Determination of $|V_{ub}|$ from Measurements of the Electron and Neutrino Momenta in Inclusive Semileptonic $B$ Decays


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We present a determination of the Cabibbo-Kobayashi-Maskawa matrix element $|V_{ub}|$ based on the analysis of semileptonic $B$ decays from a sample of $88 \times 10^6$ $\Upsilon(4S)$ decays collected with the BABAR detector at the SLAC PEP-II $e^+e^-$ storage ring. Charmless semileptonic $B$ decays are selected using measurements of the electron energy and the invariant mass squared of the electron-neutrino pair. We obtain $|V_{ub}| = (3.95 \pm 0.26^{+0.58}_{-0.42} \pm 0.25) \times 10^{-3}$, where the errors represent experimental uncertainties, heavy quark parameter uncertainties, and theoretical uncertainties, respectively.

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The study of the weak interactions of quarks has played a crucial role in the development of the standard model (SM), which embodies our understanding of the fundamental interactions. The increasingly precise measurements of $CP$ asymmetries in $B$ decays allow stringent experimental tests of the SM mechanism for $CP$ violation via the complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1]. Improved determinations of $|V_{ub}|$, the coupling strength of the $b$ quark to the $u$ quark, will improve the sensitivity of these tests.

Two observables have been used to determine $|V_{ub}|$ from inclusive semileptonic $B$ decays: the end point of the lepton momentum spectrum [2] and the mass of the accompanying hadronic system [3]. In this Letter, semileptonic $B \to X_u e \bar{\nu}$ decays are selected using a novel approach based on simultaneous requirements for the electron energy, $E_e$, and the invariant mass squared of the $e \bar{\nu}$ pair, $q^2$ [4]. The neutrino 4-momentum is reconstructed from the visible 4-momentum and knowledge of the $e^+e^-$ initial state. The dominant charm background is suppressed by selecting a region of the $q^2$–$E_e$ phase space where correctly reconstructed $B \to X_u e \bar{\nu}$ events are kinematically excluded. Background contamination in the signal region is due to resolution effects and is evaluated in Monte Carlo (MC) simulations. Theoretical calculations are applied to the measured $B \to X_u e \bar{\nu}$ partial rate to determine $|V_{ub}|$, the precision of which is limited mostly by our current knowledge of the $b$-quark mass, $m_b$.

The data used in this analysis were collected with the BABAR detector [5] at the SLAC PEP-II asymmetric-energy $e^+e^-$ storage ring. The data set consists of $88.4 \times 10^6 B\bar{B}$ pairs collected at the $\Upsilon(4S)$ resonance, corresponding to an integrated luminosity of $81.4$ fb$^{-1}$ at $\sqrt{s} = 10.58$ GeV. An additional $9.6$ fb$^{-1}$ of data were collected at center-of-mass energies $20$ MeV below the $B\bar{B}$ threshold. Off-resonance data are used to subtract the non-$B\bar{B}$ contributions from the data collected at the $\Upsilon(4S)$ resonance. To do so, the off-resonance data are scaled according to the integrated luminosity and the energy dependence of the QED cross section, and the particles are boosted to the $\Upsilon(4S)$ resonance energy. Throughout this Letter, all kinematic variables are given in the $\Upsilon(4S)$ rest frame unless stated otherwise.

The simulation of charmless semileptonic $B$ decays used in optimizing the analysis and determining reconstruction efficiencies is based on the heavy quark expansion (HQE) including $O(\alpha_s)$ corrections [6]. This calculation produces a continuous spectrum of hadronic masses, $m_X$. Subsequent hadronization is simulated using JETSET down to $2m_\pi$ [7]. Decays to low-mass hadrons ($\pi$, $\eta$, $\rho$, $\omega$, $\eta'$) are simulated separately using the ISGW2 model [8], and mixed with the nonresonant states so that the $m_X$, $q^2$, and $E_e$ spectral distributions correspond as closely as possible to the HQE calculation.

