Search for the decay $\tau^+ \to 3\pi^- 2\pi^+ 2\pi^0$
A search for the decay of the $\tau$ lepton to five charged and two neutral pions is performed using data collected by the $BABAR$ detector at the PEP-II asymmetric-energy $e^+e^-\rightarrow\tau^+\tau^-$ collider. The analysis uses $232$ fb$^{-1}$ of data at center-of-mass energies on or near the $Y(4S)$ resonance. We observe 10 events with an expected background of $6.5^{+1.4}_{-1.0}$ events. In the absence of a signal, we set the limit on the branching ratio $B(\tau^-\rightarrow 3\pi^-2\pi^+2\pi^0\nu_\tau)<3.4\times 10^{-6}$ at the 90% confidence level. This is a significant improvement over the previously established limit. In addition, we search for the decay mode $\tau^\pm\rightarrow 2\omega\pi^\mp\nu_\tau$. We observe 1 event with an expected background of $0.4_{-0.4}^{+1.0}$ events and calculate the upper limit $B(\tau^-\rightarrow 2\omega\pi^-\nu_\tau)<5.4\times 10^{-7}$ at the 90% confidence level. This is the first upper limit for this mode.

The $\tau$ decay dynamics and lead to a more stringent limit on the $\tau$ neutrino mass, if observed with sufficient statistics. This decay is allowed but suppressed due to the limited phase space of the seven-pion $\tau$ decays [3, 4]. An upper limit $B(\tau^-\rightarrow 3\pi^-2\pi^+2\pi^0\nu_\tau)<1.1\times 10^{-4}$ at the 90% confidence level (CL) has been set by the CLEO collaboration [5].

Since $\tau$ decays to five charged pions and a $\pi^0$ meson involve resonances (e.g., $\omega$ or $\eta$) [6], it is expected that the $\tau^-\rightarrow 3\pi^-2\pi^+2\pi^0\nu_\tau$ decay may also proceed through resonant subchannels. According to calculations based on isospin symmetry [7], the decay $\tau^-\rightarrow 2\omega\pi^-\nu_\tau$ is expected to be the dominant mode.

This analysis is based on data recorded with the $BABAR$ detector at the PEP-II asymmetric-energy $e^+e^-$ storage ring operated at the Stanford Linear Accelerator Center. The data sample consists of $232$ fb$^{-1}$ recorded at center-of-mass (CM) energies of $10.58$ GeV and $10.54$ GeV. With an expected cross section for $\tau$ pairs of $\sigma_{\tau\tau}=(0.89\pm 0.02)$ nb [8], the number of produced $\tau$-pair events is $N_{\tau\tau}=(206.5\pm 4.7)\times 10^6$.

The $BABAR$ detector is described in detail in Ref. [9], and only a brief description is given here. Charged-particle momenta are measured with a 5-layer double-sided silicon vertex tracker (SVT) and a 40-layer drift chamber (DCH) inside a solenoidal magnet with a 1.5 T magnetic field. A calorimeter (EMC) consisting of 6580 CsI(Tl) crystals is used to measure the energy of electrons, positrons, and photons. A ring-imaging Cherenkov detector is used to identify charged hadrons, in combination with ionization energy loss measurements in the SVT and the DCH. Muons are identified by an instrumented magnetic-flux return (IFR).

Monte Carlo (MC) simulations are used to estimate the $\tau^-\rightarrow 3\pi^-2\pi^+2\pi^0\nu_\tau$ signal efficiency and background contamination from other $\tau$ decay modes. The production of $\tau$ pairs is simulated with the KK generator [10], and nonsignal $\tau$ lepton decays are modeled with TAUOLA [11] according to measured rates [1]. The background processes $e^+e^-\rightarrow q\bar{q}$ ($q=u, d, s, c, b$) are simulated using the JETSET package [12]. Signal events are generated using phase space with a $V-A$ interaction. We find no significant variation in efficiency within the phase space. The simulation of the $BABAR$ detector is based on GEANT 4 [13].

The principal backgrounds to our signal come from $e^+e^-\rightarrow q\bar{q}$ processes and multipion $\tau$ decay modes involving at least one $\pi^0$, namely $\tau^-\rightarrow 3\pi^-2\pi^+\pi^0\nu_\tau$, $\tau^-\rightarrow 2\pi^-\pi^+\pi^0\nu_\tau$ and $\tau^-\rightarrow 2\pi^-\pi^+3\pi^0\nu_\tau$ modes. The $\tau^-\rightarrow 3\pi^-2\pi^+\pi^0\nu_\tau$ contribution comes from reconstructing an additional (fake) $\pi^0$, while the three-prong modes contribute through the $\pi^0$ decay to a photon pair and subsequent photon conversions in detector material.

