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Searches for Lepton Flavor Violation in the Decays $\tau^+ \rightarrow e^+ \gamma$ and $\tau^+ \rightarrow \mu^+ \gamma$

Searches for lepton-flavor-violating decays of a $\tau$ lepton to a lighter mass lepton and a photon have been performed with the entire data set of $963 \pm 7 \times 10^6$ $\tau$ decays collected by the BABAR detector near the $Y(4S)$, $Y(3S)$ and $Y(2S)$ resonances. The searches yield no evidence of signals and we set upper limits on...
the branching fractions of $\mathcal{B}(\tau^\pm \rightarrow e^\pm \gamma) < 3.3 \times 10^{-8}$ and $\mathcal{B}(\tau^\pm \rightarrow \mu^\pm \gamma) < 4.4 \times 10^{-8}$ at 90% confidence level.

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Amongst all the possible lepton-flavor-violating $\tau$ processes, $\tau^\pm \rightarrow \ell^\pm \gamma$ (where $\ell = e, \mu$) is predicted to be the dominant decay mode in a wide variety of new physics scenarios, with rates close to current experimental limits. Despite the existence of neutrino oscillations [1], such decays are predicted to have unobservably low rates [2] in the standard model (SM). Thus, an observation of charged lepton flavor violation would be an unambiguous signature of new physics, while improvements on existing limits will constrain many models. As the relationships between $\mu \rightarrow e \gamma, \tau \rightarrow e \gamma$ and $\tau \rightarrow \mu \gamma$ decays are model dependent, searches for both $\tau$ modes provide independent information, even in the light of the small limit of $\mathcal{B}(\mu \rightarrow e \gamma) < 1.2 \times 10^{-11}$ at 90% confidence level (C.L.) [3].

Presently, the stringent limits on the SM are $\mathcal{B}(\tau^\pm \rightarrow e^\pm \gamma) < 1.1 \times 10^{-7}$ [4] and $\mathcal{B}(\tau^\pm \rightarrow \mu^\pm \gamma) < 4.5 \times 10^{-8}$ [5] at 90% C.L., using 232.2 fb$^{-1}$ and 535 fb$^{-1}$ of $e^+e^-$ annihilation data collected near the $Y(4S)$ resonance by the BABAR and Belle experiments, respectively. This Letter reports the final result on these processes from BABAR. It utilizes the entire data set recorded by the BABAR detector at the SLAC PEP-II $e^+e^-$ storage rings, corresponding to a luminosity of 425.5 fb$^{-1}$, 28.0 fb$^{-1}$ and 13.6 fb$^{-1}$ recorded at the $Y(4S), Y(3S)$ and $Y(2S)$ resonances, and 44.4, 26, and 1.4 fb$^{-1}$ recorded at 40, 30, and 30 MeV below the resonances, respectively.

For the bulk of the data sample at the $Y(4S)$ resonance, the cross section $\sigma_{e^+e^- \rightarrow \tau+\tau^-} = (0.919 \pm 0.003) \text{nb}$ [6], determined to high precision using the KK Monte Carlo (MC) generator [7], receives negligible contribution from $Y(4S)$ due to its large decay width. But, for the remaining data at the $Y(3S)$ and $Y(2S)$ resonances, the $\tau$-pair cross section receives additional contributions of $\mathcal{B}(Y \rightarrow \tau^+\tau^-) = 2\%$, which are known only at the 10% level [8]. Including a systematic uncertainty of 0.6% on the luminosity determination, this gives a total of $N_{\tau} = (963 \pm 7) \times 10^6 \tau$ decays.

The BABAR detector is described elsewhere [9]. Charged particles are reconstructed as tracks with a 5 layer silicon vertex tracker and a 40 layer drift chamber inside a 1.5 T solenoidal magnet. A CsI(Tl) electromagnetic calorimeter is used to identify electrons and photons. A ring-imaging Cherenkov detector is used to identify charged pions and kaons. The flux return of the solenoid, instrumented with resistive plate chambers and limited streamer tubes, is used to identify muons.

