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The influence of mass-transfer variability on the growth of white dwarfs, and the implications for supernova type Ia rates

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ABSTRACT
White dwarfs (WDs) can increase their mass by accretion from companion stars, provided the mass-accretion rate is high enough to avoid nova eruptions. The accretion regimes that allow growth of the WDs are usually calculated assuming constant mass-transfer rates. However, it is possible that these systems are influenced by effects that cause the rate to fluctuate on various timescales. We investigate how long-term mass-transfer variability affects accreting WDs systems. We show that, if such variability is present, it expands the parameter space of binaries where the WD can effectively increase its mass. Furthermore, we find that the supernova type Ia (SNIa) rate is enhanced by a factor 2-2.5 to a rate that is comparable with the lower limit of the observed rates. The changes in the delay-time distribution allow for more SNIae in stellar populations with ages of a few Gyr. Thus, mass-transfer variability gives rise to a new formation channel of SNIa events that can significantly contribute to the SNIa rate. Mass-transfer variability is also likely to affect other binary populations through enhanced WD growth. For example, it may explain why WDs in cataclysmic variables are observed to be more massive than single WDs, on average.

Key words: binaries: close, symbiotic – white dwarfs – supernovae: general – novae

1 INTRODUCTION

White dwarfs (WDs) in binaries can accrete from their companion stars. Such binaries are called cataclysmic variables (CVs) if the donor stars are low-mass main-sequence stars, symbiotic binaries (SBs) if they are evolved red giants, or AM CVNs if the donor stars are low-mass Helium WDs or Helium stars. For CVs and SBs, the matter accreted by the WD consists mainly of hydrogen. As the matter piles up on the surface of the WD, it eventually reaches temperatures and densities high enough for nuclear burning.

The burning can proceed in two ways, depending on the accretion rate and the mass of the WD. For high accretion rates and WD masses, the hydrogen burning on the surface of the WD is continuous (Whelan & Iben 1973; Nomoto 1982), whereas for low accretion rates and WD masses the hydrogen is burned in thermo-nuclear runaway novae (Schatzman 1950; Starrfield, Sparks & Truran 1974). In general, the high mass-transfer rates needed for continuous surface hydrogen burning can only be reached by SBs, where high mass-transfer rates can be driven by the expansion of the evolved star and by systems with main-sequence donors more massive than the accreting WDs (Nomoto et al. 2000). The masses of WDs with high accretion rates can grow effectively, but at very high accretion rates close to the Eddington limit, the growth of the white dwarf is limited. At these rates a hydrogen red-giant-like envelope forms around the WD and hydrogen burning on top of the WD is strong enough for a wind to develop from the WD (Kato & Hachisu 1994; Hachisu, Kato & Nomoto 1996). On the other hand, at low accretion rates mass accretion on to the WD is not very efficient either, as the nova eruptions eject some or all of the accreted matter from the binary system, possibly along with some of the surface material of the WD itself (e.g. Prialnik & Kovetz 1995). The average mass-transfer rate allowing growth of the white dwarf is therefore limited to a relatively narrow range (approximately $10^{-7} - 10^{-6} M_{\odot}$ yr^{-1}).

The growth of WD masses can have important consequences. In the single-degenerate (SD) scenario for type Ia supernova (SNIa) progenitors (Whelan & Iben 1973; Nomoto 1982) the accretion on to a carbon-oxygen WD pushes the mass above the critical mass limit for WDs (close but not equal to the Chandrasekhar limit) which then explodes as a SNIa. In this scenario, it is necessary for the WD to retain several tenths of solar masses of accreted material. It is not possible to achieve such mass growth for the majority of systems with mass-transfer rates in the nova regime, even if some of the accreted matter is retained. Following this theory, the rate and delay time distribution (DTD, evolution of the rate as a function of time after a single star formation episode) can be estimated with the use of population synthesis models (e.g. Yungelson et al. 1994; Toonen, Nelemans & Portegies Zwart 2012; Bours, Toonen & Nelemans 2013). While there currently is no consensus between the models as to the shape of the DTD

(Nelemans, Toonen & Bours 2013; Bours, Toonen & Nelemans 2013), the majority of models agree on two problems: 1) There are not enough systems with high mass-transfer rates to account for all the observed SNeIa, and 2) after an age of approximately 6 – 7 Gyr, it is not possible to create SNIa explosions through this scenario, as only low-mass donors remain.

The considerations above apply to systems where the mass-transfer rate is given by the evolutionary state of the system. I.e. two binaries with the same parameters will have the same mass-transfer rate. Observations of accreting WD systems indicate that the long-term average mass-transfer rates do indeed follow the expectations (e.g. Knigge, Baraffe & Patterson 2011). However, it is possible that the mass-transfer rates are highly variable on intermediate timescales (Patterson 1984; Verbunt 1984; Warner 1987; Hameury, King & Lasota 1989). In this paper, we discuss such variability and show that it affects the evolution of accreting WD systems. In particular, the effects can be of high importance for understanding SD SNIa progenitors, as it increases the volume of the parameter space of systems that can explode as SNeIa.

2 MASS-TRANSFER VARIABILITY

Over the past decades there have been many studies discussing the theoretical and observational aspects of mass-transfer variability in WD binaries. Below we shortly review the current knowledge in order to construct models that capture the main effects of the possible variability. For a thorough review, see Knigge, Baraffe & Patterson (2011), section 4.

2.1 Theoretical considerations

In the majority of accreting WD binaries (excepting strongly magnetic WDs with low accretion rates), hydrogen rich matter is accreted through an accretion disk that deposits the matter on to the surface of the white dwarf. The matter quickly spreads over the surface of the white dwarf. What then happens depends on the rate of accretion and the resulting temperature and density structure near the surface of the white dwarf (Nomoto 1982; Nomoto et al. 2007; Shen & Bildsten 2007). At low accretion rates the temperature of the white dwarf surface remains low and the accreted hydrogen burns in an unstable manner, leading to nova eruptions that eject most (if not all) of the accreted matter. At high accretion rates the hydrogen burning is stable and the matter remains on the WD, except if the accretion rate is so high (roughly the Eddington limit) that most of the matter cannot be retained by the WD.

For the fate of the accreted matter in a system with mass-transfer variability to be different for a similar system without variability, the timescale must neither be too long nor too short. If it is too long, the properties of the binary that depend on the average long-term mass-transfer rate are affected, such as the radius of the donor star. This would be observable and would also change the whole evolution of the binary (see e.g. Knigge, Baraffe & Patterson 2011). On the other hand, if the timescale of the variability is too short, the surface temperature of the white dwarf is not adjusted to the instantaneous mass-transfer rate, which is necessary for the burning to be affected.

