We report the results of a search for $T$, $CP$, and $CPT$ violation in $B^0$-$\bar{B}^0$ mixing using an inclusive dilepton sample collected by the BABAR experiment at the PEP-II $B$ factory. Using a sample of $2.32 \times 10^6$ $PRL$ 96, 251802 (2006) PHYSICAL REVIEW LETTERS week ending 30 JUNE 2006 251802-3
$B\bar{B}$ pairs, we measure the $T$ and $CP$ violation parameter $|q/p| - 1 = (-0.8 \pm 2.7(\text{stat}) \pm 1.9(\text{syst})) \times 10^{-3}$, and the $CPT$ and $CP$ parameters $\text{Im} z = (-13.9 \pm 7.3(\text{stat}) \pm 3.2(\text{syst})) \times 10^{-3}$ and $\Delta \Gamma \times \text{Re} z = (-7.1 \pm 3.9(\text{stat}) \pm 2.0(\text{syst})) \times 10^{-3}$ ps$^{-1}$. The statistical correlation between the measurements of $\text{Im} z$ and $\Delta \Gamma \times \text{Re} z$ is 76%.

Since the first observation of $CP$ violation in 1964 [1], the neutral kaon system has provided many results probing the discrete symmetries $CPT$ and $T$ in $K^0-\bar{K}^0$ mixing [2]. Similarly, the BABAR experiment can investigate $T$, $CP$, and $CPT$ violation in $B^0-\bar{B}^0$ mixing.

The physical states (solutions of the complex effective Hamiltonian for the $B^0-\bar{B}^0$ system) [3] can be written as

\[ |B_L(\lambda)\rangle = p \sqrt{1 - z} |B^0(\lambda)\rangle + q \sqrt{1 + z} |\bar{B}^0(\lambda)\rangle, \]

\[ |B_H(\lambda)\rangle = p \sqrt{1 + z} |B^0(\lambda)\rangle - q \sqrt{1 - z} |\bar{B}^0(\lambda)\rangle, \]

where $H$ and $L$ stand for heavy and light. Under $CPT$ symmetry, the complex parameter $z$ vanishes. Similarly, $T$ invariance implies $|q/p| = 1$. Finally, $CP$ invariance requires both $|q/p| = 1$ and $z = 0$.

Inclusive dilepton events, where both $B$ mesons decay semileptonically ($b \to Xl\nu$, with $l = e$ or $\mu$), represent 4% of all $Y(4S) \to B\bar{B}$ decays and provide a very large sample with which to study $T$, $CPT$, and $CP$ violation in mixing. In the direct semileptonic neutral $B$ decay, the flavor $B^0(\bar{B}^0)$ is tagged by the charge of the lepton $l^+$ ($l^-$).

At the $Y(4S)$ resonance, neutral $B$ mesons are produced in a coherent $P$-wave state. The $B$ mesons remain in orthogonal flavor states until one decays, after which the flavor of the other $B$ meson evolves with time. Neglecting second order terms in $z$, the decay rates for the three configurations ($l^+l^+$, $l^-l^-$, and $l^+l^-$) are given by

\begin{align*}
N^{++} &\approx \frac{e^{-\Gamma_{l^+l^+}}}{2} \left| \frac{p}{q} \right|^2 \left[ \cosh(\frac{\Delta m \Delta t}{2}) - \cos(\Delta m \Delta t) \right], \\
N^{--} &\approx \frac{e^{-\Gamma_{l^-l^-}}}{2} \left| \frac{p}{q} \right|^2 \left[ \cosh(\frac{\Delta m \Delta t}{2}) - \cos(\Delta m \Delta t) \right], \\
N^{+-} &\approx \frac{e^{-\Gamma_{l^+l^-}}}{2} \left[ \cosh(\frac{\Delta \Gamma \Delta t}{2}) - 2 \text{Re} z \sinh(\frac{\Delta \Gamma \Delta t}{2}) + \cos(\Delta m \Delta t) + 2 \text{Im} z \sin(\Delta m \Delta t) \right],
\end{align*}

where $\Delta t$ is the difference between the neutral $B$ decay times, $\Delta m$ is the $B^0-\bar{B}^0$ oscillation frequency, $\Gamma$ is the average neutral $B$ decay rate and $\Delta \Gamma$ is the decay rate difference between the two physical states. The sign of $\Delta t$ has a physical meaning only for opposite-sign dileptons and is given by $\Delta t = t^+ - t^-$ where $t^+$ ($t^-$) corresponds to $l^+$ ($l^-$), respectively.

