Measurement of the $B^- \to D^0K^-$ branching fraction

MEASUREMENT OF THE $B^- \rightarrow D^{0}K^{*-}$ BRANCHING FRACTION PHYSICAL REVIEW D 73, 111104(R) (2006)

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The decays $B^- \rightarrow D^0 K^{*-}$ [1] are of interest because of their relevance to the Cabibbo-Kobayashi-Maskawa (CKM) model [2] of quark-flavor mixing. Interference effects in specific $D^0$ final states offer a means of observing direct $CP$ violation governed by the angle $\gamma = \arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$ [3], where $V$ is the CKM matrix. One way to access $\gamma$ is to measure $R_{CP\pm}$ [4]:

$$R_{CP\pm} = \frac{2}{\Gamma(B^- \rightarrow D^0 K^{*-})} \left( \frac{\Gamma(B^- \rightarrow D^0_{CP\pm} K^{(*)})}{\Gamma(B^- \rightarrow D^0 K^{*-}) + \Gamma(B^- \rightarrow D^0^{(*)} K^{*})} \right).$$

Neglecting $D^0 - \bar{D}^0$ mixing $R_{CP\pm}$ can be expressed in terms of a $CP$-conserving strong phase difference ($\delta$), the ratio of the magnitude of suppressed and favored amplitudes ($r_B$), and $\gamma$: $R_{CP\pm} = 1 \pm 2 r_B \cos \delta \cos \gamma + r_B^2$. Thus a precise determination of the $B^- \rightarrow D^0 K^{*-}$ branching fraction provides the reference for direct $CP$ violation measurements.

The decay $B^- \rightarrow D^0 K^{*-}$ was first observed by CLEO [5], and later by BABAR [6]. In this paper we present a new measurement of the branching fraction $B(B^- \rightarrow D^0 K^{*-})$ obtained with 2.7 times more data than used for the previous BABAR measurement.

This analysis uses data collected with the BABAR detector at the PEP-II $e^+e^-$ storage ring. The data corresponds to an integrated luminosity of 211 fb$^{-1}$ at the $\Upsilon(4S)$ peak ($232 \times 10^6 BB$ pairs) and 16 fb$^{-1}$ at center-of-mass energy 40 MeV below the resonance.

The BABAR detector is described in detail in [7]. We give here a brief description of the components relevant to this analysis. Charged-particle trajectories are measured by a five-layer double-sided silicon vertex tracker (SVT) and a 40-layer drift chamber (DCH) inside a 1.5 T solenoid. Charged-particle identification is achieved by combining measurements of the light detected in a ring-imaging Cherenkov device (DIRC) with measurements of the ionization energy loss ($dE/dx$) measured in the DCH and SVT. Photons are detected in a CsI(Tl) electromagnetic calorimeter (EMC) inside the coil. We use GEANT4 [8] based software to simulate the detector response and account for the varying beam and environmental conditions.

To reconstruct $B^- \rightarrow D^0 K^{*-}$ decays we select $K^{*-}$ candidates in the $K^{*-} \rightarrow K_S^0 \pi^-$ mode and $D^0$ candidates in three decay channels: $D^0 \rightarrow K^- \pi^+$, $K^- \pi^+ \pi^0$, and $K^- \pi^+ \pi^+ \pi^-$. Our event selection follows closely the one reported in [9]. $K_S^0$ candidates are formed from oppositely charged tracks assumed to be pions with a reconstructed invariant mass within 13 MeV/$c^2$ (4 standard deviations) of the known $K_S^0$ mass, $m_{K_S^0}$ [10]. The $K_S^0$ candidates are fitted so that their invariant mass equals $m_{K_S^0}$ (mass constraint). We further require their flight direction and distance to be consistent with a $K_S^0$ coming from the interaction point. The $K_S^0$ candidate’s flight path and momentum vectors must make an acute angle and the flight length in the plane transverse to the beam must be at least 3 times larger than its uncertainty. $K^-$ candidates are formed from a $K_S^0$ and a charged particle, which are required to originate from a common vertex. We select $K^-$ candidates which have an invariant mass within 75 MeV/$c^2$ of the known value [10]. Finally, since the $K^-$ in $B^- \rightarrow D^0 K^{*-}$ is polarized, we require the helicity angle $\theta_H$ to satisfy $|\cos \theta_H| \geq 0.35$, where $\theta_H$ is the angle in the $K^+$ rest frame between the daughter pion and the parent $B$ momentum. The helicity distribution discriminates well between a $B$ meson decay and an event from the $e^+e^- \rightarrow q\bar{q}(q \in \{u,d,s,c\})$ continuum, since the former is distributed as $\cos^2 \theta_H$ and the latter is almost flat.

