Measurement of the $B \to \pi \nu \bar{\nu}$ Branching Fraction and Determination of $|V_{ub}|$ with Tagged $B$ Mesons


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We report a measurement of the \( B \rightarrow \pi \ell \nu \) branching fraction based on 211 fb\(^{-1}\) of data collected with the BABAR detector. We use samples of \( B^0 \) and \( B^+ \) mesons tagged by a second \( B \) meson reconstructed in a semileptonic or hadronic decay and combine the results assuming isospin symmetry to obtain

\[
\mathcal{B}(B^0 \rightarrow \pi^- \ell^- \nu) = (1.33 \pm 0.17_{\text{stat}} \pm 0.11_{\text{syst}}) \times 10^{-4}.
\]

We determine the magnitude of the Cabibbo-Kobayashi-Maskawa matrix element \(|V_{ub}|\) by combining the partial branching fractions measured in ranges of the momentum transfer squared and theoretical calculations of the form factor. Using a recent lattice QCD calculation, we find \(|V_{ub}| = (4.5 \pm 0.5_{\text{stat}} \pm 0.3_{\text{syst}} \pm 0.2_{\text{SME}}) \times 10^{-3}\), where the last error is due to the normalization of the form factor.

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The magnitude of the Cabibbo-Kobayashi-Maskawa matrix \([1]\) element \( V_{ub} \) is a critical constraint on the unitarity triangle. Our knowledge of \(|V_{ub}|\) comes from measurements of the \( B \rightarrow u \ell \nu \) decay rate, where the hadronic system in the final state can be reconstructed either inclusively or exclusively. The precisions are limited by the uncertainties in the nonperturbative QCD calculations that are used to extract \(|V_{ub}|\) from the measured decay rates. It is therefore crucial to pursue both the inclusive and exclusive approaches, which rely on different theoretical methods, and to test their consistency.

The rate of the exclusive decay \( B \rightarrow \pi \ell \nu (\ell = e \text{ or } \mu) \) is related to \(|V_{ub}|\) through the form factor \( f_+ (q^2) \), where \( q^2 \) is the momentum transfer squared. Measurements of the \( B \rightarrow \pi \ell \nu \) branching fraction have been reported by CLEO \([2]\), BABAR \([3]\), and Belle \([4]\). In this Letter, we report a measurement in which \( B \rightarrow \pi \ell \nu \) decays are searched for in \( Y(4S) \rightarrow BB \) events that are identified by reconstruction of the second \( B \) meson (\( B_{\text{tag}} \)). The technique, which was also used in Ref. \([4]\), allows us to constrain the kinematics, reduce the combinatorics, and determine the charge of the signal \( B \). The result is an improved signal purity at the expense of the efficiency compared with the traditional measurements in which only the signal \( B \) meson is reconstructed. We perform two analyses in which \( B_{\text{tag}} \) is reconstructed in semileptonic and hadronic decays, respectively, and combine the measured partial branching fractions \( \Delta \mathcal{B} \) in ranges of \( q^2 \) with the recent form-factor calculations \([5–8]\) to determine \(|V_{ub}|\).

The measurement uses a sample of approximately 232 \( \times 10^6 \) \( BB \) pairs, corresponding to an integrated luminosity of 211 fb\(^{-1}\), recorded near the \( Y(4S) \) resonance with the BABAR detector \([9]\) at the PEP-II asymmetric-energy \( e^+ e^- \) storage rings. We use a detailed Monte Carlo (MC) simulation to estimate the signal efficiency and the signal and background distributions.

