Improved Measurements of $CP$-Violating Asymmetry Amplitudes in $B^0 \to \pi^+ \pi^-$ Decays

Institut für Physik, Universität Dortmund, D-44221 Dortmund, Germany
Institut für Kern- und Teilchenphysik, Technische Universität Dresden, D-01062 Dresden, Germany
Ecole Polytechnique, LLR, F-91128 Palaiseau, France
University of Edinburgh, Edinburgh EH9 3JZ, United Kingdom
Dipartimento di Fisica and INFN, Università di Ferrara, I-44100 Ferrara, Italy
Laboratori Nazionali di Frascati dell’INFN, I-00044 Frascati, Italy
Dipartimento di Fisica and INFN, Università di Genova, I-16146 Genova, Italy
Harvard University, Cambridge, Massachusetts 02138, USA
Physikalisches Institut, Universität Heidelberg, Philosophenweg 12, D-69120 Heidelberg, Germany
Imperial College London, London SW7 2AZ, United Kingdom
University of Iowa, Iowa City, Iowa 52242, USA
Royal Holloway and Bedford New College, University of London, Egham, Surrey TW20 0EX, United Kingdom
University of Louisville, Louisville, Kentucky 40292, USA
University of Manchester, Manchester M13 9PL, United Kingdom
University of Maryland, College Park, Maryland 20742, USA
University of Massachusetts, Amherst, Massachusetts 01003, USA
Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
McGill University, Montréal, Quebec H3A 2T8, Canada
Dipartimento di Fisica and INFN, Università di Milano, I-20133 Milano, Italy
University of Mississippi, University, Mississippi 38677, USA
Laboratoire René J. A. Lévesque, Université de Montréal, Montréal, Quebec H3C 3J7, Canada
Mount Holyoke College, South Hadley, Massachusetts 01075, USA
Dipartimento di Scienze Fisiche e INFN, Università di Napoli Federico II, I-80126 Napoli, Italy
National Institute for Nuclear Physics and High Energy Physics, NIKHEF, NL-1009 DB Amsterdam, The Netherlands
University of Notre Dame, Notre Dame, Indiana 46060, USA
University of Oregon, Eugene, Oregon 97403, USA
Dipartimento di Fisica and INFN, Università di Padova, I-35131 Padova, Italy
Laboratoire de Physique Nucléaire et de Hautes Energies, Universités Paris VI et VII, F-75252 Paris, France
University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA
Dipartimento di Fisica and INFN, Università di Perugia, I-06100 Perugia, Italy
Dipartimento di Fisica, Scuola Normale Superiore, and INFN, Università di Pisa, I-56127 Pisa, Italy
Prairie View A&M University, Prairie View, Texas 77446, USA
Princeton University, Princeton, New Jersey 08544, USA
Dipartimento di Fisica and INFN, Università di Roma La Sapienza, I-00185 Roma, Italy
University of Rostock, D-18051 Rostock, Germany
Rutherford Appleton Laboratory, Chilton, Didcot, Oxon OX11 0QX, United Kingdom
DSM/Dapnia, CEA/Saclay, F-91191 Gif-sur-Yvette, France
University of South Carolina, Columbia, South Carolina 29208, USA
Stanford Linear Accelerator Center, Stanford, California 94309, USA
Stanford University, Stanford, California 94305-4060, USA
State University of New York, Albany, New York 12222, USA
University of Tennessee, Knoxville, Tennessee 37996, USA
University of Texas at Austin, Austin, Texas 78712, USA
University of Texas at Dallas, Richardson, Texas 75083, USA
Dipartimento di Fisica Sperimentale and INFN, Università di Torino, I-10125 Torino, Italy
Dipartimento di Fisica and INFN, Università di Trieste, I-34127 Trieste, Italy
IFIC, Universitat de Valencia-CSIC, E-46071 Valencia, Spain
Vanderbilt University, Nashville, Tennessee 37235, USA
University of Victoria, Victoria, British Columbia V8W 3P6, Canada
Department of Physics, University of Warwick, Coventry CV4 7AL, United Kingdom
University of Wisconsin, Madison, Wisconsin 53706, USA
Yale University, New Haven, Connecticut 06511, USA
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We present updated measurements of the $CP$-violating parameters $S_{\pi\pi}$ and $C_{\pi\pi}$ in $B^0 \to \pi^+\pi^-$ decays. Using a sample of $227 \times 10^6$ $Y(4S) \to \BB$ decays collected with the BABAR detector at the PEP-II asymmetric-energy $e^+e^-$ collider at SLAC, we observe $467 \pm 33$ signal decays and measure $S_{\pi\pi} = -0.30 \pm 0.17(\text{stat}) \pm 0.03(\text{syst})$ and $C_{\pi\pi} = -0.09 \pm 0.15(\text{stat}) \pm 0.04(\text{syst})$.

