Measurement of Branching Fractions and Charge Asymmetries in $B^+$ Decays to $\eta\pi^+$, $\eta K^+$, $\eta\rho^+$, and $\eta\pi^+$, and Search for $B^0$ Decays to $\eta K^0$ and $\eta\omega$

(Received 23 March 2005; published 23 September 2005)
We present measurements of branching fractions and charge asymmetries for six B-meson decay modes with an \( \eta \) or \( \eta' \) meson in the final state. The data sample corresponds to \( 232 \times 10^6 \) \( \overline{B}B \) pairs collected with the BABAR detector at the PEP-II asymmetric-energy \( e^+e^- \) B Factory at SLAC. We measure the branching fractions (in units of \( 10^{-6} \)): \( \mathcal{B}(B^+ \to \eta\pi^+) = 5.1 \pm 0.6 \pm 0.3 \), \( \mathcal{B}(B^+ \to \eta K^+) = 3.3 \pm 0.6 \pm 0.3 \), \( \mathcal{B}(B^0 \to \eta K^0) = 1.5 \pm 0.7 \pm 0.1 \) (at 90\% C.L.), \( \mathcal{B}(B^+ \to \eta p^+) = 8.4 \pm 1.9 \pm 1.1 \), \( \mathcal{B}(B^0 \to \eta p^+ \to \eta \omega) = 1.0 \pm 0.5 \pm 0.2 \) (at 90\% C.L.), and \( \mathcal{B}(B^+ \to \eta\pi^+) = 4.0 \pm 0.8 \pm 0.4 \), where the first uncertainty is statistical and second systematic. For the charged modes we also determine the charge asymmetries, all found to be compatible with zero.

DOI: 10.1103/PhysRevLett.95.131803  
PACS numbers: 13.25.Hw, 11.30.Er

Charmless \( B \) decays are becoming increasingly useful to test the accuracy of theoretical predictions, for example, based on QCD factorization [1,2] or flavor SU(3) symmetry [3–5]. In this Letter we present measurements of branching fractions and, when applicable, charge asymmetries, for six charmless \( B \) decays: \( B^+ \to \eta\pi^+ \) and \( B^+ \to \eta K^+ \), which have been observed previously, and \( B^0 \to \eta K^0 \), \( B^0 \to \eta \omega \), \( B^+ \to \eta p^+ \), and \( B^+ \to \eta\pi^+ \) [6–9].

Charmless \( B \) decays with kaons as loop (“penguin”) amplitudes, while \( b \to u \) tree amplitudes are typically larger for the decays with pions and \( \rho \) mesons. However, the \( B \to \eta K \) decays are especially interesting since they are suppressed relative to the abundant \( B \to \eta'K \) decays due to destructive interference between two penguin amplitudes [10]. The Cabibbo-Kobayashi-Maskawa (CKM) suppressed \( b \to u \) tree amplitudes may interfere significantly with \( b \to s \) penguin amplitudes of similar sizes, possibly leading to large direct CP violation in \( B^+ \to \eta p^+ \), \( B^+ \to \eta \pi^+ \), and \( B^+ \to \eta'\pi^+ \) [11]; numerical estimates are available in a few cases[2,3,12]. We search for such direct CP violation by measuring the charge asymmetry \( \mathcal{A}_{\text{ch}} = (\Gamma^- - \Gamma^+)/ (\Gamma^- + \Gamma^+) \) in the rates \( \Gamma^\pm = \Gamma(B^\pm \to f^-) \) for each charged final state \( f^- \).

Finally, phenomenological fits to the branching fractions and charge asymmetries of charmless \( B \) decays can be used to understand the relative importance of tree and penguin contributions and may provide sensitivity to the CKM angle \( \gamma \) [3–5], or to the effect of non-standard-model heavy particles entering the loops [13].

The results presented here are obtained from extended unbinned maximum likelihood (ML) fits to data collected with the BABAR detector [14] at the PEP-II asymmetric \( e^+e^- \) collider [15] located at the Stanford Linear Accelerator Center. The analysis uses an integrated luminosity of \( 211 \text{ fb}^{-1} \), corresponding to \( 232 \times 10^6 \) \( \overline{B}B \) pairs, recorded at the \( Y(4S) \) resonance (center-of-mass energy \( \sqrt{s} = 10.58 \text{ GeV} \)), and follows closely the technique described in detail in Ref. [7]. The sample exceeds that of Refs. [6–8] by a factor of 2.6, and the present results supersede the corresponding ones therein.

