Measurement of the Cabibbo-Kobayashi-Maskawa Angle $\gamma$ in $B^+ \to D^{(*)} K^+$ Decays with a Dalitz Analysis of $D \to K_S^0 \pi^- \pi^+$


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CP violation in the B meson system has been clearly established in recent years [1,2]. Although these results are in good agreement with standard model expectations, other and more precise measurements of CP violation in B decays are needed to overconstrain the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [3] and search for new physics effects. The angle $\gamma$ of the unitarity triangle [4] of the CKM matrix constitutes one of these crucial measurements.

Various methods using $B^- \rightarrow D^{(*)} K^-$ [5] have been proposed to measure $\gamma$ [6–8]. Here, $\bar{D}^0$ indicates either a $D^0$ or a $\bar{D}^0$ meson and the symbol “(*)” refers to either a $D$ or $D^*$ meson. All methods exploit the fact that a $B^-$ can decay into a $D^{(*)} K^-$ ($\bar{D}^{(*)} K^-$) final state via $b \rightarrow c \bar{u}s$ ($b \rightarrow c \bar{n}c\bar{s}$) transitions. These decay amplitudes interfere when the $D^0$ and $\bar{D}^0$ decay into the same final state, which can lead to different $B^+$ and $B^-$ decay rates (direct CP violation). In this Letter we report on a measurement of $\gamma$ based on the analysis of the Dalitz distribution of the three-body decay $D^0 \rightarrow K^0_S \pi^- \pi^+$ [7,8]. The primary advantage of this method is that it involves the entire resonant structure of the three-body decay, with interference of doubly Cabibbo-suppressed (DCS), Cabibbo-allowed (CA), and CP eigenstate amplitudes, providing the sensitivity to $\gamma$. The analysis is based on an integrated luminosity of 205 fb$^{-1}$ recorded at the $\Upsilon(4S)$ resonance (corresponding to $227 \times 10^6 BB$ decays) and 9.6 fb$^{-1}$ collected at a center-of-mass (c.m.) energy 40 MeV below with the BABAR detector [9] at the SLAC PEP-II $e^-e^+$ asymmetric-energy B Factory.

The small CP asymmetry in $D$ decays allowed by the present experimental limits [10] has a negligible effect on this analysis. Thus, the $B^- \rightarrow D^{(*)} K^-$, $\bar{D}^0 \rightarrow \bar{D}^{(*)} K^-$, $D^0 \rightarrow D^{(*)} K^-$, and $\bar{D}^0 \rightarrow \bar{D}^{(*)} K^-$ decay chain amplitudes $\mathcal{A}_D(m^2_{x,y}, m^2_{z,w})$ where $m^2_x$ and $m^2_y$ are the squared invariant masses of the $K^0_S \pi^-$ and $K^0_0 \pi^+$ combinations, respectively, can be written as

$$\mathcal{A}_D(m^2_{x,y}, m^2_{z,w}) = \kappa r^\pm_B e^{i\delta^\pm_B + \gamma} \mathcal{A}_D(m^2_x, m^2_y),$$

where $r^\pm_B$ and $\delta^\pm_B$ are the amplitude ratios and relative strong phases between the amplitudes $\mathcal{A}(B^- \rightarrow \bar{D}^{(*)} K^-)$ and $\mathcal{A}(B^- \rightarrow D^{(*)} K^-)$. As a consequence of parity and angular momentum conservation in the $\bar{D}^{(*)}$ decay, the factor $\kappa$ takes the value +1 for $B^- \rightarrow \bar{D}^0 K^-$ and $B^- \rightarrow \bar{D}^{*0} K^-$, and −1 for $B^- \rightarrow \bar{D}^{*0} K^-$. We first determine $\mathcal{A}_D(m^2_x, m^2_y)$ through a Dalitz analysis of a high-statistics sample of tagged $D^0$ mesons from inclusive $D^{*+} \rightarrow D^0 \pi^+$ decays reconstructed in data. We then perform a simultaneous fit to the $|\mathcal{A}_D(m^2_x, m^2_y)|^2$ and $|\mathcal{A}_D(m^2_x, m^2_y)|^2$ distributions for the $B^- \rightarrow D^{(*)} K^-$ sample to determine the CP parameters $r^\pm_B$, $\delta^\pm_B$, and $\gamma$. We emphasize that in this analysis the Dalitz amplitude is only a means to extract the CP parameters.

