Improved Measurements of $CP$-Violating Asymmetry Amplitudes in $B^0 \to \pi^+ \pi^-$ Decays

We present updated measurements of the CP-violating parameters $S_{\pi\pi}$ and $C_{\pi\pi}$ in $B^0 \to \pi^+\pi^-$ decays. Using a sample of $227 \times 10^6 Y(4S) \to B\bar{B}$ decays collected with the BABAR detector at the PEP-II asymmetric-energy $e^+e^-$ collider at SLAC, we observe $467 \pm 33$ signal decays and measure $S_{\pi\pi} = -0.30 \pm 0.17 \text{(stat)} \pm 0.03 \text{(sys)}$ and $C_{\pi\pi} = -0.09 \pm 0.15 \text{(stat)} \pm 0.04 \text{(sys)}$.

CP violation has been established in B decays through precision measurements [1] of the angle $\beta$ of the unitarity triangle [2]. The agreement of these direct measurements with the indirect constraints [3] derived from the magnitudes of the elements of the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix [4] supports the standard model explanation of CP violation as arising from a single phase in the CKM matrix. Improving our knowledge of the remaining angles ($\alpha$ and $\gamma$) of the unitarity triangle will provide important further tests of the standard model description of CP violation.

Neutral-B decays to the CP eigenstate $\pi^+\pi^-$ can exhibit mixing-induced CP violation through interference between decays with and without $B^0 - B^0\bar{\tau}$ mixing, and direct CP violation through interference between $b \to u$ tree and $b \to d$ penguin decay processes [5]. Both effects are observable in the time evolution of the asymmetry between $B^0$ and $\bar{B}^0$ decays to $\pi^+\pi^-$, where mixing-induced CP violation leads to a sine term with amplitude $S_{\pi\pi}$ and direct CP violation leads to a cosine term with amplitude $C_{\pi\pi}$. In the absence of the penguin process, $C_{\pi\pi} = 0$ and $S_{\pi\pi} = \sin 2\alpha$, with $\alpha = \arg (-V_{ub}/V_{ub}^*)$, while significant tree-penguin interference leads to $S_{\pi\pi} = \sqrt{1 - C_{\pi\pi}^2} \sin 2\alpha_{\text{eff}}$, where $\alpha_{\text{eff}}$ is the effective value of $\alpha$ and $C_{\pi\pi} \neq 0$ if the strong phases of the tree and penguin decay amplitudes are different. The difference $\Delta\alpha_{\pi\pi} = \alpha - \alpha_{\text{eff}}$ can be determined from a model-independent analysis using the isospin-related decays $B^0 \to \pi^+\pi^-\pi^0$ and $B^0, \bar{B}^0 \to \pi^+\pi^-\pi^0$ [6,7].

The Belle collaboration recently reported [8] an observation of CP violation in $B^0 \to \pi^+\pi^-\pi^0$ decays using a data sample of $152 \times 10^6 BB$ pairs, while our previous measurement [9] on a sample of $88 \times 10^6 BB$ pairs was consistent with no CP violation. In this Letter, we report improved measurements of the CP-violating parameters $S_{\pi\pi}$ and $C_{\pi\pi}$, and corresponding constraints on $\alpha$, using a data sample comprising $227 \times 10^6 BB$ pairs collected with the BABAR detector at the PEP-II asymmetric-energy $e^+e^-$ collider at SLAC.

The BABAR detector is described in detail elsewhere [10]. The primary components used in this analysis are a charged-particle tracking system consisting of a five-layer silicon vertex tracker (SVT) and a 40-layer drift chamber surrounded by a 1.5 T solenoidal magnet, an electromagnetic calorimeter comprising 6580 CsI(Tl) crystals, and a detector of internally reflected Cherenkov light (DIRC) providing $K - \pi$ separation over the range of laboratory momentum relevant for this analysis (1.5–4.5 GeV/c).