Charged particles are detected and their momenta measured by a combination of a vertex tracker consisting of five layers of double-sided silicon microstrip detectors and a 40-layer central drift chamber, both operating in the 1.5 T magnetic field of a superconducting solenoid. We identify photons and electrons using a CsI(Tl) electromagnetic calorimeter (EMC). Charged particle identification (PID) is provided by an internally reflecting ring imaging Cherenkov detector (DIRC) covering the central region of the detector, the average energy loss \( (dE/dx) \) in the tracking devices, and by the EMC. A \( K/\pi \) separation better than 2 standard deviations \( (\sigma) \) is achieved for all momenta.

We select \( \eta, \eta', \omega, K^0, \) and \( \pi^0 \) candidates through the decays \( \eta \to \gamma\gamma(\eta_{\gamma\gamma}), \eta \to \pi^+\pi^-\pi^0(\eta_{3\pi}), \eta' \to \eta_{\gamma\gamma}\pi^+\pi^- (\eta'_{\gamma\gamma}), \eta' \to \rho^0(\eta'_{\rho}), \omega \to \pi^+\pi^-\pi^0, K^0 \to \pi^+\pi^-, \) and \( \pi^0 \to \gamma\gamma \). We impose the following requirements on the invariant mass in MeV of the particle candidates’ final states: \( 490 < m_{\gamma\gamma} < 600 \) for \( \eta_{\gamma\gamma}, 520 < m_{\pi\pi} < 570 \) for \( \eta_{3\pi}, 910 < (m_{\eta\pi\pi}, m_{\eta\gamma}) < 1000 \) for \( \eta'_{\gamma\gamma}, 735 < m_{\pi\pi\pi} < 825 \) for \( \omega, 510 < m_{\pi\pi} < 1070 \) for \( \rho^0, 470 < m_{\pi\pi} < 1070 \) for \( \rho^+, 486 < m_{\pi\pi} < 510 \) for \( K^0 \), and \( 120 < m_{\gamma\gamma} < 150 \) for \( \pi^0 \). These cuts are loose for the variables used in the ML fit described below. For \( K^0 \) candidates we require at least 3\( \sigma \) three-dimensional separation between the decay vertex and the \( e^+e^- \) collision point. For the vector resonances \( \omega \) and \( \rho^+ \) we also use the helicity-frame decay angle \( \theta_H \). The helicity frame is defined as the vector-meson rest frame with polar axis along the direction of the boost from the \( B \) rest frame. For \( \omega, \theta_H \) is the polar angle of the normal to the decay plane, and for \( \rho \) it is the polar angle of the charged daughter momentum. We define \( \mathcal{H} \equiv \cos \theta_H \) and require \(-0.75 < \mathcal{H} < 0.95 \) for \( \rho^+ \).

All tracks from resonance candidates are required to have PID consistent with pions. For the \( B^+ \) decays to \( \eta\pi^+, \eta K^+, \) and \( \eta'\pi^+ \), the primary charged track must have an associated DIRC Cherenkov angle within 3.5\( \sigma \) of the expected value for either \( \pi \) or \( K \). The discrimination between primary \( \pi \) and \( K \) is performed in the ML fits.

A \( B \)-meson candidate is characterized kinematically by the energy-substituted mass \( m_{\text{ES}} = \left( \frac{1}{2} \sqrt{s} - p_B^2 \right)^{1/2} \) and energy difference \( \Delta E = E_B - \frac{1}{2} \sqrt{s} \), where \( (E_B, p_B) \) is the \( B \)-meson 4-momentum vector, and all values are expressed in the \( Y(4S) \) frame. Signal events peak at zero for \( \Delta E \), and at the \( B \) nominal mass for \( m_{\text{ES}} \). The resolution on \( \Delta E(m_{\text{ES}}) \) is about 30 MeV (3.0 MeV). We require \( |\Delta E| \leq 0.2 \text{ GeV} \) and \( 5.25 \leq m_{\text{ES}} \leq 5.29 \text{ GeV} \).