The same-sign dilepton asymmetry $A_{T/CP}$, between the two oscillation probabilities $P(\bar{B}^0 \to B^0)$ and $P(B^0 \to \bar{B}^0)$ probes both $T$ and $CP$ symmetries and can be expressed in terms of $|q/p|:

\[ A_{T/CP} = \frac{P(\bar{B}^0 \to B^0) - P(B^0 \to \bar{B}^0)}{P(\bar{B}^0 \to B^0) + P(B^0 \to \bar{B}^0)} \]

\[ = \frac{N^{++} - N^{--}}{N^{++} + N^{--}} \]

\[ = 1 - \frac{|q/p|^4}{1 + |q/p|^4}. \]

Standard model calculations [4] predict the magnitude of this asymmetry to be at or below $10^{-3}$. A large measured value would be an indication of new physics.

Similarly, the opposite-sign dilepton asymmetry, $A_{CPT/CP}$, between events with $\Delta t > 0$ and $\Delta t < 0$ compares the $B^0 \to B^0$ and $\bar{B}^0 \to \bar{B}^0$ probabilities and is sensitive to $CPT$ and $CP$ violation. This asymmetry is given by

\[ A_{CPT/CP}(\Delta t) = \frac{P(\bar{B}^0 \to B^0) - P(\bar{B}^0 \to B^0)}{P(\bar{B}^0 \to B^0) + P(\bar{B}^0 \to B^0)} \]

\[ = \frac{N^{++}(\Delta t > 0) - N^{++}(\Delta t < 0)}{N^{++}(\Delta t > 0) + N^{++}(\Delta t < 0)} \]

\[ \approx 2 \frac{\text{Im} z \sin(\Delta m \Delta t) - \text{Re} z \sinh(\frac{\Delta \Gamma \Delta t}{2})}{\cosh(\frac{\Delta \Gamma \Delta t}{2}) + \cos(\Delta m \Delta t)}. \]

As $|\Delta \Gamma|/\Gamma \ll 1$ [3], we have $\text{Re} z \sinh(\Delta m \Delta t/2) \approx \Delta \Gamma \times \text{Re} z \times (\Delta t/2)$ and this asymmetry is not sensitive to the $CPT$-violating term $\text{Re} z$ alone, but to the product $\Delta \Gamma \times \text{Re} z$.

In this Letter, we present measurements of $|q/p|$, $\text{Im} z$ and $\Delta \Gamma \times \text{Re} z$ with a simultaneous likelihood fit to the observed $\Delta t$ distributions of same-sign and opposite-sign dilepton events. In the $\cosh(\Delta \Gamma \Delta t/2)$ term, we use $|\Delta \Gamma| = (5 \pm 3) \times 10^{-3}$ ps$^{-1}$, the value reported in Ref. [3].

This study is performed with data collected by the BABAR detector [5] at the PEP-II asymmetric-energy $B$ factory between October 1999 and July 2004. The integrated luminosity of this sample is 211 fb$^{-1}$ recorded at the $Y(4S)$ resonance (“on resonance”) (232 $\times 10^6$ $B\bar{B}$ pairs) and about 16 fb$^{-1}$ recorded 40 MeV below the $Y(4S)$ resonance (“off resonance”).

The event selection is similar to that described in Ref. [6]. Non-$B\bar{B}$ events, mainly $e^+e^- \to q\bar{q}$, ($q = u,d,s,c$) continuum events, are suppressed by applying requirements on the shape and the topology of the event.