The analysis method is similar to that used in our previous measurement of $S_{\pi\pi}$ and $C_{\pi\pi}$ [9]. We reconstruct a sample of neutral $B$ mesons ($B_{\text{rec}}$) decaying to final states with two charged tracks, and examine the remaining particles in each event to infer whether the second $B$ meson ($B_{\text{tag}}$) decayed as a $B^0$ or $\bar{B}^0$ (flavor tag). We first perform a maximum-likelihood fit that uses kinematic, event-shape, and particle-identification information to determine signal and background yields corresponding to the four distinguishable final states ($\pi^+\pi^-$, $K^+\pi^-$, $K^-\pi^+$, and $K^+K^-$). The results of this fit are described in Ref. [11], which reports the first evidence of direct $CP$ violation in $B^0 \to K^+\pi^-$ decays [12]. The $CP$ asymmetry parameters in $B^0 \to \pi^+\pi^-$ decays are then determined from a second fit including information about the flavor of $B_{\text{tag}}$ and the difference $\Delta t$ between the decay times of the $B_{\text{rec}}$ and $B_{\text{tag}}$ decays. The decay rate distribution $f_\pm(f_\mp)$ when $B_{\text{rec}} \to \pi^+\pi^-$ and $B_{\text{tag}} = B^0(\bar{B}^0)$ is given by

$$f_\pm(\Delta t) = \frac{e^{-|\Delta t|/\tau}}{4\tau} \left[ 1 \pm S_{\pi\pi} \sin(\Delta m_d \Delta t) \mp C_{\pi\pi} \cos(\Delta m_d \Delta t) \right],$$

where $\tau$ is the $B^0$ lifetime and $\Delta m_d$ is the mixing frequency due to the neutral-$B$-meson eigenstate mass difference.

The analysis begins by reconstructing two-body neutral-$B$ decays from pairs of oppositely charged tracks found within the geometric acceptance of the DIRC and originating from a common decay point near the interaction region. We reconstruct the kinematics of the $B$ candidate using the pion mass for both tracks. We require that each track have an associated Cherenkov angle ($\theta_c$) measured with at least five signal photons detected in the DIRC; the value of $\theta_c$ must agree within 4 standard deviations ($\sigma$) with either the pion or kaon particle hypothesis.

Identification of pions and kaons is primarily accomplished by including $\theta_c$ as a discriminating variable in the maximum-likelihood fit. We construct probability density functions (PDFs) for $\theta_c$ from a sample of approximately 430,000 $D^{*+} \to D^0\pi^+(D^0 \to K^-\pi^+)$ decays reconstructed in data, where $K^-\pi^+$ tracks are identified through the charge correlation with the $\pi^+$ from the $D^{*+}$ decay. Although we find no systematic difference between positive and negative $\pi^+$ tracks, the PDFs are constructed separately for $K^+$, $K^-$, $\pi^+$, and $\pi^-$ tracks as a function of momentum and polar angle using the measured and expected values of $\theta_c$ and the uncertainty.

Signal decays are identified using two kinematic variables: (1) the difference $\Delta E$ between the reconstructed...
energy of the $B$ candidate in the $e^+e^-$ center-of-mass (c.m.) frame and $\sqrt{s}/2$, and (2) the beam-energy substituted mass $m_{ES} = \sqrt{(s/2 + p_i \cdot p_B)^2/E_i^2 - p_i^2}$. Here, $\sqrt{s}$ is the total c.m. energy, and the $B$ momentum $p_B$ and the four-momentum $(E_i, p_i)$ of the $e^+e^-$ initial state are defined in the laboratory frame. We require $5.20 < m_{ES} < 5.29$ GeV/$c^2$ and $|\Delta E| < 150$ MeV. The sideband region in $m_{ES}$ is used to determine background-shape parameters, while the wide range in $\Delta E$ allows us to separate $B$ decays to all four final states in the same fit.