In order to reconstruct the $\pi^0$ of the $D^0 \rightarrow K^- \pi^+ \pi^0$ channel, we combine pairs of photons to form candidates with a total energy greater than 200 MeV and an invariant mass between 125 and 145 MeV/$c^2$. A mass-constrained fit is applied to the selected $\pi^0$ candidates. All $D^0$ candidates are mass- and vertex-constrained. Particle identification is required for the charged kaons. We select $D^0$ candidates with an unconstrained invariant mass, $m_{D^0}$, differing from the world average mass, $m_{D^0}^{PDG}$, by less than 12 MeV/$c^2$ for all channels except $K^- \pi^+ \pi^0$ where we require $-29 < m_{D^0} - m_{D^0}^{PDG} < +24$ MeV/$c^2$. To reduce combinatorial background in this channel, we further select candidates in the regions of the Dalitz plane enhanced by the $K^*$ (892), $K^{0*}$ (892) and $\rho^*$ (770) resonances using amplitudes and phases measured by the CLEO experiment [11]. In order to reduce the background from random two track combinations that have masses consistent with a $D^0$ we also require, for the $D^0 \rightarrow K^- \pi^+$ channel, $|\cos \theta_D| \leq 0.9$, where $\theta_D$ is the angle in the $D^0$ rest frame between the daughter kaon and the parent $B$ momentum. Finally, we perform a geometric fit on the $B$ candidate which constrains the $D^0$, the $K^0_S$, and the charged pion from the $K^*$ to originate from a single vertex.

To suppress continuum background we require $|\cos \theta^*_{\pi^0}| \leq 0.9$, where $\theta^*_{\pi^0}$ is defined as the angle between
the $B$ candidate momentum in the $Y(4S)$ rest frame and the beam axis. The distribution in $\cos\theta_B^q$ is flat for $q\bar{q}$ events, while for $B$ mesons it follows a $\sin^2\theta_B^q$ distribution. We also use global event shape variables to distinguish between $q\bar{q}$ continuum events which have a two-jet topology in the $Y(4S)$ rest frame and $B\bar{B}$ events which are more spherical. We require $|\cos\theta_B^q| \approx 0.9$ where $\theta_B^q$ is the angle between the thrust axes of the $B$ candidate and that of the rest of the event. We construct a linear (Fishler) discriminant [12] from $\cos\theta_B^q$ and the $L_0, L_2$ monomials (see below) describing the energy flow in the rest of the event, as in [13]. In the center-of-mass frame (CM) we define $L_j = \sum_i p_1^i \cos\theta_j^i$, where $i$ indexes the charged and neutral particles in the event once those from the $B$ candidate are removed, and $\theta_j^i$ is the angle of the CM-momentum $p_1^i$ with the thrust axis of the $B$ meson candidate.

We identify $B$ candidates using two nearly independent kinematic variables: the beam-energy-substituted mass $m_{ES} = \sqrt{(s/2 + p_0 \cdot p_B)^2/E_0^2 - p_B^2}$ and the energy difference $\Delta E = E_B - \sqrt{s}/2$, where $E$ and $p$ are energy and momentum, the subscripts $0$ and $B$ refer to the $e^+ e^-$-beam-system and the $B$ candidate in the lab frame, respectively; $s$ is the square of the CM energy, and the asterisk labels the CM frame.

