Branching Fraction and CP Asymmetries of $B^0 \to K^*_S K^0 S\_S$
We present measurements of the branching fraction and time-dependent CP-violating asymmetries in $B^0 \to K_S^0 K^+_S K^-_S$ decays based on 227\times 10^6 Y(4S) \to BB$ decays collected with the BABAR detector at the PEP-II asymmetric-energy B factory at SLAC. We obtain a branching fraction of (6.9^{+0.9}_{-0.8} \pm 0.6) \times 10^{-6}$, and CP asymmetries $C = -0.34^{+0.28}_{-0.25} \pm 0.05$ and $S = -0.71^{+0.38}_{-0.32} \pm 0.04$, where the first uncertainties are statistical and the second systematic.

The amplitude of time-dependent CP violation (CPV) predicted for $b \to c \bar{s} \bar{s}$ decays of neutral B mesons in the standard model (SM) is $\sin 2\beta$ where $\beta = \arg(-V_{cb}V_{cb}^*/V_{td}V_{td}^*)$ is the CP violating phase difference between mixing and decay amplitudes, with $V_{ij}$ the elements of the Cabibbo-Kobayashi Maskawa (CKM) quark mixing matrix [1]. This prediction has been well tested at the B factories in recent years [2]. The SM also predicts the amplitude of CPV in $b \to s\bar{q}q$ $(q = d, s)$ decays, defined as $\sin 2\beta_{eff}$, to be approximately $\sin 2\beta$. However, since $b \to s\bar{q}q$ decays are dominated by one-loop transitions that can potentially accommodate large virtual pion masses, contributions from physics beyond the SM could invalidate this prediction, making these decays especially sensitive to new physics [3]. An active program has arisen to measure $\beta_{eff}$ in as many $b \to s\bar{q}q$ “penguin” modes as possible [4]. However, many of these final states are affected by additional SM physics contributions that obscure the measurement of $\beta_{eff}$ [5], or are not CP eigenstates. Two decays to CP eigenstates that have been noted as having small theoretical uncertainties in the measurement of $\beta_{eff}$ are $B^0 \to K^0_S \phi K^0_S$ [6–8] (CP odd) and $B^0 \to K_S^0 K^0_S K^0_S$ (CP even) [9].

In this Letter we present a measurement of time-dependent CP-violating asymmetries in the decay $B^0 \to K_S^0 K^0_S K^0_S$, along with a measurement of the branching fraction (BF). Until recently the small branching fraction [10] and the absence of charged decay tracks originating at the $B^0$ decay vertex have limited the ability to extract CP parameters from $B^0 \to K_S^0 K^0_S K^0_S$. However, techniques recently developed to deal with the reconstruction of the $B^0$ decay vertex in $B^0 \to K_S^0 \pi^0$ have made this measurement possible [11].

The time-dependent CP asymmetry is obtained by measuring the proper-time difference $\Delta t \equiv t_{CP} - t_{tag}$ between a fully reconstructed decay $B^0 \to K_S^0 K^0_S K^0_S$ and the partially reconstructed tagging $B$ meson ($B_{tag}$). The asymmetry in the decay rate $f_+(f_-)$ when the tagging meson is a $B^0$ ($\bar{B}^0$) is given as

$$f_+(f_-) = \frac{e^{-|\Delta t|/\tau}}{4\tau} \times \left[1 \pm S \sin(\Delta m_d \Delta t) \mp C \cos(\Delta m_d \Delta t) \right],$$

where the parameters $C$ and $S$ describe the amount of CP violation in decay and in the interference between decay with and without mixing, respectively. Neglecting CKM-suppressed amplitudes, we expect $S = -\sin 2\beta$ and $C = 0$ in the SM.

The results presented here are based on 226.6 \pm 2.5 million $Y(4S) \to BB$ decays collected with the BABAR detector at the PEP-II asymmetric-energy $e^+e^-$ collider, located at the Stanford Linear Accelerator Center. The BABAR detector [12] provides charged-particle tracking through a combination of a five-layer double-sided silicon microstrip detector (SMT) and a 40-layer central drift chamber, both operating in a 1.5 T magnetic field. Charged kaon and pion identification is achieved through measurements of particle energy loss in the tracking system and Cherenkov angle in a detector of internally reflected Cherenkov light. A segmented CsI(Tl) electromagnetic calorimeter provides photon detection and electron identification. Finally, the instrumented flux return of the magnet allows discrimination of muons from pions.

