Precise branching ratio measurements of the decays $D^0 \to \pi^- \pi^+ \pi^0$ and $D^0 \to K^- K^+ \pi^0$ relative to the $D^0 \to K^- \pi^+ \pi^0$ decay

PRECISE BRANCHING RATIO MEASUREMENTS OF THE... PHYSICAL REVIEW D 74, 091102(R) (2006)

‡Also with Università della Basilicata, Potenza, Italy
†Also with Università di Perugia, Dipartimento di Fisica, Perugia, Italy
*Also at Laboratoire de Physique Corpusculaire, Clermont-Ferrand, France

Ecole Polytechnique, Laboratoire Leprince-Ringuet, F-91128 Palaiseau, France
University of Edinburgh, Edinburgh EH9 3JZ, United Kingdom
Università di Ferrara, Dipartimento di Fisica and INFN, I-44100 Ferrara, Italy
Laboratori Nazionali di Frascati dell’INFN, I-00044 Frascati, Italy
Università di Genova, Dipartimento di Fisica and INFN, I-16146 Genova, Italy
Harvard University, Cambridge, Massachusetts 02138, USA

Universität Heidelberg, Physikalisches Institut, Philosophenweg 12, D-69120 Heidelberg, Germany
Imperial College London, London, SW7 2AZ, United Kingdom
University of Iowa, Iowa City, Iowa 52242, USA
Iowa State University, Ames, Iowa 50011-3160, USA
Johns Hopkins University, Baltimore, Maryland 21218, USA

Universität Karlsruhe, Institut für Experimentelle Kernphysik, D-76021 Karlsruhe, Germany
Laboratoire de l’Accélérateur Linéaire, IN2P3-CNRS et Université Paris-Sud 11, Centre Scientifique d’Orsay, B.P. 34, F-91898 ORSAY Cedex, France

Lawrence Livermore National Laboratory, Livermore, California 94550, USA
University of Liverpool, Liverpool L69 7ZE, United Kingdom
Queen Mary, University of London, E1 4NS, United Kingdom

University of London, Royal Holloway and Bedford New College, Egham, Surrey TW20 0EX, United Kingdom
University of Louisville, Louisville, Kentucky 40292, USA
University of Manchester, Manchester M13 9PL, United Kingdom
University of Maryland, College Park, Maryland 20742, USA
University of Massachusetts, Amherst, Massachusetts 01003, USA

Massachusetts Institute of Technology, Laboratory for Nuclear Science, Cambridge, Massachusetts 02139, USA
McGill University, Montréal, Québec, Canada H3A 2T8
Univ. di Milano, Dipartimento di Fisica and INFN, I-20133 Milano, Italy
University of Mississippi, University, Mississippi 38677, USA
Université de Montréal, Physique des Particules, Montréal, Québec, Canada H3C 3J7
Mount Holyoke College, South Hadley, Massachusetts 01075, USA

Universitá di Napoli Federico II, Dipartimento di Scienze Fisiche e INFN, I-80126 Napoli, Italy
NIKHEF, National Institute for Nuclear Physics and High Energy Physics, NL-1009 DB Amsterdam, The Netherlands
University of Notre Dame, Notre Dame, Indiana 46556, USA
Ohio State University, Columbus, Ohio 43210, USA
University of Oregon, Eugene, Oregon 97403, USA

Università di Padova, Dipartimento di Fisica e INFN, I-35131 Padova, Italy
Universités Paris VI et VII, Laboratoire de Physique Nucléaire et de Hautes Energies, F-75252 Paris, France
University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA

Università di Perugia, Dipartimento di Fisica and INFN, I-06100 Perugia, Italy
Università di Pisa, Dipartimento di Fisica, Scuola Normale Superiore and INFN, I-56127 Pisa, Italy
Prairie View A&M University, Prairie View, Texas 77446, USA
Princeton University, Princeton, New Jersey 08544, USA

Università di Roma La Sapienza, Dipartimento di Fisica and INFN, I-00185 Roma, Italy
Universität Rostock, D-18051 Rostock, Germany
Rutherford Appleton Laboratory, Chilton, Didcot, Oxon, OX11 0QX, United Kingdom