For example, in the accretion disk instability model (e.g. Osaki 1996; Lasota 2001), the mass transfer rate is increased by a factor of approximately 10^3 – 10^5 during outbursts (observed as dwarf novae), and this model has been invoked to stabilize the hydrogen burning (King, Rolfe & Schenker 2003; Alexander et al. 2011). However, in this model, the accretion rate is only high for a very short time, and the heat and density of the accreted layer is not raised enough during the outburst to ignite (Tout 2005), as also evidenced by the lack of hydrogen burning events triggered by dwarf novae. Therefore, the layer builds up without a significant temperature increase, and when burning eventually is ignited, it is unstable and therefore leads to a nova eruption[1].

Another example regards nova eruptions. After the eruption, the temperature of the white dwarf is increased, and it is possible that for a short time the burning can be stable. Such short-lived stable surface burning triggered by nova eruptions is seen in some systems (see Schaefer & Collazzi 2010, and Sect. 2.2), but radiation losses during the quiescent periods quickly cool down the WD into the unstable burning regime[2].

Thus, if mass-transfer variability is to significantly change the surface burning, the timescale of the mass-transfer fluctuations must at least be longer than the timescale of the eruption. In this case a continuous high accretion rate after the eruption ensures that the temperature on the surface of the WD is sufficient such that the nuclear burning continues. It also means that only the variability of the rate of matter being transferred from the companion star to the WD accretion disk can be of importance to the growth of the WD (and not the variability of the transfer from the disk to the WD). Such a variability can be achieved in two ways, either through the change in the radius of the companion star, or through a change in the size of its Roche-lobe (see e.g. Knigge, Baraffe & Patterson 2011).

One way that long-term variability can be induced is through irradiation of the donor star from the accreting WD that heats the envelope of the donor star and causes it to expand slightly. An increase in the mass-transfer rate leads to stronger irradiation and therefore expansion of the donor star, whereas a decrease leads to weaker irradiation and contraction. If the effects are strong enough, the mass-transfer becomes unstable on long timescales, and the system goes through so-called irradiation-induced mass-transfer cycles (IIMTC, Podsiadlowski 1991; Hameury et al. 1993; King et al. 1996; Büning & Ritter 2004). In this theory, the mass-transfer is through a series of cycles on Myr timescales, with an off-state where there is little or no accretion, and an on-state, where the mass-transfer rate slowly increases towards a peak, and then decreases until returning to the off-state. Büning & Ritter (2004) find that the parameter space of WD binaries that are susceptible to IIMTCs is highly uncertain. CVs with relatively massive (e.g. $1M_\odot$) main-sequence donors or somewhat evolved donors with convective envelopes are most likely to be affected. Giant donors are unlikely to be affected significantly because the radius variations caused by the irradiation are small compared to the radial evolution of the envelope and the reaction to mass loss.

Another way to achieve long term mass-transfer variability is from episodic mass loss from the binary which can cause cyclic variations of the Roche-lobe radius. CVs naturally experience such mass loss events when they erupt as novae (Shara et al. 1986; MacDonald 1986). If the angular momentum loss is high compared

[1] The effect of the instability of accretion discs on the SNIa rate has been studied by Wang, Li & Han (2010) with a model similar to our model CONST. If stable burning can be maintained, the SNIa rate is increased by a factor 2-3 compared to a model without mass-transfer variability. Note that while the effect on the SNIa rate is similar to our findings, the model that we assume (model NORM-MAX) is different.

[2] Note that in the context of eq. 1 and 2, this means that $f \approx 1$ and that the majority of the mass is transferred in the off-state at low mass-transfer rates.

to the mass loss, the orbit contracts in response to the nova eruption, whereas the orbit widens if the angular momentum loss is low compared to the mass loss. The effects of this process are therefore most likely stronger in systems with extreme mass ratios, where the specific angular momentum of the two stars is very different.

2.2 Observations

The mass-transfer cycles discussed above are difficult to study observationally, as the timescales are longer than the time we have been able to monitor CVs. A useful method is by comparing systems with similar properties, as they would be expected to also have similar mass-transfer rates. Townsley & Gänsicke (2009) used the effective temperatures of the WDs to trace the mass accretion rates. In their sample there are seven non-magnetic CVs above the period gap which show a large scatter in WD effective temperatures and inferred mass-transfer rates. This might be evidence for mass-transfer variability. Below the period gap, the mass-transfer differences found by Townsley & Gänsicke (2009) are much smaller, and a similar result is found by Patterson (2011) using time-averaged accretion disk luminosities. The co-existence of dwarf novae and novae-likes at the same orbital periods adds to the case of weak mass-transfer variability below the period gap, but the evidence is not compelling.

The recurrent nova T Pyx might provide evidence for mass-transfer variability on its own. At a period of 1.83 h (Patterson et al. 1998; Uthas, Knigge & Steeghs 2010) it is clearly below the period-gap and should therefore be faint with a low mass accretion rate. However, it is observed as a recurrent nova with a very high quiescent temperature implying an accretion rate higher than $10^{-8} M_\odot\ \mathrm{yr}^{-1}$, two orders of magnitude above ordinary CVs at this period. Most likely the system is in a transient evolutionary state. Schaefer & Collazzi (2010) suggest that it was an ordinary CV until it erupted as a nova in 1866. This eruption triggered a wind-driven supersoft X-ray phase, resulting in an unusually high luminosity and accretion rate (Knigge, King & Patterson 2000). The recurrence time of the nova eruptions of T Pyx has increased, and Schaefer & Collazzi (2010) argue that the state is not self-sustaining. According to them the mass-transfer rate has decreased from about $10^{-7} M_\odot\ \mathrm{yr}^{-1}$ after the first nova eruption in 1866 to the current rate of about $10^{-8} M_\odot\ \mathrm{yr}^{-1}$. It is therefore likely that it will cease being a recurrent nova in the near future and return to the population of faint CVs.

The mass distribution of the white dwarf components in CVs may indicate mass transfer variability as well. Contradicting the accepted model of nova eruptions in CVs (see also Zorotovic, Schreiber & Gänsicke 2011), white dwarfs in CVs are significantly more massive than single white dwarfs (e.g. Warner 1995; Savoury et al. 2011). If long-term mass-transfer cycles occur in CVs, the masses of the white dwarf components could be significantly enhanced.