The event selection criteria were developed to suppress the background while maintaining high signal efficiency. Events with six charged-particle tracks and a net charge of zero are first selected. To ensure well-reconstructed tracks, each track is required to have a minimum transverse momentum of 100 MeV/c, a distance of closest approach to the interaction point in the plane transverse to the beam axis (DOCA$_{XY}$) less than 1.5 cm, and a distance of closest approach along the beam direction less than 10 cm. Four or more tracks are required to have hits in at least 12 DCH layers. Photons are reconstructed from clusters in the EMC and are required to have a minimum energy of 50 MeV, energy deposited in at least three crystals, and a lateral energy profile consistent with that of a photon. In addition, to suppress background from backscattering in the EMC, the angle between the position of a cluster and the impact point of the nearest charged track at the EMC surface, as seen from the interaction point, is required to be more than 0.08 radians.
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The $\pi^0$ mesons are reconstructed from two photon candidates passing the photon selection criteria described above. We first search for $\pi^0$ candidates with energy $E_{\gamma} > 450$ MeV and mass $113 < M_{\gamma\gamma} < 155$ MeV/c$^2$. If two or more $\pi^0$ candidates share a photon, only the one with the smallest $|M_{\gamma\gamma} - M_{\pi^0}^\text{PDG}|$, where $M_{\pi^0}^\text{PDG}$ value is taken from [1], is retained. Next, we repeat the procedure for $\pi^0$ candidates with energy $300 < E_{\gamma} < 450$ MeV and mass $120 < M_{\gamma\gamma} < 148$ MeV/c$^2$.

The $\tau$ pair is produced approximately back-to-back in the $e^+e^-$ CM frame. This allows the event to be divided into two hemispheres by a plane perpendicular to the thrust axis, where the thrust is calculated from all charged particles and photons in the event [12]. The event thrust magnitude is required to be larger than 0.9. This requirement rejects more than 90% of the $q\bar{q}$ background and the $e^+e^- \rightarrow B\bar{B}$ background is suppressed to a negligible level. Events are required to have one track in one hemisphere (the tag side) and five tracks in the other hemisphere (the signal side). To further suppress the background from $e^+e^- \rightarrow q\bar{q}$ events, we demand a well-identified electron or muon on the tag side with at most one additional photon with energy $E_{\gamma} < 500$ MeV. The combined mass of all charged particles and photons in each hemisphere is required to be less than 3 GeV/c$^2$. Finally, only events with exactly two $\pi^0$ candidates on the signal side are kept for further study. The efficiency of the two $\pi^0$ selection in the signal MC is 13.0%.

The visible energy, defined as the sum of the CM energy of the charged tracks and the reconstructed $\pi^0$ mesons, is required to be less than the CM beam energy $E_{\text{beam}} = 5.29$ GeV in each hemisphere of the event. The residual energy $E_{\text{res}}$, defined as the neutral energy on the signal side not associated with the reconstructed $\tau$ decay products, is required to be less than 300 MeV, reducing the background from $e^+e^- \rightarrow q\bar{q}$ and $\tau^- \rightarrow 2\pi^- \pi^0 \pi^0 \nu_\tau$ events.

To reconstruct the signal event, an approximation of the $\tau$ invariant mass is used:

$$M^* = 2(E_{\text{beam}} - E_{\gamma})(E_{\gamma} - P_{\gamma}) + M_{\gamma\gamma}^2,$$

where the $\tau$ neutrino is assumed to be massless and travel along the direction of the combined momentum vector $P_{\gamma\gamma}$ of the seven hadrons and its energy is taken to be the difference between $E_{\text{beam}}$ and the combined energy $E_{\gamma\gamma}$ of the hadrons in the CM system. The variable $M^*$ is called the $\tau$ pseudomass [14], and its distributions for signal and background MC events are shown in Fig. 1. The advantage of $M^*$ over the invariant mass $M_{\gamma\gamma}$ is a considerably better separation of the signal from the hadronic $q\bar{q}$ background.

We apply particle identification on the signal side, demanding four out of five tracks to be identified as pions with high probability, and apply looser identification criteria to the fifth track. This requirement significantly reduces the background from $\tau$ events with photon conversions and $e^+e^- \rightarrow q\bar{q}$ events containing kaons.

