Measurement of branching ratios and τ polarization from $\tau \rightarrow e\nu\bar{\nu}$, $\tau \rightarrow \mu\nu\bar{\nu}$, and $\tau \rightarrow \pi (K)\nu$ decays at LEP

OPAL Collaboration

From a sample of 3308 $e^+e^-\rightarrow \tau^+\tau^-$ events with an estimated background of 1.9%, we find 964 $\tau \rightarrow e\nu\bar{\nu}$, 903 $\tau \rightarrow \mu\nu\bar{\nu}$, and 309 $\tau \rightarrow \pi(K)\nu$ candidates. Efficiency and background estimates determined from both Monte Carlo and control sample studies yield the following branching ratios: $B(\tau \rightarrow e\nu\bar{\nu}) = 17.4\pm0.5$ (stat) $\pm0.4$ (sys)%, $B(\tau \rightarrow \mu\nu\bar{\nu}) = 16.8\pm0.5\pm0.4\%$, and $B(\tau \rightarrow \pi(K)\nu) = 12.1\pm0.7\pm0.5\%$. These values are in good agreement with previous measurements. The measured lepton branching ratios, when combined with the world-average measured value for the $\tau$ lifetime, yield a ratio of the $\tau$ Fermi coupling constant to that of the lighter leptons given by $G_{\tau}/G_{e,\mu} = 0.92\pm0.04$, where it is assumed $G_\tau = G_\mu = G_{e,\nu}$. The average $\tau$ polarization at the $Z^0$ resonance is measured to be $-0.01\pm0.09$ from an analysis of the momentum spectra of the electron, muon, and pion candidates, implying that the ratio of vector to axial vector couplings of the $\tau$ to the $Z^0$ is $v_\tau/a_\tau = 0.01\pm0.04$. The measurements of the average polarizations in the forward and backward hemispheres lead to the efficiency-corrected, forward-backward polarization asymmetry $A_P^{FB} = -0.22\pm0.10$, implying for the electron couplings to the $Z^0$ the ratio $v_e/a_e = 0.15\pm0.07$. Since these values for the tau and electron couplings are consistent with one another, we assume lepton universality to derive $v_\tau/a_\tau = 0.05\pm0.04$ and a value for the weak mixing angle of $\sin^2\theta_W = 0.237\pm0.009$, with no ambiguity introduced by the relative signs of $v$ and $a$. 
1. Introduction

The ample production of $\tau$ pairs in $e^+e^-$ annihilation at the $Z^0$ resonance provides a useful laboratory for studying both the decay properties of the $\tau$ particle and the electroweak couplings of the $\tau$ to the $Z^0$ boson. Measurements of the branching ratios for $\tau \rightarrow \nu \bar{\nu}$, $\tau \rightarrow \mu \bar{\nu}$, and $\tau \rightarrow \pi(K)\nu$ decays from a nearly pure sample of $\tau$ pair events are presented here. In addition, analyses of the momentum spectra of the decay particles provide measurements of the average $\tau$ polarization and of the forward-backward $\tau$ polarization asymmetry on the resonance. From these measurements relations are derived for the electroweak couplings of both the tau and the electron to the $Z^0$.

Two long-standing discrepancies add interest to the measurements of the branching ratios into the $e$, $\mu$, and $\pi$ channels. The first discrepancy is between the world-average measured $\tau \rightarrow \nu \bar{\nu}$ and $\tau \rightarrow \mu \bar{\nu}$ branching ratios and those expected from the world-average measured lifetime of the $\tau$ [1,2], an expectation based on the measured $\mu$ lifetime, assuming lepton universality in charged-current couplings. This discrepancy can be expressed in terms of the ratio of the apparent Fermi coupling constant of the $\tau$ to that (assumed identical) of the lighter leptons $e$ and $\mu$: $G_\tau/G_{e,\mu} = 0.95 \pm 0.03$. The second discrepancy, which has a greater statistical significance, is the difference between the measured inclusive $\tau \rightarrow$ one-prong branching ratio ($86.1 \pm 0.3\%$) and the sum of the measured exclusive one-prong branching ratios ($<80.2 \pm 1.4\%$, using theoretical constraints to limit poorly measured channels) [1–3]†. Both of the branching ratio discrepancies provide impetus for new measurements of exclusive $\tau$ decays.

Studies of the decay products of $\tau$ pairs produced on the $Z^0$ resonance also allow one to extract the couplings of the $\tau$ to the $Z^0$ [5–11]. An imbalance in the production of left-handed and right-handed $\tau$ particles leads, through parity violation in the $\tau$ decay, to a measurable distortion of the momentum spectra of the decay products. At the peak of the $Z^0$ resonance, the average polarization of the $\tau^-$ is expected to be:

$</p>
\[ A_{\text{pol}}^{\text{FR}}(c) \approx - \frac{3c}{3 + c^2} \lambda_e, \]

which depends only on the electron coupling constants. If lepton universality is imposed \( \lambda_s = \lambda_e \equiv \lambda \), the average polarizations \( \langle P_\tau \rangle_{F,B} \) in the forward and backward hemispheres in the ranges \([0, c]\) and \([-c, 0]\) in \( \cos \theta \) can be expressed as

\[ \langle P_\tau \rangle_F \approx -\lambda \left( 1 + \frac{3c}{3 + c^2} \right), \]

\[ \langle P_\tau \rangle_B \approx -\lambda \left( 1 - \frac{3c}{3 + c^2} \right). \]

(1)

For a sample of \( \tau \) pairs of polarization \( P \equiv \langle P \rangle \) the momentum spectra for both the \( \tau^+ \) and \( \tau^- \) should have the following distributions [5]:

\[ \frac{1}{N_\tau}(dN_\tau/dx_\tau) \approx \frac{1}{2} \left[ (5 - 9x_\tau^2 + 4x_\tau^3) + P(1 - 9x_\tau^2 + 8x_\tau^3) \right]. \]

(2)

\[ \frac{1}{N_\tau}(dN_\tau/dx_\tau) \approx 1 + P(2x_\tau - 1). \]

(3)

where \( x_\tau \) and \( x_\tau \) are the momenta of the leptons and pion normalized to the beam energy \( E_{\text{beam}} \) and where radiative corrections (discussed in section 5) and threshold effects (negligible) have been ignored. It is assumed in extracting branching ratios and polarization values that the charged-current coupling of the \( \tau \) is pure V–A, an assumption consistent with experimental observation [1,14].

In this measurement, the couplings of the \( \tau \) to the \( Z^0 \) are derived from the average polarizations for the three decay channels in both the forward and backward hemispheres, while the couplings of the electron to the \( Z^0 \) are derived from the asymmetries between polarizations measured separately in the forward and backward hemispheres. In addition, by constraining the couplings derived from the different hemisphere polarizations to be consistent with lepton universality, we obtain universal lepton couplings to the \( Z^0 \) and a value for \( \sin^2 \theta_W \).

2. The OPAL detector

The OPAL detector consists of 14 subdetectors as described below, several of which are required for reliable identification of \( \tau \) pair events and of different \( \tau \) decay channels. This measurement uses the 1990 data sample where the subdetectors necessary to the identification were fully operational. The sample corresponds to an integrated luminosity of 5.8 pb\(^{-1}\), recorded at center-of-mass energies \( (E_{\text{CM}}) \) between 88.28 and 94.28 GeV, with about half the data recorded at the peak of the \( Z^0 \) resonance.

OPAL is a general-purpose, \( 4\pi \) detector with approximate cylindrical symmetry about the \( e^+e^- \) beam axis [15]. The coordinate system is defined with \( +z \) along the \( e^- \) beam direction, where \( \theta \) and \( \phi \) are the polar and azimuthal angles, respectively. The central tracking chambers measure the momenta of charged particles over almost the entire solid angle in a uniform magnetic field of 0.435 T created by a solenoidal coil. The innermost tracking chamber is a precision vertex chamber. This is surrounded by a large-volume jet drift chamber divided into 24 azimuthal sectors, each with a radial plane of 159 axial anode sense wires, where charge deposited on each wire provides a measurement of the energy loss \( dE/dx \) of charged particles in the chamber. In the barrel region, the jet chamber is surrounded by a cylindrical array of 192 planar drift chambers which measure the \( z \)-coordinates of charged particles as they leave the jet chamber.