Lepton candidate tracks must have at least 12 hits in the drift chamber, at least one $z$-coordinate hit in the silicon vertex tracker (SVT), and a momentum in the $Y(4S)$ center-of-mass system between 0.8 and 2.3 GeV/c. Electrons are selected by requirements on the ratio of the energy deposited in the electromagnetic calorimeter to the
momentum measured in the drift chamber. Muons are identified through the energy released in the calorimeter, as well as the strip multiplicity, track continuity, and penetration depth in the instrumented flux return. Lepton candidates are rejected if their signal in the Cherenkov detector is consistent with that of a kaon or a proton. The electron and muon selection efficiencies are about 85% and 55%, with pion misidentification probabilities around 0.2% and 3%, respectively.

Electrons from photon conversions are identified and rejected with a negligible loss of efficiency for signal events. Leptons from $J/\psi$ and $B(2S)$ decays are identified by pairing them with other oppositely charged candidates of the same lepton species, selected with looser criteria. Events with at least two leptons are retained and the two highest momentum leptons in the $Y(4S)$ rest frame are used in the following.

The separation between direct leptons ($b \rightarrow l$) and background from the $b \rightarrow c \rightarrow l$ decay chain (cascade leptons) is achieved with a neural network that combines five discriminating variables: the momenta and opening angle of the two lepton candidates, and the total visible energy and missing momentum of the event, all computed in the $Y(4S)$ rest frame. Of the original sample of $232 \times 10^6$ $B\bar{B}$ pairs, $1.4 \times 10^6$ pass this dilepton selection.

Since the asymmetry $A_{T/CP}$ is expected to be small, we have determined the possible charge asymmetries induced by charge-dependent differences in the reconstruction and identification of electrons and muons. The charge asymmetries are defined by $a = (e^+ - e^-)/(e^+ + e^-)$ where $e^+$ ($e^-$) is the efficiency for positive and negative particles. As the lepton efficiencies and purities depend mainly on their momenta, we consider separately the asymmetry for the higher and lower momentum lepton, respectively, $a_{l_h}$ and $a_{l_l}$.

The charge asymmetry of track reconstruction is measured in the data by comparing tracks reconstructed using only the SVT with those passing the dilepton track selection, obtaining $a_{ch} = (0.8 \pm 0.2) \times 10^{-3}$.

The lepton identification efficiencies are measured as a function of total momentum and polar and azimuthal angles, with a control sample of radiative Bhabha events for electrons, and with a $e^+ e^- \rightarrow \mu^+ \mu^- \gamma$ control sample for muons. The misidentification probabilities are determined with control samples of kaons produced in $D^{*+} \rightarrow \pi^+ D^0 \rightarrow \pi^+ K^- \pi^+$ (and charge conjugate) decays, pions produced in $K_S \rightarrow \pi^+ \pi^- \pi^0$ decays, three-prong $\tau$ decays, and protons produced in $\Lambda$ decays.

The control samples show that the muon track reconstruction efficiency has a charge asymmetry reaching $-5 \times 10^{-3}$ and that positive kaons are $20\% - 30\%$ more likely than negative kaons to be misidentified as muons. As a consequence, in the likelihood fit (described below), we float the charge asymmetries $a_{l_h}^{\mu}$ and $a_{l_l}^{\mu}$ for direct and cascade muons.

For electrons, the charge asymmetry averaged over the signal phase space is $a_e = (0.4 \pm 0.2) \times 10^{-3}$ and we find that antiprotons with momentum $\sim 1$ GeV/c are significantly more likely than protons to be misidentified, due to annihilation with nucleons in the calorimeter material. Based on the charge asymmetry in tracking and in identification, we fix the charge asymmetry for the direct electrons with the higher momentum to $a_{l_h}^{\bar{e}} = 1.2 \times 10^{-3}$. For the lower momentum direct electrons and the cascade electrons, for which antiproton contamination is more important, we correct the initial charge asymmetry by the fraction of antiprotons estimated with $B\bar{B}$ Monte Carlo samples and the proton control sample. This gives the following charge asymmetries: $a_{l_h}^{\bar{e}} = 0.8 \times 10^{-3}$, $a_{l_l}^{\bar{e}} = 0.5 \times 10^{-3}$, and $a_{l_l}^{\text{casc}} = 0.2 \times 10^{-3}$.