DSM/Dapnia, CEA/Saclay, F-91191 Gif-sur-Yvette, France
University of South Carolina, Columbia, South Carolina 29208, USA
Stanford Linear Accelerator Center, Stanford, California 94309, USA
Stanford University, Stanford, California 94305-4060, USA
State University of New York, Albany, New York 12222, USA
University of Tennessee, Knoxville, Tennessee 37996, USA
University of Texas at Austin, Austin, Texas 78712, USA
University of Texas at Dallas, Richardson, Texas 75083, USA

Università di Torino, Dipartimento di Fisica Sperimentale and INFN, I-10125 Torino, Italy
Università di Trieste, Dipartimento di Fisica and INFN, I-34127 Trieste, Italy
IFIC, Universitat de Valencia-CSIC, E-46071 Valencia, Spain
University of Victoria, Victoria, British Columbia, Canada V8W 3P6

* Also at Laboratoire de Physique Corpusculaire, Clermont-Ferrand, France
† Also with Università di Perugia, Dipartimento di Fisica, Perugia, Italy
‡ Also with Università della Basilicata, Potenza, Italy

091102-3
Using 232 fb\(^{-1}\) of \(e^+e^-\) collision data recorded by the BABAR experiment, we measure the rates of three-body Cabibbo-suppressed decays of the \(D^0\) meson relative to the Cabibbo-favored decay, \(D^0 \rightarrow K^-\pi^+\pi^0\). We find: 
\[
\frac{b(D^0 \rightarrow K^-\pi^+\pi^0)}{b(D^0 \rightarrow K^-\pi^+\pi^0)} = (10.59 \pm 0.06 \pm 0.13) \times 10^{-2} \quad \text{and} \quad \frac{b(D^0 \rightarrow K^-\pi^+\pi^0)}{b(D^0 \rightarrow K^-\pi^+\pi^0)} = (2.37 \pm 0.03 \pm 0.04) \times 10^{-2},
\]
where the errors are statistical and systematic, respectively. These measurements are significantly more precise than the current world average measurements.

To reduce combinatorial backgrounds, we reconstruct \(D^0\) candidates in decays \(D^{\ast+} \rightarrow D^0\pi^+_s\) (\(\pi^+_s\) is a soft, low momentum charged pion and the \(CP\) conjugate decay is included) with \(D^0 \rightarrow K^-\pi^+\pi^0\), \(\pi^-\pi^+\pi^0\), and \(K^-\pi^+\pi^0\), by selecting events with at least three charged tracks and a neutral pion. Photon candidates are reconstructed from calorimeter energy deposits above 100 MeV, which are not matched to charged tracks. Neutral pions are reconstructed from pairs of photons with an invariant mass in the range 115–160 MeV/c\(^2\) and total energy in the laboratory system above 350 MeV. Charged \(K\) and \(\pi\) and \(\pi^+_s\) candidate tracks are required to be within the fiducial volumes of the tracking and PID systems; they must have at least 20 hits in the DCH and transverse momenta greater than 0.1 GeV/c. Also, they must pass PID selection criteria.

To form a \(D^0\) candidate, two oppositely charged tracks and a \(\pi^0\) are fit to a common vertex, constraining the \(\gamma\gamma\) invariant mass to the nominal \(\pi^0\) mass \cite{1} since this improves the \(\pi^0\) momentum and energy resolution. The invariant mass of the \(D^0\) candidate after the vertex fit is required to lie in the range 1.7–2.0 GeV/c\(^2\). To reduce high multiplicity events and combinatorial backgrounds, the momentum of the \(D^0\) candidate in the event’s center-of-mass frame (\(p^*\)) is required to be greater than 2.77 GeV/c (as a consequence of which the \(D^0\) candidates from \(B\) decay are removed). The selected candidates after the above requirements are combined with the \(\pi^+_s\) track to form a \(D^{\ast+}\) candidate. The \(D^0\) and the \(\pi^+_s\) are constrained to originate from the collision point; the resolution in \(\Delta m\), defined as the difference in invariant masses of the \(D^{\ast+}\) and \(D^0\) candidates, is approximately 0.3 MeV/c\(^2\) for all three modes. Only those candidates are retained for which the vertex fit to the whole decay chain, using kinematic and geometric constraints, has a \(\chi^2\) probability greater than 0.01 and \(\Delta m\) is in the range 144.9–146.1 MeV/c\(^2\). At this stage, approximately 3% of the events have multiple \(D^{\ast+} \rightarrow D^0\pi^+_s\) candidates satisfying our selection criteria, due to \(D^0\) misreconstruction or a correctly reconstructed \(D^0\) combining with a fake \(\pi^+_s\). When there is more than one candidate in an event, we select only the candidate with the lowest vertex fit \(\chi^2\) for the whole decay chain. Our selection procedures result in \(K^-\pi^+\pi^0\), \(\pi^-\pi^+\pi^0\), and
$K^-K^+\pi^0$ samples with purities 99%, 95% and 96%, respectively.