In the first analysis, we reconstruct \( B_{\text{tag}} \) in the semileptonic decay \( B \rightarrow D^{(*)} \ell \nu \). We reconstruct \( D^0 \) mesons in \( K^- \pi^+ \), \( K^- \pi^+ \pi^+ \pi^- \), \( K^- \pi^+ \pi^0 \), and \( K^0_S \pi^+ \pi^- \) decays and \( D^+ \) mesons in \( K^- \pi^+ \pi^0 \) decays \([10]\). The \( D \) mass resolution (\( \sigma \)) is between 4.6 and 12.9 MeV, depending on the decay channel. The mass of the \( D \) candidate is required to be within \( 2.6 \sigma \) and 3.0 \( \sigma \) of the expected value for the \( B^0 \) and \( B^+ \) channels, respectively. We also use a sideband sample, in which the \( D \) candidate mass is more than \( 3 \sigma \) away from the nominal value, for subtracting the combinatoric background. We reconstruct \( D^{(*)} \) mesons in \( D^0 \pi^+ \) and \( D^+ \pi^0 \) decays. The mass difference between the \( D^* \) and \( D \) is required to be within 3 MeV of the expected value \([11]\). The reconstructed \( D \) and \( D^* \) candidates are paired with a charged lepton with a center-of-mass (c.m.) momentum \(|p_L| > 0.8 \text{ GeV} \) to form a \( Y = D^{(*)}\ell \) system. If the \( D \) decay contains a charged kaon, the lepton must have the same charge as the kaon. The lepton and the \( D \) meson are required to originate from a common vertex. Assuming that only a massless neutrino escaped detection, we calculate the cosine of the angle between the \( B \) and \( Y \) momenta as

\[
\cos \theta_B Y = (2E_B E_Y - m_B^2 - m_Y^2)/(2|p_B||p_Y|),
\]

where \( m_B, m_Y, E_B, E_Y, p_B, \) and \( p_Y \) refer to the masses, c.m. energies, and momenta of \( B \) and \( Y \), respectively. For background events, \( \cos \theta_B Y \) does not correspond to the cosine of a physical angle and can extend outside \( \pm 1 \). We apply a loose selection of \( |\cos \theta_B Y| < 5 \) at this stage.

After identifying the \( B_{\text{tag}} \) meson, we require the remaining particles in the event to be consistent with a \( B \rightarrow \pi \ell \nu \) decay. Charged tracks that are not identified as a lepton or a kaon are considered charged pion candidates. Neutral pion candidates are formed from pairs of photon candidates with invariant mass between 115 and 150 MeV. For the \( B^0 \) channel, the lepton must have \(|p_L| > 0.8 \text{ GeV} \), and its charge must be opposite to that of the charged pion. The lepton charge must be opposite to that of the \( B_{\text{tag}} \) for the \( B^+ \) channel. We reject the lepton candidate if, when combined with an oppositely charged track, it is consistent with a \( J/\psi \rightarrow \ell^+ \ell^- \) decay or a photon conversion. Once the signal \( B \) candidate is identified, we require that the event contain no other charged particles and a small total c.m. energy \( E_{\text{res}} \) of the residual neutral particles. In measuring \( E_{\text{res}} \), we remove the neutral candidates that are consistent with coming from a \( D^* \rightarrow D^{(*)} \) or \( D \gamma \) decay, bremsstrahlung from an electron, or beam-related background. We require \( E_{\text{res}} < 70 \text{ MeV} \) for the \( B^0 \) channel and \( E_{\text{res}} < 250 \text{ MeV} \) for the \( B^+ \) channel, the latter being relaxed to allow for additional photons from decays of \( D^{(*)} \) and higher resonances. We calculate the cosine of the angle between the \( B \) and \( \pi \ell \) momenta as

\[
\cos \theta_{B \pi \ell} = (2E_B E_{\pi \ell} - m_B^2 - m_{\pi \ell}^2)/(2|p_B||p_{\pi \ell}|),
\]

where \( m_{\pi \ell}, E_{\pi \ell}, \) and \( p_{\pi \ell} \) are the
mass, c.m. energy, and momentum of the $\pi\ell$ system, respectively. We require $|\cos\theta_{\pi\ell}| < 5$.

Ignoring the small c.m. momentum of the $B$ meson, the invariant mass squared of the lepton-neutrino system in a $B \rightarrow \pi\ell\nu$ decay can be inferred as $q^2 = (m_B - E_\ell)^2 - |p_\pi|^2$, where $E_\ell$ and $p_\pi$ are the c.m. energy and momentum of the pion, respectively. We divide the data into three bins: $q^2 < 8$ GeV$^2$, $8 < q^2 < 16$ GeV$^2$, and $q^2 > 16$ GeV$^2$. We use simulated $B \rightarrow \pi\ell\nu$ events to estimate and to correct for the small ($<8\%$) migration between the $q^2$ bins due to resolution, which is approximately 0.8 GeV$^2$ at $q^2 = 8$ GeV$^2$ and improves with increasing $q^2$.

