Measurement of the ratio $\mathcal{B}(B^+ \to Xe\nu)/\mathcal{B}(B^0 \to Xe\nu)$

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We report measurements of the inclusive electron momentum spectra in decays of charged and neutral $B$ mesons, and of the ratio of semileptonic branching fractions $B(B^+ \to X e\nu) / B(B^0 \to X e\nu)$. These were performed on a sample of $231 \times 10^6 B\bar{B}$ events recorded with the BABAR detector at the Y(4S) resonance. Events are selected by fully reconstructing a hadronic decay of one $B$ meson and identifying an electron among the decay products of the recoiling $B$ meson. We obtain $B(B^+ \to X e\nu) / B(B^0 \to X e\nu) = 1.074 \pm 0.041_{\text{(stat)}} \pm 0.026_{\text{(syst)}}$.

The hadronic decay widths of $B^+$ and $B^0$ mesons differ because of mechanisms that depend on the flavor of the spectator quark, such as interactions involving the spectator quark or final state particles. This leads to different lifetimes $\tau_{B^+}$ and $\tau_{B^0}$ of charged and neutral $B$ mesons. We do not expect different semileptonic decay widths, since semileptonic decays do not involve the spectator quark. This means that the ratio $R_{+0} = B(B^+ \to X e\nu) / B(B^0 \to X e\nu)$ should agree with $\tau_{B^+} / \tau_{B^0}$, which can be checked experimentally.

At the Y(4S) resonance, measurements of the inclusive semileptonic branching fractions of $B^+$ and $B^0$ mesons are less precise than for an admixture of both hadrons. The reason is mainly a limitation of statistics from the small efficiency of the event tag needed to separate $B^+ B^-$ from $B^0\bar{B}^0$ events. In this paper, we use fully reconstructed hadronic $B$ decays for this separation. Combined with the high statistics of the $B$ factories, this approach allows for a precision measurement of $R_{+0}$, as already demonstrated by the Belle collaboration, measuring $R_{+0}$ with 5% uncertainty [1]. By tagging $B^0\bar{B}^0$ events with partially reconstructed $B^0 \to D^+ \ell \nu$ decays, the CLEO collaboration achieved a 14% uncertainty on $R_{+0}$ [2]. High-momentum electron tags have been used in similar analyses for the determination of $B(B \to X e\nu)$ and the electron momentum spectrum without separation of $B^0$ and $B^+$ decays [3,4].

The measurements presented here are based on data collected by the BABAR detector [5] at the PEP-II asymmetric $e^+ e^-$ storage rings and correspond to an integrated luminosity of 209 fb$^{-1}$ ($231 \times 10^6 B\bar{B}$ events) on the Y(4S) resonance and 21.6 fb$^{-1}$ at an energy 40 MeV below the resonance (off-peak). For background and efficiency corrections that cannot be measured directly from data, we use a full simulation of the detector based on GEANT4 [6]. The equivalent luminosity of the simulated event sample amounts to about 980 fb$^{-1}$ for $Y(4S) \to B\bar{B}$ events and 300 fb$^{-1}$ for nonresonant $e^+ e^- \to q\bar{q}$ ($q = u, d, s, c$) production (“continuum”).

In events with a fully reconstructed hadronic $B$ decay ($B_{\text{tag}}$), we identify electrons among the remaining tracks. To avoid large backgrounds at lower momenta, we require $p_e > 0.6$ GeV/c, where $p_e$ is the electron momentum measured in the center-of-mass frame. Depending on the electron charge $q_e$ relative to the charge $q_b$ of the bottom quark in the $B_{\text{tag}}$ candidate, each electron is assigned to either the right-sign ($q_e = -3q_b$) or to the wrong-sign sample ($q_e = 3q_b$). In events without $B^0\bar{B}^0$-mixing and a correctly reconstructed $B_{\text{tag}}$, primary electrons from semileptonic decays of the signal $B$ are the dominant source for the right-sign sample, while electrons from $B \to D\bar{X}, D \to e^- \nu_Y$ cascades populate the wrong-sign sample. We use the criteria in Ref. [4] for track selection and electron identification, and apply the same procedures for efficiency and background corrections of the right- and wrong-sign samples. In this analysis, we additionally have to correct for misreconstructed $B_{\text{tag}}$ candidates.

