP}$ samples, in the simplest picture (defined above) and with perfect tagging, are proportional to

$$e^{-\Gamma_d|\Delta t|}[1 + |\lambda_{CP}|^2 \pm (1 - |\lambda_{CP}|^2) \cos(\Delta m_d |\Delta t|) \mp 2 \text{Im}(\lambda_{CP}) \sin(\Delta m_d |\Delta t|)].$$

In the standard model we have $\lambda_{CP} = -e^{-2i\beta}$ for $J/\psi K_S^0$ with the approximation $\Delta \Gamma_d = 0$, where $\beta = \arg[-V_{cd}V_{cb}^*/\sqrt{|V_{ub}|^2 + V_{cb}^*V_{ub}}]$ is one of the angles of the triangle [11] that represents the unitarity of the quark mixing matrix $V_{ij}$. Since $|\lambda_{CP}| = 1$, the $\cos(\Delta m_d |\Delta t|)$ term is absent. Again, wrongly tagged events reduce the amplitude of the oscillatory terms.

To measure $\Delta \Gamma_d$, or $CP$, $T$, or $CPT$ violation in mixing alone we need to find small deviations from these simple patterns. Other effects that can mimic the behavior we seek must be included in the analysis. Among these are asymmetries in the response of the detector to $B^0$ and $\bar{B}^0$ decays [6] and interference between dominant and suppressed decay amplitudes to flavor eigenstates, both those that are fully reconstructed and those that contribute to tagging [6,12].

The time dependence of the $B_{CP}$ sample includes a $\sinh(\Delta \Gamma_d |\Delta t|/2)$ term that is effectively linear in $\Delta \Gamma_d$, while the flavor sample has an effective second-order sensitivity to $\Delta \Gamma_d$ through a $\cosh(\Delta \Gamma_d |\Delta t|/2)$ term. Untagged data are included in this analysis and improve our sensitivity to $\Delta \Gamma_d$ since the contributions of $\Delta \Gamma_d$-dependent terms do not depend on whether $B_{\text{tag}}$ is a $B^0$ or $\bar{B}^0$. With our sample sizes and small measured value...
of $\Delta \Gamma_d$, the $B_{CP}$ sample dominates our determination of $\Delta \Gamma_d/\Gamma_d$. While $\Delta \Gamma_d$ has been well measured previously [13–15], there is only a weak limit, $|\Delta \Gamma_d/\Gamma_d| < 0.18$ at 95% C.L. [16], on $\Delta \Gamma_d$. A recent theoretical calculation gives $\Delta \Gamma_d/\Gamma_d = -0.003$ [1].

Violation of $CP$ and $T$ in mixing leads to a difference between the $B^0 \to B^0$ and $B^0 \to B^0$ transition rates proportional to $|q/p|^4 - 1$. Our sensitivity to $|q/p|$ comes mostly from the large flavor-eigenstate sample. Previous measurements, obtained assuming $\Delta \Gamma_d = 0$, give $|q/p| - 1 = (-0.7 \pm 6.4) \times 10^{-3}$ [17]. The standard model expectation is $|q/p| - 1 = (2.5 - 6.5) \times 10^{-4}$ [2].

$CP$ violation in mixing enters the time dependence through the complex quantity

$$z = \frac{\delta m_d - \frac{i}{2} \delta \Gamma_d}{\Delta m_d - \frac{i}{2} \Delta \Gamma_d}, \quad (3)$$

where $\delta m_d$ ($\delta \Gamma_d$) is the $B^0 - \overline{B}^0$ difference of effective mass (decay-rate) expectation values for the $B^0$ and $\overline{B}^0$ flavor eigenstates. A nonzero value of either $\delta m_d$ or $\delta \Gamma_d$ is possible only if both $CP$ and $T$ are violated. The dominant contribution of $\text{Im} z$ to the time dependence is through the coefficient of $\sin(\Delta m_d t)$ for flavor eigenstates, while $\text{Re} z$ contributes primarily to the coefficients of $\cosh(\Delta \Gamma_d t/2) = 1$ and $\cos(\Delta m_d t)$ for $CP$ eigenstates. The measurement of $z$ presented here is more general than previous analyses based on $B$ decays, which obtained $\text{Im} z = 0.040 \pm 0.032 \pm 0.012$ [18], and $\text{Re} z = 0.00 \pm 0.12 \pm 0.02$, $\text{Im} z = -0.03 \pm 0.01 \pm 0.03$ [14], and complements earlier limits on the $K^0 - \overline{K}^0$ mass difference $\delta m_K/m_K < 10^{-18}$ [8].