Having identified the two $B$ mesons that decayed semileptonically, conservation of the total momentum determines the angle $\phi_B$ between the direction of the $B$ momenta and the plane defined by the $Y$ and $\pi\ell$ momenta:

$$\cos^2 \phi_B = \frac{\cos^2 \theta_{BY} + \cos^2 \theta_{B\pi\ell} + 2 \cos \theta_{BY} \cos \theta_{B\pi\ell} \cos \gamma}{\sin^2 \gamma}$$

where $\gamma$ is the angle between the $Y$ and $\pi\ell$ momenta. The variable $\cos^2 \phi_B$ satisfies $\cos^2 \phi_B \leq 1$ for correctly reconstructed signal events and is broadly distributed for the background (see Fig. 1). We use the $\cos^2 \phi_B$ distributions to extract the signal yield in the data in each $q^2$ bin. We did not require stringent cuts on $\cos \theta_{BY}$ and $\cos \theta_{B\pi\ell}$ because they are incorporated in $\cos^2 \phi_B$.

We express the data distribution as a sum of three contributions: $dN/d\cos^2 \phi_B = N_{\text{sig}} \mathcal{P}_{\text{sig}} + N_{\text{bkg}} \mathcal{P}_{\text{bkg}} + N_{\text{cmb}} \mathcal{P}_{\text{cmb}}$, where $N_c$ and $\mathcal{P}_c$ are the number of events and the probability density function (PDF) for each category $c$, defined as the signal (sig), background with correctly reconstructed $D$ mesons (bkg), and other backgrounds (cmb). The events in the $D$ mass sideband are also used in the fit to constrain the $N_{\text{cmb}} \mathcal{P}_{\text{cmb}}$ term. The PDF shapes are determined from the MC simulation. The signal PDF is a combination of a smeared step function and an exponential tail. The background PDFs are either an exponential plus a constant or a second order polynomial. The two data samples ($D$ mass peak and sideband) and the MC samples are used in an unbinned maximum likelihood fit that determines $N_{\text{sig}}$, $N_{\text{bkg}}$, $N_{\text{cmb}}$, and the PDF parameters simultaneously. Figure 1 shows the fit results summed over the $q^2$ bins. We find the signal yields and their statistical errors to be $57^{+13}_{-12}$ and $92^{+26}_{-24}$ events for the $B^0$ and $B^+$ channels, respectively.

We use simulated $B \rightarrow \pi\ell\nu$ events to estimate the signal efficiencies. Control samples are used to derive corrections for the data-MC differences in the $B_{\tag{tag}}$ reconstruction, charged and neutral particle reconstruction, and lepton identification. The largest uncertainty comes from the $B_{\tag{tag}}$ reconstruction efficiency, which is determined from a sample of events in which two nonoverlapping $B_{\tag{tag}}$ candidates are reconstructed. The efficiency correction factors for the $B_{\tag{tag}}$ reconstruction are found to be $1.00 \pm 0.07$ and $0.99 \pm 0.02$ for the $B^0$ and $B^+$ channels, respectively. The average signal efficiencies after the correction are $1.1 \times 10^{-3}$ for the $B^0$ channel and $3.0 \times 10^{-3}$ for the $B^+$ channel. The latter is larger mainly because of the higher efficiency of reconstructing a $D^0$ meson compared with a $D^+$ or $D^{*+}$ meson.

The measured branching fractions are summarized in Table I. The largest sources of systematic error [12] are the $B_{\tag{tag}}$ reconstruction efficiency (discussed above), the shape of the background $\cos^2 \phi_B$ distribution (studied with control samples that fail the signal selection criteria), and the branching fractions of the $B$ semileptonic decays other than $B \rightarrow \pi\ell\nu$ (varied within the current knowledge [11]).