The number of $D^0$ signal events in each decay mode is obtained by fitting the observed $D^0$ candidate mass distribution to the sum of signal and background components, where the latter has combinatorial contributions and reflection contributions from real three-body $D^0$ decays where a kaon (pion) is misidentified as a pion (kaon). The signal component is described by a sum of three Gaussians whose means and widths are allowed to vary. The combinatorial background is modeled by a linear function. According to the MC simulation, a large fraction of the background consists of $e^+e^-\rightarrow c\bar{c}$ events, with small contributions from processes $e^+e^-\rightarrow b\bar{b}, u\bar{u}, d\bar{d}, s\bar{s}$. The levels of various background contributions in the $\pi^-\pi^+\pi^0$ and the $K^-K^+\pi^0$ invariant mass distributions are shown in Fig. 1. Reflected $K^-\pi^+\pi^0$ events peak in the lower (upper) sideband of $m_{\pi^-\pi^+\pi^0}$ ($m_{K^-K^+\pi^0}$). The shapes of the $K^-\pi^+\pi^0$ reflections in the $\pi^-\pi^+\pi^0$ and $K^-K^+\pi^0$ invariant mass distributions are obtained from MC. The numbers of reflected events are found by making the $K^-\pi^+\pi^0$ invariant mass distributions for the $\pi^-\pi^+\pi^0$ and $K^-K^+\pi^0$ samples and fitting them. Finally, maximum-likelihood fits are performed to extract the signal yields from the data for each of the three modes. The $D^0\rightarrow K^0_S\pi^0$ decay is a Cabibbo-favored decay and a background for the $D^0\rightarrow \pi^-\pi^+\pi^0$ mode. The level of this contamination is obtained by fitting the $K^0_S$ peak in the $m_{\pi^-\pi^+}$ distribution and the number of $K^0_S\pi^0$ events is subtracted from the $\pi^-\pi^+\pi^0$ signal yield. The fitted $D^0$ candidate mass plots for the three modes are shown in Fig. 2 and the results of the fits are reported in Table I.

The event reconstruction efficiency is obtained from MC. The reconstruction efficiency for each event is calculated as a function of its position in the $D^0$ Dalitz plot. That position is calculated using track momenta from a fit which constrains the $h^-h^+\pi^0$ invariant mass to be the nominal $D^0$ mass, where $h$ is either a kaon or a pion. To correct for the differences in PID efficiency in data and MC, the ratio of these is determined for each track in bins of momentum and polar angle, and an event-by-event PID-correction factor is applied to each reconstructed event. Also, to account for differences in the $p^+$ distribution in data and MC, the reconstruction efficiency is corrected by their ratio for each event. The inverse of the calculated efficiency for each data point is taken as its weight. The average weight for each decay mode is computed by summing the weights of all events in the nominal signal regions ($\pm 3\sigma$ around the observed mean values of the $D^0$ mass distributions) and subtracting the efficiency-corrected event yields from sidebands (1.75–1.79 GeV/$c^2$ and 1.95–1.99 GeV/$c^2$), spaced almost symmetrically around the nominal $D^0$ mass) to account for background events in the signal region. For the $\pi^-\pi^+\pi^0$ mode both sidebands are used for this purpose; for the $\pi^-\pi^+\pi^0$ ($K^-K^+\pi^0$) mode only the upper (lower) sideband is used because of the $K^-\pi^+\pi^0$ reflection in the other sideband. The average weights obtained from this method are verified to be unbiased. The average reconstruction weights for $K^-\pi^+\pi^0$, $\pi^-\pi^+\pi^0$, and $K^-K^+\pi^0$ modes are $10.75\pm0.02$, $9.43\pm0.02$, and $12.61\pm0.05$ respectively, where the uncertainty is due to MC statistics.
The branching ratios are obtained from