Together, the magnetic coil and pressure vessel of the central tracking chambers have an effective thickness of about \( 2X_0/\sin \theta \) in the region \( |\cos \theta| < 0.7 \) (where \( X_0 \) is one radiation length) and are surrounded in the range \( |\cos \theta| < 0.82 \) by a barrel time-of-flight(TOF) counter array consisting of 160 scintillator bars with photomultiplier readout at both ends and, outside the TOF, by an electromagnetic calorimeter (ECAL) with a presampler. The calorimeter covers the region \( |\cos \theta| < 0.98 \) and is divided into a barrel part \((|\cos \theta| < 0.82)\) and two endcaps. Critical to this measurement is the barrel part which consists of 9440 lead-glass blocks of 24.6 radiation lengths, pointing toward the beam interaction region, each with a cross section of \( 10 \times 10 \) cm\(^2\). In the endcap there are 2264 lead-glass blocks of the same cross section but with axes parallel to the beam direction. The barrel presampler, which measures electromagnetic showers originating in the magnetic coil, consists of two layers of axial, limited-streamer tubes at a 1 cm pitch, with both wire and cathode-strip readout. The instrumented magnet return yoke serves...
as a hadron calorimeter (HCAL) and muon tracker, consisting of 9 layers of axial, limited-streamer tubes sandwiching 10 cm layers of iron, with inductive read-out of the tubes onto large pads and onto 4 mm wide aluminum strips. The detector is surrounded by four layers of (MUON) drift chambers for the detection of muons emerging from the hadron calorimeter. The hadron calorimetry in the endcaps is similar to that in the barrel, and four layers of limited-streamer tubes mounted in vertical walls transverse to the beam provide the primary muon identification in this region. The integrated luminosity is measured using low-angle Bhabha scattering with two forward detector calorimeters in the region $40 < \theta < 120$ mrad.

For this measurement the momentum resolution of the tracking detectors is $\Delta p/p \approx 9\%$ for $p_\perp \approx 45$ GeV, determined from $e^+e^- \rightarrow \mu^+\mu^-$ events. In the barrel electromagnetic calorimeter the energy resolution is $\Delta E/E \approx 3\%$ for $E \approx 45$ GeV, determined from $e^+e^- \rightarrow e^+e^-$ events. For Monte Carlo studies the OPAL detector response is simulated by a program [16] that treats in detail the detector geometry and material, as well as effects of detector resolution and efficiencies.

3. Selection of $e^+e^- \rightarrow \tau^+\tau^-$ events

In general, $\tau$ pair events are characterized by two nearly back-to-back "jets" of 1 or more charged particles, often with accompanying neutral hadrons or photons. Each jet loses energy to one or more undetected neutrinos, leading to a non-zero total momentum transverse to the beam direction and resulting in a total reconstructed energy less than $E_{CM}$.

In the vicinity of the $Z^0$ resonance, $\tau$ pair events are easier to distinguish from backgrounds than is the case at lower center of mass energies. There are four main backgrounds to consider. The first two are $e^+e^- \rightarrow e^+e^-$ and $e^+e^- \rightarrow \mu^+\mu^-$ events, which can be identified by the presence of two very high-momenta, back-to-back charged particles with the full $E_{CM}$ deposited in the electromagnetic calorimeter for $e^+e^- \rightarrow e^+e^-$ and with very little ECAL energy for $e^+e^- \rightarrow \mu^+\mu^-$. To distinguish $\tau$ pair events from these backgrounds, a detector must possess good charged-particle tracking, muon particle identification, and electromagnetic calorimetry. Hermeticity of the calorimeter ensures correct identification of $e^+e^- \rightarrow e^+e^-\gamma$ and $e^+e^- \rightarrow \mu\gamma$ events which have been a troublesome background for some previous experiments. A third background to $e^+e^- \rightarrow \tau^+\tau^-$ events comes from multihadron production; this is reduced at LEP energies by the tendency for particle multiplicity to rise with $E_{CM}$, while for $\tau$ decays, the multiplicity is constant with $E_{CM}$. A fourth background comes from two-photon processes $e^+e^- \rightarrow (e^+e^-)X$ where the final-state electron and positron escape undetected at low angles and the system X is misidentified as a low-visible-energy $\tau$ pair event. At LEP the relative size of this background is reduced compared to lower-energy experiments for two reasons. One is that $e^+e^- \rightarrow (e^+e^-)X$ lacks the $Z^0$ resonance which enhances $e^+e^- \rightarrow \tau^+\tau^-$ production. The other is that low-mass meson resonances, which contribute much to the hadronic component of X, have an energy distribution nearly independent of $E_{CM}$ for a given requirement on colinearity of the X decay products, while the total energy of $\tau$ decay products rises linearly with $E_{CM}$. The dominant components of the higher-energy X systems are from $e^+e^- \rightarrow e^+e^-e^+e^-$ and $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$ events, which are characterized in general by two charged particles with nearly equal but opposite momenta transverse to the beam. Less important backgrounds arising from cosmic rays and single-beam interactions can be suppressed with straightforward requirements on TOF, on the location of the primary event vertex, and on event topology. The consequence of the naturally reduced backgrounds to $e^+e^- \rightarrow \tau^+\tau^-$ at LEP is that high purity can be attained without sacrificing selection efficiency, and that selection requirements that strongly bias for or against certain $\tau$ decay modes are unnecessary. This substantially reduces the systematic uncertainties in branching ratio measurements due to the event selection.

In selecting $\tau$ pair events and identifying $\tau$ decay channels, only good charged tracks and electromagnetic clusters are considered. A good charged track must have a minimum momentum transverse to the beam of 100 MeV, a measured $|d_0| < 2$ cm, and a measured $|z_0| < 75$ cm, where $|d_0|$ is the distance of closest approach of the track to the beam axis, and $|z_0|$ is the displacement along the beam axis from the nominal interaction point at the point of closest ap-
approach to the beam. The track must also have at least 20 measured space points in the main jet chamber, and at least one point within 75 cm of the beam axis. In the barrel, a good ECAL cluster, which is a group of contiguous lead-glass blocks, must have a minimum energy of 100 MeV; in the endcap, the minimum energy is 200 MeV, and the shower cluster must contain at least two lead-glass blocks, no one of which may contribute more than 99% to the cluster’s energy. The ECAL cluster energies used in $\tau \rightarrow e\nu\bar{\nu}$ selection are corrected for the expected energy loss of electrons in the coil preceding the calorimeter.

The multihadron background is removed by requiring the number of good charged tracks be in the range 2 to 6 and that the number of good ECAL clusters be no greater than 10. Cosmic ray backgrounds are removed by requiring that there be at least one good charged track with a measured $|d_0| < 0.5$ cm and a measured $|z_0| < 20$ cm, and requiring that the magnitude of the average $z$ of all good tracks at their points of closest approach to the beam be less than 20 cm. In addition, there must be at least one TOF signal within 10 ns of the nominal expected value. An event is rejected if all pairs of TOF signals separated by more than 165° in azimuth have time differences greater than 10 ns.

For this analysis, it is convenient to treat each tau decay as a jet, as defined in ref. [13], where charged tracks and ECAL clusters are assigned to cones of half-angle 35°. Signals occurring in the presampler, HCAL or MUON subdetectors within the solid angle defined by a jet cone are also assigned to the jet. Only presampler signals associated to an ECAL shower are included. A $\tau$ pair candidate must contain exactly two jets, each with at least one charged track and with a total track and cluster energy exceeding 1% of the beam energy. To remove backgrounds due to two-photon processes and to remove events with extreme radiation, the acolinearity between the two charged jets must be less than 15°, where the directions of the jets are given by the momentum sums of the tracks and clusters. To avoid regions of non-uniform calorimeter response where the separation of different $\tau$ decay modes is significantly degraded, a fiducial requirement is imposed on the directions of the $\tau$ jets. The average value of $|\cos \theta|$ for the two charged jets must satisfy $|\cos \theta| < 0.68$. This requirement incurs a loss in acceptance of 41%.

