Measurement of the branching fraction and time-dependent 
CP asymmetry in the decay $B^0 \to D^{\ast+} D^{\ast-} K^0$
We study the decay $B^0 \to D^{*+}D^{*-}K_S^0$ using $(230 \pm 2) \times 10^6 \bar{B}B$ pairs collected by the BABAR detector at the PEP-II B factory. We measure a branching fraction $\mathcal{B}(B^0 \to D^{*+}D^{*-}K_S^0) = (4.4 \pm 0.4 \pm 0.7) \times 10^{-3}$ and find evidence for the decay $B^0 \to D^+\pi^-K_S^0(2536)$ with a significance of 4.6$s$. A time-dependent $CP$ asymmetry analysis is also performed to study the possible resonant contributions to $B^0 \to D^{*+}D^{*-}K_S^0$ and the sign of $\cos2\beta$. Our measurement indicates that there is a sizable resonant contribution to the decay $B^0 \to D^{*+}D^{*-}K_S^0$ from an unknown $D_{sJ}^*$ state with large width, and that $\cos2\beta$ is positive at the 94% confidence level under certain theoretical assumptions.

\[ A(t) = \frac{\Gamma_{B^0} - \Gamma_{B^0}}{\Gamma_{B^0} + \Gamma_{B^0}} = \eta_A \frac{J_{2J}}{J_0} \cos(\Delta m_d t) - \left( 2J_{1J} \frac{J_0}{J_0} \sin2\beta + \eta_A \frac{2J_{2J}}{J_0} \cos2\beta \right) \times \sin(\Delta m_d t), \]

where $s^+ \equiv m^2(D^{*+}K_S^0)$ and $s^- \equiv m^2(D^{*-}K_S^0)$, $\Gamma_{B^0}$ ($\Gamma_{B^0}$) is the decay rate for $B^0$ ($\bar{B}^0$) to $D^{*+}D^{*-}K_S^0$ at a proper time $t$ after production, $\Delta m_J$ is the mass difference between the two $B^0$ mass eigenstates, and $\eta_A = \frac{1}{3} (+1)$ for $s^+ \leq s^-$. The parameters $J_0$, $J_1$, and $J_2$ are the integrals over the half Dalitz phase space with $s^+ < s^-$. The functions $|a|^2 + |b|^2$, $|a|^2 - |b|^2$, $\text{Re}(\bar{a}a^*)$, and $\text{Im}(\bar{a}a^*)$, where $a$ and $\bar{a}$ are the decay amplitudes of $B^0 \to D^{*+}D^{*-}K_S^0$ and $\bar{B}^0 \to D^{*+}D^{*-}K_S^0$, respectively.

If the decay $B^0 \to D^{*+}D^{*-}K_S^0$ has only a nonresonant component, the parameter $J_{s2} = 0$ and $J_c$ is at the few percent level [5]. The $CP$ asymmetry can be extracted by fitting the $B^0$ time-dependent decay distribution. The measured $CP$ asymmetry is $\sin2\beta$ multiplied by a factor of $2J_{s1}/J_0$ because the final state is an admixture of $CP$ eigenstates with different $CP$ parities. In this case, the value of the dilution factor $2J_{s1}/J_0$ is estimated to be large [5], similar to the decay $B^0 \to D^{*+}D^{*-}$.

The situation is more complicated if intermediate resonances such as $D_{sJ}^*$ are present. In this case, the parameter $J_{s2}$ is nonzero and $J_c$ can be large. The resonant components are expected to be dominated by two $P$-wave excited $D_{sJ}$ states [5]. One such state is $D_{sJ}^*(2536)$ that has a narrow width and does not contribute much to $J_{s2}$. It can be easily removed by imposing a mass window requirement. The other $D_{sJ}^*$ resonant state is predicted in the quark model [6] to have a mass above the $D^{*+}K_S^0$ mass threshold with a large width. In this case, the $J_{s2}$ can be large.