Non-$B\bar{B}$ events are suppressed by requiring the ratio of the second to the zeroth Fox-Wolfram moments [7] to be less than 0.5. We reconstruct hadronic $B$ decays in very pure modes only, keeping backgrounds from misreconstructed $B_{\text{tag}}$ candidates at a low level. To cancel systematic errors related to the $B_{\text{tag}}$ reconstruction, we select similar (“twin”) modes for $B^0$ and $B^+$ decays [8]:

(I) $B^0 \to \pi(K\pi\pi)_D^+ \quad B^+ \to \pi(K\pi\pi)_D^0$

(II) $B^0 \to \pi(K\pi\pi)_D^+ \quad B^+ \to \pi(K\pi\pi)_D^0$

(III) $B^0 \to \pi(K\pi\pi)_D^+ \quad B^+ \to \pi(K\pi\pi)_D^0$

(IV) $B^0 \to \pi(K\pi\pi)_D^+ \quad B^+ \to \pi(K\pi\pi)_D^0$

(V) $B^0 \to \pi(K\pi\pi)_D^+ \quad B^+ \to \pi(K\pi\pi)_D^0$

Here $\pi$ and $K$ denote charged pions and kaons. The invariant mass of $D^0$ candidates is required to be within 15 MeV/c$^2$ of the nominal $D^0$ mass [9] for the decay $D^0 \to K\pi$ and $25$ MeV/c$^2$ for $D^0 \to K\pi\pi^0$ decays. $D^-$ candidates are accepted if the invariant mass is within 20 MeV/c$^2$ of the nominal $D^-$ mass. $D$ candidates with momenta above 2.5 GeV/c (measured in the center-of-mass frame) are rejected since they indicate non-$B\bar{B}$ events. $D^*$ candidates are built from pairs of $D^0$ candidates and charged (neutral) pions where the invariant mass difference $|M_{D^0\pi^-} - M_{D^+}|$ is within 2 MeV/c$^2$ of the nominal mass difference. In tag categories (III) and (V) we require the invariant masses $M_{\pi\pi\pi}$ and $M_{\pi\pi\pi\pi}$ to be less than 1.5 GeV/c$^2$. For further background reduction, we reject candidates where a kinematic fit with geometric constraints on the $B$ and $D$ vertices and mass constraints on the charmed mesons yields a $\chi^2$ value with a probability of less than 0.5%.
The kinematic consistency of the \(B_{\text{tag}}\) candidates is checked with two variables, the beam-energy substituted mass \(m_{\text{ES}} = (s/4 - p_T^2)^{1/2}\) and the energy difference \(\Delta E = E_B - \sqrt{s}/2\). Here \(\sqrt{s}\) refers to the total center-of-mass energy, and \(E_B\) and \(p_T\) denote the energy and momentum of the \(B_{\text{tag}}\) candidate, all quantities being measured in the center-of-mass frame. For categories (I)–(III), we require \(|\Delta E| < 50\) MeV, while the presence of an additional \(p_T^2\) in (IV) and (V) leads to asymmetric distributions in \(\Delta E\), motivating lower limits of \(\Delta E > -75\) MeV for (IV) and \(\Delta E > -100\) MeV for (V). If for a given mode more than one \(B_{\text{tag}}\) candidate satisfies these criteria, the one with the smallest \(|\Delta E|\) is selected. Figure 1 shows the \(m_{\text{ES}}\) distributions of \(B_{\text{tag}}\) candidates satisfying these selection criteria. Candidates with \(5.2 < m_{\text{ES}} < 5.25\) GeV/c\(^2\) are included in the \(B_{\text{tag}}\) sample. In \(\approx 1\%\) of all events, we find multiple \(B_{\text{tag}}\) candidates in different decay modes. Here we use all of them, correcting for the background \(B_{\text{tag}}\) candidates later.