Rejection of the $e^+e^- \rightarrow e^+e^-$ background depends on the total ECAL cluster energy and total track energy. A $\tau$ pair candidate must satisfy:

$$\sum_{i} E_{i}^{\text{clust}} \leq 0.8 E_{\text{CM}}$$

or

$$\sum_{i} E_{i}^{\text{clust}} + 0.3 \sum_{j} E_{j}^{\text{track}} \leq E_{\text{CM}},$$

where the summations run over all good clusters and tracks in the event. These requirements take advantage of the high track and shower energies associated with $e^+e^- \rightarrow e^+e^-$ events, relying mainly on the shower energy measurement which has the better resolution at high energies.

Rejection of $e^+e^- \rightarrow \mu^+\mu^-$ events depends mainly on the muon identification provided by the ECAL, HCAL, and MUON detectors and on the high momenta measured for the charged tracks. An event is rejected if it contains at least two muon candidates, and the scalar sum of the charged track momenta plus the energy of the most energetic ECAL cluster is greater than 0.60$E_{\text{CM}}$. A muon candidate must have a momentum greater than 6 GeV, and must satisfy at least one of the following 3 requirements: (1) $\geq 2$ associated signals in the four outer MUON chambers (barrel or endcap); (2) $\geq 4$ associated signals in the hadron calorimeter, with $\geq 1$ in the outer 3 layers; or (3) momentum greater than 15 GeV and associated ECAL energy less than 3 GeV.

Rejection of residual two-photon backgrounds exploits the low visible energies and very low net transverse momenta typical of $e^+e^- \rightarrow (e^+e^-)X$ events. An event is rejected if the sum of visible energies of the jets (taken for each jet as the maximum of the sum of the assigned charged track and ECAL cluster energies) is less than 3% of $E_{\text{CM}}$. Further, if the total visible energy is less than 20% of $E_{\text{CM}}$, the event is rejected if the missing transverse momenta, calculated separately for charged tracks and for ECAL clusters, are both less than 2 GeV.

These requirements select a final sample of 3308 $\tau$ pair candidates in the barrel region of the OPAL detector. From Monte Carlo studies [17] the selection efficiency is estimated to be 54.7±0.7%, in good agreement with the number of events observed, given the distribution of integrated luminosity with $E_{\text{CM}}$ and standard model expectations for $\sigma (e^+e^- \rightarrow$
Table 1
Estimated background contaminations in the 3308 $\tau$ pair candidate events.

<table>
<thead>
<tr>
<th>Background</th>
<th>Contamination (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+e^- \rightarrow e^+e^-$</td>
<td>0.2±0.2</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow \mu^+\mu^-$</td>
<td>0.8±0.9</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow q\bar{q}$</td>
<td>0.7±0.3</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow (e^+e^-)X$</td>
<td>0.2±0.2</td>
</tr>
<tr>
<td>total</td>
<td>1.9±1.0</td>
</tr>
</tbody>
</table>

$\tau^+\tau^-$. Although the overall event selection efficiency does not affect the branching ratios measured here, it is necessary to understand any biases that enhance or suppress decay modes of interest. Monte Carlo studies indicate that the above selection requirements lead to enhancements of 1.006±0.006, 0.980±0.006, and 0.978±0.008 for $\tau \rightarrow e\nu\bar{\nu}$, $\tau \rightarrow \mu\nu\bar{\nu}$, and $\tau \rightarrow \pi(K)\nu$ channels, respectively. Residual backgrounds have been estimated from Monte Carlo studies of $e^+e^- \rightarrow e^+e^-$ [18], $e^+e^- \rightarrow \mu^+\mu^-$ [17], $e^+e^- \rightarrow q\bar{q}$ [19,20], and $e^+e^- \rightarrow (e^+e^-)X$ [21] events, as shown in table 1. The total background is found to be 1.9±1.0%.

4. Branching ratio measurements

Measurements of the branching ratios for $\tau \rightarrow e\nu\bar{\nu}$, $\tau \rightarrow \mu\nu\bar{\nu}$, and $\tau \rightarrow \pi(K)\nu$ are based on counting the number of candidate jets in the $\tau$ pair sample and correcting for efficiency, backgrounds, and event selection bias. For the $\tau \rightarrow e\nu\bar{\nu}$ and $\tau \rightarrow \mu\nu\bar{\nu}$ channels the Monte Carlo identification efficiencies can be checked directly with independent, high-statistics, control samples of isolated electrons and muons from $e^+e^- \rightarrow e^+e^-$, $e^+e^- \rightarrow (e)e', e^+e^- \rightarrow \mu^+\mu^-$, and $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$ events in the data. For the $\tau \rightarrow \pi(K)\nu$ channel, no such high-statistics control sample can be obtained from the data, but a variety of indirect cross-checks allow estimation of uncertainties due to inaccuracies in Monte Carlo simulation.

4.1. Selection of $\tau \rightarrow e\nu\bar{\nu}$ candidates

$\tau \rightarrow e\nu\bar{\nu}$ candidates must satisfy the following requirements:
- The number $N_{chrg}$ of good charged tracks assigned to the candidate jet must satisfy:

$$N_{chrg} \leq 2,$$

where up to one extra track is allowed for $\tau \rightarrow e\nu\bar{\nu}$ decays in order to reduce inefficiency due to conversions from radiation emitted by the electron. When there are two tracks assigned to a jet, the higher momentum track is taken as the electron candidate.
- The energy $E_{clus}$ of the cluster associated to the electron candidate and the candidate's reconstructed track momentum $p_{tok}$ must satisfy:

$$0.7 < E_{clus}/p_{tok} < 2.0.$$
- To reject hadronic $\tau$ decays accompanied by $\pi^0 \rightarrow \gamma\gamma$, it is required that the total cluster energy assigned to the jet (excluding the cluster associated to the electron candidate track) satisfy:

$$E_{excess} < 0.04E_{beam},$$

which corresponds to a maximum excess energy of about 2 GeV. This requirement also suppresses $e^+e^- \rightarrow \tau^+\tau^-\gamma$ events.
- Further rejection of hadronic backgrounds with $\pi^0$'s present comes from requiring that the difference in azimuth $\delta \phi_{max}$ between the track and the presampler signal farthest away in azimuth but still assigned to the jet satisfy:

$$\delta \phi_{max} < 5^\circ.$$
- Backgrounds from $\tau \rightarrow \mu\nu\bar{\nu}$ and $\tau \rightarrow \pi(K)\nu$ are suppressed by requiring that the number of HCAL layers containing a signal that are assigned to the cone satisfy:

$$N_{layers}^{HC} \leq 1.$$
- The shape of the ECAL cluster associated to the track must be consistent with that expected from an electron. The minimum number of lead-glass blocks containing at least 90% of the cluster energy must satisfy:

$$N_{blk}^{90} \leq 3.$$
- To remove a small residual contamination from $e^+e^- \rightarrow e^+e^-$ events where the energy of one electron is mismeasured, an electron candidate is rejected if the opposite jet consists of a single charged particle of momentum greater than 0.75$E_{beam}$ and the acoplanarity between the electron candidate track and the opposite track is less than 0.1°. The acoplanarity is defined as the acolinearity of the track directions in the plane transverse to the beam. The resolution on the
acoplanarity is better than 0.03° for high-momentum tracks.

Finally, in order to ensure reliable electron identification with low background and well understood efficiency, the electron’s shower energy must satisfy:

$$x_e \equiv \frac{E_{\text{chls}}}{E_{\text{beam}}} > 0.05,$$

where $E_{\text{chls}}$ is the total ECAL energy in the jet cone.

After all requirements, 964 $\tau \rightarrow e\nu\bar{\nu}$ candidates are selected from the 6616 $\tau$ jets within the acceptance. Fig. 1a shows the distributions in $E_{\text{chls}}/p_{\text{trk}}$ for data and Monte Carlo expectation after all selection requirements except the requirement on $E_{\text{chls}}/p_{\text{trk}}$. Fig. 2a shows the measured distribution in $x_e$, along with the expectation from Monte Carlo, including background. The vertical dashed line indicates the cut at $x_e = 0.05$.