We further suppress photon conversions by requiring the invariant mass of each pair of oppositely charged tracks to be larger than 5 MeV/c$^2$. In addition, we apply cuts on the sums of the two lowest transverse momenta and two largest $\text{DOCA}_{XY}$ of the tracks on the signal side: $p_{t,\text{lowest1}} + p_{t,\text{lowest2}} > 0.4$ GeV/c and $\text{DOCA}_{XY}^{\text{largest1}} + \text{DOCA}_{XY}^{\text{largest2}} < 0.4$ cm.

The final event count is performed in the signal region $1.3 < M^* < 1.8$ GeV/c$^2$. According to MC studies, the signal efficiency after all cuts is $(0.66 \pm 0.05)\%$. The error is a combination of systematic and statistical uncertainties.

The systematic uncertainty on the signal efficiency includes contributions from the reconstruction of charged tracks and photons (4.3%), the reconstruction of two $\pi^0$ mesons (6.6%), and the uncertainty associated with the particle identification on the signal and tag sides (1.7%). A statistical uncertainty (1.8%) due to limited MC samples is added in quadrature to the systematic uncertainty.

The simulation of $\tau$-pair events yields a reliable estimate of their expected background contribution, verified by modifying the event selection criteria to suppress the $q\bar{q}$ background and allow for more $\tau$ events. The largest background is predicted to come from $\tau^- \rightarrow 3\pi^- 2\pi^+ 2\pi^0 \nu_\tau$ decays. For a detailed study, we use an MC sample of $\tau^- \rightarrow 3\pi^- 2\pi^+ 2\pi^0 \nu_\tau$ events corresponding to 1900 fb$^{-1}$ of data. The pseudomass spectrum of the events passing the selection criteria is fitted with a “Crystal Ball” probability density function (PDF) [15]. In order to determine the shape parameters of this PDF, we first fit a larger sample selected without tagging of the one-prong side. Using this fixed shape, we then estimate the number of $\tau^- \rightarrow 3\pi^- 2\pi^+ 2\pi^0 \nu_\tau$ events within our signal region $(1.3 < M^* < 1.8$ GeV/c$^2$) from the MC sample with the one-prong tag applied. We obtain $3.6 \pm 0.6$ events, scaled to the luminosity of 232 fb$^{-1}$, where the uncertainty is statistical only (see Fig. 2, left). Simply
counting the number of events in the signal region yields 3.2 (scaled) MC events.

The uncertainty of the $\tau^+\tau^-$ background estimate is based on the uncertainties of the fitted PDF shape parameters, namely, the central value and the width, and the correlation between them. The values of the PDF shape parameters are randomly generated according to their uncertainties expressed in the covariance matrix, and the resulting PDF is then used to estimate the number of background events in the signal region. The total uncertainty from the fitting (0.6 events, 16.7%) is added in quadrature with systematic uncertainties in the reconstruction of the tracks and neutrals, particle identification, luminosity and $\tau$-pair cross section (8.4%) and the uncertainty in the branching ratio of the $\tau^+\tau^- \rightarrow 3\pi^- 2\pi^+\pi^0\nu$ decay mode (14.9%).

An additional background contribution is expected from the $\tau^+\tau^- \rightarrow 2\pi^- 2\pi^+\pi^0\nu$ mode. Using an MC sample corresponding to 675 fb$^{-1}$ of data we estimate 0.7 ± 0.5 background events in the signal region from this source. The uncertainty is dominated by the MC statistics. Contributions from other generic $\tau$ decays are negligible. Combining both sources of the $\tau$ background, we expect a total of 4.3 ± 1.0 background events in the data.

For this analysis, a comparison of MC simulation and data has shown that the $e^+e^- \rightarrow q\bar{q}$ background contributions cannot reliably be extracted from simulation due to difficulties in modeling the fragmentation processes. The shape of the simulated pseudomass distribution appears to agree with the shape in the data, but the overall normalization does not. Therefore, the $q\bar{q}$ background is estimated directly from the data, by fitting the data pseudomass spectrum with the sum of two Gaussians. This PDF is motivated by MC studies, which show that the $e^+e^- \rightarrow (u\bar{u}, d\bar{d}, s\bar{s})$ and $e^+e^- \rightarrow c\bar{c}$ backgrounds have Gaussian pseudomass shapes with different parameters. The double-Gaussian fit to the MC pseudomass distribution of $q\bar{q}$ background is shown in Fig. 2 (right).