The \(B_{\text{tag}}\) sample can be divided into 4 components: signal, combinatorial background, \(D^{*+} \to D^{0}\) cross feed and continuum background. Correctly reconstructed \(B\) decays are called signal \(B_{\text{tag}}\) candidates, while \(B_{\text{tag}}\) candidates that contain tracks from the decay of the other \(B\) contribute to the combinatorial \(B_{\text{tag}}\) background. A special case of combinatorial background, called \(D^{*+} \to D^{0}\) cross feed, contains cross feeds between twin modes of channels (II)–(V) due to misreconstruction of a \(D^{*+}\) as a \(D^{0}\) or vice versa. Because of the low energy of the combinatorial pion, the \(m_{\text{ES}}\) distribution of this background is similar to the signal and will be treated separately from the other combinatorial \(B_{\text{tag}}\) background. The fourth component consists of \(B_{\text{tag}}\) candidates arising from continuum events and is called continuum \(B_{\text{tag}}\) background. Since the ratio of signal to background \(B_{\text{tag}}\) candidates depends on the multiplicity of the event and thus on the presence of a semileptonic decay, a precise determination of the number of signal \(B_{\text{tag}}\) candidates is crucial to avoid biases in the branching fraction measurement. Monte Carlo (MC) studies using generator information indicate that once the \(B_{\text{tag}}\), right- and wrong-sign samples have been corrected for \(B_{\text{tag}}\) background, the biases on the branching fraction measurements are below the statistical sensitivity given by the size of the MC sample, i.e. less than 0.5%.

The contributions of combinatorial and continuum \(B_{\text{tag}}\) background to the \(B_{\text{tag}}\) sample are extrapolated from the \(m_{\text{ES}}\) sideband region, \(5.2 < m_{\text{ES}} < 5.25\) GeV/c\(^2\). This requires a model of the background \(m_{\text{ES}}\) distributions over the full range, \(5.2 < m_{\text{ES}} < 5.29\) GeV/c\(^2\), which is obtained by fitting a linear combination of three functions describing the shapes of \(m_{\text{ES}}\) distributions of signal, combinatorial and continuum \(B_{\text{tag}}\) candidates to the observed \(m_{\text{ES}}\) distributions.

The shape of the combinatorial \(B_{\text{tag}}\) background \(f_{\text{cb}}(m_{\text{ES}})\) is taken from the MC simulation. For the continuum background, we use the following function [10]:

\[
    f_{\text{qg}}(m) = m\sqrt{1 - m^2}e^{-\alpha(1-m^2)}
\]

where \(m = m_{\text{ES}}/m_{\text{ES}}^{\text{max}}\) and \(m_{\text{ES}}^{\text{max}}\) is the endpoint of the \(m_{\text{ES}}\) distribution.

For a given \(B\) decay mode, the signal \(m_{\text{ES}}\) distribution is commonly described by a gaussian and a power law [11]. Since the \(B_{\text{tag}}\) signal consists of many individual decay modes, a single function of that type fails to describe our \(m_{\text{ES}}\) distribution. We have found that a more general ansatz using a gaussian shape \(f_g(x) = e^{-x^2/2}\) and a function with a similar shape near \(x = 0\), but behaving like \(e^{-x}\) for \(x \to \pm \infty\), \(f(x) = e^{-x}/(1 + e^{-x})^2\), yields a good description of our signal \(m_{\text{ES}}\) shape:

\[
    f_{\text{sig}}(\Delta) = \begin{cases} 
    \frac{C_1}{\sigma_L^2} f_g(\Delta) & \text{if } \Delta < \alpha \\
    \frac{C_1}{\sigma_L^2} f_g(\Delta) + \frac{1 - e^{-\alpha}}{\sigma_2^2} f_s(\Delta) & \text{if } \Delta \geq 0
    \end{cases}
\]