We have studied potential backgrounds from highermultiplicity $B$ decays and find them to be negligible near $\Delta E = 0$. The dominant source of background is the process $e^+e^- \rightarrow q\bar{q}(q = u, d, s, c)$, which produces a distinctive jetlike topology. In the c.m. frame, we define the angle $\theta_S$ between the sphericity axis [13] of the $B$ candidate and the sphericity axis of the remaining particles in the event. For background events, $|\cos \theta_S|$ peaks sharply near unity, while it is nearly flat for signal decays. We require $|\cos \theta_S| < 0.8$, which removes approximately 80% of this background. Additional background suppression is accomplished by a Fisher discriminant $F$ [9] based on the momentum flow relative to the $\pi^+\pi^-$ thrust axis of all tracks and clusters in the event, excluding the $\pi\pi$ pair. We use $F$ as a discriminating variable in the fit.

We use a multivariate technique [14] to determine the flavor of the $B_{tag}$ meson. Separate neural networks are trained to identify primary leptons, kaons, soft pions from $D^*$ decays, and high-momentum charged particles from $B$ decays. Events are assigned to one of five mutually exclusive tagging categories based on the estimated average mistag probability and the source of the tagging information. The quality of tagging is expressed in terms of the effective efficiency $Q = \sum_k \epsilon_k (1 - w_k)^2$, where $\epsilon_k$ and $w_k$ are the efficiencies and mistag probabilities for events tagged in category $k$. We measure the tagging performance in a data sample $B_{flav}$ of fully reconstructed neutral $B$ decays to $D^{(*)}-(\pi^+, \rho^+, a_1^0)$, and find a total effective efficiency of $Q = 29.9 \pm 0.5$. The assumption of equal tagging efficiencies and mistag probabilities for signal $\pi^+\pi^-$, $K^+\pi^-$, and $K^+K^-$ decays is validated in a detailed Monte Carlo simulation. Separate background efficiencies for the different decay modes are determined simultaneously with $S_{\pi\pi}$ and $C_{\pi\pi}$ in the fit.

The time difference $\Delta t = \Delta z/\beta \gamma c$ is obtained from the known boost of the $e^+e^-$ system ($\beta \gamma = 0.56$) and the measured distance $\Delta z$ along the beam (z) axis between the $B_{rec}$ and $B_{tag}$ decay vertices. We require $|\Delta t| < 20$ ps and $\sigma_{\Delta t} < 2.5$ ps, where $\sigma_{\Delta t}$ is the uncertainty on $\Delta t$ determined separately for each event. The resolution function for signal candidates is a sum of three Gaussians, identical to the one described in Ref. [14], with parameters determined from a fit to the $B_{flav}$ sample (including events in all five tagging categories). The background $\Delta t$ distribution is modeled as the sum of three Gaussian functions, where the common parameters used to describe the background shape for all tagging categories are determined simultaneously with the $CP$ parameters in the maximum-likelihood fit.

We use an unbinned extended maximum-likelihood fit to extract $CP$ parameters from the $B_{tag}$ sample. The likelihood for candidate $j$ tagged in category $k$ is obtained by summing the product of event yield $n_j$, tagging efficiency $\epsilon_{i,k}$, and probability $P_{i,k}$ over the eight possible signal and background hypotheses $i$ (referring to $\pi^+\pi^-$, $K^+\pi^-$, $K^+K^-$ combinations). The extended likelihood function for category $k$ is

$$L_k = \exp \left( -\sum_i n_i \epsilon_{i,k} \prod_j \left[ \sum_i n_i \epsilon_{i,k} P_{i,k}(\tilde{x}_j, \tilde{\alpha}_i) \right] \right).$$