$B^-$ candidates are formed by combining a mass-constrained $D^{(*)}$ candidate with a track identified as a kaon [9]. We accept $K^0_S \rightarrow \pi^+ \pi^-$ candidates that have a two-pion invariant mass within 9 MeV/$c^2$ of the $K^0_S$ mass [4] and a cosine of the angle between the line connecting the $D^0$ and $K^0_S$ decay vertices and the $K^0_S$ momentum (in the plane transverse to the beam) greater than 0.99. $D^0$ candidates are selected by requiring the $K^0_S \pi^+$ invariant mass to be within 12 MeV/$c^2$ of the $D^0$ mass [4]. The $\pi^0$ candidates from $D^{*0} \rightarrow D^0 \pi^0$ are formed from pairs of photons with invariant mass in the range [115, 150] MeV/$c^2$, and with photon energy greater than 30 MeV. Photon candidates from $D^{*0} \rightarrow D^0 \gamma$ are selected if their energy is $\gamma \approx 90^\circ$. The curves represent the fit projections for signal plus background (solid lines) and background (dotted lines). The peaking structure of the background is due to remaining $B^- \rightarrow \bar{D}^{(*)} \pi^-$ events.

**FIG. 1 (color online).** Distributions of $m_{bs}$ for (a) $B^- \rightarrow \bar{D}^0 K^-$, (b) $B^- \rightarrow \bar{D}^{*0}(\bar{D}^0 \pi^0) K^-$, and (c) $B^- \rightarrow \bar{D}^{*0}(\bar{D}^0 \gamma) K^-$. The events are selected if their energy is $\gamma \approx 90^\circ$. The curves represent the fit projections for signal plus background (solid lines) and background (dotted lines). The peaking structure of the background is due to remaining $B^- \rightarrow \bar{D}^{(*)} \pi^-$ events.
is greater than 100 MeV. $D^{*0} \rightarrow D^0 \pi^0(D^0 \gamma)$ candidates are required to have a $D^{*0} - D^0$ mass difference within 2.5(10) MeV/$c^2$ of its nominal value [4].

The beam-energy substituted $B$ mass $m_{ES}$ [12] (Fig. 1) and the difference $\Delta E$ between the reconstructed energy of the $B^-$ candidate and the beam energy in the $e^+e^-$ c.m. frame are used to identify signal $B^-$ decays. We require $m_{ES} > 5.2$ GeV/$c^2$ and $|\Delta E| < 30$ MeV. Since the background is dominated by random combinations of tracks arising from $e^+e^- \rightarrow q\bar{q}$, $q = [u,d,s,c]$ (continuum) events, we require $|\cos \theta^*_F| < 0.8$, where $\theta^*_F$ is the c.m. angle between the thrust axis of the $B^-$ candidate and that of the remaining particles in the event. The reconstruction efficiencies (purities in the signal region $m_{ES} > 5.272$ GeV/$c^2$) are 18\% (63\%), 5.9\% (86\%), and 81.1\% (52\%) for the $B^- \rightarrow \bar{D}^0 K^-$, $B^- \rightarrow \bar{D}^{*0} (\bar{D}^0 \pi^0) K^-$, and $B^- \rightarrow \bar{D}^{*0}(\bar{D}^0 \gamma) K^-$ decay modes, respectively. The cross feed among the different samples is negligible.

The $D^0$ decay amplitude is determined from an un-binned maximum-likelihood Dalitz fit to a high-purity (97\%) sample of 81496 $D^{*+} \rightarrow D^0 \pi^+$ decays reconstructed in 91.5 fb$^{-1}$ of data (Fig. 2). We use the isobar formalism described in Ref. [13] to express $\mathcal{A}_D$ as a sum of two-body decay-matrix elements (subscript $r$) and a nonresonant (subscript NR) contribution,

$$\mathcal{A}_D(m_1^2, m_2^2) = \sum_r a_re^{i\phi_r} \mathcal{A}_r(m_1^2, m_2^2) + a_{NR}e^{i\phi_{NR}},$$

where each term is parameterized with an amplitude $a_r$ and a phase $\phi_r$. The function $\mathcal{A}_r(m_1^2, m_2^2)$ is the Lorentz-invariant expression for the matrix element of a $D^0$ meson decaying into $K_S^0 \pi^- \pi^+$ through an intermediate resonance $r$, parameterized as a function of the position in the Dalitz plane.