In the inclusive approach used here, the $z$ coordinate of the $B$ decay point is approximated by the $z$ position of the point of closest approach between the lepton candidate and an estimate of the $Y(4S)$ decay point in the transverse plane. The $Y(4S)$ decay point is obtained by fitting the two lepton tracks to a common vertex, constrained to be consistent with the beam-spot position in the transverse plane. The proper time difference $\Delta t$ between the two $B$ meson decays is taken as $\Delta t = \Delta z/\sqrt{\gamma}$, where $\Delta z$ is the difference between the $z$ coordinates of the leptons, with the same-sign convention as for $\Delta t$, and $\beta\gamma = 0.55$ is the nominal Lorentz boost. For same-sign dileptons, the sign of $\Delta t$ is chosen randomly.

We model the contributions to our sample from $B\bar{B}$ decays using five categories of events, each represented by a probability density function (PDF) in $\Delta t$, $P_{\Delta t}^{i}$. Their shapes are determined using the $B^0\bar{B}^0$ ($n$) and $B^+\bar{B}^-$ ($c$) Monte Carlo simulation separately, with the approach described in Ref. [7].

The five categories are the following. First, the pure signal events with two direct leptons ($\text{sig}$), which are 81% of the $B\bar{B}$ events, give information on the $T$, $CPT$, and $CP$ parameters. Then, we consider two categories of cascade decays: those in which the direct lepton and the cascade lepton come from different $B$ decays ($\text{obc}$), and those in which the direct lepton and the cascade lepton stem from the same $B$ decay ($\text{sbc}$). According to $B\bar{B}$ Monte Carlo simulation, their contributions are around 9% and 4%, respectively. In addition, 3% of the dilepton events originate from the decay chain $b \rightarrow \tau^- \rightarrow l^-$ ($1d1\tau$), which tags the $B$ flavor correctly. Finally, the remaining events ($\text{oher}$) consist mainly of one direct lepton and one lepton from the decay of a charmonium resonance from the other $B$ decay.

The $\text{sig}$ event PDF, $P_{\Delta t}^{\text{sig}}$, are obtained by the convolution of an oscillatory term containing the $T$, $CPT$, and $CP$ parameters [Eq. (1)] for neutral $B$ decays (or an exponential function for charged $B$ decays) with a resolution function which is the sum of three Gaussians. The widths of the core and tail Gaussians and the fractions of the core and outlier Gaussians are free parameters in the fit. The width of the outlier Gaussian is fixed to 8 ps. The means of the Gaussians are fixed to zero [8].
The obc event PDF, \( \mathcal{P}_{\text{obc}} \), are modeled by the convolution of \( \Delta t \)-dependent terms of a form similar to those of the signal with a resolution function which takes into account the effect of the charmed meson lifetimes. Since both short-lived \( D^0 \) and \( D_s^+ \), and long-lived \( D^{+} \) mesons are involved in cascade decays, the resolution function for the long-lived and short-lived components is a double-sided exponential convolved with the sum of three Gaussians. To allow for possible outliers not present in the Monte Carlo simulation, the fraction of the third Gaussian is free in the fit. The parameterization of the sbc event PDF, \( \mathcal{P}_{\text{sbc}} \), account for the lifetimes of charmed mesons in a similar way.

The PDF for \( 1d1\tau \) events, \( \mathcal{P}_{1d1\tau} \), are similar to that of the sig events. The resolution function used takes into account the \( \tau \) lifetime and is chosen to be two double-sided exponentials convolved with two Gaussians. Finally, the PDF for the remaining events, \( \mathcal{P}_{\text{other}} \), are the convolution of an exponential function with an effective lifetime and two Gaussians.

The fractions \( f_{\text{obc}} \), \( f_{\text{sbc}} \), and \( f_{\text{other}} \) of sbc, \( 1d1\tau \) and other events, are determined directly from the \( B^0 \bar{B}^0 \) and \( B^+ \bar{B}^- \) Monte Carlo simulation. The fraction \( f_{\text{obc}} \) of obc events are fitted to the data, with the ratio \( f_{\text{obc}} / f_{\text{obc}} \) constrained to the estimate obtained with Monte Carlo samples. The fraction \( f_{+} \) of \( B^+ \bar{B}^- \) events is determined from the data themselves.