Therefore by studying the time-dependent asymmetry of $B^0 \to D^{*+}D^{*-}K_S^0$ in two different Dalitz regions, the sign of $\cos2\beta$ can be determined for a sufficiently large data set using the method described in Refs. [5,7,8]. This would allow the resolution of the $\beta \to \pi^2 - \beta$ ambiguity despite the large theoretical uncertainty of $2J_{s2}/J_0$. However, if the unknown $P$-wave $D_{sJ}$ is the newly discovered $D_{sJ}^*(2317)$ or $D_{s2J}^*(2460)$, both of which lie below the $D^{*+}K_S^0$ mass threshold, then it will not contribute to the decay $B^0 \to D^{*+}D^{*-}K_S^0$. As a result, the time-dependent analysis of $B^0 \to D^{*+}D^{*-}K_S^0$ not only has a potential to measure the sign of $\cos2\beta$, but also can help us to understand the possible structure of the excited charm meson spectrum.

In this paper, we present an improved measurement of the branching fraction of the decay $B^0 \to D^{*+}D^{*-}K_S^0$ [9] and a search for intermediate resonant decays. We also perform a time-dependent $CP$ asymmetry analysis to study the possible resonant contributions and the sign of $\cos2\beta$.

The data used in this analysis comprise $(230 \pm 2) \times 10^6 \bar{B}B$ pairs collected by the BABAR detector at the PEP-II storage rings. The BABAR detector is described in detail elsewhere [10]. We use a Monte Carlo (MC) simulation based on GEANT4 [11] to validate the analysis procedure and to study the relevant backgrounds.

We select $B^0 \to D^{*+}D^{*-}K_S^0$ decays by combining two oppositely charged $D^*$ candidates reconstructed in the modes $D^{*+} \to D^0 \pi^+$ and $D^+ \to D^0 \pi^0$ with a $K_S^0$ candidate. We include the $D^{*+}D^{-}$ combinations $(D^{*+}D^{-}, D^{*+}D^{*-})$ and $(D^0 \pi^+, D^0 \pi^0)$, but not $(D^0 \pi^0, D^- \pi^0)$ because of the small branching fraction and large backgrounds. To suppress the $e^+e^- \to q\bar{q}$ ($q = u, d, s, c$) continuum background, we require the ratio of the second and zeroth order Fox-Wolfram moments [12] to be less than 0.5.
C candidates for \( D^0 \) and \( D^+ \) mesons are reconstructed in the modes \( D^0 \rightarrow K^- \pi^+ \pi^0 \), \( K^- \pi^+ \pi^0 \), \( K^- \pi^+ \pi^+\pi^- \), and \( D^+ \rightarrow K^- \pi^+ \pi^+\pi^- \), by selecting track combinations with invariant mass within ±2σ of the nominal \( D \) masses [13]. The resolution \( \sigma \) is measured using a large data sample of inclusive \( D \) decays. It is equal to 7.0 MeV/c² for \( D^0 \rightarrow K^- \pi^+ \) decays, 13.5 MeV/c² for \( D^0 \rightarrow K^- \pi^+ \pi^0 \) decays, 5.7 MeV/c² for \( D^0 \rightarrow K^- \pi^+ \pi^+\pi^- \) decays, and 5.6 MeV/c² for \( D^+ \rightarrow K^- \pi^+ \pi^+\pi^- \) decays.