In the second analysis, we reconstruct the $B_{\tag{tag}}$ meson in a set of purely hadronic final states $B \rightarrow D^{(*)}X$. We reconstruct $D^0$ mesons in $K^-\pi^+, K^-\pi^0, K^{-}\pi^+\pi^-\pi^0$, and $K^{0}\pi^+\pi^0$ decays and $D^+$ mesons in $K^-\pi^+\pi^-\pi^0, K^-\pi^0, K^0\pi^+\pi^0, K^0\pi^0$, and $K^0\pi^+\pi^-\pi^0$ decays. The $D^*$ mesons are reconstructed in $D^0\pi^+, D^0\pi^0$, and $D^0\gamma$ decays. The hadronic system $X$ has a total charge $\pm 1$ and is composed of $n_1 \pi^+ + n_2 K^+ + n_3 \pi^0 + n_4 K^0_S$, where $n_1 + n_2 < 6, n_3 < 3$, and $n_4 < 3$. The total reconstruction efficiency for a $B^0 (B^+)$ meson is $30\% (50\%)$.

We separate correctly reconstructed $B_{\tag{tag}}$ mesons from the background using two kinematic variables: the beam-energy substituted mass $m_{\text{ES}} = \sqrt{s/4 - |p_B|^2}$ and the energy difference $\Delta E = E_B - \sqrt{s}/2$, where $\sqrt{s}$ is the c.m. energy of the $e^+e^-$ system. We select signal candidates in mode-dependent $\Delta E$ windows around zero. We apply a loose selection $5.2 < m_{\text{ES}} < 5.3$ GeV and fit the $m_{\text{ES}}$ distribution at a later stage to extract the signal yield.

After reconstructing $B_{\tag{tag}}$, we look for the signature of a $B \rightarrow \pi\ell\nu$ decay in the recoiling system. The selection criteria for the pion and lepton candidates are similar to the first analysis, except (a) the minimum $|p_\ell|$ for electrons is 0.5 GeV, and (b) the $\pi^0$ mass window is 110–160 MeV.

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**FIG. 1.** Distributions of $\cos^2 \phi_B$ of the (a) $B^0 \rightarrow \pi^-\ell^+\nu$ and (b) $B^+ \rightarrow \pi^0\ell^+\nu$ candidates. The points with error bars and the shaded histograms are the data in the $D$ mass peak and sideband, respectively. The curves are the fit results representing the total (solid), background (dashed), and “cmb” (dotted) components defined in the text. The fits were performed in bins of $q^2$, but the results shown are for the complete $q^2$ range.
We require $E_{\text{res}} < 450$ MeV for the $B^0$ channel to reduce the $B^0 \to \rho^- \ell^+ \nu$ background, and no requirement is made for the $B^+$ channel.

The full reconstruction of $B_{\text{tag}}$ allows us to determine the neutrino four-momentum precisely from the missing four-momentum $p_{\text{miss}} = p_{V(4S)} - p_{B_{\text{tag}}} - p_{\pi^-} - p_{\ell^-}$. The missing mass squared $m_{\text{miss}}^2$ peaks near zero for the signal and extends above zero for the background (see Fig. 2). We require $|m_{\text{miss}}^2| < 0.3$ GeV$^2$ for the $B^0$ channel and $-0.5 < m_{\text{miss}}^2 < 0.7$ GeV$^2$ for the $B^+$ channel, with the latter being broader and asymmetric due to the resolution of the $\pi^0$ energy measurement.

Precise knowledge of $p_{\text{miss}}$ allows us to calculate $q^2$ with small uncertainties. We divide the signal candidates into the same three $q^2$ bins as before and subtract the small bin-to-bin migration as background. In each $q^2$ bin, we obtain the number of correctly tagged events by an unbinned maximum likelihood fit to the $m_{\text{ES}}$ distribution. The PDF for the signal is determined from MC simulation as a Gaussian function joined to an exponential tail. For the background, we use a threshold function of the form $x\sqrt{1 - x^2} \exp(-\xi (1 - x^2))$, where $x = 2m_{\text{ES}}/\sqrt{s}$ and the parameter $\xi$ is allowed to float in the fit. Figure 2 shows the $m_{\text{miss}}^2$ distribution obtained by splitting the data samples in bins of $m_{\text{miss}}^2$ and repeating the $m_{\text{ES}}$ fit.