The analysis method is similar to that used in our previous measurement of $S_{\pi\pi}$ and $C_{\pi\pi}$ [9]. We reconstruct a sample of neutral B mesons ($B_{\text{rec}}$) decaying to final states with two charged tracks, and examine the remaining particles in each event to infer whether the second B meson ($B_{\text{tag}}$) decayed as a $B^0$ or $\bar{B}^0$ (flavor tag). We first perform a maximum-likelihood fit that uses kinematic, event-shape, and particle-identification information to determine signal and background yields corresponding to the four distinguishable final states ($\pi^+\pi^-, K^+\pi^-, K^-\pi^+$, and $K^+K^-$). The results of this fit are described in Ref. [11], which reports the first evidence of direct CP violation in $B^0 \to K^+\pi^-$ decays [12]. The CP asymmetry parameters in $B^0 \to \pi^+\pi^-$ decays are then determined from a second fit including information about the flavor of $B_{\text{tag}}$ and the difference $\Delta t$ between the decay times of the $B_{\text{rec}}$ and $B_{\text{tag}}$ decays. The decay rate distribution $f_+ (f_-)$ when $B_{\text{rec}} \to \pi^+\pi^-$ and $B_{\text{tag}} \to B^0(\bar{B}^0)$ is given by

$$f_\pm (\Delta t) = \frac{e^{-|\Delta t|/\tau}}{4\tau} [1 \pm S_{\pi\pi} \sin (\Delta m_d \Delta t) + C_{\pi\pi} \cos (\Delta m_d \Delta t)],$$

where $\tau$ is the $B^0$ lifetime and $\Delta m_d$ is the mixing frequency due to the neutral-B-meson eigenstate mass difference.

The analysis begins by reconstructing two-body neutral-B decays from pairs of oppositely charged tracks found within the geometric acceptance of the DIRC and originating from a common decay point near the interaction region. We reconstruct the kinematics of the B candidate using the pion mass for both tracks. We require that each track have an associated Cherenkov angle ($\theta_C$) with at least five signal photons detected in the DIRC; the value of $\theta_C$ must agree within 4 standard deviations ($\sigma$) with either the pion or kaon particle hypothesis.

Identification of pions and kaons is primarily accomplished by including $\theta_C$ as a discriminating variable in the maximum-likelihood fit. We construct probability density functions (PDFs) for $\theta_C$ from a sample of approximately $430000 D^{*+} \to D^0\pi^+ (D^0 \to K^-\pi^+)$ decays reconstructed in data, where $K^-/\pi^-$ tracks are identified through the charge correlation with the $\pi^-$ from the $D^{*+}$ decay. Although we find no systematic difference between positive and negative tracks, the PDFs are constructed separately for $K^+$, $K^-$, $\pi^+$, and $\pi^-$ as functions of momentum and polar angle using the measured and expected values of $\theta_C$ and the uncertainty.

Signal decays are identified using two kinematic variables: (1) the difference $\Delta E$ between the reconstructed
energy of the $B$ candidate in the $e^+e^-$ center-of-mass (c.m.) frame and $\sqrt{s}/2$, and (2) the beam-energy substituted mass $m_{\text{ES}} = \sqrt{(s/2 + p_\ell^2 - p_B^2)/E_\ell^2 - p_B^2}$. Here, $\sqrt{s}$ is the total c.m. energy, and the $B$ momentum $p_B$ and the four-momentum $(E_\ell, p_\ell)$ of the $e^+e^-$ initial state are defined in the laboratory frame. We require $5.20 < m_{\text{ES}} < 5.29$ GeV/$c^2$ and $|\Delta E| < 150$ MeV. The sideband region in $m_{\text{ES}}$ is used to determine background-shape parameters, while the wide range in $\Delta E$ allows us to separate $B$ decays to all four final states in the same fit.

We have studied potential backgrounds from higher-multiplicity $B$ decays and find them to be negligible near $\Delta E = 0$. The dominant source of background is the process $e^+e^- \to q\bar{q}(q = u, d, s, c)$, which produces a distinctive jetlike topology. In the c.m. frame, we define the angle $\theta_5$ between the sphericity axis [13] of the $B$ candidate and the sphericity axis of the remaining particles in the event. For background events, $|\cos \theta_5|$ peaks sharply near unity, while it is nearly flat for signal decays. We require $|\cos \theta_5| < 0.8$, which removes approximately 80% of this background. Additional background suppression is accomplished by a Fisher discriminant $F$ [9] based on the momentum flow relative to the $\pi^+\pi^-$ thrust axis of all tracks and clusters in the event, excluding the $\pi\pi$ pair. We use $F$ as a discriminating variable in the fit.