with \(\Delta = m_{\text{ES}} - \bar{m}_{\text{ES}}\) and \(\bar{m}_{\text{ES}}\) being the maximum of the \(m_{\text{ES}}\) distribution. \(C_1\), \(C_2\) and \(C_3\) are functions of the parameters \(\bar{m}_{\text{ES}}, \sigma_L, \sigma_2, \alpha\) and \(n\) to ensure that \(f_{\text{sig}}\) is continuous and differentiable at \(\Delta = 0\) and \(\Delta = \alpha\). This function, similar to the one featured in [11], describes the tails caused by the asymmetric energy resolution of neutral pions by a power law of order \(-n\) and a junction \(\alpha < 0\) where it turns into a gaussian-like shape. Fixing \(\alpha\) and \(n\) to the values obtained from a fit to MC-simulated \(m_{\text{ES}}\) distributions of signal \(B_{\text{tag}}\) candidates, we fit a linear combination of \(f_{\text{qg}}, f_{\text{cb}}\) and \(f_{\text{sig}}\) to the \(m_{\text{ES}}\) distributions observed in data, leaving all other parameters and normalizations free in the fit (Fig. 1). Due to their similar \(m_{\text{ES}}\) distributions, this method cannot distinguish between signal \(B_{\text{tag}}\) candidates and \(D^{*+} \to D^{0}\) cross feed. This background contribution is estimated from the MC simulation to be.

![FIG. 1. Fits of Eq. (1) to distributions of the energy substituted mass for (a) neutral and (b) charged \(B_{\text{tag}}\) candidates. The dotted and dashed curves indicate the fitted contributions of continuum and combinatorial \(B_{\text{tag}}\) candidates. The gray histogram displays the contribution of \(D^{*+} \to D^{0}\) background.](091105-5)
0.5% (2.6%) relative to the signal for the neutral (charged) \( B_{\text{tag}} \) sample.

To validate this extraction method, we perform the same analysis on our Monte Carlo sample and find that it reproduces the original number of signal \( B_{\text{tag}} \) candidates. Uncertainties related to the MC simulation of the combinatorial \( B_{\text{tag}} \) background are evaluated by decomposing this background into the true underlying individual exclusive decay modes, and varying their contributions by the uncertainties of their branching fractions if they are reported in [9], or \( \pm 100\% \) otherwise. This leads to an uncertainty of 1.3% on the number of \( B^0 \) and \( B^+ \) tags. Because of the different compositions of the combinatorial \( B^0 \) and \( B^+ \) backgrounds, these errors are uncorrelated. In contrast, systematic errors related to the description of the signal shape are correlated since we use similar decay modes. Here we assess the uncertainties related to the modeling of the shape for \( m_{\text{ES}} \) by repeating the fit with \( \alpha \) set to \( -\infty \), allowing an exponential function only instead of a power law to describe the tail caused by the \( \pi^0 \) energy resolution. This leads to relative uncertainties of 2.1% (2.4%) on the number of \( B^0 \) (\( B^+ \)) tags. The yields of events in which \( B_{\text{tag}} \) candidates have been found for both “twins” of decay channels (II)–(V) differ by 20% in data and MC, motivating a relative uncertainty of 20% on the \( D^{++} \to D^{*0} \) cross-feed. This adds another systematic uncertainty of 0.5% to the number of charged \( B_{\text{tag}} \) candidates. The final numbers of neutral and charged signal \( B_{\text{tag}} \) candidates are \( N_{\text{tag}} = 45420 \pm 420 \)\((\text{stat}) \pm 591(\text{u}) \pm 949(\text{c}) \) and \( N_{B^+} = 41948 \pm 463(\text{stat}) \pm 596(\text{u}) \pm 1020(\text{c}) \), where \( u \) and \( c \) denote uncorrelated and correlated systematic uncertainties, respectively. The purities of the neutral and charged \( B_{\text{tag}} \) samples are \((82.8 \pm 0.8(\text{stat}) \pm 2.8(\text{syst}))\)% and \((77.5 \pm 0.9(\text{stat}) \pm 2.9(\text{syst}))\)% respectively.