3 MODEL

3.1 Mass-transfer variability

From Sect. 2 we conclude that there are both theoretical and observational support for long-term mass-transfer variability in accreting WD binaries. To understand the effects of the mass-transfer variability on the growth of white dwarfs we need to model it. However, the observational evidence hardly constrain the theoretical models which rely on highly uncertain parameters (e.g. Büning & Ritter 2004). Here our main goal is to understand whether the effects of the variability are important, should such variability exist, rather than studying in detail the effects of a particular theoretical model. We therefore set up a number of models according to the following considerations: The mass-transfer rate cycles between two separate states, on- and off-state, with a duty cycle $\beta < 1$ representing the fraction of the time the source spends in the on-state. In other words

$$\bar{R}_{\mathrm{MT}} = \beta \bar{R}_{\mathrm{on}} + (1-\beta)\bar{R}_{\mathrm{off}}, \qquad (1)$$

where $\bar{R}_{\mathrm{MT}}$ is the average long-term mass-transfer rate, $\bar{R}_{\mathrm{on}}$ the average mass transfer rate in the on-state, and $\bar{R}_{\mathrm{off}}$ the average mass transfer rate in the off-state. In most variability models, the stars do not fill their Roche-lobe in the off-state, either because the stars shrink, or the orbit expands, and binary models show dramatic drops in the mass-transfer rate when this happens. We therefore assume that all of the accretion takes place in the on-state. Even if mass-transfer in the off-state only drops by a factor of $f \approx 10$ below the average mass-transfer rate, i.e.

$$\bar{R}_{\mathrm{off}} = \bar{R}_{\mathrm{MT}}/f, \qquad (2)$$

the fraction of mass transferred in this state is $(1-\beta)/f$ and therefore only changes our results at the percentage level. The behaviour in the on-state is probably different from system to system, but to retain the *average* long-term mass-transfer rate $\bar{R}_{\mathrm{MT}}$, the average mass transfer rate in the on-state must be

$$\bar{R}_{\mathrm{on}} = \frac{\bar{R}_{\mathrm{MT}}}{\beta} \cdot \left(1 - \frac{1-\beta}{f}\right) \simeq \bar{R}_{\mathrm{MT}}/\beta. \qquad (3)$$

We employ two model types: (model CONST) a constant mass-transfer rate, representing systems that quickly attain and keep their peak rate, and (model NORM) a lognormal probability distribution (with a standard deviation σ_e given in base e), representing systems with a gradual increase (or decrease) of the mass transfer rate, as is typically seen in the IIMTC scenario. Examples of these models are shown in Fig. 1, showing the fraction of time that a system with an average mass-transfer rate of $\bar{R}_{\mathrm{MT}} = 2.0 \times 10^{-9} M_\odot\ \mathrm{yr}^{-1}$ spends at different accretion rates.

Also shown in Fig. 1 is a model (model NORM-MAX), in which there is a maximum accretion rate that the systems can reach. The reason for this is that the models for hydrogen surface burning have a critical mass $\dot{M}_{\mathrm{crit}}$, above which mass is accreted too fast. The luminosity of the hydrogen burning at $\dot{M}_{\mathrm{crit}}$ is similar to the Eddington luminosity, and it is normally assumed that the surplus is ejected. At first the surplus piles up on the white dwarf as a "red giant" envelope, however, the envelope may interact with the binary through a common-envelope (CE) phase (Paczynski 1976) or a wind may develop from the envelope (Hachisu, Kato & Nomoto 1996). The density of the envelope material is high enough to obscure the X-rays from the hydrogen burning. As this irradiation is necessary to keep the mass-transfer rate high in the IIMTC scenario, the rate is unlikely to exceed $\dot{M}_{\mathrm{crit}}$ for long. For our lognormal model we therefore redistribute the parts of the probability density function above $\dot{M}_{\mathrm{crit}}$ to lower mass-transfer rates, modifying the function to retain the average mass-transfer rate. For model NORM-MAX it is per definition not possible to construct models with $\beta \leqslant \bar{R}_{\mathrm{MT}}/\dot{M}_{\mathrm{crit}}$ as too much time is spent at low accretion rates to reach $\bar{R}_{\mathrm{MT}}$. For these models we therefore gradually increase the duty cycle so $\beta = \bar{R}_{\mathrm{MT}}/\dot{M}_{\mathrm{crit}}$ when necessary.

We furthermore assume that for mean mass-transfer rates in the classical hydrogen burning regime (higher than a few times$10^{-7} M_\odot \mathrm{yr}^{-1}$) the variability disappears. In almost all systems with such high mass transfer rates, the mass-transfer is transferred on the thermal timescale of the donor star, which is shorter than or comparable to the timescale of the mass-transfer cycles (i.e. the star does not have the time to adjust to the heating before the heated layers are lost).

3.2 Integrated retention efficiency

The retention efficiency η is the fraction of mass transferred that is retained by the WD. This is the fraction of hydrogen that is burned stably into helium *and* where the helium is also burned stably. The fraction of mass η that is retained depends on the mass of the WD and on the accretion rate. We estimate η based on Hachisu, Kato & Nomoto (2008) for hydrogen burning and Kato & Hachisu (1999) for helium burning. It is the same prescription as the optimistic case in (Bours, Toonen & Nelemans 2013), see their Eq.5 and A1-A5. We assume that the wind-stripping effect (Hachisu, Kato & Nomoto 1999b) is not effective, i.e. $c_1 = 0$. We make one adjustment at low accretion rates where we assume that the retention factor is $\eta \leqslant 0$, corresponding to a net loss of mass from the white dwarf, with values estimated from Prialnik & Kovetz (1995). The model is shown in Fig. 2, where the final retention efficiency as a function of the mass-transfer rate is shown as the grey line. We use this model in the following analysis, but we caution that the theoretical models that this is based on are calculated assuming constant accretion rates. As discussed above, the properties of the WD (in particular the temperature) depend on the accretion history. Therefore it is not clear if these models are accurate for systems with variable accretion rates. However, we note that in the irradiation-induced mass-transfer scenario, the change in the mass-transfer rate is slow enough (timescales of Myr) that the assumption of a constant mass-transfer rate is likely to be justified.

If we know the retention efficiency as a function of the mass-transfer rate, $R_{\rm MT}$, we can find the effective retention factor for a given *mean* mass-transfer rate $\bar{R}_{\rm MT}$ for each of the models:

$$\eta_{eff} = \frac{\int_0^\infty p(R_{\rm MT}) \cdot R_{\rm MT} \cdot \eta(R_{\rm MT}) dR_{\rm MT}}{\bar{R}_{\rm MT}} \quad (4)$$

where $p(R_{\rm MT})$ is the model probability of a given mass-transfer rate $R_{\rm MT}$.

Fig. 2 shows the results of applying Eq. 4 to the examples of the mass-transfer models. By construction, all of the models conform to the shape given by the grey line in Fig. 2 in the stable burning regime (with mass-transfer rates of a few times $10^{-7} M_\odot$ to $\dot{M}_{\rm crit}$), where we assume that there is no variability. Below this range the models with mass-transfer cycles clearly distinguish themselves from the model without, as they are able to retain a significant fraction of the accreted mass at much lower mean accretion rates of about $\beta \cdot 10^{-7} M_\odot {\rm yr}^{-1}$, irrespective of the details of the model. The differences between the models are easily understood: model CONST corresponds to a simple shift of the average mass-transfer rate by a factor $1/\beta$, and the retention curve is therefore keeping its narrow shape, whereas model NORM both shifts the curve and broadens it due to the lognormal variability. The maximum is shifted slightly downwards for this model, due to the difference between the mean and the median of a lognormal model. Both curves display local minima in the retention curves near $\bar{R}_{\rm MT} \simeq 10^{-7} M_\odot$ yr^{-1}, because above this value the peaks of the mass-transfer cycles are located above $\dot{M}_{\rm crit}$. As we argue above, this is probably not realistic due to the obscuration of the X-rays from the white dwarf surface at these high accretion rates. Most likely the systems that have accretion rates in this range (approximately 10^{-8} to 10^{-7} in Fig. 2) have higher duty cycles (the depressions only appear for duty cycles $\beta \lesssim 0.1$) and/or lower peak accretion rates than assumed, and therefore also retain much of the accreted mass. Indeed in model NORM-MAX where accretion is not allowed to exceed $\dot{M}_{\rm crit}$, the retention efficiency stays high in this accretion range.

