Measurement of Time-Dependent $CP$ Asymmetries and the $CP$-Odd Fraction in the Decay $B^0 \rightarrow D^* + D^*$

The time-dependent CP asymmetry measurement in \( B^0 \rightarrow D^{*+} D^{*-} \) decay provides an important test of the standard model (SM). In the SM, CP violation arises from a complex phase in the Cabibbo-Kobayashi-Maskawa quark-mixing matrix [1]. Measurements of CP asymmetries by the BABAR [2] and Belle [3] Collaborations have firmly established this effect in the \( B^0 \rightarrow J/\psi K_S^0 \) decay [4] and related modes that are governed by the \( b \rightarrow c\bar{c}s \) transition. The \( B^0 \rightarrow D^{*+} D^{*-} \) decay is dominated by the \( b \rightarrow c\bar{c}d \) transition. Within the framework of the SM, the CP asymmetry of \( B^0 \rightarrow D^{*+} D^{*-} \) is related to \( \sin 2\beta \) when the correction due to penguin diagram contributions are neglected. The penguin-induced correction has been estimated in models based on the factorization approximation and heavy quark symmetry and was predicted to be about 2% [5]. A significant deviation of the measured \( \sin 2\beta \) from the one observed in \( b \rightarrow c\bar{c}s \) decays would be evidence for a new CP-violating interaction. The enhanced sensitivity of \( B^0 \rightarrow D^{*+} D^{*-} \) to such a process arises from its much smaller SM amplitude compared with that of the \( b \rightarrow c\bar{c}s \) transition.

The \( B^0 \rightarrow D^{*+} D^{*-} \) decay proceeds through the CP-even S and D waves and through the CP-odd P wave. In this Letter, we present an improved measurement of the CP-odd fraction [6,7] \( R_\perp \) based on a time-integrated one-dimensional angular analysis. We also present an improved measurement of the time-dependent CP asymmetry [6,7], obtained from a combined analysis of time-dependent flavor-tagged decays and the one-dimensional angular distribution of the decay products.

The data used in this analysis comprise \( 232 \times 10^6 \) \( Y(4S) \rightarrow B\bar{B} \) decays collected by the BABAR detector at the SLAC PEP-II storage ring. The BABAR detector is described in detail elsewhere [8]. We use a Monte Carlo (MC) simulation based on GEANT4 [9] to validate the analysis procedure and to study the relevant backgrounds.

We select \( B^0 \rightarrow D^{*+} D^{*-} \) decay by combining two charged \( D^* \) candidates reconstructed in the modes \( D^{*+} \rightarrow D^{0} \pi^+ \text{ and } D^{*+} \rightarrow D^{+} \pi^0 \). We include the \( D^{*+} D^{*-} \) combinations \( (D^{0} \pi^+, \bar{D}^{0} \pi^-) \text{ and } (D^{0} \pi^+, D^{-} \pi^0) \), but not \( (D^{+} \pi^0, D^{-} \pi^0) \) because of the smaller branching fraction and larger backgrounds. To suppress the \( e^+ e^- \rightarrow q\bar{q} \) (\( q = u, d, s, \text{ and } c \)) continuum background, we require the ratio of the second and zeroth order Fox-Wolfram moments [10] to be less than 0.6.

Candidates for \( D^0 \) and \( D^+ \) mesons are reconstructed in the modes \( D^0 \rightarrow K^- \pi^+, \ K^- \pi^+ \pi^0, \ K^- \pi^+ \pi^+ \pi^-, \ K_S^0 \pi^+ \pi^- \text{ and } D^+ \rightarrow K^- \pi^+ \pi^+, \ K_S^0 \pi^+ \pi^+, \ K^- K^+ \pi^+ \). The reconstructed mass of the \( D^0 (D^+) \) candidate is required to be within 20 MeV/c^2 of its nominal mass [11], except for the \( D^0 \rightarrow K^- \pi^+ \pi^0 \) candidate, where a looser requirement of 40 MeV/c^2 is applied.