The signal is characterized by a $\ell^\pm \gamma$ pair with an invariant mass and total energy in the center-of-mass (c.m.) frame ($E_{\ell^\pm \gamma}$) close to $m_{\tau} = 1.777 \text{GeV}/c^2$ [8] and $\sqrt{s}/2$, respectively. The event must also contain another $\tau$ decay, reconstructed as decaying to one or three tracks.

The dominant irreducible background comes from $\tau$-pair events containing hard photon radiation and one of the $\tau$ leptons decaying to a charged lepton. The remaining backgrounds for $\tau^\pm \rightarrow e^\pm \gamma$ and $\tau^\pm \rightarrow \mu^\pm \gamma$ decays arise from the relevant radiative processes, $e^+e^- \rightarrow e^+e^-\gamma$ and $e^+e^- \rightarrow \mu^+\mu^-\gamma$, and from hadronic $\tau$ decays where a pion is misidentified as the electron or muon.

Signal events are simulated using KK and TAUOLA [10] with measured $\tau$ branching fractions [8]. The $\mu^\pm \mu^\pm$ and $\tau^+\tau^-$ background processes are generated using KK and TAUOLA, while the $q\bar{q}$ processes are generated using JETSET [11] and EVTGEN [12]. Radiative corrections for all processes are simulated using PHOTOS [13]. The Bhabha background is studied using events with two identified electrons in the data. The two-photon background has been studied and found to be negligible. The detector response to generated particles is simulated using the GEANT4 package [14]. MC events are used to optimize the selection criteria and estimate the systematic uncertainties on the efficiency, while the background rates are estimated directly from data.

Events with two or four well reconstructed tracks and zero total charge are selected, where no track pair is consistent with being a photon conversion in the detector material. Each event is divided into hemispheres (“signal” and “tag” sides) in the c.m. frame by a plane perpendicular to the thrust axis calculated using all reconstructed charged and neutral particles [15].

The signal-side hemisphere must contain one photon with c.m. energy $E_{\gamma}^m$ greater than 1 GeV, and no other photon with energy greater than 100 MeV in the laboratory frame. The signal side must contain one track within the calorimeter acceptance with momentum in the c.m. frame less than 0.77$\sqrt{s}/2$. This track must be identified as an electron or a muon for the $\tau^\pm \rightarrow e^\pm \gamma$ or $\tau^\pm \rightarrow \mu^\pm \gamma$ search. The electron selectors have an efficiency of 96% within the fiducial coverage. For reliable muon identification, the track momentum is required to be greater than 0.7 GeV/c in the laboratory frame, above which the selection efficiency is 83%.

In the rest-frame of the $\tau^\pm$, the $\ell^\pm$ and the $\gamma$ are produced back-to-back. When boosted to the c.m. frame, kinematic considerations of two-body decays require there to be a minimum opening angle between them. The cosine of the opening angle, $\cos \theta_{\ell\gamma}$, between signal-track and signal-photon is required to be less than 0.786. The tag-side hemisphere is expected to contain a SM $\tau$ decay. A tag-side hemisphere containing a single track is
classified as \(e\) tag, \(\mu\) tag, or \(\pi\) tag if the total photon c.m. energy in the hemisphere is less than 200 MeV and the track is exclusively identified as an electron (\(e\) tag), as a muon (\(\mu\) tag), or as neither (\(\pi\) tag). Events with the tag-side track failing both the lepton selectors are classified as \(\rho\) tag if they contain at least one \(\pi^0\) candidate reconstructed from a pair of photons with invariant mass between 90 and 165 MeV/c\(^2\). If the tag-side hemisphere contains three charged tracks, all of which fail the lepton identification, it is classified as a 3h tag.

The definitions of the tag-side modes are designed to minimize the residual backgrounds from radiative QED processes. For the \(\tau^+ \rightarrow e^+\gamma\) search, very loose electron selection criteria are applied for the \(e\) tag sample. Thus, the remaining tags which fail these very loose electron criteria have small Bhabha contamination. The \(e\) tag events are used as the control sample to model the Bhabha background characteristics, and are removed from the final sample of events in the \(\tau^+ \rightarrow e^+\gamma\) search. Similarly, for the \(\tau^+ \rightarrow \mu^+\gamma\) search, very loose muon criteria are applied for the \(\mu\) tag, on which stricter kinematic requirements are later imposed with tolerable loss in signal efficiency. The other tags are required to fail these very loose muon criteria, thereby reducing dimuon backgrounds.