The $\tau \rightarrow e\nu\bar{\nu}$ selection efficiency, which is obtained from Monte Carlo study, is corrected bin-by-bin in momentum with factors derived from control samples of electrons at low $x_e$ in $e^+e^- \rightarrow (e)e\gamma$ events and electrons at $x_e \approx 1$ in $e^+e^- \rightarrow e^+e^-$ events. For the branching ratio measurement, these give an integrated correction of 0.969±0.016 to the total $\tau \rightarrow e\nu\bar{\nu}$ selection efficiency estimate, yielding a final corrected efficiency of 79.9±1.3% for $\tau$ pairs within the acceptance. Most of the correction accounts for inaccuracies in Monte Carlo simulation of ECAL shower shapes for low-$x_e$ electrons. The final corrected efficiency is shown plotted as a function of $x_e$ in fig. 3a, where radiative/ECM effects and event selection bias are included, as described in section 5.
Fig. 3. Estimated efficiencies for selection of (a) $\tau \rightarrow e\nu\bar{\nu}$, (b) $\tau \rightarrow \mu\nu\bar{\nu}$, and (c) $\tau \rightarrow \pi(K)\nu$ candidates plotted as a function of true momenta divided by $E_{\text{beam}}$. The vertical dashed line indicates the minimum momentum of accepted candidates. The efficiencies include radiative/E_{CM} effects and event selection biases. The low efficiency of the first bin for the $\tau \rightarrow e\nu\bar{\nu}$ selection is due to a loss of candidates to the region $x_e < 0.05$ which are not recovered in the unfolding procedure.

A cross-check of the efficiency estimate comes from the $dE/dx$ distribution for low-momentum tracks ($0.05 < x_e < 0.10$) failing the $\tau \rightarrow e\nu\bar{\nu}$ selection requirements. Good electron/hadron discrimination provided by the $dE/dx$ measurements reveals a loss of $4.3 \pm 0.7\%$ in the data relative to the identified candidates, in good agreement with the loss of $4.8 \pm 0.2\%$ expected from Monte Carlo simulation. Estimated backgrounds from Monte Carlo studies are shown in table 2 with a total of $5.7 \pm 1.3\%$, coming largely from hadronic decays of the $\tau$. The uncertainty on the background estimate is found primarily from comparisons of $E_{\text{clus}}/p_{\text{thik}}$ distributions in the data and Monte Carlo after inverting the selection requirements on $N^{\text{HC}}_{\text{layers}}$, $E_{\text{excess}}$, and $\delta\phi_{\text{max}}$.

4.2. Selection of $\tau \rightarrow \mu\nu\bar{\nu}$ candidates

$\tau \rightarrow \mu\nu\bar{\nu}$ candidates must satisfy the following requirements:

- The number $N_{\text{chrg}}$ of good charged tracks assigned to the candidate jet must satisfy:

$$N_{\text{chrg}} = 1.$$  

- Most $\tau \rightarrow \mu\nu\bar{\nu}$ decays within the acceptance are characterized by a small energy deposition in the ECAL and associated signals in the HCAL and MUON subdetectors consistent with the passage of a minimum-ionizing particle. In order to accept muons that enter inactive regions of the HCAL or MUON or that are accompanied by radiation, however, it is required only that a $\tau \rightarrow \mu\nu\bar{\nu}$ candidate satisfy at least two of the following three requirements:

(i) Identification by the outer four MUON chamber layers [MUID]:

$$N^{\text{MU}}_{\text{layers}} \geq 2.$$  

where $N^{\text{MU}}_{\text{layers}}$ is the total number of layers with signals associated to the track in the barrel or endcap MUON detector.

(ii) Identification by the hadron calorimeter [HCID]:

$$N^{\text{HC}}_{\text{layers}} \geq 4 \quad \text{and} \quad N^{\text{HC}}_{\text{hits/layer}} < 3,$$  

where $N^{\text{HC}}_{\text{layers}}$ is the number of HCAL layers containing signals associated to the track and where $N^{\text{HC}}_{\text{hits/layer}}$ is the total number of HCAL signals assigned to the jet divided by $N^{\text{HC}}_{\text{layers}}$.

(iii) Identification by the electromagnetic calorimeter [ECID]:

$$E_{\text{clus}} < 2 \text{ GeV},$$  

- Any associated signals in the HCAL must be consistent with the passage of a minimum-ionizing particle, even when the HCID condition is not satisfied. A $\tau \rightarrow \mu\nu\bar{\nu}$ candidate is rejected if

<table>
<thead>
<tr>
<th>Background</th>
<th>Contamination (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau \rightarrow \pi\nu$</td>
<td>$2.1 \pm 0.4$</td>
</tr>
<tr>
<td>$\tau \rightarrow \rho\nu$</td>
<td>$2.4 \pm 0.7$</td>
</tr>
<tr>
<td>$\tau \rightarrow \text{other}$</td>
<td>$0.3 \pm 0.1$</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow e^+e^-$</td>
<td>$0.3 \pm 0.3$</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow (e^+e^-)X$</td>
<td>$0.6 \pm 0.6$</td>
</tr>
<tr>
<td>total</td>
<td>$5.7 \pm 1.3$</td>
</tr>
</tbody>
</table>

Table 2

Estimated backgrounds in 964 $\tau \rightarrow e\nu\bar{\nu}$ candidates satisfying all selection requirements. The error estimates on the background estimates for other $\tau$ decays are partially correlated.

- The number $N_{\text{chrg}}$ of good charged tracks assigned to the candidate jet must satisfy:

$$N_{\text{chrg}} = 1.$$  

- Most $\tau \rightarrow \mu\nu\bar{\nu}$ decays within the acceptance are characterized by a small energy deposition in the ECAL and associated signals in the HCAL and MUON subdetectors consistent with the passage of a minimum-ionizing particle. In order to accept muons that enter inactive regions of the HCAL or MUON or that are accompanied by radiation, however, it is required only that a $\tau \rightarrow \mu\nu\bar{\nu}$ candidate satisfy at least two of the following three requirements:

(i) Identification by the outer four MUON chamber layers [MUID]:

$$N^{\text{MU}}_{\text{layers}} \geq 2.$$  

where $N^{\text{MU}}_{\text{layers}}$ is the total number of layers with signals associated to the track in the barrel or endcap MUON detector.

(ii) Identification by the hadron calorimeter [HCID]:

$$N^{\text{HC}}_{\text{layers}} \geq 4 \quad \text{and} \quad N^{\text{HC}}_{\text{hits/layer}} < 3,$$  

where $N^{\text{HC}}_{\text{layers}}$ is the number of HCAL layers containing signals associated to the track and where $N^{\text{HC}}_{\text{hits/layer}}$ is the total number of HCAL signals assigned to the jet divided by $N^{\text{HC}}_{\text{layers}}$.

(iii) Identification by the electromagnetic calorimeter [ECID]:

$$E_{\text{clus}} < 2 \text{ GeV},$$  

- Any associated signals in the HCAL must be consistent with the passage of a minimum-ionizing particle, even when the HCID condition is not satisfied. A $\tau \rightarrow \mu\nu\bar{\nu}$ candidate is rejected if

- The number $N_{\text{chrg}}$ of good charged tracks assigned to the candidate jet must satisfy:

$$N_{\text{chrg}} = 1.$$  

- Most $\tau \rightarrow \mu\nu\bar{\nu}$ decays within the acceptance are characterized by a small energy deposition in the ECAL and associated signals in the HCAL and MUON subdetectors consistent with the passage of a minimum-ionizing particle. In order to accept muons that enter inactive regions of the HCAL or MUON or that are accompanied by radiation, however, it is required only that a $\tau \rightarrow \mu\nu\bar{\nu}$ candidate satisfy at least two of the following three requirements:

(i) Identification by the outer four MUON chamber layers [MUID]:

$$N^{\text{MU}}_{\text{layers}} \geq 2.$$  

where $N^{\text{MU}}_{\text{layers}}$ is the total number of layers with signals associated to the track in the barrel or endcap MUON detector.