Backgrounds arise primarily from random combinations in continuum \( e^+e^- \to q\bar{q} (q = u, d, s, c) \) events. To reject
these events we make use of the angle $\theta$ between the thrust axis of the $B$ candidate in the $Y(4S)$ frame and that of the rest of the charged tracks and neutral clusters in the event. The distribution of $|\cos\theta|$ is sharply peaked near 1 for combinations drawn from jetlike $q\bar{q}$ pairs, and nearly uniform for the almost isotropic $B$-meson decays; we require $|\cos\theta| < 0.9 (\leq 0.65$ for the higher-background $B^\pm \pi^\mp$). Further discrimination from continuum in the ML fit is obtained from a Fisher discriminant $F$ that is described in detail elsewhere [7].

To reject $B \to K^*\gamma$ background we require that photons have energies less than 2.4 GeV. For $\eta_{\gamma\gamma}$, $\eta_{\gamma\gamma}K^0_S$ we discriminate against charmless two-body decays with $\pi^0$ by removing $\eta_{\gamma\gamma}$ candidates that share a photon with any $\pi^0$ candidate having momentum between 1.9 and 3.1 GeV in the $Y(4S)$ frame.

Multiple candidates are found in less than 30% of the events, in which case we choose the candidate with the smallest value of a $\chi^2$ constructed from the deviations of the daughter resonance masses from their nominal values.

We use Monte Carlo (MC) simulation [16] for an initial estimate of the residual $B\bar{B}$ background and to identify the few (mostly charmless) decays that may survive the candidate selection and have characteristics similar to the signal. We find these contributions to be negligible for several of our modes. Where they are not negligible, namely, for $\eta_{\gamma\gamma}\pi^0$, $\eta_{\gamma\gamma}K^0_S$, $\eta_{\rho\rho}$, and $\eta_{\delta\delta}$, we include a component in the ML fit to account for them. Nonresonant backgrounds are not included in the fit, as they are found from MC calculations to be negligible after the resonance mass and helicity cuts.

We obtain yields and $A_{ch}$ for each decay chain from a ML fit with the following input observables: $\Delta E$, $m_{ES}$, $F$, and $m_{res}$ (the mass of the $\eta$, $\eta'$, $\rho^+$, or $\omega$ candidate). For $\omega$ and $\rho^+$ decays we also use $H$ and, for charged modes with a primary charged track, the PID variables $S_\pi$ and $S_K$, defined as the number of standard deviations between the measured DIRC Cherenkov angle and that expected for pions and kaons, respectively.

For each event $i$, hypothesis $j$ (signal, continuum background, $B\bar{B}$ background), and flavor $k$ (primary $\pi^+$ or $K^+$), we define the probability density function (PDF)

$$P_{jk} = P_j(m_{ES})P_j(\Delta E_i, S_i)P_j(F)P_j(m_{res}, H).$$

(1)

The bracketed variables $S$ and $H$ pertain to modes with a primary charged track or vector resonance daughters, respectively. Known correlations between $\Delta E_i$ and $S_i$, and between $m_{res}$ and $H$, are included in the PDF. The likelihood function is

$$\mathcal{L} = \exp\left(-\sum_{j,k} Y_{jk}\right)\prod_{j,k} Y_{jk} P_{jk},$$

where $Y_{jk}$ is the yield of events of hypothesis $j$ and flavor $k$,

to be found by maximizing $\mathcal{L}$. $N$ is the number of events in the sample. Free parameters of the fit are the signal and background yields, between 4 and 9 $q\bar{q}$ background PDF parameters (see below), and for charged modes the signal and $q\bar{q}$ background charge asymmetries.