The requirement of an identified electron leads to significantly lower \( B_{\text{tag}} \) backgrounds, as shown in Fig. 2 for the right-sign sample. For high electron momenta \( (p_e > 1 \text{ GeV}/c) \), the purities are 96% (98%) for the right-sign (wrong-sign) samples, with combinatorial \( B_{\text{tag}} \) candidates being the dominant background, while for decreasing electron momenta, the purities decrease to 90% because of an increasing amount of continuum-background. As for the full \( B_{\text{tag}} \) sample, we estimate these backgrounds from the \( m_{\text{ES}} \) sideband region. The background estimates are performed separately for each sample as functions of \( p_e \).

Because of low statistics, we do not determine the shape of the \( m_{\text{ES}} \) distribution of misidentified \( B_{\text{tag}} \) candidates from a fit, but use the MC predictions instead. The systematic errors due to the shape of the combinatorial background and \( D^{*+} \to D^{*0} \) cross feed are evaluated in the same way as for the \( B_{\text{tag}} \) sample. Comparing the yields of like- and unlike-sign electrons with \( p_e > 0.6 \text{ GeV}/c \) in events with \( B_{\text{tag}} \) candidates satisfying \( m_{\text{ES}} > 5.2 \text{ GeV}/c^2 \) in off-peak data and the MC simulation, we estimate the systematic uncertainty on the continuum contribution to be 20%.

Figure 3 shows the momentum spectra of right- and wrong-sign electrons in events with a charged \( B_{\text{tag}} \) candidate, together with the estimated \( B_{\text{tag}} \) background. This figure also displays the background contributions of electrons from photon conversions, \( \pi^0 \to \gamma e^+ e^- \) Dalitz decays and misidentified hadrons. These backgrounds are identified and corrected for as in [3,4]. Corrections for
TABLE I. Electron yields for the four samples and corrections with statistical and systematic errors.

<table>
<thead>
<tr>
<th></th>
<th>$B^0$ tags, right-sign</th>
<th>$B^0$ tags, wrong-sign</th>
<th>$B^+$ tags, right-sign</th>
<th>$B^+$ tags, wrong-sign</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5.27 &lt; m_{ES}(B_{tag}) &lt; 5.29$ GeV/$c^2$</td>
<td>3461 ± 59</td>
<td>1943 ± 44</td>
<td>4074 ± 64</td>
<td>1070 ± 33</td>
</tr>
<tr>
<td>$B_{tag}$ background</td>
<td>198 ± 16 ± 40</td>
<td>135 ± 13 ± 27</td>
<td>320 ± 24 ± 64</td>
<td>114 ± 12 ± 23</td>
</tr>
<tr>
<td>$B</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table><p>ightarrow e^+e^-$ | 55 ± 14 ± 8 | 87 ± 17 ± 12 | 66 ± 14 ± 10 | 83 ± 16 ± 11 |
| $\pi^0ightarrow \gamma e^+e^-$ | 31 ± 14 ± 7 | 25 ± 12 ± 5 | 36 ± 14 ± 7 | 47 ± 16 ± 9 |
| fake $e$ | 29 ± 1 ± 8 | 21 ± 1 ± 4 | 37 ± 1 ± 12 | 16 ± 1 ± 2 |</p>

Yield before and after $e$ efficiency correction
3149 ± 64 ± 42 | 1674 ± 51 ± 30 | 3616 ± 71 ± 66 | 810 ± 41 ± 27 |
3443 ± 70 ± 71 | 1842 ± 56 ± 50 | 3947 ± 78 ± 96 | 898 ± 46 ± 41 |