(ii) Identification by the hadron calorimeter [HCID]:

$$N^{\text{HC}}_{\text{layers}} \geq 4 \quad \text{and} \quad N^{\text{HC}}_{\text{hits/layer}} < 3,$$  

where $N^{\text{HC}}_{\text{layers}}$ is the number of HCAL layers containing signals associated to the track and where $N^{\text{HC}}_{\text{hits/layer}}$ is the total number of HCAL signals assigned to the jet divided by $N^{\text{HC}}_{\text{layers}}$.

(iii) Identification by the electromagnetic calorimeter [ECID]:

$$E_{\text{clus}} < 2 \text{ GeV},$$  

- Any associated signals in the HCAL must be consistent with the passage of a minimum-ionizing particle, even when the HCID condition is not satisfied. A $\tau \rightarrow \mu\nu\bar{\nu}$ candidate is rejected if
$N_{\text{hit}}^{\text{HC}}/\text{layer} \geq 3$ and $N_{\text{layers}}^{\text{HC}} \geq 3$.

Some residual background from hadronic decays accompanied by $\pi^0$ production is suppressed by requiring $M_{\text{track-ecal}} < 0.3$ GeV,

where $M_{\text{track-ecal}}$ is the invariant mass of the charged track (assuming a $\pi^\pm$ hypothesis) and all ECAL clusters in the jet (assuming a $\gamma$ hypothesis). In the calculation, 0.5 GeV is subtracted from the energy and momentum of the ECAL cluster nearest the charged track, to account for the energy deposition expected from a minimum-ionizing particle. This invariant mass measures not only the angular spread of multiple clusters within a jet but also the angular difference between the track direction and the centroid of its associated ECAL cluster.

In order to suppress residual $e^+e^-\rightarrow \mu^+\mu^-$ contamination, a $\tau \rightarrow \mu\nu\bar{\nu}$ candidate is rejected if the opposite jet consists of exactly one charged track, consistent with being a muon, that satisfies

$$p_{\text{track}} + E_{\text{clus}} - 0.5 \text{ GeV} \geq 0.8E_{\text{beam}},$$

where 0.5 GeV is the average ECAL energy deposition of a minimum ionizing particle. A track is considered consistent with being a muon if it satisfies any one of the following three criteria:

(i) Identification according to MUID defined above.
(ii) Identification according to HCID defined above.
(iii) Passage through a geometric region where neither the hadron calorimeter nor the outer MUON chambers are fully active.

Contamination ($\approx 0.5\%$) of the $\tau$ pair sample arises from $e^+e^-\rightarrow \mu^+\mu^-$ events where one or both muons travel near an anode wire plane in the jet chamber where there is some degradation of reconstructed momentum resolution, causing a small fraction of the $e^+e^-\rightarrow \mu^+\mu^-$ events to fail the $0.6E_{\text{CM}}$ energy threshold requirement necessary for correct identification. Although unimportant to the $\tau \rightarrow e\nu\bar{\nu}$ and $\tau \rightarrow \pi(K)\nu$ analyses, the contamination, which is not modelled well by the Monte Carlo, would introduce a large uncertainty in the background fraction of $\tau \rightarrow \mu\nu\bar{\nu}$ events. To avoid this uncertainty, $\tau \rightarrow \mu\nu\bar{\nu}$ candidates must satisfy:

$$|\phi_{\text{track}} - \phi_{\text{anode}}| > 0.3^\circ,$$

where $\phi_{\text{track}}$ is the azimuth direction of the charged track at its closest approach to the beam, and $\phi_{\text{anode}}$ is the azimuth of the nearest anode plane of the central jet chamber.

Finally, in order to ensure reliable muon identification with low background and well understood efficiency, the muon candidates must satisfy:

$$x_{\mu} \equiv \frac{p_{\text{track}} + E_{\text{clus}} - 0.5 \text{ GeV}}{E_{\text{beam}}} > 0.05,$$

where $E_{\text{clus}}$ is added to the track momentum in defining $x_{\mu}$ to reduce sensitivity to radiative effects in the extraction of $\tau$ polarization.

After all requirements, 903 $\tau \rightarrow \mu\nu\bar{\nu}$ candidates are selected from the 6616 $\tau$ jets within the acceptance. Fig. 4 shows distributions in $N_{\text{layers}}^{\text{HC}}$ and $N_{\text{layers}}^{\text{MU}}$ for data and Monte Carlo expectation, both before and after the requirements of HCID or MUID are imposed.

![Fig. 4. Distributions in $N_{\text{layers}}^{\text{HC}}$ and $N_{\text{layers}}^{\text{MU}}$ (a),(b) before and (c),(d) after the HCID and MUID requirements are imposed on the $\tau \rightarrow \mu\nu\bar{\nu}$ candidates. Tracks appearing in (a) satisfy ECID or MUID requirements and the requirement $N_{\text{hit}}^{\text{HC}}/\text{layer} < 3$. Tracks appearing in (b) satisfy ECID or HCID requirements. In each figure the data are indicated by the points with error bars, the expected signal from Monte Carlo by the open histogram, and the expected background from Monte Carlo by the shaded histogram. Monte Carlo predictions are normalized to the world-average value for $B(\tau \rightarrow \mu\nu\bar{\nu})$. The arrows indicate the cut values.](image-url)
The excess of Monte Carlo tracks with $N_{\text{MC layers}} = 0$ in figs. 4a, c has a negligible effect on the $\tau \rightarrow \mu \nu \bar{\nu}$ selection efficiency, and its effect on the background estimate is included in the uncertainty estimated below. Fig. 2b shows the measured distribution in $x_\mu$, along with the expectation from Monte Carlo, including background.

The $\tau \rightarrow \mu \nu \bar{\nu}$ selection efficiency for all requirements is estimated to be $82.2 \pm 1.7\%$ for $\tau$ pairs within the acceptance, where the Monte Carlo efficiency estimates, which are nearly independent of $x_\mu$, have been checked using control samples from the data. The control samples consist of muons at low $x_\mu$ from $e^+e^- \rightarrow e^+e^- + \mu^+\mu^-$ events and muons at $x_\mu \approx 1$ from $e^+e^- \rightarrow \mu^+\mu^-$ events. The dominant errors on the total efficiency estimate arise from uncertainties in the effects of the fiducial requirements on $\cos \theta$ (1.3%), and on the azimuthal distance from the jet-chamber anode wire planes (1.1%). The efficiency as a function of $x_\mu$ is shown in fig. 3b, where radiative/ECM effects and event selection bias are included. The event selection bias introduces a slight momentum dependence in the efficiency because of the $e^+e^- \rightarrow \mu^+\mu^-$ rejection requirements. Estimated backgrounds from Monte Carlo studies are shown in table 3 with a total of $3.0 \pm 0.6\%$, coming largely from $\tau \rightarrow e\nu \bar{\nu}$ decays.

The estimated uncertainty on the background is found from comparison of the distributions of $N_{\text{MC layers}}$ and $E_{\text{clus}}$ in the data and Monte Carlo for final $\tau \rightarrow \mu \nu \bar{\nu}$ candidates and from study of the $N_{\text{MC layers}}$ and $N_{\text{hit/layer}}$ distributions for $\tau$ decays with an enhanced hadronic contribution.

### Table 3

Estimated backgrounds in 903 $\tau \rightarrow \mu \nu \bar{\nu}$ candidates satisfying all selection requirements.