The yields for the $K\pi$ final state are parametrized as $n_{K\pi\pi^0} = n_{K\pi}(1 - A_{K\pi})/2$, where $A_{K\pi}$ is the direct-$CP$-violating asymmetry [11]. The probabilities $P_{i,k}$ are evaluated as the product of PDFs for each of the independent variables $\tilde{x}_j = (m_{ES}, \Delta E, F, \theta^+_S, \theta^-_S, \Delta t)$ with parameters $\tilde{\alpha}_i$, where $\theta^+_S$ and $\theta^-_S$ are the Cherenkov angles for the positively and negatively charged tracks. The $\Delta t$ PDF for signal $\pi^+\pi^-$ decays is given by Eq. (1) modified to include the mistag probabilities for each tag category, and convolved with the signal resolution function. The $\Delta t$ PDF for signal $K\pi$ decays takes into account $B^0 - \bar{B}^0$ mixing and the correlation between the charge of the kaon and the flavor of $B_{tag}$. We fix $\tau$ and $\Delta m_d$ to their world-average values [15]. The total likelihood $\mathcal{L}$ is the product of likelihoods for each tagging category, and the free parameters are determined by maximizing the quantity $\ln \mathcal{L}$.

The fit proceeds in two steps. First, the signal and background yields and $K\pi$ charge asymmetries are determined in a separate fit that does not use flavor tagging or $\Delta t$ information [11]. Out of a fitted sample of 68 030 events, we find $n_{\pi\pi} = 467 \pm 33$, $n_{K\pi} = 1606 \pm 51$, and $n_{KK} = 3 \pm 12$ decays, and measure $A_{K\pi} = -0.133 \pm 0.030$, where all errors are statistical only. We next add the flavor tagging and $\Delta t$ information and perform a fit for $S_{\pi\pi}$ and $C_{\pi\pi}$. We fix the signal and background yields and charge asymmetries to values determined in the first fit, and fix the signal parameters describing flavor tagging and $\Delta t$ resolution function parameters to the values determined in the $B_{flav}$ sample. By fixing these parameters we reduce the total number of free parameters by 30 relative to our previous analysis [9]. A total of 46 parameters are left free in the fit, including 12 parameters describing the background PDFs for $m_{ES}$, $\Delta E$, and $F$; 8 parameters describing the background $\Delta t$ PDF; 12 background flavor-tagging efficiencies; 12 background flavor-tagging efficiency asymmetries; and $S_{\pi\pi}$ and $C_{\pi\pi}$. The fit yields

$S_{\pi\pi} = -0.30 \pm 0.17$ (stat) $\pm 0.03$ (syst),

$C_{\pi\pi} = -0.09 \pm 0.15$ (stat) $\pm 0.04$ (syst),
where the correlation between $S_{\pi\pi}$ and $C_{\pi\pi}$ is $-1.6\%$, and the correlations with all other free parameters are less than $1\%$. These values are consistent with, and supersede, our previously published measurements [9].

We use the event-weighting technique described in Ref. [16] to check the agreement between PDFs and data for signal $\pi^+\pi^-$ candidates. For Figs. 1(a)–1(c), we perform a fit excluding the variable being plotted, and the covariance matrix is used to determine a weight that each event is signal, not background. The resulting distributions (points with errors) are normalized to the signal yield (467) and can be directly compared with the PDFs (solid curves) used in the fit for $S_{\pi\pi}$ and $C_{\pi\pi}$. In Fig. 1(d), we use a similar technique to compare the $F$ distribution based on the probability to be a $q\bar{q}$ event with the PDF used for background events. Figure 2 shows distributions of $\Delta t$ for signal $\pi^+\pi^-$ events with $B_{tag}$ tagged as $B^{0}$ or $\bar{B}^{0}$, and the asymmetry as a function of $\Delta t$ using the same event-weighting technique. The $\chi^2$/n.d.o.f. for the distributions in Fig. 2 are (a) 17.3/12, (b) 11.3/12, and (c) 9.6/6, indicating satisfactory agreement in all three plots.