For models NORM and NORM-MAX the retention efficiency stays above zero well below $\bar{R}_{\rm MT} = 10^{-9} M_\odot$ yr^{-1}, despite the systems spending more time in accretion states with negative retention efficiencies, because even if they only spend a short time at high $R_{\rm MT}$, the fraction of mass accreted in this regime is still considerable. We believe that model NORM-MAX captures the behaviour of IIMTCs best, such as the ones modelled by Büning & Ritter (2004), because the formation of a WD envelope at high mass transfer rates is likely to quench the irradiation process (see also Sect. 3). We conclude that for all of the models the WDs can effectively grow down to average mass-transfer rates a factor of β lower than in the standard scenario without variability, irrespective of the specific shape of the variability (assuming that the mass-transfer rate does not exceed $\dot{M}_{\rm crit}$).

4 APPLICATION TO BINARY STELLAR EVOLUTION

Our models of mass-transfer cycles from Sect. 3 significantly modify and enhance the mass retention efficiency of accreting white dwarfs (see Fig. 2). This can have a significant effect on the characteristics of the population of accreting white dwarf binaries, e.g. the distribution of WD masses in cataclysmic variables. Furthermore, the growth of carbon-oxygen white dwarfs is important for understanding the rate of SNIa and their delay-time distribution in the single-degenerate channel, which we study here as an example of the implications of mass-transfer variability.

In the traditional picture without variability, the systems that can become type Ia supernovae are distributed in two regions ("islands") in the plane of the two parameters - orbital period and secondary mass - just after the formation of the WD (Li & van den Heuvel 1997; Hachisu, Kato & Nomoto 1999b; Han & Podsiadlowski 2004). One of the islands consists of progenitors where the companion has evolved to a giant before commencing the mass transfer. As mentioned above, these systems are not likely to be susceptible to IIMTCs (Büning & Ritter 2004). The other island consists of main-sequence or slightly evolved donors. If the mass of the donor star is much higher than the white dwarf mass, the mass transfer is dynamically or tidally unstable, leading to a merger of the two stars, leading to a natural upper mass limit to the island. The lower mass limit is determined by the fact that when the mass of the donor star comes near to the mass of the white dwarf, the mass transfer rate drops below the stable surface hydrogen burning limit. This typically happens around $1.5M_\odot$, and since the vast majority of CO WDs are born below $1M_\odot$, the initial mass of the donor star must be above approximately $2M_\odot$.

In the models with mass-transfer variability it is possible to retain the matter accreted at lower mass-transfer rates. This does not affect the upper limit to the donor mass, since this limit is determined by the stability of mass transfer at high rates. However, the lower mass limit of the donor star is likely to be affected. Therefore an increased retention at low mass-transfer rates allows WDs in binaries with lower donor masses to grow, and therefore allows systems with lower initial donor masses to become type Ia supernovae.

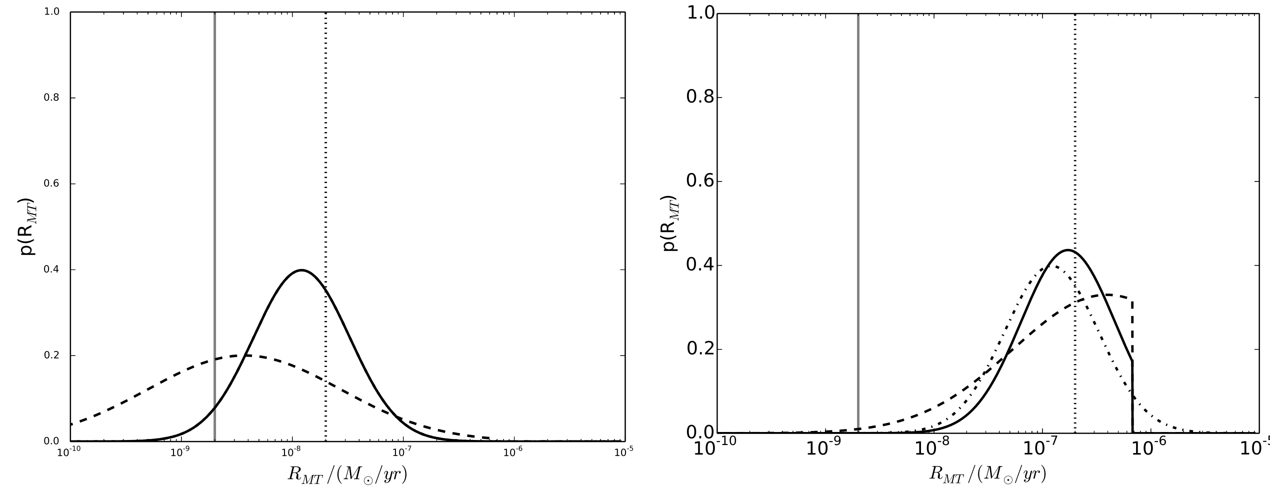

Figure 1. Example of the mass-transfer variability models, for an average mass-transfer rate of $\bar{R}_{\rm MT} = 2.0 \times 10^{-9} M_\odot$ yr^{-1} (grey line). The lines show the fraction of time that the system spends with a mass-transfer rate between $R_{\rm MT}$ and $R_{\rm MT} + {\rm d}R_{\rm MT}$ in the on-state for each of the models. The black lines indicate models with duty cycles of $\beta = 0.1$ on the left and $\beta = 0.01$ on the right. The dotted line is model CONST, the dash-dotted line is model NORM with $\sigma_e = 1$, and the solid line model NORM-MAX with $\sigma_e = 1$. The dashed line is also model NORM-MAX with with a larger spread $\sigma_e = 2$.