For the signal and $B\bar{B}$ background components we determine the PDF parameters from simulation. For background from continuum (and nonpeaking combinations from $B$ decays) we obtain the PDF from $(m_{ES}, \Delta E)$ sideband data for each decay chain, before applying the fit to data in the signal region; we refine this PDF by letting as many of its parameters as feasible vary in the final fit. The fitted $q\bar{q}$ background PDF parameters are found in close agreement with the values obtained from the fits to sideband data. We parametrize each of the functions $P_{ij}(m_{ES})$, $P_{ij}(\Delta E)$, $P_{ij}(F)$, $P_{ij}(S)$, and the peaking components of $P_{ij}(m_{res})$ with either a Gaussian, the sum of two Gaussians, or an asymmetric Gaussian function as required to describe the distribution. Slowly varying distributions (mass, energy, or helicity-angle for combinatorial background) are represented by one or a combination of linear, quadratic, and phase-space motivated functions [7]. The peaking and combinatorial components of the $\omega$ and $\rho^+$ mass spectra each have their own $H$ shapes. Control samples with similar topologies as our signal modes (e.g., $B \to D(K\pi\pi)\pi$) are used to verify or adjust the simulated resolutions evaluated from MC calculations [7].

Before applying the fitting procedure to the data we subject it to several tests. In particular, we evaluate possible biases in the yields from our neglect of small residual correlations among discriminating variables in the signal and charmless $B\bar{B}$ background PDFs. The bias is determined by fitting ensembles of simulated $q\bar{q}$ experiments drawn from the PDF into which we have embedded the expected number of signal and $B\bar{B}$ background events, randomly extracted from the fully simulated MC samples. We measure the correlations in the data, which are dominated by $q\bar{q}$, and find them to be negligibly small. The measured biases are listed in Table I.