The \( K_S^0 \) candidates are reconstructed from two oppositely charged tracks with an invariant mass within 20 MeV/c^2 of the nominal \( K_S^0 \) mass. The \( \chi^2 \) probability of the \( \pi^- \pi^+ \) vertex fit must be greater than 0.1%. Charged kaon candidates are required to be inconsistent with the pion hypothesis, as inferred from the Cherenkov angle measured by the Cherenkov detector and the ionization energy loss measured by the charged-particle tracking system [8]. Neutral pion candidates are formed from two photons detected in the electromagnetic calorimeter [8], each with energy above 30 MeV. The mass of the pair must be within 30 MeV/c^2 of the nominal \( \pi^0 \) mass, and their summed energy is required to be greater than 200 MeV. In addition, a mass-constrained fit is applied to the \( \pi^0 \) candidates for further analysis.

The \( D^0 \) and \( D^+ \) candidates are subject to a mass-constrained fit prior to the formation of the \( D^{*+} \) candidates. A slow \( \pi^+ \) from \( D^{*+} \) decay is required to have a momentum in the \( Y(4S) \) center-of-mass (c.m.) frame less than 450 MeV/c. A slow \( \pi^0 \) from \( D^{*+} \) must have a momentum between 70 and 450 MeV/c in the c.m. frame. No requirement on the photon-energy sum is applied to the \( \pi^0 \) candidates from the \( D^{*+} \) decays.

For each \( B^0 \rightarrow D^{*+} D^{*-} \) candidate, we construct a likelihood function [12] \( \mathcal{L}_{\text{mass}} \) from the masses and mass uncertainties of the \( D \) and \( D^* \) candidates. The likelihood \( \mathcal{L}_{\text{mass}} \) is calculated as the product of the likelihoods for the \( D \) and \( D^* \) candidates. The \( D \) mass resolution is modeled by a Gaussian whose variance is determined on a candidate-by-candidate basis. The \( D^- \rightarrow D^{*+} \) mass difference resolution is modeled by a double-Gaussian distribution whose parameters are determined from simulated events. The values of \( \mathcal{L}_{\text{mass}} \) and the difference of the \( B^0 \) candidate energy \( E_B \) from the beam energy \( E_{\text{Beam}} \), \( \Delta E \equiv E_B - E_{\text{Beam}} \), in the \( Y(4S) \) c.m. frame are used to reduce the combinatoric background further. From the simulated events, the maximum allowed values of \( -\ln \mathcal{L}_{\text{mass}} \) and \( |\Delta E| \) are optimized for each individual
The energy-substituted mass, \( m_{ES} = \sqrt{E^2_{\text{beam}} - p_B^2} \), where \( p_B \) is the \( B^0 \) candidate momentum in the \( Y(4S) \) c.m. frame, is used to extract the signal yield from the events satisfying the aforementioned selection. We select the \( B^0 \) candidates that have \( m_{ES} \approx 5.23 \text{ GeV}/c^2 \). In cases where more than one \( B^0 \) candidate is reconstructed in an event, the candidate with the smallest value of \( -\ln L_{\text{mass}} \) is chosen. A fit to the \( m_{ES} \) distribution with a probability density function (PDF) given by the sum of a Gaussian shape for the signal and an ARGUS [13] function for the background yields \( 391 \pm 28 \) (stat) signal events. In the region of \( m_{ES} > 5.27 \text{ GeV}/c^2 \), the signal purity is approximately 70%.

In the transversity basis [14], we define the following three angles: the angle \( \theta_1 \) between the momentum of the slow pion from the \( D^{*+} \) and the opposite direction of flight of the \( D^{*+} \) in the \( D^{*-} \) rest frame; the polar angle \( \theta_u \) and azimuthal angle \( \phi_u \) of the slow pion from the \( D^{*+} \) defined in the \( D^{*+} \) rest frame, where the opposite direction of flight of the \( D^{*-} \) is chosen as the \( x \) axis, and the \( z \) axis is defined as the normal to the \( D^{*-} \) decay plane.