$$\frac{B(D^0 \rightarrow \pi^- \pi^+ \pi^0)}{B(D^0 \rightarrow K^- \pi^+ \pi^0)} = \frac{N_{\pi^- \pi^+ \pi^0} \times W_{\pi^- \pi^+ \pi^0}}{N_{K^- \pi^+ \pi^0} \times W_{K^- \pi^+ \pi^0}}, \tag{1}$$

$$\frac{B(D^0 \rightarrow K^- K^+ \pi^0)}{B(D^0 \rightarrow K^- \pi^+ \pi^0)} = \frac{N_{K^- K^+ \pi^0} \times W_{K^- K^+ \pi^0}}{N_{K^- \pi^+ \pi^0} \times W_{K^- \pi^+ \pi^0}}, \tag{2}$$

where $N$ and $W$ stand for the number of signal events detected and the average weight, respectively.

After accounting for the cancellation of common uncertainties, the most important sources of systematic uncertainties in the branching ratios are reported in Table II. The finite statistics of the MC samples used to obtain reconstruction efficiencies contributes a small uncertainty. The uncertainty due to the $\Delta m$ selection is estimated by repeating the analysis with different selection windows. The systematic uncertainty due to the background subtraction procedure is obtained by repeating the analysis using $c\bar{c}$ Monte Carlo data and subtracting statistically comparable "true" background events in the signal region. The uncertainty caused by Dalitz plot binning effects in the efficiency calculation is estimated by varying the bin-size. The effect of the modeling of the background probability distribution function on the signal yield is studied by repeating the fits to the $D^0$ candidate mass distributions with exponential and polynomial combinatorial background functions. The systematic uncertainty due to differences in the $p^*$ distribution in data and MC is estimated from the uncertainty in the calibration of this effect. Charged-particle identification studies in the data lead to small corrections applied to each track in the simulation. A large control sample of data and MC is studied separately to determine the residual PID uncertainties. Uncertainty due to potential differences in charged-particle tracking efficiencies in data and MC originating from an imprecise knowledge of different kaon and pion nuclear interaction cross sections and from the approximations used in our material model simulation, is conservatively assigned.

As a consistency check, the analysis is performed separately for $D^0$ and $\bar{D}^0$ events in different ranges of the $D^0$ candidate laboratory momenta to look for systematic variations as a function of charge or momentum outside the levels accounted for in the estimation of statistical and systematic uncertainties. The analysis is repeated for different data run periods and on the MC sample treated as data. As yet another cross-check, the branching ratios are measured by directly fitting the efficiency-corrected histograms of the $D^0$ invariant mass distributions and then taking the ratio of the yields obtained from the fit. The results from all these cross-checks are consistent with the results of the main analysis.

Using Eqs. (1) and (2), we obtain the following results for the branching ratios:

$$\frac{B(D^0 \rightarrow \pi^- \pi^+ \pi^0)}{B(D^0 \rightarrow K^- \pi^+ \pi^0)} = (10.59 \pm 0.06 \pm 0.13) \times 10^{-2}, \tag{3}$$

$$\frac{B(D^0 \rightarrow K^- K^+ \pi^0)}{B(D^0 \rightarrow K^- \pi^+ \pi^0)} = (2.37 \pm 0.03 \pm 0.04) \times 10^{-2}, \tag{4}$$

where the errors are statistical and systematic, respectively. The previous most precise measurements for these branching ratios are $(8.40 \pm 3.11) \times 10^{-2}$ [2,10] and $(0.95 \pm 0.26) \times 10^{-2}$ [3], respectively. We note that the second result differs significantly from the current world average value [11]. As we consider events with any level of FSR as parts of our signals, the ratios we measure correspond to those of the so-called "bare" decay rates discussed, for example, in Ref. [12]. Using the world average value [11] for the $D^0 \rightarrow K^+ K^- \pi^0$ branching fraction, we obtain,

$$B(D^0 \rightarrow \pi^- \pi^+ \pi^0) = (1.493 \pm 0.008 \pm 0.018 \pm 0.053) \times 10^{-2}, \tag{5}$$