In those events where we find more than one acceptable $B$ candidate (less than 25% of selected events depending on the $D^0$ mode), we choose the one with the smallest $x^2$ formed from the differences of the measured and world average $D^0$ and $K^-$ masses scaled by the mass resolution which includes the experimental resolution and, for the $K^{*-}$, its natural width. Simulations show that no bias is introduced by this choice and the correct candidate is picked at least 80% of the time. According to simulation of signal events, the total reconstruction efficiencies are: 13.3%, 4.6%, and 9.0% for the $D^0 \rightarrow K^- \pi^+$, $K^- \pi^+ \pi^0$, and $K^- \pi^+ \pi^- \pi^0$ modes, respectively.

To study $B\bar{B}$ backgrounds we look at sideband regions away from the signal region in $\Delta E$ and $m_{D^{0\rho}}$. The $\Delta E$ distributions are centered around zero for signal with a resolution between 11 and 13 MeV for all three channels. We define a signal region $|\Delta E| < 25$ MeV. We also define a $\Delta E$ sideband in the intervals $-100 \leq \Delta E \leq -60$ MeV and $60 \leq \Delta E \leq 200$ MeV. The lower limit ($-100$ MeV) is chosen to avoid selecting a region of high background coming from $B^- \rightarrow D^0 K^{*-}$. In this $\Delta E$ sideband we see no significant evidence of a background peaking near the $B$ mass in $m_{ES}$ which could leak into the signal region. The sideband region in $m_{D^{0\rho}}$ is defined by requiring that this quantity differs from the $D^0$ mass peak by more than 4 standard deviations. It provides sensitivity to doubly-peaking background sources that mimic signal both in $\Delta E$ and $m_{ES}$. This pollution comes from either charmed or charmless $B$ meson decays that do not contain a true $D^0$. Since many of the possible contributions to this background are not well known, we attempt to measure its size by including the $m_{D^{0\rho}}$ sideband in the fit described below.

An unbinned extended maximum likelihood fit to $m_{ES}$ distributions in the range $5.2 \leq m_{ES} \leq 5.3$ GeV/$c^2$ is used to determine the event yields. For signal modes, the $m_{ES}$ distributions are described by a Gaussian function $G$ centered at the $B$ mass with resolution ($\sigma$), averaged over the three $D^0$ decay modes, of 2.7 MeV/$c^2$. For each $D^0$ decay mode $k (= 1, 2, 3)$ we determine the mean and sigma of the Gaussian $G_k$ by fitting to the data. The combinatorial background in the $m_{ES}$ distribution is modeled with a threshold function $A_k$ [14]. Its shape is governed by one parameter $\xi_k$ that is left free in the fit for each $D^0$ decay mode. We fit simultaneously $m_{ES}$ distributions of nine samples: the $K^- \pi^+$, the $K^- \pi^0 \pi^0$ and $K^- \pi^+ \pi^- \pi^+$ samples for (i) the $\Delta E$ signal region, (ii) the $m_{D^{0\rho}}$ sideband and (iii) the $\Delta E$ sideband. We fit three probability density functions (PDF) weighted by the unknown event yields. For the $\Delta E$ sideband, we use $A_k$. For the $m_{D^{0\rho}}$ sideband we use $N_{n_D} A_k + N_{D^{0\rho}} G_k$, where $G_k$ accounts for the doubly-peaking $B$ decays. For the signal region PDF we use $N_{n_D} A_k + N_{D^{0\rho}} G_k + N_{sig} \cdot G_k$, where $\kappa$ is the ratio of the $m_{D^{0\rho}}$ signal-window to sideband widths and $N_{sig}$ is the number of $B^- \rightarrow D^0 K^{*-}$ signal events. The $\Delta E$ sideband sample helps define the shape of the background function $A_k$. We assume that the $B$ decays found in the

FIG. 1 (color online). Distributions of $m_{ES}$ in the signal region for $B^- \rightarrow D^0 K^{*-}$ decays where $D^0 \rightarrow K^- \pi^+$ (top), $K^- \pi^+ \pi^0$ (middle), and $K^- \pi^+ \pi^- \pi^+$ (bottom). The dashed curve indicates the contribution from the combinatorial background and the peaking $B$-background which is estimated from a simultaneous fit to the $D^0$ sideband (not shown).
TABLE I. Results from the fit and quantities used to derive the $B^- \to D^0 K^-$ branching fraction. For each channel we give the event yield resulting from the fit, the efficiency, and the branching fraction measurement, in units of $10^{-6}$, derived using Eq. (1). The uncertainties are statistical only.