The \( K_S^0 \) candidates are reconstructed from two oppositely charged tracks with an invariant mass within 15 MeV/c² of the nominal \( K_S^0 \) mass [13], which is equivalent to slightly less than 5σ of the measured \( K_S^0 \) mass resolution. The \( \chi^2 \) probability of the \( \pi^+ \pi^- \) vertex fit must be greater than 0.1%. To reduce combinatorial background, we require the measured proper decay time of the \( K_S^0 \) to be greater than 3 times its uncertainty. Charged kaon candidates, except for the one in the decay \( D^0 \rightarrow K^- \pi^+ \), are required to be inconsistent with the pion hypothesis, as inferred from the Cherenkov angle measured by the Cherenkov detector and the ionization energy loss measured by the charged-particle tracking system [10]. Neutral pion candidates are formed from pairs of photons detected in the electromagnetic calorimeter [10], each with energy above 30 MeV. The mass of the pair must be within 30 MeV/c² of the nominal \( \pi^0 \) mass, and their summed energy is required to be greater than 200 MeV. In addition, a mass-constrained fit is applied to the \( \pi^0 \) candidate.

The \( D^0 \) and \( D^+ \) candidates are subject to a mass-constrained fit prior to the formation of the \( D^{*+} \) candidates. The slow \( \pi^+ \) from the \( D^{*+} \) decay is required to have a momentum in the \( \Upsilon(4S) \) center-of-mass (CM) frame less than 450 MeV/c. The slow \( \pi^0 \) from the \( D^{*+} \) must have a momentum between 70 and 450 MeV/c in the CM frame. No requirement on the photon-energy sum is applied to the \( \pi^0 \) candidates from the \( D^{*+} \) decays. The \( D^{*+} \) mass is required to be within 4 MeV/c² of the nominal \( D^{*+} \) mass, corresponding to slightly more than 3σ of the measured \( D^{*+} \) mass resolution.

For each \( B^0 \rightarrow D^{*+} D^{*-} K_S^0 \) candidate, we calculate the difference of the \( B^0 \) candidate energy \( E^B \) from the beam energy \( E_{\text{Beam}} \), \( \Delta E \equiv E^B - E_{\text{Beam}} \), in the CM frame. In order to reduce the combinatorial background further, \( |\Delta E| \) is required to be less than 25 MeV, which is equivalent to 2.5σ of the measured \( \Delta E \) resolution.

The beam energy-substituted mass, \( m_{\text{ES}} = \sqrt{E_{\text{Beam}}^2 - p_B^2} \), where \( p_B \) is the \( B^0 \) candidate momentum in the CM frame, is used to extract the signal yield from the events satisfying the aforementioned selection. We select \( B^0 \) candidates with \( m_{\text{ES}} \approx \frac{5.23}{\text{GeV}} \). On average we have 1.25 \( B^0 \) candidates per event. If more than one candidate is selected in an event, we retain the one with the smallest \( |\Delta E| \). Studies using MC samples show that this procedure results in the selection of the correct \( B^0 \) candidate more than 95% of the time. The total probability density function (PDF) is the sum of the signal and background components. The signal \( m_{\text{ES}} \) PDF is modeled by a Gaussian function and the combinatorial background is described by an ARGUS [14] function. MC studies show that there is a small peaking background from \( B^+ \rightarrow D^{*0} D^{*+} K_S^0 \) in which a \( D^0 \) originating from a \( D^{*0} \) decay is combined with a random soft \( \pi^- \) to form a \( D^{*-} \) candidate. The peaking background is described by the same PDF as the signal, its fraction with respect to the signal yield is fixed to be 1.4%, determined from the MC simulation. An unbinned maximum likelihood (ML) fit to the \( m_{\text{ES}} \) distribution yields 201 ± 17(stat) signal events, where the mean and width of the signal Gaussian, as well as the ARGUS shape parameters, are allowed to float in the fit. In the region of \( m_{\text{ES}} > 5.27 \text{ GeV/c}^2 \), the signal purity is approximately 79%. The fit result is shown in Fig. 1(a).

