$R_b$: Experimental Results

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The measurements leading to the current preliminary world average value for

$$R_b \equiv \frac{\Gamma(Z^0 \rightarrow b\bar{b})}{\Gamma(Z^0 \rightarrow \text{hadrons})} = 0.2202 \pm 0.0009 \pm 0.0013 \quad (\text{for } R_c = 0.172)$$

are presented. The main systematic uncertainties and the averaging technique for the measurements are shortly discussed. An outlook on possible future improvements is given.

1. Introduction

The quantity $R_b \equiv \frac{\Gamma(Z^0 \rightarrow b\bar{b})}{\Gamma(Z^0 \rightarrow \text{hadrons})}$ has the distinct feature compared to most other electroweak observables to be sensitivity to couplings that are specific to the third generation, i.e. couplings that are sensitive to mass or involve other particles that couple specifically to the third generation. This makes $R_b$ one of the quantities where effects of, for example, Super Symmetry could be observed first. However, the deviations of $R_b$ from the Standard Model value that are predicted by, again for example, the Minimal Supersymmetric Standard Model are only of the order 1%. With the large amount of $Z^0$ bosons provided to the ALEPH, DELPHI, L3 and OPAL experiments by the LEP collider and to the SLD experiment by the SLC a measurement of $R_b$ with an accuracy below 1% has been achieved and the uncertainty can be further reduced in the future. The present, preliminary, combined results from all relevant measurements of $R_b$ shows an interesting discrepancy with the Standard Model prediction. It is therefore important to examine the experimental situation in some detail before embarking on its consequences for the Standard Model or its possible extensions.

2. The $R_b$ Measurements

$R_b$ is measurement by selecting a sample of hadronic $Z^0$ decays and subsequently determining the fraction of $Z^0 \rightarrow b\bar{b}$ events in this hadronic decay sample.

2.1. $\Gamma(Z^0 \rightarrow \text{hadrons})$: The Denominator

The selection of hadronic $Z^0$ decays is based on requiring a minimum number of tracks and energy clusters in the calorimeter, sufficient visible energy and a small energy imbalance in the detector. Usually also a cut is applied to the polar angle of the thrust axis to ensure the event is well contained in the fiducial volume of the detector and sometimes a cut is made on the thrust value to enhance the back-to-back nature of the quark and anti-quark in the event (important for the double b-tagging method described later.)

There are three things to worry about in the hadronic event sample selection. Firstly hadronic events produced from a non-pure $Z^0$ state, i.e. from $\gamma$ exchange or $\gamma-Z^0$ interference. To correct for these $\gamma$ exchange effects a shift of $+0.0003$ to $R_b$ is applied as derived from calculations in the Standard Model [1,2]. Secondly the background from non-hadronic $Z^0$ decays, e.g. from $Z^0 \rightarrow \tau^+\tau^-$ and from $2\gamma$ interactions. This background is estimated from the Monte Carlo and is of the order of 0.1%, for both $\tau$ and $2\gamma$ events. Each experiment corrects for this background. Lastly, there is a flavour bias introduced by the hadronic event selection, i.e. relatively more non-$b$ events get rejected than $b$ events. This bias is of order a few per mille and is estimated from the Monte Carlo and corrected for by each experiment.

The hadronic events selection for $R_b$ is similar to that for the measurement of the hadronic peak
cross section, $\sigma_{\text{had}}^0$, and the ratio of the hadronic to lepton decay width of the $Z^0$, $R_t \equiv \Gamma(Z^0 \rightarrow \text{hadrons})/\Gamma(Z^0 \rightarrow \ell\ell)$. Both of these quantities are in excellent agreement with the Standard Model predictions [1]. This boosts confidence that the hadronic event selection is not biasing the $R_b$ determination.

2.2. $\Gamma(Z^0 \rightarrow b\bar{b})$: Tagging $b$ Quarks

Before discussing the tagging of $b$ quarks it should be noted that the measurement of $\Gamma(Z^0 \rightarrow b\bar{b})$ has an ambiguity. This is illustrated by considering the process $Z^0 \rightarrow b\bar{g} \rightarrow b\bar{u}u$ and $Z^0 \rightarrow u\bar{g} \rightarrow u\bar{b}b$, where the latter does not contribute to $\Gamma(Z^0 \rightarrow b\bar{b})$ but still has $b\bar{b}$ in the final state. Fortunately the practical implication of this ambiguity is negligible due to the small $g \rightarrow b\bar{b}$ branching rate [3] and the much reduced tagging efficiency for $b$ quarks from gluon splitting due to their soft momentum spectrum.