<table>
<thead>
<tr>
<th>Background</th>
<th>Contamination (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau \rightarrow \pi \nu$</td>
<td>$2.3 \pm 0.5$</td>
</tr>
<tr>
<td>$\tau \rightarrow K\nu$</td>
<td>$0.2 \pm 0.2$</td>
</tr>
<tr>
<td>$\tau \rightarrow \rho\nu$</td>
<td>$0.1 \pm 0.1$</td>
</tr>
<tr>
<td>$\tau \rightarrow \text{other}$</td>
<td>$0.1 \pm 0.1$</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow \mu^+\mu^-$</td>
<td>$0.1 \pm 0.1$</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow e^+e^- + \mu^+\mu^-$</td>
<td>$0.3 \pm 0.3$</td>
</tr>
<tr>
<td>total</td>
<td>$3.0 \pm 0.6$</td>
</tr>
</tbody>
</table>

#### 4.3. Selection of $\tau \rightarrow \pi (K)\nu$ candidates

Separating $\tau \rightarrow \pi (K)\nu$ from other $\tau$ decays is more difficult than for $\tau \rightarrow e\nu \bar{\nu}$ and $\tau \rightarrow \mu \nu \bar{\nu}$. Pions leave various signatures, depending on whether they interact hadronically in the magnetic coil, in the lead glass or in the hadron calorimeter. Those interacting in the coil give rise to measured ECAL energies with much larger fluctuations than seen for $\tau \rightarrow e\nu \bar{\nu}$. Furthermore, it is difficult to remove all of the potentially large $\tau \rightarrow \mu \nu \bar{\nu}$ background by imposing requirements on HCAL and outer MUON chamber signals without creating a bias against pions that interact deep in the hadron calorimeter. A consequence of these difficulties is a greatly reduced efficiency for $\tau \rightarrow \pi (K)\nu$ selection in order to avoid large systematic errors on the efficiency and background estimates.

$\pi (K)\nu$ candidates must satisfy the following requirements:

- The number $N_{\text{chrg}}$ of good charged tracks assigned to the jet cone must satisfy:

  $$N_{\text{chrg}} = 1.$$  

- To reject $\tau \rightarrow e\nu \bar{\nu}$ and other backgrounds with large associated ECAL energy, the energy $E_{\text{clus}}$ of the cluster associated to the pion candidate and the candidate's reconstructed track momentum $p_{\text{track}}$ must satisfy:

  $$E_{\text{clus}}/p_{\text{track}} < 0.8.$$  

- To reject hadronic decays accompanied by $\pi^0 \rightarrow \gamma\gamma$, it is required that the total cluster energy in the jet (excluding the cluster associated to the pion candidate tracks) satisfy:

  $$E_{\text{excess}} < 0.02E_{\text{beam}}.$$  

and that the difference in azimuth between the track and the presampler cluster farthest away (as defined in the $\tau \rightarrow e\nu \bar{\nu}$ selection) satisfy:

  $$\delta\phi_{\text{max}} < 0.5^\circ.$$  

This is a more severe requirement than that used in identifying $\tau \rightarrow e\nu \bar{\nu}$. It results in a loss of 33% in $\tau \rightarrow \pi (K)\nu$ selection efficiency.

- Removing the background from $\tau \rightarrow \mu \nu \bar{\nu}$ depends critically upon the HCAL and the MUON chambers, which both have some geometric gaps in acceptance. Only $\tau \rightarrow \pi (K)\nu$ candidates in the region of full response of the outer MUON chambers are accepted in
this measurement. If a \( \tau \to \pi(K)\nu \) candidate enters a region where HCAL is also fully active, then the candidate is rejected if

\[
N_{\text{HC/MU}}^{\text{layers}} \geq 3
\]

or

\[(N_{\text{HC/MU}}^{\text{layers}} = 2 \text{ and } N_{\text{hits/layer}}^{\text{HC}} < 3),\]

where \( N_{\text{HC/MU}}^{\text{layers}} \) is the number of layers containing signal out of a possible seven, consisting of the outer three HCAL layers and all four outer MUON chamber layers. If a \( \tau \to \pi(K)\nu \) candidate enters a region where HCAL is not fully active, then the candidate is rejected if

\[
N_{\text{MU}}^{\text{layers}} \geq 2.
\]

where \( N_{\text{MU}}^{\text{layers}} \) is the number of associated outer MUON chamber layers with a signal.

Finally, in order to ensure good understanding of the pion selection efficiency and the backgrounds, the pion candidate's track momentum must satisfy:

\[
x_\pi \equiv p_{\text{track}}/E_{\text{beam}} > 0.05.
\]

After all requirements, 309 \( \tau \to \pi(K)\nu \) candidates are selected from the 6616 \( \tau \) jets within the acceptance. Fig. 1b shows the distributions in \( E_{\text{ch}}/p_{\text{track}} \) for data and for Monte Carlo expectation after all selection requirements except the requirement on \( E_{\text{ch}}/p_{\text{track}} \). Inaccuracies in Monte Carlo simulation of high-energy \( \pi \) interactions in the lead-glass calorimeter lead to a relative shift between the data and Monte Carlo distributions in \( E_{\text{ch}}/p_{\text{track}} \). The loose requirement of \( E_{\text{ch}}/p_{\text{track}} < 0.8 \), however, minimizes the sensitivity of the efficiency estimate to this discrepancy. In calculating the final branching ratio and polarization values, conservative systematic errors have been assigned to the uncertainty due to this discrepancy. Fig. 2c shows the measured distribution in \( x_\pi \), along with the expectation from Monte Carlo, including background.

The \( \tau \to \pi(K)\nu \) selection efficiency is estimated from Monte Carlo simulation and plotted as a function of \( x_\pi \) in fig. 3c. The efficiency decreases with increasing \( x_\pi \), mainly because of the requirements on \( E_{\text{excess}} \) and \( \delta \phi_{\text{max}} \), with a less pronounced dependence due to the \( E_{\text{ch}}/p_{\text{track}} \) requirement. The accuracy of the Monte Carlo simulation of these momentum dependences has been verified from comparisons of distributions in these variables for several ranges of \( x_\pi \). A somewhat more direct but statistically limited check of the efficiency loss due to the HCAL and MUON detector requirements and the dependence of that loss on momentum is obtained from control samples of hadronic \( \tau \) decays in the data that contain one or more \( \pi^0 \)'s. None of the efficiency checks reveals a statistically significant disagreement between the data and Monte Carlo expectation. Therefore no correction is applied to the Monte Carlo efficiency estimates, but a conservative estimate of 3.4% is assigned to the relative error on the efficiency, limited primarily by the low statistics of the comparisons. The uncertainty in the total efficiency is included in the final systematic error on \( B(\tau \to \pi(K)\nu) \), and the uncertainty on the momentum dependence is included in the final systematic error on the \( \tau \) polarization measured from \( \tau \to \pi(K)\nu \) decays. The overall efficiency is estimated to be 37.7\pm1.3% for \( \tau \) pairs within the geometric acceptance. Estimated backgrounds from Monte Carlo studies are shown in table 4 with a total of 6.0\pm1.6%, coming largely from other hadronic decays of the \( \tau \). These background estimates have been cross-checked using the variable \( M_{\text{track}}-E_{\text{cal}} \) defined in the \( \tau \to \mu\nu\bar{\nu} \) selection. The effect on backgrounds from electrons and other hadrons of the strict requirement on the presampler \( \delta \phi_{\text{max}} \) has been checked with control samples of \( e^+e^- \to (e)e\gamma \) and \( \tau \to e\nu\bar{\nu} \) decays (selected with tight requirements on \( E_{\text{ch}}/p_{\text{track}} \) and checked also by inverting the requirement on \( E_{\text{excess}} \). Estimates of background from \( \tau \to \mu\nu\bar{\nu} \) decay have been checked directly with control samples of \( e^+e^- \to e^+e^-\mu^+\mu^- \) and \( e^+e^- \to \mu^+\mu^- \) events.