To suppress non-\(\tau\) backgrounds with missing momentum along the beam direction due to initial and final state photon radiation, we require that the polar angle \(\theta_{\text{miss}}\) of the missing momentum be inside the detector acceptance, i.e., \(-0.76 < \cos\theta_{\text{miss}} < 0.92\).

The total c.m. momentum of all tracks and photon candidates on the tag side is required to be less than 0.77\(\sqrt{s}/2\) for \(e\), \(\mu\), \(\pi\) tags and less than 0.9\(\sqrt{s}/2\) for \(\rho\) and 3h tags. The tag-side pseudomass [16] is required to be less than 0.5 GeV/c\(^2\) for \(e\), \(\mu\), \(\pi\) tags and less than 1.777 GeV/c\(^2\) for \(\rho\) and 3h tags.

The mass squared \(m_{\gamma
\text{miss}}^2\) of the missing particles on the tag side is calculated using the tag-side tracks and photon candidates and assuming that in the c.m. frame, the tag-side angle is constant that of the signal \(\tau\) and that its energy is \(\sqrt{s}/2\). To reduce backgrounds, we require \(m_{\gamma
\text{miss}}^2 > -0.25 \text{ GeV}/c^2\) for \(e\) and \(\mu\) tags, \(m_{\gamma
\text{miss}}^2 < 0.25 \text{ GeV}/c^2\) for \(\pi\) and 3h tags, and \(m_{\gamma
\text{miss}}^2 < 0.50 \text{ GeV}/c^2\) for \(\rho\) tags.

For radiative Bhabha and di-muon events, the expected photon energy in the c.m. frame \((E_{\gamma
\text{c.m.}})^\text{obs} = \sqrt{|\sin(\theta_1 + \theta_2)|^2 \sum_{\gamma \gamma}^2 + \sin^2(\theta_1 \theta_2)}^2\), where \(\pi - \theta_1\) and \(\pi - \theta_2\) are the angles the photon momentum makes with the signal-track and the total observed tag-side momentum, respectively. Also, for such events, we expect the cosine of the opening angle, \(\theta_{\text{recoil}}\), between the signal-track and the total observed tag-side momentum in the reference frame obtained by removing the signal photon from the c.m. frame to peak at \(-1\). To suppress these backgrounds, we remove events having reconstructed photon energy consistent with the expected value, i.e., \(|E_{\gamma
\text{c.m.}} - (E_{\gamma
\text{c.m.}})^\text{exp}| = |\Delta E_{\gamma}| < 0.125\sqrt{s} \text{ and } \cos\theta_{\text{recoil}} < -0.975\) in \(e\) and \(\mu\) tags for the \(\tau^+ \rightarrow \mu^+\gamma\) search. No such criteria are necessary for the \(\tau^+ \rightarrow e^+\gamma\) search according to the optimization procedure.

To further suppress the remaining backgrounds, neural net (NN) based discriminators are employed separately for each tag and for each data set taken at values of \(\sqrt{s}\) near the \(Y(4S), Y(3S), \) and \(Y(2S)\) resonances. Six observables are used as input to the NN: the total tag-side momentum divided by \(\sqrt{s}/2\), \(m_{\gamma
\text{miss}}^2, \Delta E_{\gamma}/\sqrt{s}, \cos\theta_{\text{recoil}}\), \(\cos\theta_{\gamma}\), and the transverse component of missing momentum relative to the collision axis. The NN based discriminators improve the signal to background ratios for the two searches by factors of 1.4 and 1.3, respectively.