TABLE I. Number of observed signal events and the central value and resolution of the $D^0$ candidate mass distribution obtained from fit. The central value and the resolution are, respectively, the average mean and rms width of three Gaussians in the signal weighted by their fit-fractions. The errors are statistical only.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Number of signal events ($S$)</th>
<th>Central value of $D^0$ mass (GeV/c$^2$)</th>
<th>Resolution (MeV/c$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^-\pi^+\pi^0$</td>
<td>$505660 \pm 750$</td>
<td>$1.8646 \pm 0.0002$</td>
<td>$16.0 \pm 0.5$</td>
</tr>
<tr>
<td>$\pi^-\pi^+\pi^0$</td>
<td>$60426 \pm 343$</td>
<td>$1.8637 \pm 0.0004$</td>
<td>$17.4 \pm 0.8$</td>
</tr>
<tr>
<td>$K^-K^+\pi^0$</td>
<td>$10773 \pm 122$</td>
<td>$1.8649 \pm 0.0004$</td>
<td>$13.5 \pm 1.0$</td>
</tr>
</tbody>
</table>

TABLE II. Summary of systematic uncertainties. The total systematic uncertainty was obtained by adding the individual contributions in quadrature.

<table>
<thead>
<tr>
<th>Systematics</th>
<th>$B(D^0 \rightarrow \pi^- \pi^+ \pi^0)$</th>
<th>$B(D^0 \rightarrow K^- \pi^+ \pi^0)$</th>
<th>$B(D^0 \rightarrow K^- K^+ \pi^0)$</th>
<th>$B(D^0 \rightarrow K^- \pi^+ \pi^0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MC statistics</td>
<td>$0.27%$</td>
<td>$0.47%$</td>
<td>$0.30%$</td>
<td>$0.90%$</td>
</tr>
<tr>
<td>$\Delta m$ selection</td>
<td>$0.30%$</td>
<td>$0.90%$</td>
<td>$0.60%$</td>
<td>$0.90%$</td>
</tr>
<tr>
<td>Bg. Subtraction</td>
<td>$0.11%$</td>
<td>$0.24%$</td>
<td>$0.24%$</td>
<td>$0.13%$</td>
</tr>
<tr>
<td>Efficiency binning</td>
<td>$0.06%$</td>
<td>$0.02%$</td>
<td>$0.07%$</td>
<td>$0.84%$</td>
</tr>
<tr>
<td>Bg. PDF model</td>
<td>$0.60%$</td>
<td>$0.60%$</td>
<td>$0.07%$</td>
<td>$\cdots$</td>
</tr>
<tr>
<td>$p^*$ difference</td>
<td>$1.25%$</td>
<td>$1.73%$</td>
<td>$1.07%$</td>
<td>$\cdots$</td>
</tr>
</tbody>
</table>

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$B(D^0 \rightarrow K^- K^+ \pi^0) = (0.334 \pm 0.004 \pm 0.006 \pm 0.012) \times 10^{-2}$, \hspace{1cm} (6)

where the errors are statistical, systematic, and due to the uncertainty of $B(D^0 \rightarrow K^- \pi^+ \pi^0)$ respectively.

The decay rate for each process can be written as:

$$\Gamma = \int d\Phi |\mathcal{M}|^2,$$ \hspace{1cm} (7)

where $\Gamma$ is the decay rate to a particular three-body final state, $\mathcal{M}$ is the decay matrix element, and $\Phi$ is the phase space. Integrating over the Dalitz plot assuming a uniform phase space density, the above equation can be written as:

$$\Gamma = \langle |\mathcal{M}|^2 \rangle \times \Phi,$$ \hspace{1cm} (8)

where $\langle |\mathcal{M}|^2 \rangle$ is the average value of $|\mathcal{M}|^2$ over the Dalitz plot and the three-body phase space, $\Phi$ is proportional to the area of the Dalitz plot. For the three signal decays $\Phi$ is in the ratio $\pi^- \pi^+ \pi^0; K^- \pi^+ \pi^0; K^- K^+ \pi^0 = 5.05:3.19:1.67$. Combining the statistical and systematic errors, we find:

$$\frac{\langle |\mathcal{M}|^2 \rangle(D^0 \rightarrow \pi^- \pi^+ \pi^0)}{\langle |\mathcal{M}|^2 \rangle(D^0 \rightarrow K^- \pi^+ \pi^0)} = (6.68 \pm 0.04 \pm 0.08) \times 10^{-2}$$ \hspace{1cm} (9)