Table I summarizes the values of $a_r$ and $\phi_r$ obtained using a model consisting of 16 two-body elements comprising 13 distinct resonances and accounting for efficiency variations across the Dalitz plane and the small background contribution. For $r = \rho(770)$, $\rho(1450)$ we use the functional form suggested in Ref. [14], while the remaining resonances are parameterized by a spin-dependent relativistic Breit-Wigner distribution. For intermediate states with a $K^+$, the regions of interference between DCS and CA decays are particularly sensitive to $\gamma$, and we include the DCS component when a significant contribution is expected. In addition, we find that the inclusion of the scalar $\pi\pi$ resonances $\sigma$ and $\sigma'$ significantly improves the quality of the fit [15]. Since the two $\sigma$ resonances are not well established and are only introduced to improve the description of our data, the uncertainty on their existence is considered in the systematic errors. We estimate the goodness of fit through a two-dimensional $\chi^2$ test and obtain $\chi^2 = 3824$ for 3054–32 degrees of freedom.

We simultaneously fit the $B^- \rightarrow \bar{D}^{*0} K^-$ samples using an un-binned extended maximum-likelihood fit to extract the $CP$-violating parameters along with the signal and background yields. Three different background compo-

![FIG. 2 (color online). (a) The $D^{*+} \rightarrow K_S^0 \pi^- \pi^+$ Dalitz distribution from $D^{*+} \rightarrow D^0 \pi^+$ events, and projections on (b) $m_1^2$, (c) $m_2^2$, and (d) $m_2^2$. The curves are the fit projections.](image-url)
nents are considered: continuum events, $B^- \rightarrow \bar{D}^{(*)0} \pi^-$ and $Y(4S) \rightarrow BB$ (other than $B^- \rightarrow \bar{D}^{(*)0} \pi^-$) decays. In addition to $m_{ES}$, the fit uses $\Delta E$ and a Fisher discriminant [12] to distinguish signal from $B^- \rightarrow \bar{D}^{(*)0} \pi^-$ and continuum background, respectively. The log-likelihood is

$$\ln L = -\sum_{c} N_c + \sum_{j} \ln \left[ \sum_{c} N_c \mathcal{P}_c(\tilde{\xi}_j) \mathcal{P}_c^{Dalitz}(\tilde{\eta}_j) \right].$$

where $\tilde{\xi}_j = \{m_{ES}, \Delta E, F\}$, and $\tilde{\eta}_j = (m^2, m^2_z)$, characterize the event $j$. Here, $\mathcal{P}_c(\tilde{\xi})$ and $\mathcal{P}_c^{Dalitz}(\tilde{\eta})$ are the probability density functions (PDF's), and $N_c$ the event yield for signal or background component $c$. For signal events, $\mathcal{P}_c^{Dalitz}(\tilde{\eta})$ is given by $|\mathcal{A}_c^{Dalitz}(\tilde{\eta})|^2$ corrected by the efficiency variations. All PDF shape parameters used to describe signal, continuum, and $B^- \rightarrow \bar{D}^{(*)0} \pi^-$ components are determined directly from $B^- \rightarrow \bar{D}^{(*)0}K^-$ and $B^- \rightarrow \bar{D}^{(*)0}\pi^-$ signal, sideband regions, and off-peak data, and are fixed in the final fit for CP parameters and event yields. Only the $m_{ES}$, $\Delta E$, and Dalitz PDF's for $BB$ background events are determined from a detailed Monte Carlo simulation. $B^- \rightarrow \bar{D}^{(*)0} \pi^-$ candidates have been selected using criteria similar to those applied for $B^- \rightarrow \bar{D}^{(*)0}K^-$ but requiring the bachelor pion not to be consistent with the kaon hypothesis.