To extract the $q\bar{q}$ background in the signal region, we subtract the expected $\tau$ background contribution from the data pseudomass distribution, and fit the resulting histogram in the range $1.8 < M^* < 3.3$ GeV/c$^2$ with a double-Gaussian PDF whose means and sigmas are allowed to float. To avoid experimenter bias, this fit is performed “blind”, with the data in the signal region hidden. The fit function is then extrapolated below 1.8 GeV/c$^2$ and its integral between 1.3 and 1.8 GeV/c$^2$ yields the $q\bar{q}$ background estimate in the data, 2.2 events.

To calculate the statistical uncertainty of the $q\bar{q}$ background estimate we vary the number of events in each bin of the data $q\bar{q}$ pseudomass spectrum above 1.8 GeV/c$^2$ according to its Poisson error and refit the resulting histogram for a new estimate. The statistical uncertainty of 1.0 events is extracted from the variance of the distribution of the generated $q\bar{q}$ background estimates. Variations in the functional form of the fit PDF are taken into account as a systematic uncertainty of 0.7 events. The total uncertainty is calculated by adding the statistical and systematic uncertainties in quadrature. Thus, the $q\bar{q}$ background estimate is $2.2^{+1.0}_{-1.0}$ events.

To validate the $e^+e^- \rightarrow q\bar{q}$ background estimate method, we apply it to a $\tau$-event-free data sample, obtained by requiring at least 3 photons with energies greater than 300 MeV on the tag side not associated with a $\pi^0$. This requirement effectively suppresses $\tau$ events to a negligible level and provides a clean $q\bar{q}$ sample in the data. Comparison between the expected and observed $q\bar{q}$ background levels for this sample shows good agreement, 11.8 predicted background events vs 12 observed.

Another cross-check we perform is the branching ratio measurement of the $\tau^+\tau^- \rightarrow 3\pi^- 2\pi^+\pi^0\nu$ decay mode using the same selection criteria (except for demanding only one $\pi^0$ on the signal side instead of two) as described above. The measured branching ratio is consistent with the Particle Data Group’s value [1].

Combining the background estimates from $\tau$ and $q\bar{q}$ events, we calculate a total of $6.5^{+2.0}_{-1.4}$ background events.
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Figure 3 illustrates the final pseudomass spectrum of the data, along with the expected background PDF. We observe 10 events in the signal region and conclude that there is no evidence for the $\tau^- \to 3\pi^- 2\pi^+ 2\pi^0 \nu_{\tau}$ decay.

The upper limit for the $\tau^- \to 3\pi^- 2\pi^+ 2\pi^0 \nu_{\tau}$ decay branching ratio is calculated from

$$\mathcal{B}(\tau^- \to 3\pi^- 2\pi^+ 2\pi^0 \nu_{\tau}) < \frac{\lambda_{N_{\text{signal}}}}{2 \times N_{\tau\tau} \times \epsilon},$$

where $\lambda_{N_{\text{signal}}}$ is the upper limit on the number of signal events at the 90% CL. This number is obtained using a limit calculator program [16] that follows the Cousins and Highland approach [17] of incorporating systematic uncertainties into the upper limit, using the numbers of expected background and observed events, as well as the uncertainties on the background, signal efficiency and the number of $\tau$ pairs. We find $\lambda_{N_{\text{signal}}} = 9.2$ events and $\mathcal{B}(\tau^- \to 3\pi^- 2\pi^+ 2\pi^0 \nu_{\tau}) < 3.4 \times 10^{-6}$ at the 90% CL. Table I summarizes the results of this analysis.

In addition to this inclusive result, we also search for the resonant decay mode $\tau^- \to 2\omega \pi^- \nu_{\tau}$ with the subsequent decay $\omega \to \pi^- \pi^0 \pi^0$, which is predicted to be the main channel for the $\tau^- \to 3\pi^- 2\pi^+ 2\pi^0 \nu_{\tau}$ decay [7]. The $\tau^- \to 2\omega \pi^- \nu_{\tau}$ mode has a much narrower allowed pseudoscalar mass range ($1.7 < M^* < 1.8$ GeV/$c^2$) due to its kinematics. For the same reason, the background level is expected to be much smaller. The event selection is reoptimized for this analysis. Photons are required to have a minimum energy of 50 MeV, energy deposited in at least two crystals and a lateral energy profile consistent with that of a photon. Reconstructed $\pi^0$ candidates must have energies above 200 MeV. The $\omega$ resonance is reconstructed as a $\pi^- \pi^- \pi^0$ combination with an invariant mass of $0.76 < M_{\pi^- \pi^- \pi^0} < 0.80$ GeV/$c^2$.