<table>
<thead>
<tr>
<th>Background</th>
<th>Contamination (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau \to e\nu\bar{\nu} )</td>
<td>0.8\pm0.2</td>
</tr>
<tr>
<td>( \tau \to \mu\nu\bar{\nu} )</td>
<td>0.4\pm0.3</td>
</tr>
<tr>
<td>( \tau \to \rho\nu )</td>
<td>3.0\pm1.0</td>
</tr>
<tr>
<td>( \tau \to \text{other} )</td>
<td>1.7\pm0.5</td>
</tr>
<tr>
<td>total</td>
<td>6.0\pm1.6</td>
</tr>
</tbody>
</table>
**4.4. Calculation of branching ratios**

Branching ratios for the three channels $\tau \rightarrow e\nu\nu$, $\tau \rightarrow \mu\nu\nu$, and $\tau \rightarrow \pi(K)\nu$ are calculated from the following expression:

$$B(\tau \rightarrow x\nu) \equiv \frac{\frac{1}{\epsilon_{\tau \rightarrow x\nu}} \frac{N_{\text{bias}}^{\tau \rightarrow x\nu}}{F_{\text{bias}}^{\tau \rightarrow x\nu}}}{1 - f_{\text{background}}^{\tau \rightarrow x\nu} - f_{\text{background}}^{\tau \rightarrow x\nu}}$$

$$\times \left(1 - \frac{f_{\text{background}}^{\tau \rightarrow x\nu}}{1 - f_{\text{background}}^{\tau \rightarrow x\nu}}\right)$$

$$\times \left(1 - \frac{f_{\text{background}}^{\tau \rightarrow x\nu}}{1 - f_{\text{background}}^{\tau \rightarrow x\nu}}\right)$$  \(4\)

where $N_{\text{candidate}}^{\tau \rightarrow x\nu}$ is the number of selected $\tau \rightarrow x\nu$ candidates, $N_{\text{candidate}}^{\tau \rightarrow x\nu} = 6616$ is the number of $\tau$ jets within the acceptance, $f_{\text{background}}^{\tau \rightarrow x\nu}$ is the estimated background fraction in the $\tau \rightarrow x\nu$ sample from other $\tau$ decay modes and from non-$\tau$ sources, $f_{\text{background}}^{\tau \rightarrow x\nu} = 1.9\pm1.0\%$ is the estimated contamination of non-$\tau$ events in the $\tau$ pair sample, $\epsilon_{\tau \rightarrow x\nu}$ is the estimated efficiency for selecting $\tau \rightarrow x\nu$ decays within the acceptance, and $F_{\text{bias}}^{\tau \rightarrow x\nu}$ is the relative enhancement of $\tau \rightarrow x\nu$ decays in the $\tau$ pair sample. Values of $N_{\text{candidate}}^{\tau \rightarrow x\nu}$, $f_{\text{background}}^{\tau \rightarrow x\nu}$, $f_{\text{background}}^{\tau \rightarrow x\nu}$, $\epsilon_{\tau \rightarrow x\nu}$, and $F_{\text{bias}}^{\tau \rightarrow x\nu}$ are shown in Table 5 for the three decay channels.

The measured branching ratio values are

$$B(\tau \rightarrow e\nu\nu) = 17.4 \pm 0.5 \pm 0.4\%,$$

$$B(\tau \rightarrow \mu\nu\nu) = 16.8 \pm 0.5 \pm 0.4\%,$$

$$B(\tau \rightarrow \pi(K)\nu) = 12.1 \pm 0.7 \pm 0.5\%,$$

where the first error is statistical and the second systematic. The estimated systematic errors are calculated from the errors on the backgrounds, efficiencies, and bias factors listed in Table 5. As a cross-check, the variations of the final branching ratios have been studied as selection variable cut values are varied within a range given by the experimental resolutions on the variables. The observed variations agree well with the expectation from the above systematic errors. Additional cross-checks of numbers of observed events with two identified decays from among these three channels confirm the self-consistency of the measured branching ratios. It has also been confirmed that the numbers of $\tau$ jets identified as both an electron and a pion or as both a muon and a pion are small (<1%) and in good agreement with expectation from Monte Carlo.

These measured branching ratio values agree well with the world averages of 17.7±0.4%, 17.8±0.4, and 11.7±0.5 compiled by the Particle Data Group [1]. From the ratio $B(\tau \rightarrow \mu\nu\nu) / B(\tau \rightarrow e\nu\nu)$, one derives from this measurement (after correcting for the reduced phase space of the $\tau \rightarrow \mu\nu\nu$ decay) the ratio of the electron and muon charged couplings: $G_{\mu}/G_{e} = 0.99 \pm 0.06$, consistent with unity. If these couplings are constrained to be identical, then the electron and muon branching ratios from this measurement can be used to derive $G_{\mu}/G_{e,\mu} = 0.92 \pm 0.04$, using the world-average values for the $\tau$ and $\mu$ lifetimes and masses [1].

**5. Measurement of $\tau$ polarization**

The measurement of $\tau$ polarization depends on fitting the momentum distributions of the decay products in the three channels considered to the expectation for a given polarization. A number of corrections must be made to the observed spectra, however, before the fit can be performed. The corrections account for backgrounds, efficiency, momentum resolution, radiative effects, $E_{\text{CM}}$ dependence, and event selection bias. For each channel, the following formulation is used:
N_{i}^{corr} = C_{i}^{bias} C_{i}^{rad/ECM} \times \frac{1}{\epsilon_{i}} \sum_{j} A_{ij} N_{j}^{meas} (1 - f_{j}^{bkgd}), \quad (5)

where \(N_{i}^{corr}\) is the final unfolded, corrected number of candidates in true momentum bin \(i\), \(N_{j}^{meas}\) is the number of candidates in observed momentum bin \(j\), \(\epsilon_{i}\) is the estimated efficiency in true momentum bin \(i\), \(f_{j}^{bkgd}\) is the estimated background in measured bin \(j\), \(A_{ij}\) is the unfolding matrix that transforms measured momenta into true momenta, \(C_{i}^{rad/ECM}\) is the correction factor for radiative effects and \(ECM\) dependence, and \(C_{i}^{bias}\) is the correction factor for event selection bias.

The unfolding matrix \(A_{ij}\) has been calculated directly from the bin-to-bin migrations observed in Monte Carlo \(\tau\) pair events with full detector simulation. The systematic errors due to uncertainties in \(A_{ij}\) are estimated to be about 3% for all three decay channels and arise from the slight dependence of \(A_{ij}\) on the value of \(\tau\) polarization used in Monte Carlo generation. The corrections for radiative effects and \(ECM\) dependence have been derived from high-statistics Monte Carlo studies of \(\tau\) pair events generated without detector simulation. Because calorimeter energy is used in defining \(x_{e}\) for the \(\tau \rightarrow e\nu\bar{\nu}\) channel and because both charged track momentum and calorimeter energy are used for defining \(x_{\mu}\) in the \(\tau \rightarrow \mu\nu\bar{\nu}\) channel, the radiative distortions to the underlying Born-level momenta distributions in the leptonic channels are greatly reduced compared to those expected from measurements based on charged track momenta alone [11]. The correction procedure entailed generating two samples of \(4 \times 10^{5}\) Monte Carlo \(\tau\) pair events. One sample was generated on the peak of the \(Z^{0}\) resonance with initial-state, final-state, and internal decay radiation disabled. The other sample was generated with full radiation effects enabled at seven different \(ECM\) values, with the generated number of events at each energy weighted according to the integrated luminosity accumulated by OPAL at that energy in 1990. The correction factors \(C_{i}^{rad/ECM}\) are calculated as the ratio of the normalized differential momentum spectra of the two samples, where in the second sample nearby photons were added to the charged particle momenta. The estimated errors on \(\langle P_{t}\rangle\) due to uncertainties in \(C_{i}^{rad/ECM}\) are less than 1% for \(\tau \rightarrow e\nu\bar{\nu}\) and \(\tau \rightarrow \mu\nu\bar{\nu}\), and about 2% for \(\tau \rightarrow \pi(K)\nu\). The corrections \(C_{i}^{bias}\) are calculated directly from the Monte Carlo \(\tau\) pair sample with full detector simulation and are most important at very low momenta because of \(e^{+}e^{-} \rightarrow (e^{+}e^{-})X\) rejection and at very high momenta because of \(e^{+}e^{-} \rightarrow e^{+}e^{-}\) and \(e^{+}e^{-} \rightarrow \mu^{+}\mu^{-}\) rejection. The estimated errors on \(\langle P_{t}\rangle\) due to uncertainties in \(C_{i}^{bias}\) are about 1% for the three channels.

Applying eq. (5) to the observed momentum spectra in fig. 2 leads to the unfolded, corrected spectra shown in fig. 5 (data points). The average polarization is extracted from each of these spectra by a \(\chi^{2}\) fit to the expected distributions given by eqs. (2) and (3), where bin-to-bin correlations due to the unfolding procedure are taken into account. As a consistency check, the Monte Carlo sample of \(\tau\) pairs with full detector simulation was subjected to the same unfolding.