The CP fit yields $282 \pm 20, 90 \pm 11$, and $44 \pm 8$ signal $\bar{D}^0K^-$, $D^{(*)0}(\bar{D}^0\pi^0)K^-$, and $\bar{D}^{(*)0}(\bar{D}^0\gamma)K^-$ candidates, respectively, consistent with expectations based on measured branching fractions and efficiencies estimated from Monte Carlo simulation. The results for the CP-violating parameters $Z_{\pm}^{(s)} = (x_{\pm}^{(s)}, y_{\pm}^{(s)})$, where $x_{\pm}^{(s)}$ and $y_{\pm}^{(s)}$ are defined as the real and imaginary parts of the complex amplitude ratios $r_{B^-}^{(s)}e^{i(\delta_{B^-})}$, respectively, are summarized in Table II. Here, $r_{B^-}^{(s)}$ is the amplitude ratio between the amplitudes $b \rightarrow u$ and $b \rightarrow c$, separately for $B^+$ and $B^-$. The only nonzero statistical correlations involving the CP parameters are for the pairs $z_-, z_+ \equiv z_{\pm}$, and $z^*_\pm$, which amount to $3\%$, $6\%$, $-17\%$, and $-27\%$, respectively. The $z^{(s)}_{\pm}$ variables are more suitable fit parameters than $r_{B^-}^{(s)}, \delta_{B^-}^{(s)}$, and $\gamma$ because they are better behaved near the origin, especially in low-statistics samples. Figures 3(a) and 3(b) show the one- and two-standard deviation confidence-level contours (statistical only) in the $z^{(s)}_\pm$ planes for $\bar{D}^0K^-$ and $\bar{D}^{(*)0}K^-$, and separately for $B^-$ and $B^+$. The separation between the $B^-$ and $B^+$ regions in these planes is an indication of direct CP violation.

The largest single contribution to the systematic uncertainties in the CP parameters comes from the choice of the Dalitz model used to describe the $D^{0} \rightarrow K^0_s \pi^+ \pi^-$ decay amplitudes. To evaluate this uncertainty we use the nominal Dalitz model (Table I) to generate large samples of pseudoexperiments. We then compare experiment by experiment the values of $Z_{\pm}^{(s)}$ obtained from fits using the nominal model and a set of alternative models. We find that removing different combinations of $K^*$ and $\rho$ resonances (with low fit fractions), or changing the functional form of the resonance shapes, has little effect on the total $\chi^2$ of the fit, or on the values of $Z_{\pm}^{(s)}$. However, models where one or both of the $\sigma$ and $\rho$ resonances are removed lead to a significant increase in the $\chi^2$ of the fit. We use the average variations of $Z_{\pm}^{(s)}$ corresponding to this second set of alternative models as the systematic uncertainty due to imperfect knowledge of $\mathcal{A}_D$.

The experimental systematic uncertainties include the errors on the $m_{ES}$, $\Delta E$, and $F$ PDF parameters for signal and background, the uncertainties in the knowledge of the Dalitz distribution of background events, the efficiency variations across the Dalitz plane, and the uncertainty in the fraction of events with a real $D^0$ produced in a back-top-

| TABLE II. CP-violating parameters $Z_{\pm}^{(s)}$ obtained from the CP fit to the $B^- \rightarrow \bar{D}^{(*)0}K^-$ samples. The first error is statistical, the second is the experimental systematic uncertainty, and the third reflects the Dalitz model uncertainty. |

<table>
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<tr>
<th>$x_{-}^{(s)}$</th>
<th>$y_{-}^{(s)}$</th>
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<tbody>
<tr>
<td>$z_-$</td>
<td>$0.08 \pm 0.07 \pm 0.03 \pm 0.02$</td>
</tr>
<tr>
<td>$z_+$</td>
<td>$-0.13 \pm 0.07 \pm 0.03 \pm 0.03$</td>
</tr>
<tr>
<td>$z^*_-$</td>
<td>$-0.13 \pm 0.09 \pm 0.03 \pm 0.02$</td>
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<tr>
<td>$z^*_+$</td>
<td>$0.14 \pm 0.09 \pm 0.03 \pm 0.02$</td>
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FIG. 3 (color online). Contours at 39.3% (dark) and 86.5% (light) confidence level (statistical only) in the $z^{(s)}_{\pm}$ planes for (a) $\bar{D}^0K^-$ and (b) $\bar{D}^{(*)0}K^-$, separately for $B^-$ (thick and solid lines) and $B^+$ (thin and dotted lines). Projections in the $r_{B^-}^{(s)}-\gamma$ planes of the five-dimensional one-standard (dark) and two-standard (light) deviation regions, for (c) $\bar{D}^0K^-$ and (d) $\bar{D}^{(*)0}K^-$. |
back configuration with a negatively charged kaon. Less significant systematic uncertainties originate from the imprecise knowledge of the fraction of real $D^{0}$'s, the invariant mass resolution, and the statistical errors in the Dalitz amplitudes and phases from the fit to the tagged $D^{0}$ sample. The possible effect of $CP$ violation in $B^{-} \rightarrow D^{(*)0}\pi^{-}$ decays and $B\bar{B}$ background was found to be negligible.