Hadronic events containing an identified electron with energy $2.1 < E_e < 2.8$ GeV are selected. Radiative $B\bar{B}$ events rejected using the criteria given in Ref. [9] and electrons from $J/\psi \to e^+e^-$ decays are vetoed. The total visible 4-momentum, $p_{\text{vis}}$, is determined using charged tracks emanating from the collision point, identified pairs of charged tracks from $K^0_S \to \pi^+\pi^-$, $\Lambda \to p\pi^-$, and $\gamma \to e^+e^-$, and energy deposits in the electromagnetic calorimeter. Each charged particle is assigned a mass hypothesis based on particle identification information. Calorimeter clusters unassociated with a charged track and with a lateral energy spread consistent with electromagnetic showers are treated as photons.

Additional requirements are made to improve the quality of the neutrino reconstruction and suppress contributions from $e^+e^- \to q\bar{q}$ continuum events. We form the missing 4-momentum, $p_{\text{miss}} = p_{e^+\bar{\nu}} - p_{\text{vis}}$, where $p_{e^+\bar{\nu}}$ is the 4-momentum of the initial state. For each event we require (1) no additional identified $e$ or $\mu$; (2) $-0.95 < \cos\theta_{\text{miss}} < 0.8$, where $\theta_{\text{miss}}$ is the polar angle of the missing 3-momentum; (3) $0.0 < E_{\text{miss}} - |p_{\text{miss}}| < 0.8$ GeV, where $E_{\text{miss}}$ is the missing energy in the event; (4) $|p_{\text{miss}}| < 2.5$ GeV and (5) $|\cos\theta_{\hat{T}}| < 0.75$, where $\theta_{\hat{T}}$ is the angle between the electron momentum and the thrust vector of the remaining particles in the event.

The measured $|p_{\text{miss}}|$ differs from the true neutrino momentum due to additional particles that escape detection. Therefore, a bias correction, $p_r = p_{\text{miss}}(0.804 - 0.078/|p_{\text{miss}}|)$, is derived from the simulation. Since the resolution on $|p_{\text{miss}}|$ is superior to that of $E_{\text{miss}}$, we set $p_r = (p_r, |p_r|)$ and $q^2 = (p_e + p_{\nu})^2$. Defining $\eta_z =\sqrt{(1 \pm \beta)/(1 \mp \beta)}$, where $\beta \approx 0.06$ is the velocity of the $B$ meson in the $\Upsilon(4S)$ frame, the maximum kinematically allowed hadronic mass squared for a given $E_e$ and $q^2$ is $s_h^{\text{max}} = m_B^2 + q^2 - 2m_B(E_e\eta_+ + q^2\eta_+/4E_e)$ for $\pm E_e > \pm\sqrt{q^2}\eta_z$, and $s_h^{\text{max}} = m_B^2 + q^2 - 2m_B\sqrt{q^2}$ otherwise. We require $s_h^{\text{max}} < 3.5$ GeV$^2 = m_{D^*}^2$; no $B \to X_u e \bar{\nu}$ decays can
have values of $s_h^{\text{max}}$ below this limit before accounting for resolution. The requirements on $E_e$ and $s_h^{\text{max}}$ and criteria (1)–(5) were chosen to minimize the total (experimental and theoretical) expected uncertainty $\sigma(|V_{ub}|)/|V_{ub}|$.

The quality of the neutrino reconstruction is evaluated using a control sample ($De\bar{\nu}$) consisting of the decays $B \rightarrow D^0 e\bar{\nu}(X)$, where kinematic criteria result in the $X$ system typically being no more than a $\pi$ or $\gamma$ from a $D^* \rightarrow D^0 X$ transition. The $D^0$ is reconstructed in the $K^-\pi^+$ decay mode, and we require $|p_{D^0}| > 0.5 \text{ GeV}$ and $E_e > 1.4 \text{ GeV}$. The $D^0 e$ combination must satisfy $-2.5 < \cos \theta_{B_D e} < 1.1$, where $\cos \theta_{B_D e} = (2E_B E_{D e} - m_B^2 - m_{D e}^2)/(2|p_B||p_{D e}|)$ is the cosine of the angle between the vector momenta of the $B$ and the $D^0 e$ system assuming the only missing particle in the $B$ decay was a single neutrino. After the combinatorial background is subtracted using $D^0$ mass sidebands, the selected sample consists primarily ($\approx 95\%$) of $B \rightarrow D^0 e\bar{\nu}$ and $B \rightarrow D^* e\bar{\nu}$ decays. The control sample selection makes no requirements on the other $B$ in the event, and can therefore be used to study the impact of the modeling of the other $B$ on the neutrino reconstruction. Since the unreconstructed $X$ system in the $B \rightarrow D^0 e\bar{\nu}(X)$ decays carries away little energy, a good estimate (rms $\approx 0.2 \text{ GeV}$) of the neutrino energy can be obtained from the known $B$ energy and the measured $D^0$ and $e$ energies, $E_{D e}^{\text{MC}}$. A second estimate of the neutrino energy is constructed from the visible momentum as described previously. Subtracting the first estimate from the second gives the distribution shown in Fig. 1, where the criteria (1)–(5) described above have been imposed. We find good agreement between data and MC calculations; the average (rms) is 0.066 GeV (0.366 GeV) for data and 0.072 GeV (0.365 GeV) for simulated events.