The last component of the dilepton sample originates from non-\( B\bar{B} \) events, and has been estimated using off-resonance data to be \( f_{\text{cont}} = (3.1 \pm 0.1)\% \) of the data set. This PDF is modeled using off-resonance events with looser cuts and on-resonance events that fail the continuum-rejection cuts. The charge asymmetries \( a_{\text{cont}}^{\text{dir}} \) obtained with the two samples are consistent with zero at the 1\% level and thus are fixed to zero in the likelihood.

The \( T/CP \) and \( CPT/CP \) violation parameters are extracted from a binned maximum likelihood fit to the events that pass the dilepton selection. The likelihood \( \mathcal{L} \) combines the detector-related charge asymmetries and the time-dependent PDFs described previously. As the charge asymmetries are significantly different for electrons and muons, we split the sample into four lepton combinations: \( e_e, e_\mu, \mu_e, \mu_\mu \), in which the first lepton has the higher momentum.

The likelihood is given by

\[
\mathcal{L}(\Delta t) = \left( 1 + q_1 a_{1}^{\text{cont}} \right) \left( 1 + q_2 a_{2}^{\text{cont}} \right) f_{\text{cont}} \mathcal{P}_{\text{cont}} + (1 - f_{\text{cont}}) \left[ f_{+} \mathcal{P}_{\text{bcb}} + (1 - f_{+}) \mathcal{P}_{\text{bcb}} \right]
\]

where \( q_1, q_2, l_1, \) and \( l_2 \) are the charges and the flavors (\( e, \mu \)) of the two leptons.

The likelihood fit gives \( |q/p| - 1 = (0.8 \pm 2.7) \times 10^{-3}, \) \( \text{Imz} = (-13.9 \pm 7.3) \times 10^{-3}, \) and \( \Delta \Gamma \times \text{Rez} = (-7.1 \pm 3.9) \times 10^{-3} \text{ ps}^{-1}. \) The correlation between the measurements of \( \text{Imz} \) and \( \Delta \Gamma \times \text{Rez} \) is 76\%. If we fix \( \Delta \Gamma = 0, \) we obtain \( \text{Imz} = (-3.7 \pm 4.6) \times 10^{-3}. \) The fitted fractions of \( B^+ \bar{B}^- \) and \( \text{obc} \) events are \( f_{+} = (59.1 \pm 0.3)\% \) and \( f_{\text{obc}} = (10.7 \pm 0.1)\%. \) Figure 1 shows the \( A_{T/CP} \) asymmetry between \( (l^+, l^+) \) and \( (l^-, l^-) \) dileptons defined in Eq. (2) and the \( A_{CPT/CP} \) asymmetry between \( (l^+, l^-) \) dileptons with \( \Delta t > 0 \) and \( \Delta t < 0 \) defined in Eq. (3).

There are several sources of systematic uncertainty in these measurements. To determine their magnitude, we vary each source of systematic effect by its known or estimated uncertainty, and take the resulting deviations in the measured parameters as its error.

For \( |q/p|, \) the most important systematic uncertainties are due to the correction of electron charge asymmetries. A \( 1.4 \times 10^{-3} \) deviation of \( |q/p| \) is observed by shifting simultaneously the electron charge asymmetries by \( 1.0 \times 10^{-3} \) which corresponds to the uncertainty estimated with Monte Carlo and control samples. The systematic uncertainty related to the charge asymmetry due to the tracking is estimated by randomly discarding 0.16\% of the negative tracks from our data sample. This fraction has been determined from an independent data control sample. A \( 1.0 \times 10^{-3} \) deviation of \( |q/p| \) is observed. Similarly, a possible
1% charge asymmetry for non-$B \bar{B}$ backgrounds induces a systematic uncertainty of $0.6 \times 10^{-3}$.