Interference effects between the amplitudes for dominant decays of flavor eigenstates (e.g., $B^0 \to D^- \pi^+$) and for doubly Cabibbo-Kobayashi-Maskawa-suppressed (DCS) decays (e.g., $\overline{B}^0 \to D^- \pi^+$) are analogous to the interference familiar in decays to $CP$ eigenstates [12]. They thus affect, in particular, the sin($\Delta m_d t$) terms and have the potential to obscure a similar contribution from $\text{Im} z$. The size of the DCS interference relative to the dominant $B^0$ decay is governed by $\lambda_{B^0}$ and $\lambda_{B^0}$, for $B_{flav}$ and $B_{tag}$ states, respectively. These parameters are defined analogously to $\lambda_{CP}$, and we expect $|\lambda_{B^0}| = |q/p| \times |V_{ud}V_{cd}^*/V_{ub}V_{cb}| \approx 0.02 |q/p|$ [6]. There are similar interference contributions from DCS amplitudes for $\overline{B}^0$ decays, governed by $\lambda_{\overline{B}^0}$ and $\lambda_{\overline{B}^0}$. We write $\lambda_{B^0} = 1/\lambda_{B^0}$, so $|\lambda_{B^0}| = 0.02 |p/q|$. The $B_{flav}$ and $B_{tag}$ samples are ensembles of final states that each contribute to the expected decay-rate distributions with different amplitudes. We find that, working to first order in the small quantities $|\lambda_{B^0}|$, $|\lambda_{B^0}|$, and $|q/p| - 1$, the cumulative effect of each ensemble does not modify the expected decay-rate distributions, once $\lambda_{B^0}$ and $\lambda_{B^0}$ are reinterpreted as effective parameters.

We combine all the data for the $CP$ eigenstates, taking into account the $CP$ eigenvalue of the final state. We assume $|\mathcal{A}_{CP}/\mathcal{A}_{CP}| = 1$ (but vary this ratio as a system-
model, while the imaginary parts and magnitudes of these effective parameters are treated as independent variables. For all sets of nonleptonic flavor eigenstates analyzed, the magnitude of each $|\lambda|$ is fixed to 0.02 (up to a factor $|q|/l$ or $|p|/q$) but Im $\lambda/|\lambda|$ is left unconstrained. The decay model uses 26 more parameters to model the effects of experimental $\Delta t$ resolution (10), $B^0/\bar{B}^0$ tagging capability (11), and reconstruction and tagging efficiencies (5). An additional 22 parameters model the levels and $\Delta t$ dependence of backgrounds. A total of 58 free parameters are determined with a simultaneous unbinned maximum-likelihood fit to the $\Delta t$ distributions of $CP$ and flavor-eigenstate samples [6].

Table I summarizes the results of fits allowing ($z$ free) or not allowing ($z = 0$) $CPT$ violation in $B^0/\bar{B}^0$ oscillations. The largest statistical correlations involving the parameters of interest are between $|q|/l$ and parameters modeling $B^0/\bar{B}^0$ asymmetries in reconstruction efficiency and mistag probabilities, and between Im $z$ and the DCS contributions to $B_{\text{tag}}$ decay amplitudes. The fitted values of $\Delta m_d$ and Im $\lambda_{CP}/|\lambda_{CP}|$ are consistent with recent $B$ Factory measurements [9,10,13,15]. When $z$ is fixed, the value of Im $\lambda_{CP}/|\lambda_{CP}|$ decreases by 0.011, equal to 15% of the statistical uncertainty on Im $\lambda_{CP}/|\lambda_{CP}|$ which is consistent with the correlations observed in the fit with $z$ free, while the value of uncertainty in $\Delta m_d$ are unchanged. No statistically significant $B^0/\bar{B}^0$ differences in reconstruction and tagging efficiencies are observed.