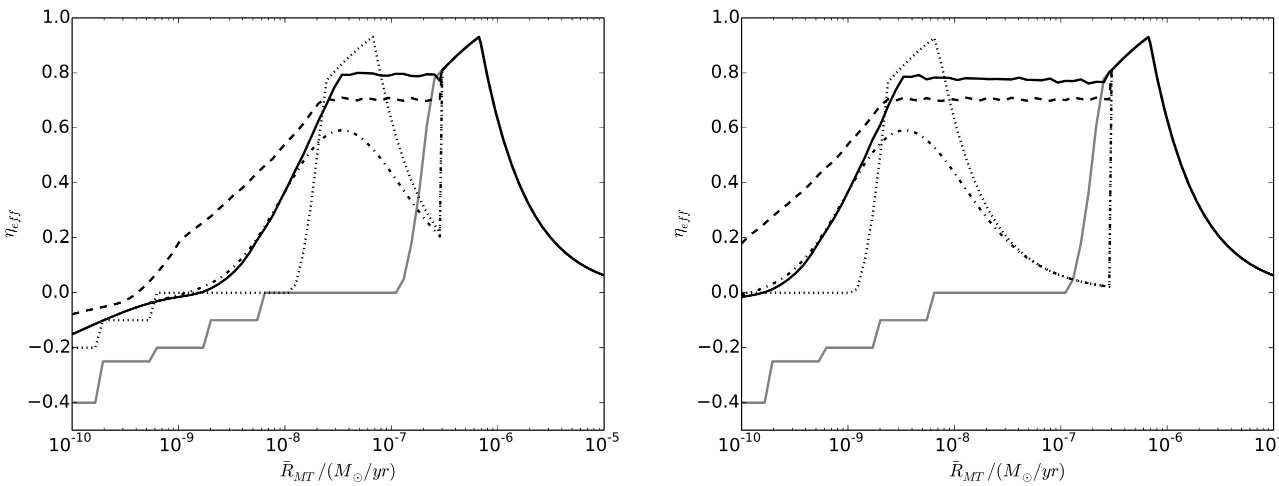

Figure 2. Effective retention efficiency as a function of average mass-transfer rate for different mass-transfer models for a $1.3M_\odot$ WD accretor. The grey line shows a model without mass-transfer variability. Model CONST is shown as a dotted line and model NORM with $\sigma_e = 1$ is shown as the dash-dotted line. The solid black and dashed lines are model NORM-MAX with $\sigma_e = 1$ and $\sigma_e = 2$ respectively. On the left models with a duty cycle of $\beta = 0.1$ are shown and on the right $\beta = 0.01$.

The limits depend on the strengths and shape of the mass-transfer variability, but also on the evolution of the donor star and its reaction to the mass loss. To better understand what our results mean for the evolution of binaries with WDs, we have calculated binary evolutionary sequences with the binary population synthesis code SeBa (Portegies Zwart & Verbunt 1996; Nelemans et al. 2001; Toonen, Nelemans & Portegies Zwart 2012; Toonen & Nelemans 2013). Our goal is to understand how the possible long-term variability affects the evolution of accreting WD binaries that might become type Ia supernovae. We therefore compare evolutionary tracks computed with a standard SeBa model to tracks where the accretion efficiency has been modified by variability. For the model without mass transfer variability, the retention efficiency η as depicted by the grey line in Fig. 2 is adopted in SeBa (see also Sect. 3.2). For model NORM-MAX, the standard retention efficiency is additionally modified to

$$\eta = \begin{cases} 0.8 & \text{if } \beta\dot{M}_{\rm ST} < \bar{R}_{\rm MT} < \dot{M}_{\rm ST} \\ 0.8(\log(\bar{R}_{\rm MT}) - \log(\beta\dot{M}_{\rm ST})) & \text{if } 0.1\beta\dot{M}_{\rm ST} < \bar{R}_{\rm MT} < \beta\dot{M}_{\rm ST} \\ 0 & \text{if } \bar{R}_{\rm MT} < 0.1\beta\dot{M}_{\rm ST} \end{cases} \tag{5}$$

where

$$\bar{R}_{\rm MT} < \dot{M}_{\rm st} = 3.1 \cdot 10^{-7}\left(\frac{M_{\rm WD}}{{\rm M}_\odot} - 0.54\right). \tag{6}$$

As can be seen from Fig. 2, other variability models might give somewhat smaller effects if the peak accretion rate is not limited. For each model, we make the simplifying assumption that the effective retention η_{eff} only depends on $\bar{R}_{MT}$ and the mass of the WD, i.e. that the shape and strength of the IIMTCs are the same irrespective of the properties of the donor star. This is clearly unrealistic. However, the goal of our study is to understand *if* the variability is likely to impact the SNIa population properties, and to indicate