Candidates for $B^0 \to K^0_S K^0_S K^0_S$ are formed by combining three $K^0_S$ candidates in an event. We reconstruct $K^0_S \to \pi^+ \pi^-$ candidates from pairs of oppositely charged tracks. The two-track combinations must form a vertex with a $\pi^+ \pi^-$ invariant mass within 12 MeV/c^2 (about 4\sigma) of the nominal $K^0_S$ mass [13], a reconstructed flight distance between 0.2 and 40.0 cm from the beam spot in the plane transverse to the beam, and an angle between the transverse flight direction and the transverse momentum vector of less than 200 mrad. For each B candidate two nearly independent kinematic variables are computed, namely, the beam-energy-substituted mass $m_{ES} = (s/2 + p_t \cdot p_B)^2/E_B^2 + p_B^2$, and the energy difference $\Delta E = E_B - \sqrt{s}/2$. Here, $(E_B, p_B)$ is the four vector of the initial $e^+e^-$ system, $\sqrt{s}$ is the center-of-mass energy, $p_B$ is the reconstructed momentum of the $e^+e^-$ rest frame. For signal decays, the $m_{ES}$ distribution peaks near the $B^0$ mass with an rms deviation of about 2.5 MeV/c^2 and the $\Delta E$ distribution peaks near zero with an rms deviation of about 14 MeV. We select candidates within the window $5.22 < m_{ES} < 5.30$ GeV/c^2 and $-120 < \Delta E < 120$ MeV, which includes the signal peak and a “sideband” region for background characterization.

The sample of $B^0 \to K^0_S K^0_S K^0_S$ candidates is dominated by random $K^0_S K^0_S K^0_S$ combinations from $e^+e^- \to q\bar{q}$ $(q = u, d, s, c)$ fragmentation. Monte Carlo (MC) studies show that contributions from other B meson decays can be neglected. We exploit topological observables to discriminate the jetlike $e^+e^- \to q\bar{q}$ events from the more uniformly distributed BB events. In the $Y(4S)$ rest frame we compute the angle $\theta^*_T$ between the thrust axis of the $B^0$
candidate and that of the remaining particles in the event. While $|\cos \theta_T|$ is highly peaked near 1 for $e^+e^- \rightarrow q\bar{q}$ events, it is nearly uniformly distributed for $BB$ events. We require $|\cos \theta_T| < 0.9$, eliminating ~68% of the background. In addition, we use a Fisher discriminant variable ($F$), based on the momenta and angles of tracks in the event [11], in the maximum-likelihood fit described below.

For the 1.4% of events with more than one candidate we select the combination with the smallest $\chi^2 = \sum (n_i - m_i + m_{K_S}^2/\sigma_{m_i})^2$, where $m_i$ ($m_{K_S}^2$) is the measured (nominal $K_S^0$) mass and $\sigma_{m_i}$ is the estimated uncertainty on the mass of the $i$-th $K_S^0$ candidate. We also remove all $B^0$ candidates that have a $K_S^0 K_S^0$ mass combination within 3$\sigma$ (45 MeV/$c^2$) of the $\chi_{c0}$ or $\chi_{c2}$ mass. While we expect few $\chi_{c0}$ and $\chi_{c2}$ $K_S^0 K_S^0$ decays in our final sample, these are $b \rightarrow c\bar{c}s$ decays that would bias the CP-asymmetry measurement.

We extract the results from unbinned maximum-likelihood fits to the kinematic, event-shape ($F$) and $\Delta t$ variables. We maximize the logarithm of an extended likelihood function

$$L = e^{-(N_S + N_B)} \times \prod_{i=1}^{N_F} [N_S P_S^i + N_B P_B^i],$$

where $P_S$ and $P_B$ are the probability density functions (PDFs) for signal (S) and continuum background (B), $N_F$ is the total number of events, and $N_S$ and $N_B$ are the event yields to be determined from the fit. The product is over the selected events. The observables are sufficiently uncorrelated that we can construct the likelihoods as the products of one-dimensional PDFs. The PDFs for signal are parameterized from signal MC events. For background PDFs we determine the functional form from data in the sideband regions of the other observables where backgrounds dominate. We include these regions in the fitted sample and simultaneously extract the parameters of the background PDFs along with the fit results.