To correct for variations in signal efficiency across the \( D^{*+} D^{*-} K_S^0 \) Dalitz plane, we calculate the branching fraction using the \( s\text{Plots} \) method [15]:

\[
B = \sum \frac{w_{\text{sig}}(m_{\text{ES},i})}{N_{BB} \cdot \epsilon_i \cdot B_{\text{sub}}},
\]

where the sum is over all events \( i \), \( \epsilon_i \) is the efficiency estimated from the simulated events in the vicinity of each data point in the Dalitz plane, \( B_{\text{sub}} \) is the product of the branching fractions of the subdecays, and \( w_{\text{sig}} \) is an event-dependent signal weight that is defined as [15]

\[
w_{\text{sig}}(m_{\text{ES},i}) = \frac{\sum_{j=1}^{N_j} V_{\text{sig},j} P_j(m_{\text{ES},i})}{\sum_{j=1}^{N_j} N_j P_j(m_{\text{ES},i})},
\]

which is calculated from the yield \( N_j \) of the \( j \)th PDF component \( P_j \) in the fit, and the covariance matrix elements \( V_{\text{sig},j} \) between the signal yield \( N_{\text{sig}} \) and \( N_j \). The \( N_j \) is the number of PDF components in the fit.

We investigate the production of intermediate resonances by examining the invariant mass distribution of the \( D^{*\pm} \) and \( K_S^0 \) combinations. Figure 1(b) shows the projected distribution of \( m(D^{*\pm} K_S^0) \) from \( B^0 \rightarrow D^{*+} D^{*-} K_S^0 \) signal events after efficiency correction using the \( s\text{Plots} \) technique. A peak is seen at the value of \( D^{*+} \) (2536) mass. We do not observe evidence of the \( D^{*+} \) (2536) mass. The events tend to cluster toward lower values of \( m(D^{*\pm} K_S^0) \) (below about 2.9 GeV/c²), in contrast to the phase-space model, as shown in Fig. 1(b).

To extract the signal yield of \( B^0 \rightarrow D^{*+} D^{*-} \) (2536), we perform an unbinned ML fit to the \( \Delta m = m(D^{*\pm} K_S^0) - m(D^{*\pm}) - m(K_S^0) \) distribution in the region \( m_{\text{ES}} > 5.27 \text{ GeV/c}^2 \) with a PDF given by the sum of a Gaussian shape for the signal and a threshold function \( \Delta m^a \exp(b \Delta m) \) for the background. The mean and width of the signal Gaussian, as well as the background PDF parameters \( a \) and \( b \), are allowed to float in the fit. The fit yields 12.3 ± 4.0(stat) signal events, as shown in Fig. 1(c).
consistent signal yields of different background parametrizations. All of these give
with the MC simulation. We repeat the fit in different
yield. The fitted signal mean and width are consistent
uncertainty corresponding to the estimate of the signal
uncertainty and has little contribution from the systematic
none. The significance is dominated by the statistical un-

where the first uncertainty is the statistical and the second
parametrizations. All of these give consistent signal yields of $D_{s1}(2536)$. We also examine the
Delta m distribution in the $m_{ES} \leq 5.27 \text{ GeV}/c^2$ region, and see no peaking structure.

The systematic uncertainties of the branching fraction
measurements are dominated by the uncertainty of the
charged track reconstruction efficiency (10.7%). Other sources also contribute to the systematic errors, such as
the kaon particle identification efficiency (3.9%), $\pi^0$ recon-
struction efficiency (3.5%), branching fractions of the
$D$ decays (5.8%), determination of the number of $B\bar{B}$ in
the data sample (1.1%), event selection criteria (5.0%), and
the estimate of the peaking background fraction (1.8%).
The measured branching fraction is

$$B(B^0 \to D^{*-} D^{*-} K^0_s) = (4.4 \pm 0.4 \pm 0.7) \times 10^{-3},$$

where the first uncertainty is the statistical and the second
is systematic. Our result is in good agreement with the
previous BABAR measurement [9]. We also measure the
intermediate resonant decay branching fraction and find

$$B(B^0 \to D^{*-} D_{s1}^+(2536)) \times B(D_{s1}^+(2536) \to D^{*-} K^0_s)$$

$$= (4.1 \pm 1.3 \pm 0.6) \times 10^{-4}.$$  

The fraction of the decay $B^0 \to D^{*-} D^{*-} K^0_s$ through the
intermediate $D_{s1}^+(2536)$ resonance is measured to be $0.092 \pm 0.024(\text{stat}) \pm 0.001(\text{syst})$.