Hadrons containing $b$ quarks distinguish themselves from other hadrons at $e^+e^-$ colliders in their mass, in their hard fragmentation, and especially in their decay properties, such as relatively long lifetimes, high decay multiplicities and large momenta of their decay products. These properties are used in the various $b$ tagging techniques that are employed.

Lepton tag

Leptons with both high momentum ($P_T$) and large momentum with respect to the jet-axis of the jet containing the lepton ($P_T$) indicate the semileptonic decay of a $b$-flavoured hadron. Results for lepton tags are reported by all four LEP experiments [4–9].

Lifetime tags

These are based either on identifying a secondary vertex that is displaced from the primary vertex and is then called a decay length tag [9], or on a number of tracks that have a large impact parameter and are therefore not compatible with coming from the primary vertex. In both techniques the multiplicity of either the secondary vertex, or the number of tracks with a large impact parameter also play an important role. Also in both techniques the decay length or impact parameter is signed in such a way that positive values correspond to decays of particles with a positive lifetime. Negative values for these parameters are then identified with resolution effects on tracks that come from the primary vertex. An approach to the impact parameter tag first followed by ALEPH [10] and later also by DELPHI [6] uses the joint probability of a set of tracks to be compatible with the primary vertex, $P = \prod_{j=0}^{N} (-\log \Pi_j)^{1/2}$, with $\Pi_j = \prod_{i=1}^{N} \mathcal{P}_T$, and $\mathcal{P}_T$, the probability for track $i$ to come from the primary vertex.

Mass tag

Recently the SLD collaboration has presented results for a mass tag of $b$-flavour [11]. To enhance the sensitivity, information on missing momentum is added to the raw invariant mass that is obtained from the tracks that are reconstructed in one secondary vertex according to $M = \sqrt{\mathcal{P}_T^2 + \mathcal{P}_T^2} + |P_T|$, where $P_T$ is the transverse momentum of the tracks from the secondary vertex with respect to the axis that connects the primary and secondary vertex.

Multivariate tags

Tags for $b$-flavour are also formed by combining several of the tags above with more elaborate information on the event structure into one discriminant using a neural net or a canonical discriminant formalism [6,12,13].

2.3. The Double Tagging Method

Once the $b$ tag is applied the efficiency and purity of the tagged event sample has to be determined. The purity of the tagged samples are usually quite high, above 90%, and by cutting harder the purity can be increased to close to 100% for some of the tags that have been presented (of course at the cost of a low efficiency.) The efficiency is not so easy to determine, as it is never close to 100%. To solve this problem the fact that in $Z^0$ decay always a $b$ quark and an $\bar{b}$ quark are produced simultaneously and almost always back-to-back can be employed. To do this the event is split into two hemispheres along the thrust axis. In most of the cases the $b$ quarks will line up with the thrust axis and in this way there will be one $b$ quark in each hemisphere. By now applying the $b$ tag to each hemisphere both the
b tagging efficiency and the fraction of b events in the original sample, $R_b$, can be determined simultaneously using the equations:

$$f_b = \epsilon_b R_b + \epsilon_c R_c + \epsilon_{uds} R_{uds},$$

$$f_d = C_b \epsilon_b^2 R_b + C_c \epsilon_c^2 R_c + C_{uds} \epsilon_{uds}^2 R_{uds},$$

where $f_b \equiv N_b/(2N_{tot})$ is the number of hemispheres with a b tag divided by the total number of hemispheres and $f_d \equiv N_d/N_{tot}$ the number of events with a b tag in both hemispheres divided by the total number of events. The tagging efficiency for flavour $i$ ($i=uds,c,b$) is given by $\epsilon_i$ and $R_{uds}$ and $R_{uds}$ are defined for c and uds quarks similarly to $R_b$ for b quarks. In the double tag equation, hemisphere correlation factors, $C_i$ ($i=uds,c,b$ again), are added to account for tagging efficiency correlations between the two hemispheres in the same event.

Generalisations of the double tagging method are also used, such as the mixed tag method [6,12] and the winning margin idea [6].