We compute the branching fraction for each decay chain by subtracting the fit bias from the measured yield, and dividing the result by the efficiency and the number of produced $B\bar{B}$ pairs [7]. We assume equal decay rates of the $Y(4S)$ to $B^+B^-$ and $B^0\bar{B}^0$ in Table I we show for each decay mode the measured branching fraction together with the event yield and efficiency, and $A_{ch}$ when applicable. The purity is the ratio of the signal yield ($Y_S$) to the effective background plus signal ($Y_{B eff} + Y_S$), which we estimate as the square of the uncertainty in the signal yield ($Y_{B eff}^2 + Y_S^2 = \sigma_Y^2$). The statistical uncertainties in the signal yield and $A_{ch}$ are taken as the change in the central value when the quantity $-2\ln\mathcal{L}$ increases by one unit from its minimum value. The significance is taken as the square root of the difference between the value of $-2\ln\mathcal{L}$ (with systematic uncertainties included) for zero signal and the value at its minimum.
TABLE I. Fitted signal yield $Y_S$ in events (ev.), estimated purity $P$, measured bias (see text), detection efficiency $e$, daughter branching fraction product ($\prod B_i$), significance $S$ (with systematic uncertainties included), measured branching fraction $B$, and signal charge asymmetry $A_{\text{ch}}$ for each mode. The quantities in parentheses are 90\% C.L. upper limits.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$Y_S$ (ev.)</th>
<th>$P$ (%)</th>
<th>Bias (ev.)</th>
<th>$e$ (%)</th>
<th>$\prod B_i$ (%)</th>
<th>$S(\sigma)$</th>
<th>$B(10^{-6})$</th>
<th>$A_{\text{ch}}$</th>
</tr>
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<tbody>
<tr>
<td>$\eta_{\gamma}\pi^+$</td>
<td>153$^{+24}_{-22}$</td>
<td>30</td>
<td>$+$7</td>
<td>33</td>
<td>39</td>
<td>7.9</td>
<td>$4.8^{+0.8}_{-0.7}$</td>
<td>$-0.04 \pm 0.14$</td>
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<tr>
<td>$\eta_{\pi}\pi^+$</td>
<td>76$^{+15}_{-16}$</td>
<td>32</td>
<td>$+$6</td>
<td>24</td>
<td>23</td>
<td>5.6</td>
<td>$5.6^{+1.3}_{-1.2}$</td>
<td>$-0.32 \pm 0.20$</td>
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<tr>
<td>$\eta\pi^+$</td>
<td>9.7</td>
<td>$5.1 \pm 0.6 \pm 0.3$</td>
<td>$-0.13 \pm 0.12 \pm 0.01$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta_{\gamma}\eta K^+$</td>
<td>116$^{+21}_{-19}$</td>
<td>29</td>
<td>$+$8</td>
<td>32</td>
<td>39</td>
<td>6.1</td>
<td>$3.6 \pm 0.7$</td>
<td>$-0.19 \pm 0.16$</td>
</tr>
<tr>
<td>$\eta_{3\pi}K^+$</td>
<td>37$^{+13}_{-12}$</td>
<td>24</td>
<td>$+$5</td>
<td>23</td>
<td>23</td>
<td>2.8</td>
<td>$2.6^{+1.1}_{-1.0}$</td>
<td>$-0.22 \pm 0.33$</td>
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<tr>
<td>$\eta\eta K^+$</td>
<td>6.7</td>
<td>$3.3 \pm 0.6 \pm 0.3$</td>
<td>$-0.20 \pm 0.15 \pm 0.01$</td>
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<tr>
<td>$\eta_{\gamma}K^0$</td>
<td>17$^{+9}_{-7}$</td>
<td>27</td>
<td>$+$3</td>
<td>28</td>
<td>14</td>
<td>2.3</td>
<td>$1.6^{+1.0}_{-0.9}$</td>
<td></td>
</tr>
<tr>
<td>$\eta_{3\pi}K^0$</td>
<td>5$^{+5}_{-3}$</td>
<td>28</td>
<td>$+$1</td>
<td>21</td>
<td>8</td>
<td>1.4</td>
<td>$1.1^{+1.3}_{-0.9}$</td>
<td></td>
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<tr>
<td>$\eta K^0$</td>
<td>2.6</td>
<td>$1.5 \pm 0.7 \pm 0.1 (&lt;2.5)$</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>$\eta_{\gamma}\omega$</td>
<td>13$^{+7}_{-6}$</td>
<td>32</td>
<td>$+$1</td>
<td>14</td>
<td>35</td>
<td>2.5</td>
<td>$1.1^{+0.6}_{-0.5}$</td>
<td></td>
</tr>
<tr>
<td>$\eta_{3\pi}\omega$</td>
<td>2$^{+7}_{-5}$</td>
<td>6</td>
<td>$-$1</td>
<td>11</td>
<td>20</td>
<td>0.6</td>
<td>$0.6^{+0.1}_{-0.0}$</td>
<td></td>
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<tr>
<td>$\eta\omega$</td>
<td>2.5</td>
<td>$1.0 \pm 0.5 \pm 0.2 (&lt;1.9)$</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$\eta_{\gamma}\rho^+$</td>
<td>126$^{+34}_{-32}$</td>
<td>12</td>
<td>$+$18</td>
<td>16</td>
<td>39</td>
<td>3.7</td>
<td>$7.3^{+2.4}_{-2.2}$</td>
<td>$0.10 \pm 0.23$</td>
</tr>
<tr>
<td>$\eta_{3\pi}\rho^+$</td>
<td>65$^{+22}_{-20}$</td>
<td>15</td>
<td>$+$3</td>
<td>11</td>
<td>23</td>
<td>3.4</td>
<td>$10.6^{+3.7}_{-3.5}$</td>
<td>$-0.14 \pm 0.31$</td>
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<tr>
<td>$\rho^+$</td>
<td>4.7</td>
<td>$8.4 \pm 1.9 \pm 1.1$</td>
<td>$0.02 \pm 0.18 \pm 0.02$</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$\eta_{\pi\pi}\pi^+$</td>
<td>69$^{+13}_{-12}$</td>
<td>42</td>
<td>$+$9</td>
<td>27</td>
<td>18</td>
<td>5.6</td>
<td>$5.5^{+1.2}_{-1.1}$</td>
<td>$0.09 \pm 0.18$</td>
</tr>
<tr>
<td>$\eta_{\rho\pi}\pi^+$</td>
<td>30$^{+16}_{-15}$</td>
<td>13</td>
<td>$+$9</td>
<td>17</td>
<td>30</td>
<td>1.4</td>
<td>$1.8^{+1.3}_{-1.2}$</td>
<td>$0.58 \pm 0.44$</td>
</tr>
<tr>
<td>$\eta\pi^+$</td>
<td>5.4</td>
<td>$4.0 \pm 0.8 \pm 0.4$</td>
<td>$0.14 \pm 0.16 \pm 0.01$</td>
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</table>

For each mode the measurements for separate daughter decays are combined by adding the values of $-2 \ln L$ as functions of branching fraction, taking proper account of the correlated and uncorrelated systematic uncertainties described below [7]. For $\eta\omega$ and $\eta K^0$ we quote 90\% confidence level (C.L.) upper limits, taken to be the branching fraction below which lies 90\% of the total of the likelihood integral in the positive branching fraction region.