We subsequently perform a time-dependent analysis
using the event sample described previously. In the time-
dependent analysis, we require that the invariant mass of the
$D^{*\pm}$ and $K^0_s$ combination be larger than 2.55 GeV/$c^2$
in order to reject the narrow $D_{s1}^+(2536)$ resonant decays.

For the time-dependent CP analysis, we use information
from the other $B$ meson in the event to tag the initial flavor
of the fully reconstructed $B^0 \to D^{*-} D^{*-} K^0_s$ candidate. The
decay rate $f_+(f_-)$ for a neutral $B$ meson accompanied by a
$B^0(B^0)$ tag is given by

$$f_\pm(\Delta t) \propto e^{-|\Delta t|/\tau_{B^0}} \left[ (1 \mp \Delta \omega) \pm (1 - 2\omega) \times \left[ \eta_y \frac{J_c}{J_0} \cos(\Delta m_j \Delta t) - \frac{2J_{s2}}{J_0} \sin 2\beta \right. \right.$$  

$$+ \left. \eta_y \frac{2J_{s2}}{J_0} \cos 2\beta \sin(\Delta m_j \Delta t) \right],$$

where $\Delta t = t_{\text{rec}} - t_{\text{tag}}$ is the difference between the proper
decay time of the reconstructed signal $B$ meson ($B_{\text{rec}}$) and
that of the tagging $B$ meson ($B_{\text{tag}}$). $\tau_{B^0}$ is the $B^0$ lifetime,
and $\Delta m_j$ is the mass difference determined from the $B^0 -$
$B^0$ oscillation frequency [13]. The average mistag probability $\omega$
describes the effect of incorrect tags, and $\Delta \omega$ is the
difference between the mistag rate for $B^0$ and $B^0$.

The technique used to measure the CP asymmetry is
analogous to that used in previous BABAR measurements
as described in Refs. [16,17]. We calculate the time interval
$\Delta t$ between the two $B$ decays from the measured separation
$\Delta z$ between the decay vertices of $B_{\text{rec}}$ and $B_{\text{tag}}$ along the
collision ($z$) axis [16]. The $z$ position of the $B_{\text{rec}}$ vertex is
determined from the charged daughter tracks. The $B_{\text{tag}}$
decay vertex is determined by fitting charged tracks not
belonging to the $B_{\text{rec}}$ candidate to a common vertex, employing
constraints from the beam-spot location and the $B_{\text{rec}}$
momentum [16]. Only events with a $\Delta t$ uncertainty
less than 2.5 ps and a measured $|\Delta t|$ less than 20 ps are accepted.
We perform a simultaneous unbinned maximum
likelihood fit to the $\Delta t$ and $m_{ES}$ distributions to extract the
CP asymmetry. The signal PDF in $\Delta t$ is given by Eq. (4)
convolved with an empirical $\Delta t$ resolution function [16].
Both the signal mistag probability and $\Delta t$ resolution
function are determined from a sample of neutral $B$ decays to
flavor eigenstates, $B_{\text{flav}}$.  

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The background $\Delta t$ distributions are parametrized with an empirical description that includes zero and nonzero lifetime components [16]. We also allow the nonzero lifetime background to have effective CP asymmetries and let them float in the likelihood fit. The fits to the data yield
\[
\frac{J_c}{J_0} = 0.76 \pm 0.18 \text{(stat)} \pm 0.07 \text{(syst)}
\]
\[
\frac{2J_{s1}}{J_0} \sin 2\beta = 0.10 \pm 0.24 \text{(stat)} \pm 0.06 \text{(syst)}
\]
\[
\frac{2J_{s2}}{J_0} \cos 2\beta = 0.38 \pm 0.24 \text{(stat)} \pm 0.05 \text{(syst)}.
\]

Figure 2 shows the $\Delta t$ distributions and asymmetries in yields between $B^0$ and $\bar{B}^0$ tags, overlayed with the projection of the likelihood fit result. The effective CP asymmetries in the background are found to be consistent with zero within statistical uncertainties. As a cross-check, we also repeat the fit by allowing the $B^0$ lifetime to float. The obtained $B^0$ lifetime is in a good agreement with its world average [13] within the statistical uncertainty.