Reconstruction of both $\omega$ mesons suppresses the background and therefore further selection cuts can be substantially loosened to increase the signal efficiency. The conversion veto and the $E_{\text{vis}}$ cuts are not used. In addition, we allow one charged particle of any type on the tag side, and only loose pion identification is required on the signal side. As a result, the $\tau^- \to 2\omega \pi^- \nu_{\tau}$ efficiency for this selection is $(1.53 \pm 0.13)\%$. The uncertainty is a combination of systematic and statistical uncertainties, as described above for the inclusive $\tau^- \to 3\pi^- 2\pi^+ 2\pi^0 \nu_{\tau}$ analysis.

The background is estimated from MC simulation (see Fig. 4). As in the inclusive analysis, while there is a discrepancy between the data and MC $q\bar{q}$ yields, the shape of the $q\bar{q}$ pseudomass spectrum agrees with the data. As a result of the study we expect negligible $q\bar{q}$ contribution in the signal region. The uncertainty on the $q\bar{q}$ background estimate is calculated using the same technique described for the inclusive $\tau^- \to 3\pi^- 2\pi^+ 2\pi^0 \nu_{\tau}$ analysis. The total expected $q\bar{q}$ background is $0.0^{+0.1}_{-0.0}$ events. An additional contribution comes from the $\tau^- \to \omega 2\pi^- \pi^- \nu_{\tau}$ mode. Out of $530 \text{ fb}^{-1}$ of MC simulated $\tau^- \to \omega 2\pi^- \pi^- \nu_{\tau}$ events, only 1 event is found in the signal region. Thus, we expect $0.4^{+1.0}_{-0.4}$ events in $232 \text{ fb}^{-1}$ of data.

We find 1 event passing the selection criteria in $232 \text{ fb}^{-1}$ of data, which is consistent with the expected background. We calculate the upper limit of the $\tau^- \to 2\omega \pi^- \nu_{\tau}$ decay branching ratio using the limit calculator [16], which yields $\lambda_{N_{\text{signal}}} = 3.4$ events at the 90% CL. The upper limit for the decay is $\mathcal{B}(\tau^- \to 3\pi^- 2\pi^+ 2\pi^0 \nu_{\tau}) < 5.4 \times 10^{-7}$, is significantly lower than for the inclusive decay $\tau^- \to 3\pi^- 2\pi^+ 2\pi^0 \nu_{\tau}$. Table II summarizes the results of the $\tau^- \to 2\omega \pi^- \nu_{\tau}$ search.

In conclusion, we present results of a search for the $\tau^- \to 3\pi^- 2\pi^+ 2\pi^0 \nu_{\tau}$ and $\tau^- \to 2\omega \pi^- \nu_{\tau}$ decay modes using $232 \text{ fb}^{-1}$ of data collected by the BABAR detector. No evidence for these decays is found. We calculate $\mathcal{B}(\tau^- \to 3\pi^- 2\pi^+ 2\pi^0 \nu_{\tau}) < 3.4 \times 10^{-6}$ at the 90% CL, improving the existing experimental limit for this mode.
by more than a factor of 30. The upper limit for the decay, \( \mathcal{B}(\tau^- \rightarrow 2\omega \pi^- \nu_\tau) < 5.4 \times 10^{-7} \), is reported here for the first time.

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**TABLE II.** Signal efficiency, expected background, observed data events, and the upper limit of the \( \tau^- \rightarrow 2\omega \pi^- \nu_\tau \) decay at the 90% CL.

<table>
<thead>
<tr>
<th>( N_{\tau\tau} )</th>
<th>((206.5 \pm 4.7) \times 10^6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau^- \rightarrow 2\omega \pi^- \nu_\tau ) efficiency</td>
<td>((1.53 \pm 0.13)%)</td>
</tr>
<tr>
<td>Expected ( \tau^+\tau^- ) background</td>
<td>(0.4^{+1.0}_{-0.4}) events</td>
</tr>
<tr>
<td>Expected ( q\bar{q} ) background</td>
<td>(0.0^{+0.1}_{0.0}) events</td>
</tr>
<tr>
<td>Expected total background</td>
<td>(0.4^{+1.0}_{-0.4}) events</td>
</tr>
<tr>
<td>Observed events</td>
<td>1</td>
</tr>
<tr>
<td>( \mathcal{B}(\tau^- \rightarrow 2\omega \pi^- \nu_\tau) )</td>
<td>(&lt; 5.4 \times 10^{-7})</td>
</tr>
</tbody>
</table>

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[2] Throughout this paper, whenever a mode is given its charge conjugate is also implied.