Fig. 5. Final, unfolded, corrected momentum distributions for (a) \(\tau \rightarrow e\nu\bar{\nu}\), (b) \(\tau \rightarrow \mu\nu\bar{\nu}\), and (c) \(\tau \rightarrow \pi(K)\nu\) candidates. Data are indicated by points with error bars. The solid curves show the distributions corresponding to the fitted polarizations for each channel. The dashed/dotted curves show the distributions corresponding to values of polarization at +/- 2 standard deviations from the fitted values. The error bars shown are the square roots of the diagonal terms of the error matrices for the unfolded distributions, but in fitting for polarization, full account is taken of the correlations among the unfolded bins, described by the off-diagonal terms of the error matrices.
correction, and fitting procedures. For the e, \( \mu \), and \( \pi \) channels the obtained values for \( \langle P_t \rangle \) are \(-0.12 \pm 0.05\), \(-0.16 \pm 0.05\), and \(-0.18 \pm 0.04\%), respectively, in good agreement with the generation value of \(-0.15\).

Applying the full procedure to the observed data, we obtain the following values for the average \( \tau \) polarization:

\[
\langle P_t \rangle (\tau \rightarrow e\nu \bar{\nu}) = +0.20 \pm 0.13 \pm 0.08,
\]

\[\chi^2/DOF = 6.9/9,\]

\[
\langle P_t \rangle (\tau \rightarrow \mu\nu \bar{\nu}) = -0.17 \pm 0.16 \pm 0.10,
\]

\[\chi^2/DOF = 12.4/9,\]

\[
\langle P_t \rangle (\tau \rightarrow \pi(K)\nu) = -0.08 \pm 0.10 \pm 0.07,
\]

\[\chi^2/DOF = 14.3/9,\]

where the first error is statistical and the second is systematic. The statistical error on the polarization measured for the \( \tau \rightarrow \mu\nu \bar{\nu} \) decays is larger than that for \( \tau \rightarrow e\nu \bar{\nu} \) because the muon energy is determined mainly by the measured momentum of a charged track, for which the resolution at high momentum is poorer than that for the shower energy of an electron. Systematic errors include contributions from limited Monte Carlo statistics (\(\pm 5\%\)), uncertainties due to the unfolding and correction procedure (\(\pm 4\%\)) and uncertainties in efficiency and background estimates (\(\pm 4\% - 8\%\)). Taking the weighted average of these polarizations yields a combined value for the average polarization of

\[
\langle P_t \rangle = -0.01 \pm 0.09,
\]

where a common systematic error of 0.03 due to uncertainties in the unfolding procedure has been assumed in the averaging. For the \( \tau \) couplings to the \( Z^0 \) this implies

\[
v_\tau/a_\tau = 0.01 \pm 0.04.
\]

The measured average polarization and forward-backward polarization asymmetry yield values for \( v_\tau/a_\tau \) and \( v_e/a_e \) that are consistent with lepton universality. If lepton universality is now imposed by constraining the six statistically independent polarization measurements in the forward and backward hemispheres for the three decay channels to obey eqn. (1), one finds the lepton couplings:

\[
v/a = 0.05 \pm 0.04
\]

and in the improved Born approximation of the standard model.

![Fig. 6. Unfolded momentum distributions of candidates in the forward and backward hemispheres for the (a),(b) \( \tau \rightarrow e\nu \bar{\nu} \), (c),(d) \( \tau \rightarrow \mu\nu \bar{\nu} \), and (e),(f) \( \tau \rightarrow \pi(K)\nu \) channels. The data points and curves have the same meanings as in fig. 5.](image)
Table 6 Average $\tau$ polarizations observed in both the forward and backward hemispheres together and in each hemisphere separately for the $\tau \rightarrow e\nu\bar{\nu}$, $\tau \rightarrow \mu\nu\bar{\nu}$, and $\tau \rightarrow \pi(K)\nu$ channels in the region $|\cos \theta| < 0.68$. Shown also are the observed forward–backward polarization asymmetries for each channel, derived from the differences in forward and backward polarizations in the region $|\cos \theta| < 0.68$ without efficiency corrections. The final row of the table lists the implied polarization asymmetries for $|\cos \theta| < 1.0$ after correction for efficiency and geometric acceptance. The last column of the table lists the corresponding expectations for $\sin^2 \theta_W = 0.234$, the central value obtained from OPAL measurements of leptonic partial widths and forward–backward charge asymmetries [13], in the improved Born approximation, assuming the standard model with minimal Higgs structure.

<table>
<thead>
<tr>
<th>Channel</th>
<th>$P_{\tau}$</th>
<th>$P_{\tau}^{FB}$</th>
<th>$P_{\tau}^{FB, \text{corr}}$</th>
<th>$\sin^2 \theta_W = 0.234$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau \rightarrow e\nu\bar{\nu}$</td>
<td>$+0.20 \pm 0.13 \pm 0.08$</td>
<td>$-0.17 \pm 0.16 \pm 0.10$</td>
<td>$-0.08 \pm 0.10 \pm 0.07$</td>
<td>$-0.13$</td>
</tr>
<tr>
<td>$\tau \rightarrow \mu\nu\bar{\nu}$</td>
<td>$+0.06 \pm 0.19 \pm 0.08$</td>
<td>$-0.24 \pm 0.23 \pm 0.10$</td>
<td>$-0.35 \pm 0.14 \pm 0.07$</td>
<td>$-0.20$</td>
</tr>
<tr>
<td>$\tau \rightarrow \pi(K)\nu$</td>
<td>$+0.30 \pm 0.19 \pm 0.08$</td>
<td>$-0.12 \pm 0.23 \pm 0.10$</td>
<td>$+0.13 \pm 0.14 \pm 0.07$</td>
<td>$-0.05$</td>
</tr>
</tbody>
</table>

$\sin^2 \theta_W = 0.237 \pm 0.009$,

where there is no ambiguity introduced by the relative signs of $\nu$ and $a$, as is present in measurements based on lepton forward–backward charge asymmetries [13].

These results are consistent with previous measurements of $\tau$ polarization at lower values of $E_{CM}$ [22–24]. The results are also in good agreement with previous measurements by OPAL [13] and other LEP and SLC experiments of electron and tau neutral-current couplings from leptonic partial widths $\Gamma(Z^0 \rightarrow \ell^+\ell^-)$ and forward–backward charge asymmetries.

6. Summary

We have obtained the following $\tau$ decay branching ratios: $B(\tau \rightarrow e\nu\bar{\nu}) = 17.4 \pm 0.5$ (stat) $\pm 0.4$ (sys)%, $B(\tau \rightarrow \mu\nu\bar{\nu}) = 16.8 \pm 0.5 \pm 0.4 \%$, and $B(\tau \rightarrow \pi(K)\nu) = 12.1 \pm 0.7 \pm 0.5 \%$. The measured lepton branching ratios, when combined with the world-average measured value for the $\tau$ lifetime, yield a ratio of the $\tau$ Fermi coupling constant to that of the lighter leptons given by $G_\tau/G_{e,\mu} = 0.92 \pm 0.04$, where it is assumed $G_e = G_\mu \equiv G_{e,\mu}$. These results are consistent with previous measurements and increase slightly the statistical significance of both the discrepancy between the $\tau$ lifetime and leptonic branching ratios and the discrepancy between the inclusive and the sum of the exclusive one-prong branching ratios of the $\tau$.

From an analysis of the momentum spectra of the decay products, assuming V–A charged current couplings of the $\tau$, we obtain a combined average $\tau$ polarization at the peak of the $Z^0$ resonance of $-0.01 \pm 0.09$, implying $v_\tau/a_\tau = 0.01 \pm 0.04$. From the spectra of identified decays in the forward and backward hemispheres, we obtain a value for the efficiency-corrected, forward–backward polarization asymmetry of $-0.22 \pm 0.10$, implying $v_\tau/a_\tau = 0.15 \pm 0.07$. Imposing lepton universality and combining the forward and backward hemisphere measurements leads to $v_\tau/a_\tau = 0.05 \pm 0.04$ or $\sin^2 \theta_W = 0.237 \pm 0.009$, where there is no ambiguity introduced by the relative signs of $\nu$ and $a$.

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