As a consistency check on the $\Delta t$ resolution function, we take advantage of the large number of $K\pi$ signal decays in the $B_{rec}$ sample to perform a $B^{0}-\bar{B}^{0}$ mixing analysis. Floating $\tau$ and $\Delta m_{d}$ along with $S_{\pi\pi}$, $C_{\pi\pi}$, and $A_{K\pi}$, we find values consistent with the world averages ($\tau = 1.60 \pm 0.04$ ps and $\Delta m_{d} = 0.523 \pm 0.028$ ps$^{-1}$), and $CP$ parameters consistent with the nominal fit results. This test gives us confidence that the $\Delta t$ measurement is unbiased.

The dominant sources of systematic uncertainty for $S_{\pi\pi}$ arise from uncertainty on the shape of the background $\Delta t$ distribution (0.016), and on the alignment of the SVT (0.01) and the run-by-run position of the $BB$ production point (0.01). The systematic uncertainty on $C_{\pi\pi}$ is dominated by potential bias from doubly Cabibbo-suppressed decays of the $B_{tag}$ meson (0.023) [17], and uncertainties on the non-$\Delta t$ PDF parameters (0.015), the mistag fractions (0.013), and the position of the $BB$ production point (0.01). Contributions to the systematic uncertainty arising from knowledge of the signal $\Delta t$ resolution function, $\Delta m_{d}$, $\tau$, and possible differences in vertexing and $B$-flavor tagging between the $\pi^+\pi^-$ and $B_{tag}$ samples have all been evaluated and found to be less than 0.01 for both $S_{\pi\pi}$ and $C_{\pi\pi}$. Uncertainties on the signal and background yields and $K\pi$ asymmetries are negligible for both $S_{\pi\pi}$ and $C_{\pi\pi}$. Finally, we verify that we are sensitive to nonzero values of $S_{\pi\pi}$ and $C_{\pi\pi}$ by fitting a large sample of Monte Carlo simulated signal decays with large values of the $CP$ parameters. Although the fit results are consistent with the generated values within the statistical precision of the sample, we assign the sum in quadrature of the statistical uncertainty and the difference between the fitted and generated values as a conservative systematic error accounting for potential bias in the fit procedure (0.013 for $S_{\pi\pi}$ and 0.007 for $C_{\pi\pi}$). The total systematic uncertainty is calculated by summing in quadrature the individual contributions.

![FIG. 1 (color online). Distributions of (a) $m_{ES}$, (b) $\Delta E$, and (c) $F$ for signal $\pi^+\pi^-$ events (points with error bars), and (d) the distribution of $F$ for $q\bar{q}$ background events, using the weighting technique described in Ref. [16]. Solid curves represent the corresponding PDFs used in the fit.](image1)

![FIG. 2 (color online). Distributions of the decay-time difference $\Delta t$ using the event-weighting technique described in the text. The top two plots show events where $B_{tag}$ is identified as (a) $B^{0}$ ($n_{ge}$) or (b) $\bar{B}^{0}$ ($n_{ge}$), where the solid curves indicate the signal PDFs used in the fit. (c) The asymmetry (points with errors), defined as $(n_{ge} - n_{\bar{g}e})/(n_{ge} + n_{\bar{g}e})$, for signal events in each $\Delta t$ bin, and the projection of the fit (solid curve).](image2)
Using the model-independent isospin analysis [6] (neglecting electroweak penguin amplitudes) and the technique described in Ref. [3], we display in Fig. 3 the confidence level (C.L.) derived from the measured values of $S/.0025/.0025$ and $C/.0025/.0025$ reported here, and the results for $S/.0011/.0025$ determined in Ref. [7]. Values of $\alpha$ in the range $[29\,^{\circ}, 61\,^{\circ}]$ are excluded at the 90% C.L.

In summary, we present improved measurements of the $CP$-violating asymmetry amplitudes $S/.0025/.0025$ and $C/.0025/.0025$, which govern the time distributions of $B_0 \rightarrow \pi^+ \pi^-$ decays. We find $S_{\pi\pi} = -0.30 \pm 0.17 \pm 0.03$ and $C_{\pi\pi} = -0.09 \pm 0.15 \pm 0.04$, which are consistent with our previous measurements. These results do not confirm the observation of large $CP$ violation reported in Ref. [8].

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*Also with Università della Basilicata, Potenza, Italy.
†Deceased.

[12] This result was confirmed by Y. Chao et al. (Belle Collaboration), Phys. Rev. Lett. 93, 191802 (2004).