We use a multivariate technique [14] to determine the flavor of the $B_{\text{tag}}$ meson. Separate neural networks are trained to identify primary leptons, kaons, soft pions from $D^*$ decays, and high-momentum charged particles from $B$ decays. Events are assigned to one of five mutually exclusive tagging categories based on the estimated average mistag probability and the source of the tagging information. The quality of tagging is expressed in terms of the effective efficiency $Q = \sum_i \epsilon_i/(1 - w_i)^2$, where $\epsilon_i$ and $w_i$ are the efficiencies and mistag probabilities for events tagged in category $i$. We measure the tagging performance in a data sample $B_{\text{flav}}$ of fully reconstructed neutral $B$ decays to $D^{*0}$($\pi^+\pi^-$, $\rho^+\rho^-$, $a_1^0$), and find a total effective efficiency of $Q = 29.9 \pm 0.5$. The assumption of equal tagging efficiencies and mistag probabilities for signal $\pi^+\pi^-$, $K^+\pi^-$, and $K^+K^-$ decays is validated in a detailed Monte Carlo simulation. Separate background efficiencies for the different decay modes are determined simultaneously with $S_{\pi\pi}$ and $C_{\pi\pi}$ in the fit.

The time difference $\Delta t = \Delta z/B\gamma c$ is obtained from the known boost of the $e^+e^-$ system ($B\gamma = 0.56$) and the measured distance $\Delta z$ along the beam ($z$) axis between the $B_{\text{rec}}$ and $B_{\text{tag}}$ decay vertices. We require $|\Delta t| < 20$ ps and $\sigma_{\Delta t} < 2.5$ ps, where $\sigma_{\Delta t}$ is the uncertainty on $\Delta t$ determined separately for each event. The resolution function for signal candidates is a sum of three Gaussians, identical to the one described in Ref. [14], with parameters determined from a fit to the $B_{\text{flav}}$ sample (including events in all five tagging categories). The background $\Delta t$ distribution is modeled as the sum of three Gaussian functions, where the common parameters used to describe the background shape for all tagging categories are determined simultaneously with the $CP$ parameters in the maximum-likelihood fit.

We use an unbinned extended maximum-likelihood fit to extract $CP$ parameters from the $B_{\text{rec}}$ sample. The likelihood for candidate $j$ tagged in category $k$ is obtained by summing the product of event yield $n_i$, tagging efficiency $\epsilon_{i,k}$, and probability $P_{i,k}$ over the eight possible signal and background hypotheses $i$ (referring to $\pi^+\pi^-$, $K^+\pi^-$, $K^0\pi^+$, and $K^+K^-$ combinations). The extended likelihood function for category $k$ is

$$L_k = \exp\left(-\sum_i n_i \epsilon_{i,k}\right) \prod_j \left[ \sum_i n_i \epsilon_{i,k} P_{i,k}(\hat{x}_j, \hat{a}_i) \right].$$

The yields for the $K\pi$ final state are parametrized as $n_{K^+\pi^-} = n_{K^0\pi^+}(1 \pm A_{\text{K}\pi})/2$, where $A_{\text{K}\pi}$ is the direct-$CP$-violating asymmetry [11]. The probabilities $P_{i,k}$ are evaluated as the product of PDFs for each of the independent variables $\hat{x}_j = (m_{\text{ES}}, \Delta E, F, \theta_5^+, \theta_5^-, \Delta t)$ with parameters $\hat{a}_i$, where $\theta_5^+$ and $\theta_5^-$ are the Cherenkov angles for the positively and negatively charged tracks. The $\Delta t$ PDF for signal $\pi^+\pi^-$ decays is given by Eq. (1) modified to include the mistag probabilities for each tag category, and convolved with the signal resolution function. The $\Delta t$ PDF for signal $K\pi$ decays takes into account $B^0 - \bar{B}^0$ mixing and the correlation between the charge of the kaon and the flavor of $B_{\text{tag}}$. We fix $\tau$ and $\Delta m_d$ to their world-average values [15]. The total likelihood $L$ is the product of likelihoods for each tagging category, and the free parameters are determined by maximizing the quantity $\ln L$.