A frequentist (Neyman) construction of the confidence regions of $p = (r_{B}, \delta_{B}, \beta_{B}, \gamma)$ based on the constraints on $Z_{\pm}^{(e)}$ has been adopted [4]. Using a large number of pseudoexperiments corresponding to the nominal $CP$ fit model but with many different values of the $CP$ fit parameters, we construct an analytical (Gaussian) parameterization of the PDF of $Z_{\pm}^{(e)}$ as a function of $p$. For a given $p$, the five-dimensional confidence level $C = 1-\alpha$ is calculated by integrating over all points in the fit parameter space closer (larger PDF) to $p$ than the fitted data values. The one- (two-)standard deviation region of the $CP$ parameters is defined as the set of $p$ values for which $\alpha$ is smaller than 3.7% (45.1%).

Figures 3(c) and 3(d) show the two-dimensional projections in the $r_{B}^{(e)}-\gamma$ planes, including systematic uncertainties, for $D^{*0}K^{-}$ and $D^{0}\bar{K}^{-}$. The figures show that this Dalitz analysis has a twofold ambiguity, $(\gamma, \delta_{B}^{(e)}) \rightarrow (\gamma + 180^\circ, \delta_{B}^{(e)} + 180^\circ)$. The significance of direct $CP$ violation, obtained by evaluating $C$ for the most probable $CP$ conserving point, corresponds to 1.6, 2.1, and 2.4 standard deviations, for $D^{*0}K^{-}$ and $D^{0}\bar{K}^{-}$, and their combination, respectively. Similar results are obtained using a Bayesian technique with uniform $a$ priori probability distributions for $r_{B}^{(e)}$, $\delta_{B}^{(e)}$, and $\gamma$.

In summary, we have measured the direct $CP$-violating parameters in $B^{-} \rightarrow D^{(*)0}\pi^{-}$ using a Dalitz analysis of $D^{0} \rightarrow K_{S}^{0}\pi^{+}\pi^{-}$ decays, obtaining $r_{B} = 0.12 \pm 0.08 \pm 0.03 \pm 0.04 \ [0.028]$, $r_{B} = 0.17 \pm 0.10 \pm 0.03 \pm 0.03 \ [0.035]$, $\delta_{B} = (104 \pm 45_{-21}^{+17+16}_{-10}+14_{-13}^{+15}_{-10})^\circ$, and $\gamma = (70 \pm 31_{-10}^{+12}_{-10}+14_{-12}^{+15}_{-10})^\circ \ [12_{-9}^{+12}_{-9}, 137_{-9}^{+12}_{-9}]$. The first error is statistical, the second is the experimental systematic uncertainty, and the third reflects the Dalitz model uncertainty. The values inside square brackets indicate the two-standard deviation intervals. The results for $\gamma$ from $B^{-} \rightarrow D^{*0}K^{-}$ and $B^{-} \rightarrow D^{0}\bar{K}^{-}$ alone are $(70 \pm 38)^\circ$ and $(71 \pm 35)^\circ$, respectively (statistical errors only). The constraint on $\gamma$ is consistent with that reported by the Belle Collaboration [8], which has a slightly better statistical precision since our $r_{B}^{(e)}$ constraint favors smaller values.

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‡Deceased.
[5] Reference to the charge-conjugate state is implied here and throughout the text unless otherwise stated.
[15] The $\sigma$ and $\sigma'$ masses and widths are determined from the data. We find (in MeV/$c^2$) $M_{\sigma} = 484 \pm 9$, $\Gamma_{\sigma} = 383 \pm 14$, $M_{\sigma'} = 1014 \pm 7$, and $\Gamma_{\sigma'} = 88 \pm 13$. Errors are statistical.