In Fig. 1 we show projections onto $m_{ES}$ and $\Delta E$ of subsamples enriched with a mode-dependent threshold requirement on the signal likelihood (computed without the variable plotted) that optimizes the sensitivity.

The systematic uncertainties are dominated by our knowledge of the signal and $B\bar{B}$ PDF modeling, the fit bias correction, and the neutral selection efficiency. The PDF modeling error is largely included in the statistical uncertainty since most background parameters are free in the fit. The uncertainties in the signal PDF parameters are estimated from the consistency of fits to MC samples and data in control modes with similar final states. Varying the signal PDF parameters within these errors, we estimate the mode-dependent uncertainties to be 1–8 events.

The uncertainty in the fit bias correction is taken to be half of the correction. Similarly we estimate the uncertainty from modeling the $B\bar{B}$ backgrounds by taking half of the difference between the signal yield fitted with and without the $B\bar{B}$ background component (0–7 events).

Uncertainties in the reconstruction efficiency, found from auxiliary studies on inclusive control samples [7], include 0.6\% per primary track, 0.8\% per track from a resonance, 1.5\% per photon, and 2.1\% for a $K_S^0$. Our
estimate of the systematic uncertainty in the number of $B\bar{B}$ pairs is 1.1%. Published data [17] provide the uncertainties in the $B$-daughter product branching fractions (1%–3%). The uncertainties in the efficiency of the event selection are 1% (4% in $B^+ \to \eta\rho^0\pi^+$) for the requirement on $\cos\theta_\pi$ and 1% for PID. Using large inclusive kaon and $B$-decay samples, we find a systematic uncertainty for $\mathcal{A}_\text{ch}$ of 1.1%, due mainly to the dependence of reconstruction efficiency on the charge, for the high momentum pion from $B^+ \to \eta\pi^+$, $\eta K^+$ and $\eta'\pi^+$, and 2% for the softer charged pion from the $\rho^+$ in $B^+ \to \eta\rho^+$.

In this Letter, we have presented improved measurements of branching fractions for six charmless $B$-meson decays. All are in agreement with previous measurements. The previously unobserved $B^+ \to \eta\rho^+$ and $B^+ \to \eta'\pi^+$ decay modes are seen with significance $4.7\sigma$ and $5.4\sigma$, respectively. The branching fractions for these are smaller than for the corresponding $|\Delta S| = 1$ modes $B^+ \to \eta K^{*+}$ and $B^+ \to \eta K^+$, reflecting the importance of penguin amplitudes in transitions for which they are CKM favored. Our result for $B^+ \to \eta'\pi^+$ is consistent with a small value of the nonleading flavor-singlet amplitude $S_{u\bar{u}}$ [4], which in turn implies a small phenomenological correction for the value of $\sin 2\beta$ measured in $B^0 \to \eta'K^0$ decays [4,18]. For the charged modes we find values of $\mathcal{A}_\text{ch}$ consistent with zero; absolute values as large as 0.30 or more are not excluded with our present statistics.

We are grateful for the excellent luminosity and machine conditions provided by our PEP-II colleagues, and for the substantial dedicated effort from the computing organizations that support BABAR. We wish to acknowledge support from the University of Colorado Undergraduate Research Opportunities Program. The collaborating institutions wish to thank SLAC for its support and kind hospitality. This work is supported by DOE and NSF (USA), NSERC (Canada), IHEP (China), CEA and CNRS-IN2P3 (France), BMBF and DFG (Germany), INFN (Italy), FOM (The Netherlands), NFR (Norway), MIST (Russia), and PPARC (United Kingdom). Individuals have received support from CONACyT (Mexico), A. P. Sloan Foundation, Research Corporation, and Alexander von Humboldt Foundation.

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