The signal side of the correctly tagged events may not be a $B \to X\ell\nu$ decay. Contributions from this type of background are estimated with the MC simulation, as indicated by shaded histograms in Fig. 2, which are scaled to match the data in the sideband region $1 < m_{\text{miss}}^2 < 4$ GeV$^2$. After background subtraction, we find signal yields of $31 \pm 7$ and $26 \pm 7$ events for the $B^0$ and $B^+$ channels, respectively, where the errors are statistical.

Instead of estimating the absolute signal efficiency, we normalize the signal yield to the number of inclusive $B$ semileptonic decays, $B \to X\ell\nu$, in the recoil of $B_{\text{tag}}$. The reconstruction efficiencies of $B_{\text{tag}}$ and of the lepton cancel to first order in the ratio between the yields of the signal and normalization samples. The inclusive branching fraction $\mathcal{B}(B \to X\ell\nu)$ is taken as $10.73 \pm 0.28\%$ [11]. The yield of the normalization sample is extracted by a fit to the $m_{\text{ES}}$ distribution. The component of the background that peaks in the $m_{\text{ES}}$ distribution is estimated from the MC simulation and subtracted. Efficiency differences between the signal and normalization samples are estimated with the MC simulation, and the corresponding corrections are applied to the result.

The measured branching fractions are summarized in Table I. The largest source of systematic error is the limited statistics of the signal MC sample. Other significant sources include the modeling of the signal PDF (studied with alternative fitting methods), photon-energy measurement, $\pi^0$ reconstruction, muon identification, and the branching fractions of nonsignal $B \to X_{\ell\nu}$ decays.

We take weighted averages of the measured partial branching fractions in each $q^2$ bin. The results for the $B^0$ and $B^+$ channels are consistent with the isospin relation $\Gamma(B^0 \to \pi^- \ell^+ \nu) = 2\Gamma(B^+ \to \pi^0 \ell^+ \nu)$ and the lifetime ratio $\tau_B^-/\tau_B^0 = 1.081 \pm 0.015$ [11], with $x^2 = 5.2$ for 3 degrees of freedom. Assuming isospin symmetry, we combine the $B^0$ and $B^+$ channels and express the results as the $B^{0/+}$ branching fraction in the last row in Table I. The overall $x^2$ is 10.2 for 9 degrees of freedom.

We extract $|V_{ub}|$ from the partial branching fractions $\Delta\mathcal{B}$ using $|V_{ub}| = \sqrt{\Delta\mathcal{B}/(\tau_B^- \Delta \xi)}$, where $\tau_B^- = (1.536 \pm 0.014) \text{ ps}$ [11] is the $B^-$ lifetime and $\Delta \xi = \Delta \Gamma/|V_{ub}|^2$ is the normalized partial decay rate predicted by the form-factor calculations. We use the light-cone sum rules calcula-

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**TABLE I.** Partial and total branching fractions, in units of $10^{-4}$, measured with the semileptonic and hadronic tag analyses. The $q^2$ ranges are in GeV$^2$. The errors are statistical and systematic. The combined results are expressed as $B^0 \to \pi^- \ell^+ \nu$ branching fractions.