The $De\bar{\nu}$ control sample is also used to improve the modeling of the $B \rightarrow X_e e\bar{\nu}$ decays. After relaxing the $\cos \theta_{B_D e}$ requirements and subtracting continuum and combinatorial backgrounds, we perform a binned $\chi^2$ fit to the $De\bar{\nu}$ sample in the variables $|p_{D e}|$, $E_e$, and $\cos \theta_{B_D e}$. The fit determines scale factors for the MC components $B \rightarrow De\bar{\nu}$, $B \rightarrow D^* e\bar{\nu}$, and other contributions (85\% of which are decays to $D^{*\ast}$ states), while keeping the total $B \rightarrow X_e e\bar{\nu}$ branching fraction fixed to the measured value [10]. The fit increases the $B \rightarrow D e\bar{\nu}$ and $B \rightarrow D^* e\bar{\nu}$ branching fractions to 2.29\% and 6.02\% (2.48\% and 6.52\%) for neutral (charged) $B$ mesons, respectively, while decreasing the remaining contributions. By design, these revised branching fractions respect isospin symmetry and are used in the determination of the background.

Two control samples are used to reduce the sensitivity of the efficiency and background estimates to details of the simulation: the $De\bar{\nu}$ control sample described above, but with $E_e > 2.0 \text{ GeV}$; and events satisfying the normal selection criteria but having $s_h^{\text{max}} > 4.25 \text{ GeV}^2$, a sample with <5\% signal decays. Efficiencies $e_{\text{Data}}^{De\bar{\nu}}$ and $e_{\text{MC}}^{De\bar{\nu}}$ are calculated separately in data and MC calculations as the ratio of $De\bar{\nu}$ candidates satisfying criteria (1)–(5) to the total $De\bar{\nu}$ sample. The $B \rightarrow X_e e\bar{\nu}$ signal efficiency is multiplied by the ratio of these efficiencies to reduce sensitivity to background. The $s_h^{\text{max}} > 4.25 \text{ GeV}^2$ sideband region is used to normalize the simulated $s_h^{\text{max}}$ distribution to the data, reducing sensitivity to background normalization uncertainties.

We determine a partial branching fraction $\Delta B(E_h, s_h^{\text{max}}) = B(B \rightarrow X_e e\bar{\nu})f_u$, unfolded for detector effects. The acceptance, $f_u$, is the fraction of $B \rightarrow X_e e\bar{\nu}$ decays in the region of interest, $E_e > 2.0 \text{ GeV}$ and $s_h^{\text{max}} < 3.5 \text{ GeV}^2$, where $E_e$ and $s_h^{\text{max}}$ are the true (generated) values in the $B$ meson rest frame. Slightly lower values are accepted for $E_e$ than for $E_e$ to account for the boost of the $B$ meson and to increase the efficiency times acceptance for $B \rightarrow X_e e\bar{\nu}$ decays can be written as $e_u = e_{\text{sig}} f_u + e_{\text{bkg}} (1 - f_u)$, where $e_{\text{sig}}$ ($e_{\text{bkg}}$) is the efficiency for an event inside (outside) the region of interest to be reconstructed and pass our selection criteria. We calculate the partial branching fraction as follows:

$$\Delta B = \frac{N_{\text{cand}} - M_{\text{bkg}}}{2N_{BB} e_{\text{sig}}^{\text{MC}} e_{\text{sig}}^{\text{MC}}} \left[ 1 + \frac{1 - f_u}{f_u} \frac{e_{\text{bkg}}^{\text{MC}}}{e_{\text{sig}}^{\text{MC}}} \right]^{-1}, \quad (1)$$

where $N_{\text{cand}}$ and $N_{\text{side}}$ refer to the number of candidates in the signal and $s_h^{\text{max}}$ sideband regions of the data, $M_{\text{bkg}}$ and $M_{\text{side}}$ refer to background in the signal region and the yield in the sideband region in simulated events, and $2N_{BB}$ is the number of $B$ mesons produced from $Y(4S) \rightarrow BB$ decays. Since the resulting ratio of $e_{\text{bkg}}^{\text{MC}}/e_{\text{sig}}^{\text{MC}}$ is small, $\Delta B$ depends only weakly on the model used to determine $f_u$.