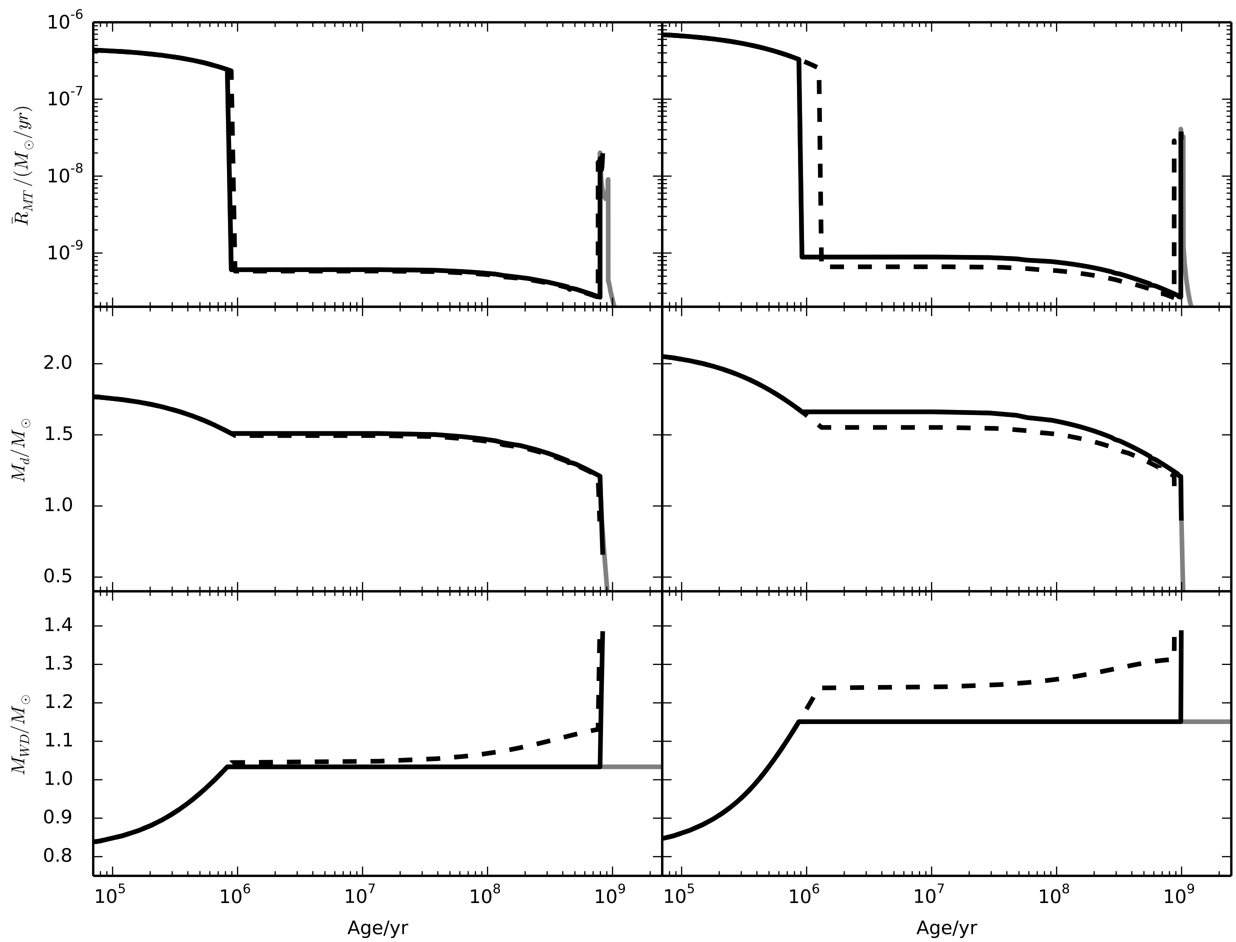

Figure 3. Binary tracks for two WD binaries starting mass transfer while the donor star is on the main sequence. The initial mass of the WD is $0.8M_\odot$, the initial orbital separation is $4.5R_\odot$, and the initial mass of the donor star is $1.8M_\odot$ (left) and $2.1M_\odot$ (right). The panels show the evolution of the (time-averaged) mass-transfer rate $\bar{R}_{\rm MT}$ (upper), the donor mass M_d (middle) and the WD mass M_{WD}(lower), for three different models: SeBa standard (grey solid), model NORM-MAX with $\beta = 0.1$ and $\sigma_e = 1$ (black solid) and model NORM-MAX with $\beta = 0.01$ and $\sigma_e = 1$ (black dashed).

what the possible effects might be, and the assumption is sufficient for this.

In Fig. 3 we show the results of evolving two WD binaries with close main-sequence companions according to the standard SeBa model (grey solid lines) and model NORM-MAX with $\sigma_e = 1$, and $\beta = 0.1$ (black solid lines) and $\beta = 0.01$ (black dashed lines). The systems behave similarly after the initial contact, when the mass-transfer rate is high. When it drops below the standard surface burning regime, differences appear, not just in the WD growth (bottom panel), but also in the time-averaged mass-transfer rate itself (top panel). This is because the matter ejected from the system carries angular momentum, which can strongly affect the evolution of the binary orbit. We assume that the matter that can not be accreted by the WD leaves the system with the specific orbital angular momentum of the WD. Note that this is the only way in which we allow the time-averaged mass transfer rate to vary in our models.

The main point of Fig. 3 is that for both systems of the standard model the mass of the WD never reaches the critical explosion mass (approximately $1.4M_\odot$, for this model the initial companion mass must be about $2.3M_\odot$ for the WD to reach this mass), whereas the variability models do reach the explosion mass.

5 BINARY POPULATION SYNTHESIS

In the previous sections we have shown that mass-transfer variability has the potential to significantly change the parameter space of initial WD binaries that can become type Ia supernovae, towards both lower-mass donor stars as well as lower-mass white dwarfs. To understand how this can affect the population of type Ia supernovae, we use the binary population synthesis (BPS) code SeBa to model the evolution of SNIa progenitors according to different mass-transfer variability models.

In SeBa, stars are evolved from the zero-age main sequence (ZAMS) until remnant formation, and, at every timestep, processes such as stellar winds, mass transfer, angular momentum loss, magnetic braking and gravitational radiation are taken into account with appropriate recipes. Magnetic braking (Verbunt & Zwaan 1981) is based on Rappaport, Verbunt & Joss (1983). The initial stellar population is generated with a Monte-Carlo approach according to appropriate distribution functions. Initial primary masses are drawn from 0.95-10$M_\odot$ from a Kroupa IMF (Kroupa, Tout & Gilmore 1993) and secondary masses from a flat mass ratio distribution between 0 and 1. The semi-major axis of the binary is drawn from a power law distribution with an exponent of -1 (Abt 1983), ranging

Table 1. Time-integrated SNIa rates in the SD channel for different mass-transfer variability models and the common envelope prescriptions in units of $10^{-4}M_{\odot}{}^{-1}$.

	γ-prescription	$\alpha_{\rm CE}$-prescription
No variability	0.59	0.79
Model NORM-MAX (β = 0.1)	1.2	1.6
Model NORM-MAX (β = 0.01)	1.4	2.0
Observed	4– > 34[1]	

[1] Maoz, Sharon & Gal-Yam (2010); Graur & Maoz (2013), see also Sect. 6 for a discussion on the observed rates.

from 0 to $10^6R_{\odot}$ and the eccentricity from a thermal distribution, ranging from 0 to 1 (Heggie 1975). Furthermore, solar metallicities are assumed. For the normalization of the simulation, a binary fraction of 50% is assumed and an extended range of primary masses between 0.1-100$M_{\odot}$.

The CE-phase (Paczynski 1976) plays an essential role in binary evolution in the formation of close binaries with compact objects. Despite of the importance of the CE-phase and the enormous efforts of the community, we still do not understand the phenomenon in detail. To take into account the uncertainty in the CE-phase in our models, we differentiate between two CE-models. The canonical CE-formalism is the $\alpha_{\rm CE}$-formalism (Tutukov & Yungelson 1979; Webbink 1984) that is based on the energy budget of the binary system. The $\alpha_{\rm CE}$-parameter describes the efficiency with which orbital energy $E_{\rm orb}$ is consumed to unbind the CE, i.e.

$$\frac{GM_1M_{1,\rm e}}{\lambda R} = \alpha_{\rm CE}(E_{\rm orb,init} - E_{\rm orb,final}), \quad (7)$$

where M_1, $M_{1,\rm env}$ and R are the mass, envelope mass and radius of the donor star and λ is the envelope structure parameter (de Kool, van den Heuvel & Pylyser 1987). Based on the evolution of double WDs, Nelemans et al. (2000) derives a value of $\alpha_{\rm CE}\lambda = 2$, which we have assumed here.

An alternative CE-prescription was introduced by Nelemans et al. (2000) in order to explain the observed distribution of double WDs systems. The γ-formalism of CE-evolution is based on the angular momentum balance. The γ-parameter describes the efficiency with which orbital angular momentum is used to expel the CE according to:

$$\frac{J_{\rm b,init} - J_{\rm b,final}}{J_{\rm b,init}} = \gamma\frac{\Delta M_1}{M_1 + M_2}, \quad (8)$$

where $J_{\rm b}$ is the orbital angular momentum of the binary, and M_2 is the mass of the companion. We assume $\gamma = 1.75$ (Nelemans et al. 2001). Although assuming the γ-prescription in BPS codes leads to a significant improvement in the synthetic double WD population, the physical mechanism remains unclear. Recently Woods et al. (2010, 2012) proposed that double WDs can be formed by stable, non-conservative mass transfer between a red giant and a main-sequence star. The effect on the orbit is a modest widening, with a result not unlike the γ-description. For a review on CE-evolution, see Webbink (2008) and Ivanova et al. (2013).