$B ightarrow (D_s^0 ightarrow \tau ightarrow e)$
97 ± 10 ± 11 | 21 ± 4 ± 2 | 116 ± 11 ± 13 | 0 |
$B ightarrow D_s ightarrow e$ | 85 ± 11 ± 31 | 18 ± 5 ± 7 | 131 ± 14 ± 43 | 0 |
$B ightarrow D_s ightarrow e$ | 60 ± 8 ± 25 | 12 ± 4 ± 5 | 96 ± 10 ± 16 | 0 |

$B ightarrow J/\psi (2S) ightarrow e$ | 22 ± 5 ± 1 | 23 ± 5 ± 1 | 17 ± 4 ± 1 | 19 ± 4 ± 1 |
$D^{0} ightarrow D^{++}$ cross feefd | 9 ± 3 ± 5 | 4 ± 2 ± 2 | 45 ± 7 ± 22 | 29 ± 5 ± 15 |

Net $e$ yield
3170 ± 73 ± 82 | 1764 ± 57 ± 51 | 3542 ± 81 ± 109 | 850 ± 47 ± 45 |


dN_{B^0}^{D^0} / dp = dN_{B^0 \rightarrow Xe\nu} / dp (1 - \chi_m) + dN_{B^0 \rightarrow D^+ \rightarrow Xe\nu} / dp \chi_m,

dN_{B^0}^{D^0} / dp = dN_{B^0 \rightarrow Xe\nu} / dp \chi_m + dN_{B^+ \rightarrow D^+ \rightarrow Xe\nu} / dp (1 - \chi_m),

with $\chi_m = (0.188 \pm 0.003)$ [9] being the $B^0\overline{B}^0$ mixing parameter. The primary electron spectrum $dN_{B^0 \rightarrow Xe\nu} / dp$ of neutral $B$ decays derived from these equations is shown in Fig. 4, together with $dN_{B^+ \rightarrow Xe\nu} / dp$, after normalizations to the respective number of tags.

We integrate these spectra between $p_{\text{min}} = 0.6$ GeV/$c$ and 2.5 GeV/$c$ and apply corrections for geometrical acceptance ($\epsilon_{\text{geom}} = 85\%$) and the small loss of electrons due to bremsstrahlung in the detector material ($\epsilon_{\text{brems}} = 97.4 \pm 0.1\%$) to obtain the partial branching fractions $B(B^0 \rightarrow Xe\nu(\gamma)) = B(B^0 \rightarrow Xe\nu(\gamma), p_e > p_{\text{min}})$ for decays with any number of photons in the final state:

![FIG. 4](image-url)
The ratio of branching fractions, $R_{+0}(1.0 \text{ GeV/c}) = 1.074 \pm 0.049$, is consistent with $\tau_{B^+}/\tau_{B^0} = 1.071 \pm 0.009$ from direct measurements [9]. From this we conclude that the semileptonic decay widths of charged and neutral $B$ mesons agree to a precision of 5%, $\Gamma(B^+ \rightarrow Xe\nu)/\Gamma(B^0 \rightarrow Xe\nu) = 1.003 \pm 0.047$.

We are grateful for the excellent luminosity and machine conditions provided by our PEP-II colleagues, and for the substantial dedicated effort from the computing organizations that support BABAR. The collaborating institutions wish to thank SLAC for its support and kind hospitality. This work is supported by DOE and NSF (USA), NSERC (Canada), IHEP (China), CEA and CNRS-IN2P3 (France), BMBF and DFG (Germany), INFN (Italy), FOM (The Netherlands), NFR (Norway), MIST (Russia), and PPARC (United Kingdom). Individuals have received support from the A.P. Sloan Foundation, Research Corporation, and Alexander von Humboldt Foundation.