Signal decays are identified by two kinematic variables: the energy difference \(\Delta E = E_{\gamma
\text{c.m.}} - \sqrt{s}/2\) and the beam-energy constrained \(\tau\) mass \((m_{\text{EC}})\), obtained from a kinematic fit after requiring the c.m. \(\tau\) energy to be \(\sqrt{s}/2\) and after assigning the origin of the \(\gamma\) candidate to the point of closest approach of the signal-lepton track to the \(e^+e^-\) collision axis. The distributions of these two variables have a small correlation arising from initial- and final-state radiation. For signal MC events, the \(m_{\text{EC}}\) and \(\Delta E\) distributions are centered at \(m_{\tau}\) and small negative values, respectively, where the shifts from zero for the latter are due to radiation and photon energy reconstruction effects. The mean and standard deviations of the \(m_{\text{EC}}\) and \(\Delta E\) distributions for the reconstructed signal MC events are presented in Table I. The data events falling within a 3\(\sigma\) ellipse in the \(m_{\text{EC}}\) vs \(\Delta E\) plane, centered around the reconstructed peak positions as obtained using signal MC, are not examined until all optimization and systematic studies have been completed. The selections are optimized to yield the smallest expected upper limits [17] for observing events inside a 2\(\sigma\) signal ellipse under background-only hypotheses.

The distributions of events in \(m_{\text{EC}}\) vs \(\Delta E\) are shown in Fig. 1. To study signal-like events, a grand signal box (GSB) is defined as \(m_{\text{EC}} \in [1.55, 2.05] \text{ GeV}/c^2\) and

<table>
<thead>
<tr>
<th>Decay modes</th>
<th>(&lt;m_{\text{EC}}&gt;(\text{MeV}/c^2))</th>
<th>(\sigma(m_{\text{EC}})(\text{MeV}/c^2))</th>
<th>(&lt;\Delta E&gt;(\text{MeV}))</th>
<th>(\sigma(\Delta E)(\text{MeV}))</th>
<th>2(\sigma) signal ellipse obs</th>
<th>(\epsilon) (exp)</th>
<th>UL ((\times 10^{-8}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\tau^+ \rightarrow e^+\gamma)</td>
<td>1777.3</td>
<td>8.6</td>
<td>-21.4</td>
<td>42.1</td>
<td>0</td>
<td>1.6 ± 0.4</td>
<td>3.9 ± 0.3</td>
</tr>
<tr>
<td>(\tau^+ \rightarrow \mu^+\gamma)</td>
<td>1777.4</td>
<td>8.3</td>
<td>-18.3</td>
<td>42.2</td>
<td>2</td>
<td>3.6 ± 0.7</td>
<td>6.1 ± 0.5</td>
</tr>
</tbody>
</table>
The two searches, where the quoted statistical errors are due to summed over background types only, we expect the $C_28$ events are obtained from separate maximum likelihood fits to 41 and 2053 data events survive for the $\tau^\pm \rightarrow \mu^\pm \gamma$ channel. These agree to within 2.4% and 1.7% with the numbers of background MC events observed. The signal-track arises from a real electron or muon in 96% and 82% of the background MC events for the two searches.

The shape of the Bhabha component is obtained using the data samples having $\cos \theta_{\text{recoil}} < -0.8$ from events selected in the $e$ tag sample for the $\tau^\pm \rightarrow e^\pm \gamma$ search, while the shapes of $\mu^\pm$, $\tau^\pm$, $\tau^\pm \tau^\mp$ and $q\bar{q}$ PDFs are obtained from their respective MC samples.

The fractions of events for each background type are obtained from separate maximum likelihood fits to 41 and 105 events inside the FB, respectively, for the two searches. We find $(70 \pm 15)\%$ and $(90 \pm 8)\%$ of the background events are $\tau$-pair events. By integrating the total PDF summed over background types only, we expect $(1.6 \pm 0.3)$ and $(3.6 \pm 0.4)$ events inside the $2\sigma$ signal ellipse for the two searches, where the quoted statistical errors are due to the sizes of the fitted data set.

As a cross check, we integrate the total PDF over four $2\sigma$ ellipses inside the FB, whose centers are shifted by $\pm 5\sigma$ or by $\pm 9\sigma$ along $m_{\text{EC}}$ only. The numbers of observed events in each of these neighboring regions and their sums are consistent with the expected numbers of events, which are shown along with their statistical errors in Table II.