<table>
<thead>
<tr>
<th></th>
<th>$K^- \pi^+$</th>
<th>$K^- \pi^+ \pi^0$</th>
<th>$K^- \pi^+ \pi^0 \pi^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield</td>
<td>144 ± 13</td>
<td>185 ± 19</td>
<td>195 ± 18</td>
</tr>
<tr>
<td>Efficiency</td>
<td>13.30%</td>
<td>4.60%</td>
<td>8.99%</td>
</tr>
<tr>
<td>$\mathcal{B}(B^- \to D^0 K^-)$</td>
<td>5.15 ± 0.47</td>
<td>5.65 ± 0.57</td>
<td>5.24 ± 0.49</td>
</tr>
</tbody>
</table>

$m_{D^0}$ sidebands have the same final states as the signal so we use the same Gaussian shape for the doubly-peaking $B$ background.

The fit results are shown graphically in Fig. 1 and numerically in Table I. For each channel $k$, a measurement $\mathcal{B}_k$ of the branching fraction $\mathcal{B}(B^- \to D^0 K^-)$ is derived as follows:

$$\mathcal{B}_k = \frac{N(D^0 \to X_k) \cdot f}{N_{B^+} \cdot e_k \cdot \mathcal{B}(K^- \to X_k) \cdot \mathcal{B}(D^0 \to X_k)}, \quad (1)$$

where $N(D^0 \to X_k)$ is the event yield from the fit, $f$ the fraction of $K^+$ in the sample (discussed below), $N_{B^+}$ is the number of charged $B$ mesons in the data sample, $e_k$ is the efficiency to reconstruct $B^- \to D^0 K^+$ when $D^0 \to X_k$, $\mathcal{B}(K^- \to X_k)$ and $\mathcal{B}(D^0 \to X_k)$ are the branching fractions of the $K^+$ and the $D^0$. We have assumed equal production of pairs of neutral and charged $B$ mesons in $\Upsilon(4S)$ decay.

Systematic effects arise from the difference between the actual detector response for the data and the simulation model for the Monte Carlo. Here the main effects stem from the modeling of the tracking efficiency (1.2–1.3% per track), the $K^0_S$ reconstruction efficiency (2% per $K^0_S$), the $\pi^0$ reconstruction efficiency for the $K^- \pi^+ \pi^0$ channel (3%) and the efficiency and misidentification probabilities from the particle identification (2% per kaon). A study of a high-statistics $B^- \to D^0 \pi^-$ control sample shows excellent agreement between the data and Monte Carlo sample except for the distributions of $\Delta E$ and the continuum-suppression Fisher discriminant. For these variables, differences of up to $(2.5 \pm 1.1\%$ are measured between the data and Monte Carlo. Suitable corrections to the efficiencies are therefore applied and systematic errors assigned. The $K^{+}$ helicity angle distributions differ significantly between data and simulation because of the nonresonant background under the $K^{+}$ peak. We describe below how we subtract this background. For the pure $K^+$ events, we estimate that the residual discrepancy between data and simulation in the helicity to be less than 1.6%. We determine using simulations that the $m_{ES}$ signal PDFs deviate from the single Gaussian shape by less than 0.1%. Substantial systematic uncertainties come from the measured $D^0$ branching fractions [10] and the number of $B^\pm$ pairs in the sample.