![Figure 2](image)

FIG. 2. (a) The distribution of $\Delta t$ in the region $m_{ES} > 5.27$ GeV/c$^2$ for $B^0$ ($\bar{B}^0$) tag candidates in the half Dalitz space $s^+ < s^-$ ($\eta_S = -1$). The solid (dashed) curve represents the fit projections in $\Delta t$ for $B^0$ ($\bar{B}^0$) tags. (b) The raw asymmetry $(N_{s^+} - N_{s^-})/(N_{s^+} + N_{s^-})$, as functions of $\Delta t$, where $N_{s^+}$ ($N_{s^-}$) is the number of candidate with $B^0$ ($\bar{B}^0$) tag. (c) and (d) contain the corresponding information for the other half Dalitz space $s^+ > s^-$ ($\eta_S = +1$).

The sources and estimates of systematic uncertainties are summarized in Table I. Since the signal reconstruction efficiency is not uniform over the entire Dalitz space, the different CP components may not have the same acceptance. Therefore the measured parameters will deviate slightly from their true values. We estimate the possible bias using the signal MC weighted according to the expected theoretical Dalitz distributions in Ref. [5]. Because of the lack of knowledge of the unknown $D_s^+$ state, we vary its mass and width over a wide range. The largest bias of the measured parameters $J_c/J_0$, $(2J_{s1}/J_0) \sin 2\beta$, and $(2J_{s2}/J_0) \cos 2\beta$ are taken as the corresponding systematic uncertainties on the acceptance effect.

The other systematic uncertainties arise from the possible backgrounds that tend to peak under the signal and their CP asymmetries, the assumed parametrization of the $\Delta t$ resolution function, the possible differences between the $B_{flav}$ and $B^0 \rightarrow D^{**} D_{s1}^{(*)} K_{S}^0$ tagging performances, knowledge of the event-by-event beam-spot position, and the possible interference between the suppressed $b \rightarrow \bar{u}c\bar{d}d$ amplitude and the favored $b \rightarrow c\bar{u}d\bar{d}$ amplitude for some tag-side decays [18]. They also include the systematic uncertainties from the finite MC sample used to verify the fitting method. All the systematic uncertainties are found to be much smaller than the statistical uncertainties.

In summary, we have reported an improved branching fraction measurement of the decay $B^0 \rightarrow D^{*+} D^{*-} K_S^0$ that supersedes the previous BABAR result [9]. We also find evidence for the decay $B^0 \rightarrow D^{*-} D_{s1}^+(2536)$ with 4.6$\sigma$ significance. A time-dependent CP asymmetry analysis has also been performed. The measured $J_c/J_0$ is significantly different from zero, which may indicate that there is a sizable resonant contribution to the decay $B^0 \rightarrow D^{*+} D^{*-} K_S^0$ from an unknown $D_s^+$ state with large width, according to Ref. [5]. We measure that $(2J_{s2}/J_0) \cos 2\beta = 0.38 \pm 0.24 \text{(stat)} \pm 0.05 \text{(syst)}$. Under the assumption that there is a significant broad resonant contribution to the decay $B^0 \rightarrow D^{*+} D^{*-} K_S^0$, it implies that the sign of $\cos 2\beta$ is preferred to be positive at the 94% confidence level if the theoretical parameter $J_{s2}/J_0$ is positive, as predicted in Ref. [5].

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[4] We imply charge conjugate modes throughout the paper.