The fit proceeds in two steps. First, the signal and background yields and $K\pi$ charge asymmetries are determined in a separate fit that does not use flavor tagging or $\Delta t$ information [11]. Out of a fitted sample of 68,030 events, we find $n_{\pi\pi} = 467 \pm 33$, $n_{K\pi} = 1606 \pm 51$, and $n_{KK} = 3 \pm 12$ decays, and measure $A_{\text{K}\pi} = -0.133 \pm 0.030$, where all errors are statistical only. We next add the flavor tagging and $\Delta t$ information and perform a fit for $S_{\pi\pi}$ and $C_{\pi\pi}$. We fix the signal and background yields and charge asymmetries to values determined in the first fit, and fix the signal parameters describing flavor tagging and $\Delta t$ resolution function parameters to the values determined in the $B_{\text{flav}}$ sample. By fixing these parameters we reduce the total number of free parameters by 30 relative to our previous analysis [9]. A total of 46 parameters are left free in the fit, including 12 parameters describing the background PDFs for $m_{\text{ES}}$, $\Delta E$, and $F$; 8 parameters describing the background $\Delta t$ PDF; 12 background flavor-tagging efficiencies; 12 background flavor-tagging efficiency asymmetries; and $S_{\pi\pi}$ and $C_{\pi\pi}$. The fit yields

$$S_{\pi\pi} = -0.30 \pm 0.17(\text{stat}) \pm 0.03(\text{syst}),$$

$$C_{\pi\pi} = -0.09 \pm 0.15(\text{stat}) \pm 0.04(\text{syst}).$$
where the correlation between $S_{\pi\pi}$ and $C_{\pi\pi}$ is $-1.6\%$, and the correlations with all other free parameters are less than $1\%$. These values are consistent with, and supersede, our previously published measurements [9].

We use the event-weighting technique described in Ref. [16] to check the agreement between PDFs and data for signal $\pi^+\pi^-$ candidates. For Figs. 1(a)–1(c), we perform a fit excluding the variable being plotted, and the covariance matrix is used to determine a weight that each event is signal, not background. The resulting distributions (points with errors) are normalized to the signal yield (467) and can be directly compared with the PDFs (solid curves) used in the fit for $S_{\pi\pi}$ and $C_{\pi\pi}$. In Fig. 1(d), we use a similar technique to compare the $\mathcal{F}$ distribution based on the probability to be a $q\bar{q}$ event with the PDF used for background events. Figure 2 shows distributions of $\Delta t$ for signal $\pi^+\pi^-$ events with $B_{\text{tag}}$ tagged as $B^0$ or $\bar{B}^0$, and the asymmetry as a function of $\Delta t$ using the same event-weighting technique. The $\chi^2$/n.d.o.f. for the distributions in Fig. 2 are (a) 17.3/12, (b) 11.3/12, and (c) 9.6/6, indicating satisfactory agreement in all three plots.

As a consistency check on the $\Delta t$ resolution function, we take advantage of the large number of $K\pi$ signal decays in the $B_{\text{rec}}$ sample to perform a $B^0 - \bar{B}^0$ mixing analysis. Floating $\tau$ and $\Delta m_s$ along with $S_{\pi\pi}$, $C_{\pi\pi}$, and $A_{K\pi}$, we find values consistent with the world averages ($\tau = 1.60 \pm 0.04$ ps and $\Delta m_s = 0.523 \pm 0.028$ ps$^{-1}$), and $CP$ parameters consistent with the nominal fit results. This test gives us confidence that the $\Delta t$ measurement is unbiased.