The observed number of signal events must be corrected for the nonresonant $K^0_S \pi^-$ pairs under the $K^{+}$. When we remove the requirement on the $K^{+}$ helicity distribution (Fig. 2) of the selected events manifests a forward-backward asymmetry that indicates an interference with a $K^0_S \pi^-$ background [9,15]. We model the $K^0_S \pi^-$ system with a $P$-wave and an $S$-wave component. The $P$-wave mass dependence is described by a relativistic Breit-Wigner while the $S$-wave piece is assumed to be a complex constant. This model is fitted to the data and shown in Fig. 2 along with an estimate of the combinatorial background. Neglecting higher resonances, the number of $K^0_S \pi^-$ peaking background events is $(4 \pm 1\%$ of the total measured number of signal events. We do not quote a systematic error on the contributions of the neglected partial waves (non-$K^+$ $P$-wave and higher order waves) since their expected rates in the $K \pi$ mass window are far below that of the $S$-wave [15]. In Fig. 3 we see that a...
TABLE II. Systematic uncertainties. $X_k$ refers to the $D^0$ decay modes given in the columns. $B_{K^\pm}$ is the branching fraction of the $K^\pm \rightarrow K_S^0 \pi^\pm$, $K_S^0 \rightarrow \pi^+ \pi^- \pi^0$ decay chain.

<table>
<thead>
<tr>
<th>Source</th>
<th>$K^- \pi^+$</th>
<th>$K^- \pi^+ \pi^0$</th>
<th>$K^- \pi^- \pi^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tracking efficiency</td>
<td>3.8%</td>
<td>3.8%</td>
<td>6.3%</td>
</tr>
<tr>
<td>$\pi^0$ efficiency</td>
<td>-</td>
<td>3.1%</td>
<td>-</td>
</tr>
<tr>
<td>Particle Identification</td>
<td>2.0%</td>
<td>2.0%</td>
<td>2.0%</td>
</tr>
<tr>
<td>$K_S^0$ efficiency</td>
<td>1.6%</td>
<td>1.9%</td>
<td>1.8%</td>
</tr>
<tr>
<td>cos$\beta/(K^-)$</td>
<td>1.6%</td>
<td>1.6%</td>
<td>1.6%</td>
</tr>
<tr>
<td>Fisher</td>
<td>1.1%</td>
<td>1.1%</td>
<td>1.1%</td>
</tr>
<tr>
<td>$\Delta E$</td>
<td>1.9%</td>
<td>1.8%</td>
<td>2.0%</td>
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<td>$m_{ES}$ PDF shape</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.1%</td>
</tr>
<tr>
<td>Number of $B^-$</td>
<td>1.1%</td>
<td>1.1%</td>
<td>1.1%</td>
</tr>
<tr>
<td>Simulation statistics</td>
<td>0.9%</td>
<td>1.4%</td>
<td>1.0%</td>
</tr>
<tr>
<td>$B_{K^-}$ [10]</td>
<td>0.2%</td>
<td>0.2%</td>
<td>0.2%</td>
</tr>
<tr>
<td>$B(D^0 \rightarrow X_k)$ [10]</td>
<td>2.4%</td>
<td>6.2%</td>
<td>4.2%</td>
</tr>
<tr>
<td>$K_S^0$ $\pi^- S$-wave subtraction</td>
<td>1.1%</td>
<td>1.1%</td>
<td>1.1%</td>
</tr>
<tr>
<td>Total systematic error</td>
<td>6.1%</td>
<td>9.0%</td>
<td>8.7%</td>
</tr>
</tbody>
</table>

The first error is statistical and the second is systematic. We have compared the results from this analysis using the same data set as in our previously published analysis [6]. The two analyses use different selection criteria and therefore find different numbers of events. The results from the two analyses are consistent to within a half of a (statistical) standard deviation. We have also calculated the branching fraction for the two data sets obtained since the previous analysis. The measurement in each set is consistent with, although lower than the value obtained in [6]. This result supersedes our previously published result.

In summary, we have measured the branching fraction of the decay $B^- \rightarrow D^0 K^-$ in the $D^0 K_S^0 \pi^0$ final state and observed the interference of the $K^-$ with a small non-resonant $K_S^0 \pi^0$ background.

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