The time-dependent angular distribution of the decay products is given in Ref. [15]. Taking into account the detector angular acceptance efficiency and integrating over the decay time and the angles \( \theta_1 \) and \( \phi_u \), we obtain a one-dimensional differential decay rate:

\[
\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_u} = \frac{9}{32\pi} \left[ (1 - R_\perp) \sin^2 \theta_u \left( \frac{1 + \alpha}{2} I_0(\cos \theta_u) + \frac{1 - \alpha}{2} I_\parallel(\cos \theta_u) \right) + 2R_\perp \cos^2 \theta_u \times I_\perp(\cos \theta_u) \right],
\]

where \( R_\perp = |A_0|^2/(|A_\parallel|^2 + |A_0|^2 + |A_\perp|^2) \), \( \alpha = (|A_0|^2 - |A_\parallel|^2)/(|A_\parallel|^2 + |A_0|^2) \), \( A_\parallel \) is the amplitude for longitudinally polarized \( D^* \) mesons, \( A_\parallel \) and \( A_\perp \) are the amplitudes for parallel and perpendicular transversely polarized \( D^* \) mesons. The three efficiency moments, \( I_k (k = 0, \parallel, \perp) \), are defined as

\[
I_k(\cos \theta_u) = \int d \cos \theta_1 d \phi_u g_k(\theta_1, \phi_u) \epsilon(\theta_1, \theta_u, \phi_u), \quad (2)
\]

where \( g_0 = 4 \cos^2 \theta_1 \cos^2 \phi_u \), \( g_\parallel = 2 \sin^2 \theta_1 \sin^2 \phi_u \), \( g_\perp = \sin^2 \theta_1 \), and \( \epsilon \) is the detector efficiency. The efficiency moments are parametrized as second-order even polynomials of \( \cos \theta_u \). Their parameter values are determined from the MC calculation and are subsequently fixed in the likelihood fit to the differential decay distribution of \( \cos \theta_u \). In fact, the three \( I_k \) functions deviate only slightly from a constant, making the distribution, Eq. (1), nearly independent of the amplitude ratio \( \alpha \).

The \( CP \)-odd fraction \( R_\perp \) is measured in a simultaneous unbinned maximum likelihood fit to the \( \cos \theta_u \) and the \( m_{ES} \) distribution. The background shape is modeled as an even second-order polynomial in \( \cos \theta_u \), while the signal PDF is given by Eq. (1). The finite detector resolution of the \( \theta_u \) measurement is modeled as a double Gaussian plus a small tail component that accounts for misreconstructed events. The parametrization of the \( \theta_u \) resolution function is fixed from the MC simulation and subsequently used to convolve the PDF in the maximum likelihood fit. Since the angle \( \theta_u \) is calculated with the slow pion from the \( D^{*+} \), we categorize events into three types: \( D^{*+} \to (D^0 \pi^+, D^0 \pi^-) \), \( (D^0 \pi^+, D^- \pi^-) \), and \( (D^0 \pi^+, D^0 \pi^-) \), each with different signal-fraction parameters in the likelihood fit. Their angular efficiency moments and \( \cos \theta_u \) resolutions are also separately determined from the MC simulation. The other parameters determined in the likelihood fit are the \( \cos \theta_u \) background-shape parameter, three \( m_{ES} \) parameters (\( \sigma \) and mean of the signal Gaussian, and the ARGUS shape parameter \( \kappa \)), as well as \( R_\perp \). The fit to the data yields

\[
R_\perp = 0.125 \pm 0.044 \text{(stat)} \pm 0.007 \text{(syst)}. \quad (3)
\]

The projections of the fitted result onto \( m_{ES} \) and \( \cos \theta_u \) are shown in Fig. 1.

In the fit described above, the value of \( \alpha \) is fixed to zero. We estimate the corresponding systematic uncertainty by varying its value from \(-1\) to \(+1\) and find negligible change (less than 0.002) in the fitted value of \( R_\perp \). Other systematic uncertainties arise from the parametrization of the angular resolution, the determination of the efficiency moments, and the background parametrization. The total systematic uncertainty on \( R_\perp \) is 0.007, significantly smaller than the statistical error.