### 2.4. Uncertainties in Double Tagging

In the double tagging equations $f_b$ and $f_d$ are measured from the data, and $R_b$ and $\epsilon_b$ are solved from the equations, leaving the remaining quantities to be determined in other ways. In addition the normalisation condition $R_{uds} = 1 - R_b - R_c$ can be applied. The applied b tags usually lead to very high purities and especially the fraction of uds events selected by these tags is very low. From this it can be seen that even large changes in $C_c$ and $C_{uds}$ will not affect the outcome for $R_b$ and $\epsilon_b$. This leaves as important quantities to be determined $\epsilon_c$, $\epsilon_{uds}$, $R_c$ and $C_b$ and these will be the main sources of systematic uncertainties in the determination of $R_b$.

The efficiencies $\epsilon_c$ and $\epsilon_{uds}$

The fraction of uds events tagged by pure b tags is very low and the uncertainties due to the modelling of light quark events are therefore not very important. The most significant problems from light flavour events are the estimates of long lived hadron ($K, \Lambda, \ldots$) abundancies, the light quark fragmentation and resolution effect (especially the tails of the resolution function.)

More problematic, and in fact the largest contribution to the systematic uncertainty, is the systematic error due to knowledge of the charm sector. The biggest problem are the relative production rates of $D^0, D^+, D_s$ and $\Lambda_c$ hadrons [14], followed by the $D$ meson and $\Lambda_c$ baryon decay rates in various combinations of $\phi, K, \pi$ mesons and protons, the $D$ decay multiplicity, and the lifetimes of the different weakly decaying charmed hadrons (see ref [1] for a discussion). It also seems worth noting that the average D charged decay multiplicity that is used, and which is a critical parameter in the charm background determination, has only been measured by the MARKIII collaboration [15].

The c quark fraction $R_c$

The value of $R_c$ is measured separately from basically three methods. $R_c$ can be obtained from fits to lepton $P$ and $P_T$ spectra (where charm semileptonic decay produced leptons at large momentum, $P$, and low transverse momentum w.r.t. the jet axis, $P_T$) [4,5]. $R_c$ can be obtained from the tagging of $D^*$ mesons with large momentum. This $D^*$ charm tag, with variations like tagging just the slow $\pi$, is also used in double tagging methods similar to the one presented above for $R_b$ [16,17]. Lastly $R_c$ can be obtained from charm counting in which the absolute production rates of $D^+, D^0, D_s$ and $\Lambda_c$ from c quark fragmentation are measured [16,18]. These four hadron species represent nearly all those that are produced or are eventually decayed into in $Z^0 \to c\bar{c}$ events. The results for $R_c$ are summarised in Fig. 1.

The correlation factor $C_b$

Tagging efficiency correlations can arise from several sources. Hard gluon radiation can cause two b quarks to go in the same thrust hemisphere in which case the double tag assumption clearly breaks down, giving a negative contribution to $C_b$. Gluon radiation also affects the b-hadron boost giving a positive contribution to $C_b$. The correlations from gluon radiation are estimated from the Monte Carlo, but work is in progress to study these effects also from data.

Misestimates of the primary vertex position or its uncertainty can give rise to positive or negative correlations. The effects are to a large extent testable from the data themselves.

Non-uniform tagging efficiencies in the detector
ALEPH (90+91) lepton fit
0.165±0.005±0.020

DELPHI (91+92) lepton fit
0.162±0.008±0.0209

DELPHI (91-94 prel) double tag
0.162±0.009±0.009

OPAL (92-94 prel) double tag
0.155±0.013±0.017

OPAL (90-92 prel) single D°
0.148±0.008±0.014

DELPHI (91-94 prel) D°+D°+D*+A/
0.164±0.011±0.013

OPAL (90-93 prel) D°+D°+D°+A/
0.166±0.011±0.014

Average
0.1598±0.0069

Table 1. Results for $R_c$.

give rise to positive correlations. These correlations can be estimated from the data by studying the efficiency variation as a function of a number of quantities, such as polar and azimuthal angle.

Efficiency correlations are hard to study exclusively from the data and therefore care has to be taken to include all possible effects when breaking down the problem. It should also be noted that the overall correlation is a convolution of the individual sources and not a straight sum or product.