We have used data and Monte Carlo samples to validate our analysis technique. Tests with large, parametrized Monte Carlo samples demonstrate that the observed statistical uncertainties and correlations are consistent with expectations. Analyses of Monte Carlo samples generated with a detailed detector simulation verify that the analysis procedure is unbiased. Fits to data subsamples selected by tagging category, running period, and $B_{\text{rec}}$ decay mode give consistent results. Changes to the algorithms used to estimate $\Delta t$ and $\sigma_{\Delta t}$ or to their allowed ranges also have no statistically significant effect. Fits to samples of charged $B$ decays, in which no oscillations are present, give the expected results.

We identify four general sources of systematic uncertainty with the contributions shown in Table II for the fit in which $z$ is free [6]. The first is possible bias in the event selection and fit method: we see no evidence of such bias when analyzing Monte Carlo samples and assign the statistical uncertainty of these checks as a systematic uncertainty on the final results. The second is the $\Delta t$ measurement. The choice of parametrization of the resolution function dominates this uncertainty, but assumptions about the beam spot and detector alignment contribute as well. Assumptions about the properties of signal $Y(4S) \rightarrow B_{\text{rec}}B_{\text{tag}}$ decays include the values of the lifetime, $|\mathcal{A}_{CP}/\mathcal{A}_{CP}|$, and DCS parameters, and are the third source of systematic uncertainty. Uncertainties in the size and $\Delta t$ distributions of background (BG) events incorrectly identified as $Y(4S) \rightarrow B_{\text{rec}}B_{\text{tag}}$ make small contributions to the systematic uncertainties.

Different sources dominate the systematic uncertainty for each parameter. Most systematic uncertainties are determined with data and will decrease with additional statistics. The largest single source of uncertainty is the contribution of the DCS parameters to $\text{Re} \lambda_{CP}/|\lambda_{CP}|$, and it is estimated by varying the DCS phase parameters over their full allowed range, and $|\mathcal{A}_{B^0}/\mathcal{A}_{B^0}|$ and $|\mathcal{A}_{B^0}/\mathcal{A}_{B^0}|$ over the range 0–0.04. Systematic uncertainties on $\text{sgn}(\text{Re} \lambda_{CP})\Delta \Gamma_d/\Gamma_d$ and $|q|/l$ for the analysis assuming $z = 0$ were evaluated similarly as $\pm 0.018$ and $\pm 0.011$, respectively.

Using the world-average value of $\Delta m_d$ [8], we derive the value $\text{sgn}(\text{Re} \lambda_{CP})\Delta \Gamma_d/\Gamma_d = -0.011 \pm 0.049(\text{stat}) \pm 0.024(\text{syst})$, corresponding to the range $[-0.112, 0.091]$ at the 90% confidence level, from the fit results with $z$ free. The limit on $CP$ and $T$ violation in oscillations is independent of and consistent with our previous measurement based on an analysis of inclusive dilepton events [20]. Using Eq. (3) and taking the world-average $B_d$ mass [8],

![FIG. 1. Favored regions at 68% confidence level in the $(|q|/l - 1, |z|)$ plane determined by this analysis and by the BABAR measurement of the dilepton asymmetry [20]. Labels reflect the requirements that both $CP$ and $T$ be violated if $|q|/l \neq 1$ and that both $CP$ and $CPT$ be violated if $|z| \neq 0$. The dilepton measurement constrains $|q|/l$ without assumptions on the value of $|z|$. The standard model expectation of $|q|/l - 1 = (2.5 - 6.5) \times 10^{-4}$ is obtained from Ref. [2].](image)

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we derive $|\delta m_d|/m_{B_d} < 1.0 \times 10^{-14}$ and $-0.156 < \delta \Gamma_d/\Gamma_d < 0.042$ at the 90% confidence level. Figure 1 shows the results of the fit with $Z$ free in the $(|q/p| - 1,|Z|)$ plane, compared to the previous $BABAR$ measurement of $|q/p|$, and to standard model expectations.

Conventional analyses of oscillations and $CP$ violation in the $B_d$ system neglect possible contributions from several sources that are expected to be small in the standard model. This analysis includes these effects and finds results consistent with standard model expectations. While the standard model predictions for $|q/p|$, and $Z$ are well below our current sensitivity, higher-precision measurements may still bring surprises.

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[5] Charge conjugation is implied throughout this Letter, unless explicitly stated otherwise.