For the branching fraction fit we use only the kinematic and event-shape variables ($P_{BF} = P(m_{KS}) P(\Delta E) P(F)$). There are two yields and six continuum PDF parameters floated in the fit. There are 721 $K_S^0 K_S^0 K_S^0$ candidates that pass all the above criteria, and the fit to this data yields $N_S = 88 \pm 10$ events and $N_B = 633 \pm 26$ events. Figure 1 shows the $m_{KS}$ and $\Delta E$ distributions for these events with the results of the fit plotted as curves. As a check we also add a fit component for random combinatorial $B$ background, with PDF parameters determined from large MC samples. This fit finds 14 $\pm$ 11 candidates assigned to the $B$ background. These candidates come from the continuum background; the signal yield changes by less than one candidate. A signal reconstruction efficiency of 5.6% is derived from a large MC sample in which the $K_S^0$ reconstruction efficiency is carefully matched with that observed in large hadronic data samples. Assuming equal production rates of $B^0 \bar{B}^0$ and $B^+ B^-$, we determine $B(B^0 \rightarrow K_S^0 K_S^0 K_S^0) = (6.9^{+0.5}_{-0.4} \pm 0.6) \times 10^{-6}$.

The largest systematic error (5%) for the branching fraction measurement comes from our uncertainty on the efficiency of reconstructing $K_S^0 \rightarrow \pi^+ \pi^- \pi^0$ decays [14]. We determine uncertainties of 4% for the effect of the candidate selection cuts and 5% for the parametrization of the PDFs used in the fit. The remaining uncertainties, including possible errors in modeling the $K_S^0 K_S^0 K_S^0$ Dalitz plot distribution in determining the signal efficiency, combine to 2%.

The CP-fit PDF for a given tagging category is $P_{CP} = P_{BF} P(\Delta t, \sigma_{\Delta t}) e^{i \phi}$ where $e^{i \phi}$ is the tagging efficiency for tag category $c$. The total likelihood $L$ is the product of likelihoods for each tagging category, and the free parameters are determined by maximizing the quantity $\ln L$. Along with the CPV asymmetries $S$ and $C$, the fit extracts $e^{i \phi}$ for the background and other background parameters. The background PDFs include parameters for the $\Delta t$-resolution function $R$ and for asymmetries in the rate of $B^0$ versus $\bar{B}^0$ tags. We extract 25 parameters from the CP fit.

We use a neural network to determine the flavor of the $B_{\text{tag}}$ meson from kinematic and particle-identification information [15]. Each event is assigned to one of six mutually exclusive tagging categories, designed to combine flavor tags with similar performance and $\Delta t$ resolution. We parameterize the performance of this algorithm with a data sample ($B_{\text{flav}}$) of fully reconstructed $B^0 \rightarrow D^{(*)-} \pi^+ / \rho^+ / a_1^+$ decays. The effective tagging efficiency obtained from this sample is $Q = \Sigma \varepsilon e^{(1 - 2w^2)^2} = 0.305 \pm 0.004$, where $e^{i \phi}$ and $\varepsilon$ are the efficiencies and mistag probabilities, respectively, for events tagged in category $c$.

We compute the proper-time difference $\Delta t = (z_{CP} - z_{\text{tag}})/\gamma \beta c$ using the known boost of the $e^+ e^-$ system and the measured $\Delta z = z_{CP} - z_{\text{tag}}$, the difference of the reconstructed decay vertex positions of the $B^0 \rightarrow K_S^0 K_S^0 K_S^0$ and $B_{\text{tag}}$ candidate along the boost direction ($z$). A
description of the inclusive reconstruction of the $B_{\text{tag}}$ vertex is given in Ref. [16]. For the $B^0 \rightarrow K_S^0 K_S^0 K_S^0$ decay, where no charged particles are present at the decay vertex, we constrain the $B$ meson production vertex to the interaction point (IP) in the transverse plane using a geometric fit. The position and size of the interaction region are determined on a run-by-run basis from the spatial distribution of vertices from two-track events. The uncertainty on the IP position, which follows from the size of the interaction region, is about 150 $\mu$m horizontally and 4 $\mu$m vertically. The uncertainty on $z_{CP}$, a convolution of the interaction region and the vertex of the $B^0 \rightarrow K_S^0 K_S^0 K_S^0$ decay, is about 75 $\mu$m. The uncertainty on $z_{\text{tag}}$ is about 200 $\mu$m and thus the uncertainty in $\Delta z$ is dominated by the uncertainty in the vertex of the tagging decay. The resulting resolution is comparable to that in $B^0 \rightarrow J/\psi K_S^0$ [11].