$$\frac{\langle |\mathcal{M}|^2 \rangle(D^0 \rightarrow K^- K^+ \pi^0)}{\langle |\mathcal{M}|^2 \rangle(D^0 \rightarrow \pi^- \pi^+ \pi^0)} = (4.53 \pm 0.06 \pm 0.08) \times 10^{-2}$$ \hspace{1cm} (10)

$$\frac{\langle |\mathcal{M}|^2 \rangle(D^0 \rightarrow K^- K^+ \pi^0)}{\langle |\mathcal{M}|^2 \rangle(D^0 \rightarrow \pi^- \pi^+ \pi^0)} = (6.78 \pm 0.14 \pm 0.21) \times 10^{-1}.$$ \hspace{1cm} (11)

To the extent that the differences in the matrix elements are only due to Cabibbo-suppression at the quark level, the ratios of the matrix elements squared for singly Cabibbo-suppressed decays to that for the Cabibbo-favored decay should be approximately $\sin^2 \theta_C = 0.05$ and the ratio of the matrix elements squared for the two singly Cabibbo-suppressed decays should be unity. The deviations from this naive picture are less than 35% for these three-body decays. In contrast, the corresponding ratios may be calculated for the two-body decays $D^0 \rightarrow \pi^- \pi^+$, $D^0 \rightarrow K^- \pi^+$, and $D^0 \rightarrow K^- K^+$. Using the world average values for two-body branching ratios [1], the ratios of the matrix elements squared for two-body Cabibbo-suppressed decays, corresponding to Eqs. (9)–(11), are, respectively, 0.034 ± 0.001, 0.111 ± 0.002, and 3.53 ± 0.12. Thus the naive Cabibbo-suppression model works well for three-body decays but not so well for two-body decays.

In summary, we have measured the ratios of the decay rates for the three-body singly Cabibbo-suppressed decays $D^0 \rightarrow \pi^- \pi^+ \pi^0$ and $D^0 \rightarrow K^- K^+ \pi^0$ relative to that for the Cabibbo-favored decay $D^0 \rightarrow K^- \pi^+ \pi^0$. This constitutes the most precise measurement for these channels to date. The average squared matrix elements for both of the singly Cabibbo-suppressed decays are roughly a factor of $\sin^2 \theta_C$ smaller than that for the Cabibbo-favored decay and are therefore, in contrast to the corresponding two-body modes, consistent with the naive expectations.

We are grateful for the extraordinary contributions of our PEP-II colleagues in achieving the excellent luminosity and machine conditions that have made this work possible. The success of this project also relies critically on the expertise and dedication of the computing organizations that support BABAR. The collaborating institutions wish to thank SLAC for its support and the kind hospitality extended to them. This work is supported by the US Department of Energy and National Science Foundation, the Natural Sciences and Engineering Research Council (Canada), Institute of High Energy Physics (China), the Commissariat à l’Energie Atomique and Institut National de Physique Nucléaire et de Physique des Particules (France), the Bundesministerium für Bildung und Forschung and Deutsche Forschungsgemeinschaft (Germany), the Istituto Nazionale di Fisica Nucleare (Italy), the Foundation for Fundamental Research on Matter (The Netherlands), the Research Council of Norway, the Ministry of Science and Technology of the Russian Federation, Ministerio de Educación y Ciencia (Spain), and the Particle Physics and Astronomy Research Council (United Kingdom). Individuals have received support from the Marie-Curie IEF program (European Union) and the A. P. Sloan Foundation.

[4] Reference to the charge-conjugate decays is implied throughout the text, unless stated otherwise.

[10] P. Rubin et al. (CLEO Collaboration), Phys. Rev. Lett. 96, 081802 (2006) has measured the branching ratio of $D_0 \to \pi^- \pi^0 \pi^0$ relative to $D_0 \to K^- \pi^+$ decay.

[11] C. Cawlfield et al. (CLEO Collaboration), Phys. Rev. D 74, 031108 (2006), in a recent cross-check for the branching fraction of $D^0 \to K^- K^+ \pi^0$ in course of an amplitude analysis of this decay, has found a value which is consistent with the one obtained from our measurement.