We subsequently perform a combined analysis of the \( \cos \theta_u \) distribution and the time dependence to extract the

FIG. 1. Measured distribution of \( m_{ES} \) (left) and of \( \cos \theta_u \) in the region \( m_{ES} > 5.27 \text{ GeV}/c^2 \) (right). The solid line is the projection of the fit result. The dotted line represents the background component.
time-dependent \( CP \) asymmetry, using the event sample described previously. We use information from the other 
\( B \) meson in the event to tag the initial flavor of the fully reconstructed \( B^0 \rightarrow D^{*+}D^{*-} \) candidate.

The decay rate \( f_\pm(f_-) \) for a neutral \( B \) meson accompanied by a \( B^0(\bar{B}^0) \) tag is given by

\[
f_\pm(\Delta t, \cos \theta_{\text{tr}}) \propto e^{-|A_0|/2} \left[ G(1 \mp \Delta \omega) \mp (1 - 2 \omega) \right] \times \left[ F \sin(\Delta m_d \Delta t) + H \cos(\Delta m_d \Delta t) \right],
\]

(4)

where \( \Delta t = t_{\text{rec}} - t_{\text{tag}} \) is the difference between the proper decay time of the reconstructed \( B \) meson (\( B_{\text{rec}} \)) and that of the tagging \( B \) meson (\( B_{\text{tag}} \)), \( \tau_{B^0} \) is the \( B^0 \) lifetime, and \( \Delta m_d \) is the mass difference determined from the \( B^0 - \bar{B}^0 \) oscillation frequency \([11]\). The average mistag probability \( \omega \) describes the effect of incorrect tags, and \( \Delta \omega \) is the difference between the mistag rate for \( B^0 \) and \( \bar{B}^0 \). The \( G, F, \) and \( H \) coefficients are defined as

\[
G = (1 - R_\perp) \sin^2 \theta_{\text{tr}} + 2 R_\perp \cos^2 \theta_{\text{tr}},
\]

\[
F = (1 - R_\perp) S_\perp \sin^2 \theta_{\text{tr}} - 2 R_\perp S_{\perp} \cos^2 \theta_{\text{tr}},
\]

\[
H = (1 - R_\perp) C_\perp \sin^2 \theta_{\text{tr}} + 2 R_\perp C_{\perp} \cos^2 \theta_{\text{tr}},
\]

(5)

where we allow the three transversity amplitudes to have different \( \lambda_k = (q/p)(A_k/A_0) \) \( (k = 0, \perp, \perp) \) \([15]\) due to possibly different penguin-to-tree amplitude ratios, and define the \( CP \) asymmetry \( C_k = 1 - |\lambda_k|^2/1 + |\lambda_k|^2 \), \( S_k = 2 \text{Im}(A_k)/1 + |\lambda_k|^2 \). Here we also have

\[
C_+ = \frac{C_{||} |A_{||}|^2 + C_0 |A_0|^2}{|A_{||}|^2 + |A_0|^2}, \quad S_+ = \frac{S_{||} |A_{||}|^2 + S_0 |A_0|^2}{|A_{||}|^2 + |A_0|^2}.
\]

(6)

In the absence of penguin contributions, we expect that \( C_0 = C_\parallel = C_\perp = 0 \), and \( S_\perp = S_{\perp} = -\sin 2\beta \).

In Eq. (4), the small angular acceptance effects are not incorporated, but absorbed into the “effective” value of \( R_\perp \), which is left free to vary in the final fit. No bias is seen in the resulting values of \( C_+ \), \( C_\perp \), \( S_+ \), and \( S_\perp \) in MC simulation.