Table II lists the contributions to the systematic errors. These results are in agreement with [1,3,4]. For the ratio of branching fractions, $R_{+0}(p_{\text{min}}) = \mathcal{B}(B^+ \rightarrow Xe\nu, p_e > p_{\text{min}})/\mathcal{B}(B^0 \rightarrow Xe\nu, p_e > p_{\text{min}})$, the result is $R_{+0}(0.6 \text{ GeV/c}) = 1.067 \pm 0.041_{(\text{stat})} \pm 0.033_{(\text{syst})}$. For higher values of $p_{\text{min}}$, the statistical error increases, while the systematic error decreases. At $p_{\text{min}} = 1 \text{ GeV/c}$, the combined statistical and systematic error is minimal, leading to our final result

$$R_{+0}(1.0 \text{ GeV/c}) = 1.074 \pm 0.041_{(\text{stat})} \pm 0.026_{(\text{syst})}.$$ 

In summary, we have used electrons in $\Upsilon(4S)$ decays tagged by a fully reconstructed hadronic $B$ decay to measure the inclusive semileptonic branching fractions of $B^0$ and $B^+$ mesons. The ratio of branching fractions, $R_{+0}(1.0 \text{ GeV/c}) = 1.074 \pm 0.049$, is consistent with $\tau_{B^+}/\tau_{B^0} = 1.071 \pm 0.009$ from direct measurements [9]. From this we conclude that the semileptonic decay widths of charged and neutral $B$ mesons agree to a precision of 5%, $\Gamma(B^+ \rightarrow Xe\nu)/\Gamma(B^0 \rightarrow Xe\nu) = 1.003 \pm 0.047$.

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Table II. Breakdown of systematic errors on partial branching fractions $\hat{\mathcal{B}}$ and the ratio $R_{+0}$. Contributions in the upper part of this table are taken to be uncorrelated for $B^0$ and $B^+$.

<table>
<thead>
<tr>
<th>$p_{\text{min}}$ [GeV/c]</th>
<th>$\Delta \hat{\mathcal{B}}^0$ [10^{-3}]</th>
<th>$\Delta \hat{\mathcal{B}}^+$ [10^{-3}]</th>
<th>$\Delta R_{+0}$ [10^{-3}]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>$N_{\text{tags}}$ (uncorr.)</td>
<td>0.125</td>
<td>0.139</td>
<td>0.020</td>
</tr>
<tr>
<td>$B_{\text{tag}}$ background</td>
<td>0.080</td>
<td>0.122</td>
<td>0.014</td>
</tr>
<tr>
<td>$B \rightarrow D$</td>
<td>0.080</td>
<td>0.041</td>
<td>0.011</td>
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<tr>
<td>$B \rightarrow D_s$</td>
<td>0.100</td>
<td>0.119</td>
<td>0.016</td>
</tr>
<tr>
<td>$\chi_{n}$</td>
<td>0.038</td>
<td></td>
<td>0.004</td>
</tr>
<tr>
<td>$D^{*-} \rightarrow D^{*0}$</td>
<td>0.014</td>
<td>0.064</td>
<td>0.004</td>
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<tr>
<td>$B \rightarrow \tau$</td>
<td>0.019</td>
<td>0.020</td>
<td>0.003</td>
</tr>
<tr>
<td>$N_{\text{tags}}$ (corr.)</td>
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<td>0.250</td>
<td>0.004</td>
</tr>
<tr>
<td>$e$ eff.</td>
<td>0.135</td>
<td>0.143</td>
<td></td>
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<tr>
<td>track eff.</td>
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<td>0.090</td>
<td></td>
</tr>
<tr>
<td>$D, D_s, \tau \rightarrow e$</td>
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<td>0.030</td>
<td></td>
</tr>
<tr>
<td>conversion, Dalitz</td>
<td>0.024</td>
<td>0.039</td>
<td>0.001</td>
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<tr>
<td>fake $e$</td>
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<td>0.027</td>
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</tr>
</tbody>
</table>

* * *

[8] Charge conjugation is implied throughout the paper.