To obtain the systematic errors on the numbers of expected background events, we fit the $m_{\text{EC}}$ distributions of 32 and 81 data events inside the $\pm 2\sigma$ band in $\Delta E$ over the GSB region but outside the blinded $3\sigma$ ellipse. Varying degrees of polynomial functions are used to model the $m_{\text{EC}}$ distributions, which are then integrated to obtain the number of expected events inside the $2\sigma$ ellipse. The largest deviations between the predictions from one-dimensional and two-dimensional fits are used to set the total uncertainties of 0.4 and 0.7 events on the background estimates.

The systematic uncertainties in the signal selection and reconstruction efficiencies for $\tau^\pm \rightarrow e^\pm \gamma$ and $\tau^\pm \rightarrow \mu^\pm \gamma$ decays due to the modeling of the variables entering the NN are 2.7% and 1.8%, respectively. Those due to the photon reconstruction efficiency are 1.8% for both decays, while those due to the signal-lepton track identification are 2.3% and 2.7%, respectively. The contributions due to the uncertainty in the signal-track momentum and signal-photon energy scale and resolution, estimated by varying the peak position and resolution of the $m_{\text{EC}}$ and $\Delta E$ distributions, are 6.4% and 6.2%, respectively. Other systematic uncertainties totaling less than 1.5% for both signal decay modes include those arising from trigger and filter efficiencies, tracking efficiencies, and the beam-energy scale and spread. We use approximately $10^6$ MC events per channel, resulting in a negligible systematic uncertainty due to MC statistics. Although the signal MC has been modeled using a flat phase space model, the efficiencies are insensitive to this assumption as demonstrated by considering the two extreme cases of $V-A$ and $V+A$ forms of interaction for the signal MC. All contributions to the systematic uncertainties are added in quadrature to give

FIG. 1. The GSB and the $2\sigma$ ellipse for $\tau^\pm \rightarrow e^\pm \gamma$ (left) and $\tau^\pm \rightarrow \mu^\pm \gamma$ (right) decays in the $m_{\text{EC}}$ vs $\Delta E$ plane. Data are shown as dots and contours containing 90% (50%) of signal MC events are shown as light- (dark-) shaded regions.

$\Delta E \in [-1.0, 0.5]$ GeV. Outside the blinded $3\sigma$ ellipse, 1389 data events survive in the GSB for the $\tau^\pm \rightarrow e^\pm \gamma$ channel, and 2053 data events survive for the $\tau^\pm \rightarrow \mu^\pm \gamma$ channel. These agree to within 2.4% and 1.7% with the numbers of background MC events observed. The signal-track arises from a real electron or muon in 96% and 82% of the background MC events for the two searches.

A Fit Box (FB) region is defined as $m_{\text{EC}} \in [1.6, 2.0]$ GeV/c$^2$ and $\Delta E \in [-0.14, 0.14]$ GeV, excluding the blinded $3\sigma$ ellipse. The $m_{\text{EC}}$ vs $\Delta E$ distributions of events inside the FB are modeled by two-dimensional probability density functions (PDFs) summed over all background event types. The PDFs have correlations built in using Gaussian weights with an adaptive kernel estimation procedure [18]. The shape of the Bhabha component is obtained using the data samples having $\cos \theta_{\text{recoil}} < -0.8$ from events selected in the $e$ tag sample for the $\tau^\pm \rightarrow e^\pm \gamma$ search, while the shapes of $\mu^\pm$, $\tau^\pm \tau^\mp$, and $q\bar{q}$ PDFs are obtained from their respective MC samples.

The fractions of events for each background type are obtained from separate maximum likelihood fits to 41 and 105 events inside the FB, respectively, for the two searches. We find $(70 \pm 15)\%$ and $(90 \pm 8)\%$ of the background events are $\tau$-pair events. By integrating the total PDF summed over background types only, we expect $(1.6 \pm 0.3)$ and $(3.6 \pm 0.4)$ events inside the $2\sigma$ signal ellipse for the two searches, where the quoted statistical errors are due to the sizes of the fitted data set.

As a cross check, we integrate the total PDF over four $2\sigma$ ellipses inside the FB, whose centers are shifted by $\pm 5\sigma$ or by $\pm 9\sigma$ along $m_{\text{EC}}$ only. The numbers of observed events in each of these neighboring regions and their sums are consistent with the expected numbers of events, which are shown along with their statistical errors in Table II.