Figure 2 shows the electron energy and $s_h^{\text{max}}$ distributions after cuts have been applied to all variables except the one being displayed. The discrepancy observed between data and MC calculations for $E_e < 1.95 \text{ GeV}$ is covered by the systematic error on the $B \rightarrow X_e e\bar{\nu}$ modeling. The yields and efficiencies are given in Table I. We find

$$\Delta B(2.0, 3.5) = (3.54 \pm 0.33 \pm 0.34) \times 10^{-4}, \quad (2)$$

where the uncertainties are statistical and systematic, re-
their measured range. Furthermore, the form factors for $B \rightarrow D \bar{e} \bar{\nu}$ [13] and $B \rightarrow D^* \bar{e} \bar{\nu}$ [14] were varied within their uncertainties, and the composition of the $D^{**}$ states was modified to include only narrow resonances, broad resonances, or Goity-Roberts decays [15]; the effect of these variations is reduced by the fit to the $D \bar{e} \bar{\nu}$ control sample. The modeling of $D$ decays was varied based on the measurements reported in Ref. [16]; the variation in the $D \rightarrow K_L^0 \phi$ branching fractions dominates the uncertainty. The modeling of $B \rightarrow X_e e \bar{\nu}$ decays is sensitive to the resonance structure at low mass. The branching fractions of $B \rightarrow (\pi, \rho, \omega, \eta, \eta') e \bar{\nu}$ were varied as follows: $\pi$: $\pm 30\%$; $\rho$: $\pm 30\%$; $\omega$: $\pm 40\%$; simultaneously $\eta$ and $\eta'$: $\pm 100\%$.

We extract $|V_{ub}| = [\Delta \mathcal{B}/(\Delta \xi \tau_B)]^{1/2}$ using $\tau_B = 1.604$ $\pm$ 0.012 ps [16]. The normalized partial rate, $\Delta \xi$, computed in units of $\Delta \Gamma/|V_{ub}|^2$, is taken from Ref. [17], in which the leading terms in the HQE of the $B \rightarrow X_e e \bar{\nu}$ spectra are computed at next-to-leading order, and power corrections are included at $O(\alpha_s)$ for the leading shape function (SF) and at tree level for subleading SFs. The values used for the heavy quark parameters, $m_b = 4.61 \pm 0.08$ GeV and $\mu_b^2 = 0.15 \pm 0.07$ GeV$^2$, with a correlation coefficient of $-0.4$, are based on fits to $B \rightarrow X_e e \bar{\nu}$ moments [18], translated to the shape-function scheme of Ref. [19].

We find $|V_{ub}| = (3.95 \pm 0.26^{+0.58}_{-0.25}) \times 10^{-3}$ for $\bar{E}_e > 2.0$ GeV, where the errors represent experimental, heavy quark parameters, and theoretical uncertainties, respectively. The latter include estimates of the effects of subleading SFs [20], variations in the matching scales used in the calculation, and weak annihilation [21]. No uncertainty is assigned for possible quark-hadron duality violation. The determination of $|V_{ub}|$ is limited primarily by our knowledge of $m_b$. An approximate dependence is $|V_{ub}(m_b)| = |V_{ub}(m_0)|[1 + 7(m_b - m_0)/m_0]$, where

![Graph](image)
$m_0 = 4.61$ GeV. The sensitivity to higher moments of the SF is weak; the change in $|V_{ub}|$ when varying $\mu_0^2$ from 0.03 to 0.35 GeV$^2$ with $m_b$ fixed is 2%, and the impact of using alternative SF parametrizations [22] is <2%. The overall precision on the above result surpasses that of Refs. [2,3], but is comparable to determinations of $|V_{ub}|$ that have become available while this Letter was nearing completion [23].

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