Simulation studies show that the procedure we use to determine the vertex for a $B^0 \rightarrow K_S^0 K_S^0 K_S^0$ decay provides an unbiased estimate of $z_{CP}$. The estimate of the $\Delta t$ error in an event reflects the strong dependence of the $z_{CP}$ resolution on the number of SVT layers traversed by the $K_S^0$ decay daughters. However, essentially all events have at least one $K_S^0$ candidate for which both tracks have at least one hit in the inner three layers. In this case the mean $\Delta t$ resolution is comparable to that in decays in which the vertex is directly reconstructed from charged particles originating at the $B$ decay point [16]. For a small fraction (0.1%) of the signal events, at least one $K_S^0$ has tracks with hits in the outer two SVT layers (at radii from 3.2 to 5.4 cm). In this case the mean $\Delta t$ resolution is nearly 2 times worse but the event can still be used in the $CP$ fit. Events with $\sigma_{\Delta t} > 2.5$ ps or $|\Delta t| > 20$ ps are excluded from the $CP$ fit.

The resolution function $R$ is parameterized as the sum of a “core” and a “tail” Gaussian distribution, each with a width and mean proportional to $\sigma_{\Delta t}$, and a third Gaussian with a mean of zero and a width fixed at 8 ps [16]. We have verified with MC simulation that the parameters of $R$ for $B^0 \rightarrow K_S^0 K_S^0 K_S^0$ decays are similar to those obtained from the $B_{\text{flav}}$ sample. Therefore, we extract these parameters from a fit to the $B_{\text{flav}}$ sample. We find that the $\Delta t$ distribution of background candidates is well described by a delta function convolved with a resolution function having the same functional form as that for the signal. The parameters of the background function are determined in the fit.

The fit including $\Delta t$ and tagging information yields $S = -0.71^{+0.38}_{-0.32} \pm 0.04$ and $C = -0.34^{+0.28}_{-0.25} \pm 0.05$. Fixing $C = 0$ we obtain $\sin2\beta = -S = 0.79^{+0.29}_{-0.36} \pm 0.04$. Figure 2 shows distributions of $\Delta t$ for $B^0$-tagged and $B^0$-tagged events, and the asymmetry $A(\Delta t) = (N_{B^0} - N_{\bar{B}^0})/(N_{B^0} + N_{\bar{B}^0})$, obtained by making a likelihood ratio cut to remove the background component.

Systematic uncertainties on the $CP$ parameters are given in Table I. The systematic errors are evaluated with large samples of simulated $B_{\text{flav}}$ and $B^0 \rightarrow K_S^0 K_S^0 K_S^0$ decays. We employ the difference in resolution function parameters extracted from these samples to vary the resolution function parameters extracted from the $B_{\text{flav}}$ sample in data. We also perform fits to the simulated $B^0 \rightarrow K_S^0 K_S^0 K_S^0$ signal with parameters obtained either from signal or $B_{\text{flav}}$ events to account for any potential bias due to the vertexing technique. Several SVT misalignment scenarios are applied to the simulated $B^0 \rightarrow K_S^0 K_S^0 K_S^0$ events to estimate detector effects. We consider large variations, several times the IP size, of the IP position and resolution and find they have negligible impact. Asymmetries in the rate of $B^0$ versus $\bar{B}^0$ tags in the background events, which are free parameters in the fit, are fixed to zero as a systematic uncertainty. The systematic error due to correlations in the fit variables is extracted from a fit to a sample of randomly selected signal MC events added to background events from a parametrized MC calculation. We allow for

![FIG. 2 (color online). Distributions of $\Delta t$ for background subtracted events for $B_{\text{tag}}$ tagged as (a) $B^0$ or (b) $\bar{B}^0$, and (c) the asymmetry $A(\Delta t)$. We use a likelihood ratio cut that removes 96% of the background while retaining 95% of the signal.](image-url)
the possible interference between the suppressed $\bar{b} \rightarrow \bar{u}c\bar{d}$ and the favored $b \rightarrow c\bar{d}u$ amplitude for some tagside $B$ decays [17]. Finally, we include a systematic uncertainty to account for imperfect knowledge of the PDFs used in the fit. Most of the uncertainties on the PDFs are statistical and some are associated with data and MC differences. As an additional check, a $B$ background component is added to the $CP$ fit and we find the variation of the asymmetries to be negligible.

In summary, we have measured the $B^0 \rightarrow K_S^0 K^+$ branching fraction and the time-dependent CPV asymmetries. The BF measurement is in good agreement with previous measurements [10]. The measurements of $S$ and $C$ are in good agreement with the SM expectation.

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Note added.—We became aware of a submission by the Belle Collaboration [18] on the same subject, while this paper was under review.

*Also with Università della Basilicata, Potenza, Italy
†Deceased