3. Combination of Results

Particular care is taken to combine the results from the different experiments and different methods [1]. The fitting procedure is described in detail in [19] and makes use of the best linear unbiased estimate technique, minimizing a $\chi^2$ function that is defined in the usual way as a function of differences between measured and fitted quantities and the covariance matrix of the measurements. All source of systematic errors are detailed for each method by each experiment in a uniform way. All correlations are implemented in the off-diagonal elements of the covariance matrix or by adding terms to the $\chi^2$ function explicitly. Also the statistical correlations between different measurements are taken into account. Since the heavy flavour electroweak parameters depend, explicitly or implicitly, on each another, a fit is performed simultaneously for all these parameters.

At the moment a total of 8 quantities are fitted: the $b$ and $c$ quark partial widths $R_b$, $R_c$, the forward backward asymmetries for $b$ and $c$ events at the $Z^0$ peak $A_{FB}^{bb}(0)$ and $A_{FB}^{cc}(0)$, the semileptonic and cascade branching ratios $BR(b \rightarrow \ell)$ and $BR(b \rightarrow c \rightarrow \ell)$, the average $B$ mixing $\chi$, and the product fragmentation probability times branching ratio $P(c \rightarrow D^{*+}) \times BR(D^{*+} \rightarrow X^0 D^0)$.

The results for $R_b$ are summarised in Fig. 2, where the average is given for the case where $R_c$ is fixed to the Standard Model value. Note that this average $R_b$ value contains preliminary measurements. When $R_c$ is floated freely in the combined fit $R_b = 0.2211 ± 0.0016$ is obtained with $R_c = 0.1598 ± 0.0069$. A simplified list of uncertainties for the preliminary $R_b$ average is:

<table>
<thead>
<tr>
<th>Source</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistical</td>
<td>0.000977</td>
</tr>
<tr>
<td>Uncorrelated systematic</td>
<td>0.000818</td>
</tr>
<tr>
<td>Charm sector common systematic</td>
<td>0.000922</td>
</tr>
<tr>
<td>Other common systematic</td>
<td>0.000459</td>
</tr>
<tr>
<td>Total uncertainty on $R_b$</td>
<td>0.001639</td>
</tr>
</tbody>
</table>

where uncorrelated systematic uncertainties are from sources that do not correlate between the experiments. Common systematic uncertainties are from sources that are correlated between the experiments. Other common systematic uncertainties refer to all common systematic uncertainties that do not involve the charm sector.

4. Outlook and Conclusion

The preliminary average of $R_b = 0.2202 ± 0.0009 ± 0.0013$ that has been presented here has a 3σ discrepancy with the Standard Model prediction. This discrepancy did not arise suddenly in time, but is a stable development of the measurement over years as illustrated in Fig. 3.

Although the present precision on $R_b$ is already at the sub-percent level, even better precisions can be expected in the future. For the $R_b$ value presented here only roughly half the available data has been used and a small improvement

$^2$Also some more extended fits including the $b$ and $c$ quark asymmetries $A_{FB}^{b,c}$ and the off-peak backward backwar $d$ and $c$ quark asymmetries $A_{FB}^{b,c}(\pm 2)$ and $A_{FB}^{c}(\pm 2)$, are performed all giving slightly different, but consistent, results for $R_b$. 
in the statistical error can be expected. However, even more progress is expected for the systematic uncertainty.

The systematic uncertainty is dominated by the knowledge of the charm sector. Most of the information for this sector is taken from lower energy $e^+e^-$ collider experiments: CLEO, ARGUS, MARKIII and DELCO [14]. Measurements of production rates and branching ratios of charmed hadrons are also available from LEP [18,20]. These measurements are in agreement with the lower energy data, are more precise and the accuracy is expected to improve some more in the future. A special effort is made at the moment to understand hemisphere tagging efficiency correlations. Although the systematic checks on the $R_b$ measurements are constantly being refined, no significant problems have shown up with respect to the values quoted here. Therefore the presented preliminary $R_b$ average should be considered reliable to the best of our current knowledge.

Acknowledgements

Many thanks go to the members of the ALEPH, DELPHI, L3, SLD and OPAL collaborations and in particular those who participate in the LEP Heavy Flavour Electroweak Working Group.

The current precise measurement of $R_b$ would not have been possible without the excellent performance of the LEP and SLC accelerators and those who contributed to delivering all those $Z^0$ events are gratefully acknowledged.

The organisers of SUSY96 are thanked for a well organised and stimulating conference.

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