To obtain the systematic errors on the numbers of expected background events, we fit the $m_{\text{EC}}$ distributions of 32 and 81 data events inside the $\pm 2\sigma$ band in $\Delta E$ over the GSB region but outside the blinded $3\sigma$ ellipse. Varying degrees of polynomial functions are used to model the $m_{\text{EC}}$ distributions, which are then integrated to obtain the number of expected events inside the $2\sigma$ ellipse. The largest deviations between the predictions from one-dimensional and two-dimensional fits are used to set the total uncertainties of 0.4 and 0.7 events on the background estimates.

The systematic uncertainties in the signal selection and reconstruction efficiencies for $\tau^\pm \rightarrow e^\pm \gamma$ and $\tau^\pm \rightarrow \mu^\pm \gamma$ decays due to the modeling of the variables entering the NN are 2.7% and 1.8%, respectively. Those due to the photon reconstruction efficiency are 1.8% for both decays, while those due to the signal-lepton track identification are 2.3% and 2.7%, respectively. The contributions due to the uncertainty in the signal-track momentum and signal-photon energy scale and resolution, estimated by varying the peak position and resolution of the $m_{\text{EC}}$ and $\Delta E$ distributions, are 6.4% and 6.2%, respectively. Other systematic uncertainties totaling less than 1.5% for both signal decay modes include those arising from trigger and filter efficiencies, tracking efficiencies, and the beam-energy scale and spread. We use approximately $10^6$ MC events per channel, resulting in a negligible systematic uncertainty due to MC statistics. Although the signal MC has been modeled using a flat phase space model, the efficiencies are insensitive to this assumption as demonstrated by considering the two extreme cases of $V-A$ and $V+A$ forms of interaction for the signal MC. All contributions to the systematic uncertainties are added in quadrature to give
total relative systematic uncertainties on the efficiencies of 7.7% and 7.4% for $\tau^\pm \rightarrow e^\pm \gamma$ and $\tau^\pm \rightarrow \mu^\pm \gamma$ decays, respectively.

We observe 0 and 2 events for the $\tau^\pm \rightarrow e^\pm \gamma$ and $\tau^\pm \rightarrow \mu^\pm \gamma$ searches inside the $2\sigma$ signal ellipse, respectively. As there is no evidence for a signal, we set a frequentist upper limit calculated using $B^{90}_{UL} = N^{90}_{UL}/(N,e)$ to be $B(\tau^\pm \rightarrow e^\pm \gamma) < 3.3 \times 10^{-8}$ and $B(\tau^\pm \rightarrow \mu^\pm \gamma) < 4.4 \times 10^{-8}$ at 90% C.L., where $e$ is the signal efficiency inside the $2\sigma$ signal ellipse and $N^{90}_{UL}$ is the 90% C.L. upper limit on the number of signal events, estimated using the POLE program [19]. The upper limits which include all systematic uncertainties, are presented in Table I, along with signal efficiencies and numbers of observed and expected background events. These results supersede previous BABAR results [4,20], reducing the upper limits by factors of 3.3 and 1.5, respectively, and are the most stringent limits on lepton flavor violation in $\tau^\pm \rightarrow e^\pm \gamma$ and $\tau^\pm \rightarrow \mu^\pm \gamma$ decays.

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*Deceased.
†Present address: Temple University, Philadelphia, PA19122, USA.
‡Also with Università di Perugia, Dipartimento di Fisica, Perugia, Italy.
§Also with Università di Roma La Sapienza, I-00185 Roma, Italy.
¶Present address: University of South Alabama, Mobile, AL 36688, USA.
¶¶Also with Laboratoire de Physique Nucléaire et de Hautes Energies, IN2P3/CNRS, Université Pierre et Marie Curie-Paris6, Université Denis Diderot-Paris7, F-75252 Paris, France.
**Also with Università di Sassari, Sassari, Italy.


[2] B. W. Lee and R. E. Shrock, Phys. Rev. D 16, 1444 (1977). A value of $\Delta m^2_{31} \approx 3 \times 10^{-3}$ (eV/c$^2$)$^2$ [1] and maximal mixing implies $B(\tau^\pm \rightarrow \mu^\pm \gamma) \approx O(10^{-54})$.