The widths of the first and second Gaussian of the resolution function for the obs and sbc categories as well as the pseudolifetime for the $l_d l \tau$ and other categories are varied separately by 10%. This variation is motivated by the comparison of the fitted parameters of the signal resolution function obtained on $B \bar{B}$ Monte Carlo samples and on data. The fractions of the short-lived and long-lived charmed meson components for obs and sbc are varied by 10%.

We have also varied the parameters $\Delta m, \tau_{B^0}$, and $\tau_{B^+}$ independently within their known uncertainties [9], and $\Delta \Gamma$ from $10^{-5}$ to 0.1 ps$^{-1}$. Finally, one of the dominant systematic uncertainties on $\Delta \Gamma \times \text{Re} \zeta$ is the imperfect knowledge of the absolute $z$ scale of the detector and the residual uncertainties in the SVT local alignment, for which we estimate an error of $1.2 \times 10^{-3}$ ps$^{-1}$.

For each parameter, the total systematic uncertainty is the sum in quadrature of the estimated systematic uncertainties from each source, as summarized in Table I. When we assume $\Delta \Gamma = 0$, the systematic uncertainty for $\text{Im} \zeta$ is $2.9 \times 10^{-3}$.

If we compare our results to $\Delta \Gamma \times \text{Re} \zeta = 0.0$ and $\text{Im} \zeta = 0.0$ (no CPT violation case), the $\chi^2$ is 3.25 for 2 degrees of freedom, which is consistent with CPT invariance at 19.7% confidence level. Finally, assuming $\Delta \Gamma = 0$, we obtain $\text{Im} \zeta = (-3.7 \pm 4.6(\text{stat}) \pm 2.9(\text{syst.})) \times 10^{-3}$.

In summary with the 1999–2004 data ($232 \times 10^6 B \bar{B}$ pairs), we have performed a simultaneous likelihood fit of same-sign and opposite-sign dileptons. We measure the independent parameters governing $CP$ and $T$ violation, and the $CP$ and $CPT$ violation parameters. The results are:

$$|q/p| - 1 = (-0.8 \pm 2.7(\text{stat}) \pm 1.9(\text{syst.})) \times 10^{-3},$$
$$\text{Im} \zeta = (-13.9 \pm 7.3(\text{stat}) \pm 3.2(\text{syst.})) \times 10^{-3},$$
$$\Delta \Gamma \times \text{Re} \zeta = (-7.1 \pm 3.9(\text{stat}) \pm 2.0(\text{syst.})) \times 10^{-3} \text{ ps}^{-1}.$$

These measurements are a clear improvement over previous results [3,10]. The new measurement of $|q/p|$ is consistent with the standard model predictions [4].

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### Table I. Summary of systematic uncertainties for $|q/p|$, $\text{Im} \zeta$, and $\Delta \Gamma \times \text{Re} \zeta$ measurements.

| Systematic effects                          | $\sigma(|q/p|) \times 10^{-3}$ | $\sigma(\text{Im} \zeta) \times 10^{-3}$ | $\sigma(\Delta \Gamma \times \text{Re} \zeta) \times 10^{-3}$ ps$^{-1}$ |
|--------------------------------------------|-------------------------------|----------------------------------------|------------------------------------------------------------------|
| Charge asymmetry of non-$B \bar{B}$ background | 0.6                           | 0.0                                    | 0.0                                                              |
| Charge asymmetry in tracking               | 1.0                           | 0.0                                    | 0.0                                                              |
| Charge asymmetry of electrons              | 1.4                           | 0.0                                    | 0.0                                                              |
| PDF modeling                               | 0.3                           | 2.5                                    | 1.2                                                              |
| Fraction of background components          | 0.2                           | 0.4                                    | 0.1                                                              |
| $\Delta m, \tau_{B^0}, \tau_{B^+}$ and $\Delta \Gamma$ | 0.2                           | 1.9                                    | 1.1                                                              |
| SVT alignment                              | 0.5                           | 0.6                                    | 1.2                                                              |
| Total                                      | 1.9                           | 3.2                                    | 2.0                                                              |

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