<table>
<thead>
<tr>
<th>$q^2$</th>
<th>$B^0$ Semileptonic</th>
<th>$B^0$ Hadronic</th>
<th>$B^0$ Average</th>
<th>$B^+$ Semileptonic</th>
<th>$B^+$ Hadronic</th>
<th>$B^+$ Average</th>
<th>Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q^2 &lt; 8$</td>
<td>0.50 ± 0.16 ± 0.05</td>
<td>0.33 ± 0.14 ± 0.04</td>
<td>0.29 ± 0.15 ± 0.04</td>
<td>0.83 ± 0.22 ± 0.08</td>
<td>1.12 ± 0.25 ± 0.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$8 &lt; q^2 &lt; 16$</td>
<td>0.09 ± 0.10 ± 0.02</td>
<td>0.33 ± 0.15 ± 0.05</td>
<td>0.65 ± 0.20 ± 0.13</td>
<td>0.42 ± 0.18 ± 0.05</td>
<td>1.07 ± 0.27 ± 0.15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q^2 &gt; 16$</td>
<td>0.38 ± 0.12 ± 0.04</td>
<td>0.33 ± 0.10 ± 0.03</td>
<td>0.47 ± 0.13 ± 0.06</td>
<td>0.72 ± 0.16 ± 0.06</td>
<td>1.19 ± 0.20 ± 0.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>0.18 ± 0.08 ± 0.02</td>
<td>0.45 ± 0.13 ± 0.05</td>
<td>0.10 ± 0.12 ± 0.04</td>
<td>0.63 ± 0.16 ± 0.06</td>
<td>0.73 ± 0.18 ± 0.08</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**FIG. 2.** Distributions of $m_{\text{miss}}^2$ of the (a) $B^0 \to \pi^- \ell^+ \nu$ and (b) $B^+ \to \pi^0 \ell^+ \nu$ candidates. The points with error bars are the data. The histograms represent, from the lightest to the darkest, the MC simulation of the $B \to \pi\ell\nu$ signal, $b \to u\ell\nu$, $b \to c\ell\nu$, and other backgrounds. The arrows indicate the regions in which the signals are extracted.
branching fraction calculations. The first two errors on $|V_{ub}|$ come from the statistical and systematic uncertainties of the partial branching fractions. The third errors correspond to the uncertainties on $\Delta \zeta$ due to the form-factor calculations and are taken from Refs. [5–8].

| $q^2$ (GeV$^2$) | $\Delta \zeta$ (ps$^{-1}$) | $|V_{ub}|$ (10$^{-3}$) |
|-----------------|-----------------|-----------------|
| Ball-Zwicky [5] | $<16$ | $5.44 \pm 1.43$ | $3.2 \pm 0.2 \pm 0.1^{+0.5}_{-0.4}$ |
| Gulez et al. [6] | $>16$ | $1.46 \pm 0.35$ | $4.5 \pm 0.5 \pm 0.3^{+0.7}_{-0.5}$ |
| Okamoto et al. [7] | $>16$ | $1.83 \pm 0.50$ | $4.0 \pm 0.5 \pm 0.3^{+0.7}_{-0.5}$ |
| Abada et al. [8] | $>16$ | $1.80 \pm 0.86$ | $4.1 \pm 0.5 \pm 0.3^{+1.6}_{-0.7}$ |

In conclusion, we have measured the $B \to \pi \ell \nu$ branching fraction as a function of $q^2$ using tagged $B$ meson samples and have extracted $|V_{ub}|$. The measured total branching fraction $B(B^0 \to \pi^- \ell^+ \nu) = (1.33 \pm 0.17_{\text{stat}} \pm 0.11_{\text{sys}}) \times 10^{-4}$ has the smallest systematic uncertainty among the existing measurements [2–4] thanks to the superior signal purity, and the overall precision is comparable to the best. Using theoretical calculations of the form factor, we obtain values of $|V_{ub}|$ ranging between $3.2 \times 10^{-3}$ and $4.5 \times 10^{-3}$. As an example, the recently published unquenched lattice QCD calculation [6] gives $|V_{ub}| = (4.5 \pm 0.5_{\text{stat}} \pm 0.3_{\text{sys}}^{+0.7}_{-0.5FF}) \times 10^{-3}$. Improvement will be possible with additional data combined with more precise form-factor calculations.

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[10] Charge conjugate states are implied throughout this Letter.
[12] See EPAPS Document No. E-PRLTAO-97-050647 for two tables containing (i) the systematic errors of the measured partial branching fractions ($B \to \pi \ell \nu$) and (ii) the error matrices. For more information on EPAPS, see http://www.aip.org/pubservs/epaps.html.