6 RESULTS AND DISCUSSION

Fig. 4 and 5 shows the systems that become type Ia supernovae in the diagram of orbital period - donor mass at the birth of the WD according to SeBa. Fig. 4a and 5a shows the distribution of classical SD SNIa progenitors. Most systems have low-mass donor stars and relatively long periods in accordance with Hachisu, Kato & Nomoto (2008) and Claeys et al. (2014). Fig. 4b and 5b show how the parameter space of systems that can become type Ia supernovae is extended for model NORM-MAX with a duty cycle of $\beta = 0.1$, and Fig. 4c and 5c for a lower duty cycle of $\beta = 0.01$. These four figures show that the parameter space of SNIa progenitors extends to lower donor masses when mass-transfer cycles are taken into account.

The time-integrated number of SNIa events is about $10^{-4}M_{\odot}{}^{-1}$, see Table 1. When taking into account mass-transfer variability according to model NORM-MAX, the rate is increased by a factor of 2 and 2.5 for $\beta = 0.1$ and $\beta = 0.01$ respectively, compared to the standard model of non-variable mass-transfer rates. The integrated rates of Table 1 are based on approximately 1000-3000 SNIa progenitors in the BPS simulation. The DTDs (assuming a single burst of star formation at $t = 0$) from all models show a strong decline with time (see Fig. 6). When mass-transfer variability is included in our simulations, the DTDs are affected at delay times from about 100Myr to a Hubble time. However the shape of the DTDs has not changed significantly.

The effect of including mass-transfer variability on the SNIa rate is mild, even though the retention efficiency of WD accretion is greatly enhanced in our models. The extra systems that become SNeIa due to mass-transfer variability is limited, compared to the number of extra ZAMS systems that are born with secondaries in the extended mass range. Our study shows that the reason for this is that, as the mass of the secondary decreases, it becomes harder to create close binaries with massive WDs. As the initial binary mass ratio is higher for these systems, the orbital separation is decreased more during the first mass-transfer episode from the WD progenitor to the secondary star, and most of the lower-mass secondaries end up being too close to survive until the formation of the WD. Most of the systems that do survive experience Roche-lobe overflow from the WD progenitor (primary) when it has become a helium star, which significantly increases their donor mass and therefore decreases the evolutionary timescale. This speed-up means that they cannot explode as the very delayed supernovae that could be expected of low-mass secondaries, but rather on relatively short timescales below 1 Gyr.

From galaxy cluster measurements and cluster iron abundances, Maoz & Mannucci (2012) and Maoz, Sharon & Gal-Yam (2010) find an observed integrated rate of $(18 - 29) \cdot 10^{-4}\ M_{\odot}^{-1}$ and a lower limit of $34 \cdot 10^{-4}\ M_{\odot}^{-1}$, respectively. Furthermore Maoz & Mannucci (2012) find that the delay time distribution that roughly follows a t^{-1} power-law shape. Neither the integrated rate from the standard model nor from the variability models is consistent with these observations. Recent measurements in volumetric surveys however have shown lower rates; $(4.4 \pm 0.2) - (5.0 \pm 0.2) \cdot 10^{-4}\ M_{\odot}^{-1}$ by Perrett & et al. (2012), $(13 \pm 1.5) \cdot 10^{-4}\ M_{\odot}^{-1}$ by Maoz, Mannucci & Brandt (2012), and $(4 - 12) \cdot 10^{-4}\ M_{\odot}^{-1}$ by Graur & Maoz (2013). It is unclear if the different observed integrated rates are due to systematic effects (for example overestimation of the cosmic star formation history or over-correction of dust extinction) or if there is a real enhancement of SNeIa in cluster galaxies (see also Maoz, Mannucci & Brandt 2012). With these recent observations of the integrated rate, the long-standing problem of BPS studies predicting too low SNIa rates has reduced. The SNIa rates of our most optimistic models of low duty cycles are comparable with the lowest observed integrated rates, but the corresponding synthetic DTD shows a stronger decline with time than the observed DTDs.

The increase in the SNIa rate in the mass-transfer variability models compared to the standard model is limited by the formation of close binaries with low mass companions. This depends on our understanding of binary evolution. A comprehensive comparison of four BPS codes (including SeBa, see Toonen et al. 2014) showed that differences between the predictions of BPS codes for low- and intermediate-mass stars are not caused by numerical effects in the codes, but by different assumptions for phases in stellar and binary evolution that are not understood well. When these assumptions are equalized, the synthetic populations of the four BPS codes are similar. Important assumptions (or uncertain processes) for the SD channel are the retention efficiency for WD accretion and CE-evolution (Bours, Toonen & Nelemans 2013; Toonen & Nelemans 2013; Toonen et al. 2014; Claeys et al. 2014). Bours, Toonen & Nelemans (2013) shows that the effect of different retention efficiencies can effect the SNIa rate by a factor 3-4 to even more than a factor 100, which explains for a large degree the large disagreement in the predictions of the SD SNIa rate by different BPS studies. Regarding the poorly understood common-envelope phase, we have shown that mass-transfer variability can effect the SNIa rate to a comparable degree as CE-evolution (Ruiter, Belczynski & Fryer 2009; Mennekens et al. 2010; Bours, Toonen & Nelemans 2013; Claeys et al. 2014, this paper). Especially now that the gap between observed and synthetic SNIa rates has decreased, it is important to take uncertainties in binary evolution such as retention efficiency, CE-evolution and mass-transfer variability into account.

7 CONCLUSIONS

We have studied the effect of mass-transfer variability on accreting WDs in binary companion stars. Long-term mass-transfer variability can be induced by e.g. irradiation of the donor star by the accreting WD or by cyclic variations of the Roche lobe from mass loss episodes (Knigge, Baraffe & Patterson 2011). The timescale of the variability should be longer than the thermal timescale of the non-degenerate surface layer of the WD so that the surface burning is affected. On the other hand, the timescale of the mass-transfer cycles should not be too long, such that the binary is not affected in any observable way (e.g. strong bloating of donor stars by irradiation). Currently observations hardly constrain the theoretical models of mass-transfer variability (e.g. Büning & Ritter 2004) and therefore we have constructed a number of models rather than studying the details of a particular mass-transfer variability model. We show that long-term mass-transfer variability can significantly affect the accretion process and retention efficiency of mass transfer towards WDs.

Mass-transfer variability and accompanying enhanced retention efficiencies is likely to impact the properties of accreting WD binaries. We find that irrespective of the specific shape of the mass-transfer variability, for all variability models the WDs can effectively grow down to average mass-transfer rates a factor of β lower than in the standard scenario without variability. As an example, we study the evolution of SNIa progenitors from the single-degenerate channel. We find that if mass-transfer cycles take place, the parameter space of systems that become SNIa events is increased towards low mass donor stars. Furthermore we find that the integrated SNIa rate increases by a factor of about 2-2.5, which is comparable with the lower limit of the observed rates (see Maoz & Mannucci 2012; Perrett & et al. 2012; Maoz, Mannucci & Brandt 2012; Graur & Maoz 2013). Variability models in which the maximum mass-transfer rate is not limited affect the SNIa rate less. In conclusion, mass-transfer cycles potentially lead to a new formation channel of SNIa events that can significantly contribute to the SNIa rate.