The dominant sources of systematic uncertainty for $S_{\pi\pi}$ arise from uncertainty on the shape of the background $\Delta t$ distribution (0.016), and on the alignment of the SVT (0.01) and the run-by-run position of the $B\bar{B}$ production point (0.01). The systematic uncertainty on $C_{\pi\pi}$ is dominated by potential bias from doubly Cabibbo-suppressed decays of the $B_{\text{tag}}$ meson (0.023) [17], and uncertainties on the non-$\Delta t$ PDF parameters (0.015), the mistag fractions (0.013), and the position of the $B\bar{B}$ production point (0.01). Contributions to the systematic uncertainty arising from knowledge of the signal $\Delta t$ resolution function, $\Delta m_s$, $\tau$, and possible differences in vertexing and $B$-flavor tagging between the $\pi^+\pi^-$ and $B_{\text{tag}}$ samples have all been evaluated and found to be less than 0.01 for both $S_{\pi\pi}$ and $C_{\pi\pi}$.

Uncertainties on the signal and background yields and $K\pi$ asymmetries are negligible for both $S_{\pi\pi}$ and $C_{\pi\pi}$. Finally, we verify that we are sensitive to nonzero values of $S_{\pi\pi}$ and $C_{\pi\pi}$ by fitting a large sample of Monte Carlo simulated signal decays with large values of the $CP$ parameters. Although the fit results are consistent with the generated values within the statistical precision of the sample, we assign the sum in quadrature of the statistical uncertainty and the difference between the fitted and generated values as a conservative systematic error accounting for potential bias in the fit procedure (0.013 for $S_{\pi\pi}$ and 0.007 for $C_{\pi\pi}$). The total systematic uncertainty is calculated by summing in quadrature the individual contributions.

![Graphs](attachment:graphs.png)

FIG. 1 (color online). Distributions of (a) $m_{ES}$, (b) $\Delta E$, and (c) $\mathcal{F}$ for signal $\pi^+\pi^-$ events (points with error bars), and (d) the distribution of $\mathcal{F}$ for $q\bar{q}$ background events, using the weighting technique described in Ref. [16]. Solid curves represent the corresponding PDFs used in the fit.

FIG. 2 (color online). Distributions of the decay-time difference $\Delta t$ using the event-weighting technique described in the text. The top two plots show events where $B_{\text{tag}}$ is identified as (a) $B^0 (n_{ge})$ or (b) $\bar{B}^0 (n_{ge})$, where the solid curves indicate the signal PDFs used in the fit. (c) The asymmetry (points with errors), defined as $(n_{ge} - n_{\bar{g}e})/(n_{ge} + n_{\bar{g}e})$, for signal events in each $\Delta t$ bin, and the projection of the fit (solid curve).
Using the model-independent isospin analysis [6] (neglecting electroweak penguin amplitudes) and the technique described in Ref. [3], we display in Fig. 3 the confidence level (C.L.) derived from the measured values of \( S/0.0025/0.0025 \) and \( C/0.0025/0.0025 \) reported here, and the results for \( \Delta - 0.11 \) determined in Ref. [7]. Values of \( \Delta \) for which the solid line lies below the dashed line are excluded at 90% C.L.

In summary, we present improved measurements of the CP-violating asymmetry amplitudes \( S/0.0025/0.0025 \) and \( C/0.0025/0.0025 \), which govern the time distributions of \( B^0 \rightarrow \pi^+ \pi^- \) decays. We find \( S_{\pi\pi} = -0.30 \pm 0.17 \pm 0.03 \) and \( C_{\pi\pi} = -0.09 \pm 0.15 \pm 0.04 \), which are consistent with our previous measurements. These results do not confirm the observation of large CP violation reported in Ref. [8].

We are grateful for the excellent luminosity and machine conditions provided by our PEP-II colleagues, and for the substantial dedicated effort from the computing organizations that support BABAR. The collaborating institutions wish to thank SLAC for its support and kind hospitality. This work is supported by DOE and NSF (USA), NSERC (Canada), IHEP (China), CEA and CNRS-IN2P3 (France), BMBF and DFG (Germany), INFN (Italy), FOM (The Netherlands), NFR (Norway), MIST (Russia), and PPARC (United Kingdom). Individuals have received support from CONACYT (Mexico), A.P. Sloan Foundation, Research Corporation, and Alexander von Humboldt Foundation.

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[12] This result was confirmed by Y. Chao et al. (Belle Collaboration), Phys. Rev. Lett. 93, 191802 (2004).