The technique used to measure the \( CP \) asymmetry is analogous to previous \( BABAR \) measurements as described in Ref. \([16]\). Only events with a \( \Delta t \) uncertainty less than 2.5 ps and a measured \( |\Delta t| \) less than 20 ps are accepted. We performed a simultaneous unbinned maximum likelihood fit to the \( \cos \theta_{\text{tr}}, \Delta t, \) and \( m_{\text{ES}} \) distributions to extract the \( CP \) asymmetry. The signal PDF in \( \theta_{\text{tr}} \) and \( \Delta t \) is given by Eq. (4). The signal mistag probability is determined from a sample of neutral \( B \) decays to flavor eigenstates, \( B_{\text{flav}} \). In the likelihood fit, the expression in Eq. (4) is convolved with an empirical \( \Delta t \) resolution function determined from the \( B_{\text{flav}} \) sample. The \( \theta_{\text{tr}} \) resolution is accounted for in the same way as described previously.

The background \( \Delta t \) distributions are parametrized with an empirical description that includes prompt and non-prompt components. We allow the nonprompt background to have two free parameters, \( C_{\text{eff}} \) and \( S_{\text{eff}} \), the effective \( CP \) asymmetries, in the likelihood fit. The background shape in \( \theta_{\text{tr}} \) is modeled as an even second-order polynomial in \( \cos \theta_{\text{tr}} \), much as it is in the time-integrated angular analysis.

The fit to the data yields

\[
C_+ = 0.06 \pm 0.17(\text{stat}) \pm 0.03(\text{syst}),
\]

\[
C_\perp = -0.20 \pm 0.96(\text{stat}) \pm 0.11(\text{syst}),
\]

\[
S_+ = -0.75 \pm 0.25(\text{stat}) \pm 0.03(\text{syst}),
\]

\[
S_\perp = -1.75 \pm 1.78(\text{stat}) \pm 0.22(\text{syst}).
\]

(7)

Figure 2 shows the \( \Delta t \) distributions and asymmetries in yields between \( B^0 \) and \( \bar{B}^0 \) tags, overlaid with the projection of the likelihood fit result. Because the \( CP \)-odd fraction is small, we have rather large statistical uncertainties for the measured \( C_\perp \) and \( S_\perp \) values. For comparison, we repeat the fit with the assumption that both \( CP \)-even and \( CP \)-odd states have the same \( CP \) asymmetry. We find that \( C_+ = C_\perp = 0.03 \pm 0.13(\text{stat}) \pm 0.02(\text{syst}) \), and \( S_+ = S_\perp = -0.69 \pm 0.23(\text{stat}) \pm 0.03(\text{syst}) \). In both cases, the effective \( CP \) asymmetries in the background are found to be consistent with zero within the statistical uncertainties.

The systematic uncertainties on \( C_+, C_\perp, S_+, \) and \( S_\perp \) arise from the amount of possible backgrounds that tend to peak under the signal and their \( CP \) asymmetry, the assumed parametrization of the \( \Delta t \) resolution function, the possible differences between the \( B_{\text{flav}} \) and \( B_{CP} \) mistag fractions, knowledge of the event-by-event beam-spot
position, and the possible interference between the suppressed \( \bar{b} \to \bar{u}c\bar{d} \) amplitude and the favored \( b \to c\bar{u}d \) amplitude for some tagside decays \[17\]. It also includes the systematic uncertainties from the finite MC calculation sample used to verify the fitting method. In general, all of the systematic uncertainties are found to be much smaller than the statistical uncertainties.

In summary, we have reported measurements of the \( CP \)-odd fraction and time-dependent \( CP \) asymmetries for the decay \( B^0 \to D^{\ast\ast+}D^{\ast\ast-} \). The measurement supersedes the previous BABAR result \[6\], with more than 50\% reduction in the statistical uncertainty, and indicates that \( B^0 \to D^{\ast\ast+}D^{\ast\ast-} \) is mostly \( CP \) even. The time-dependent asymmetries are found to be consistent with the SM predictions within the statistical uncertainty.

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*Also at Università di Perugia, Dipartimento di Fisica, Perugia, Italy.
†Deceased.

[4] We imply charged conjugate modes throughout the Letter.