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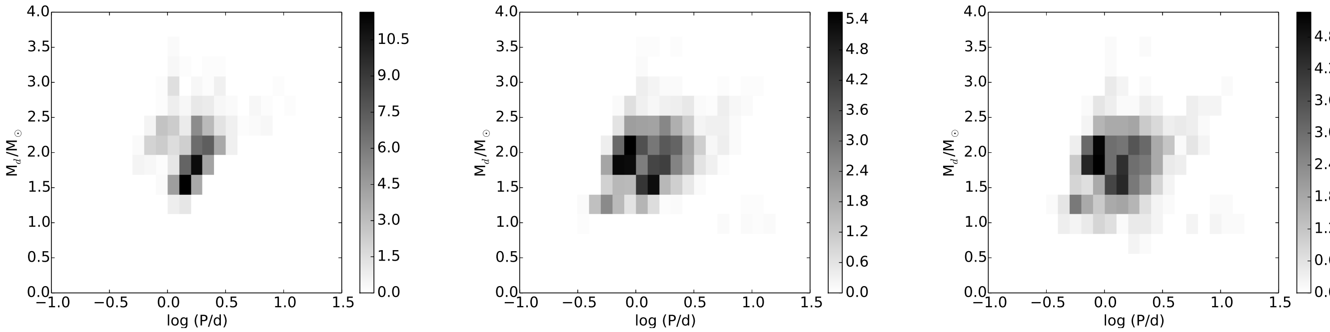
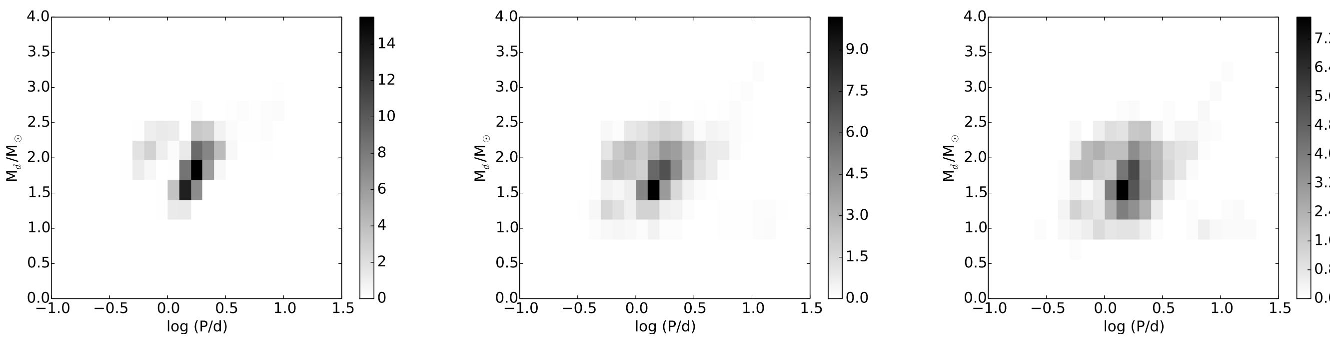


(a) (b) (c)

Figure 4. Donor mass vs. orbital period after the last mass-transfer event in which the WD is formed for the SD SNIa progenitors assuming the γ-algorithm with γ = 1.75 for three different mass-transfer models. On the left a model without mass-transfer variability, in the middle model NORM-MAX with β = 0.1 and on the right model iii with a duty cycle of β = 0.01. The intensity of the grey scale corresponds to the density of objects on a linear scale in percentages of all SD SNIa progenitors of the corresponding model.



(a) (b) (c)

Figure 5. Donor mass vs. orbital period after the last mass-transfer event in which the WD is formed for the SD SNIa progenitors assuming the $\alpha_{\rm CE}$-algorithm with $\alpha_{\rm CE}\lambda$ = 2. On the left a model without mass-transfer variability, in the middle model NORM-MAX with β = 0.1 and on the right model NORM-MAX with a duty cycle of β = 0.01. The intensity of the grey scale corresponds to the density of objects on a linear scale in percentages of all SD SNIa progenitors of the corresponding model.

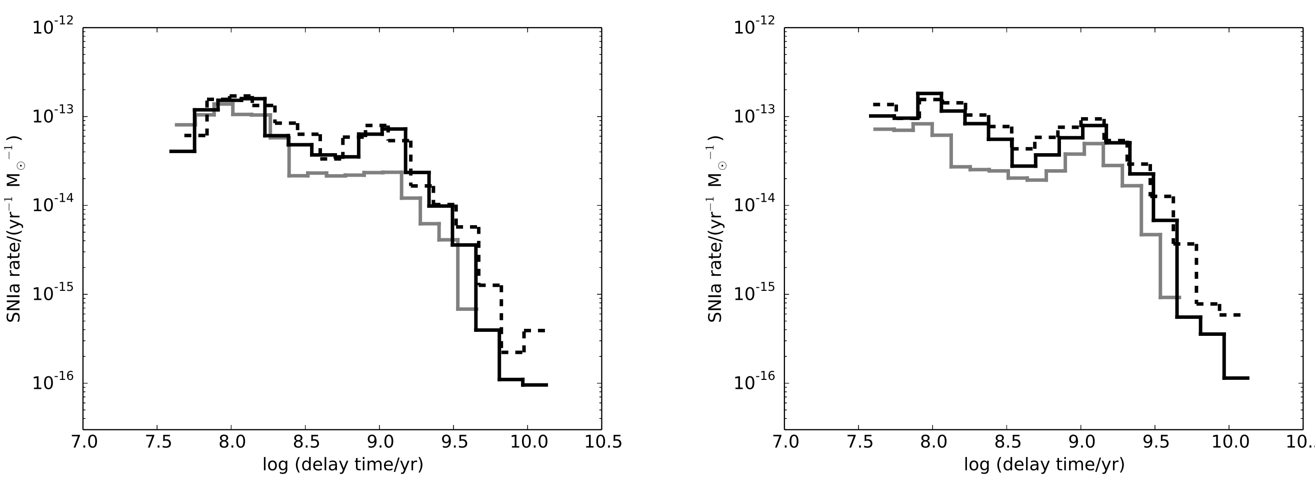


Figure 6. Delay time distribution of SNIa events from the SD channel for three different mass-transfer models. The black lines indicate model NORM-MAX with β = 0.1 (solid) and β = 0.01 (dashed). The grey line shows a model without mass-transfer variability. On the left assuming the γ-algorithm with γ = 1.75 and on the right assuming the $\alpha_{\rm CE}$-algorithm with $\alpha_{\rm CE}\lambda$ = 2.

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