A Measurement of the Forward-Backward
Asymmetry of $e^+e^- \rightarrow b\bar{b}$
Applying a Jet Charge Algorithm
to Lifetime Tagged Events

The OPAL Collaboration

This note describes preliminary OPAL results and is intended primarily
for members of the collaboration.

Abstract

The forward-backward asymmetry of $e^+e^- \rightarrow Z^0 \rightarrow b\bar{b}$ has been measured using approximately 2.15 million selected hadronic $Z^0$ decays collected at the LEP $e^+e^-$ collider with the OPAL detector. A lifetime tag technique was used to select an enriched $b\bar{b}$ event sample. The measurement of the $b\bar{b}$ asymmetry was then performed using a jet-charge algorithm to determine the direction of the primary quark. Values of:

\[
A_{FB}^b(\sqrt{s} = 89.52\text{GeV}) = 0.062 \pm 0.034 \pm 0.002
\]
\[
A_{FB}^b(\sqrt{s} = 91.25\text{GeV}) = 0.0963 \pm 0.0067 \pm 0.0046
\]
\[
A_{FB}^b(\sqrt{s} = 92.94\text{GeV}) = 0.172 \pm 0.028 \pm 0.008
\]

were measured, where the first error is statistical and the second is systematic in each case. Assuming the Standard Model form for the couplings, these correspond to an effective value of the weak mixing angle of:

\[
\sin^2 \theta_W^{\text{eff}} = 0.2313 \pm 0.0012 \pm 0.0006
\]

with $M_{\text{top}} = 196^{+38}_{-35}^{+16}_{-19} \text{GeV}/c^2$, where again the first error is statistical and the second is systematic. This note supersedes PN127.
1 Introduction

The differential cross-section for the production of fermion-antifermion pairs in $e^+e^-$ annihilation can be expressed as:

$$\frac{d\sigma}{d\cos\theta} \propto 1 + \cos^2\theta + \frac{8}{3} A_{FB} \cos\theta$$  \hspace{1cm} (1)

where $\theta$ is the angle between the directions of the outgoing fermion and incoming electron, neglecting mass and higher order terms. This form makes explicit the resulting forward-backward asymmetry, $A_{FB}$, which is defined by $A_{FB} = (\sigma_F - \sigma_B)/(|\sigma_F + \sigma_B|)$, where $\sigma_F$ and $\sigma_B$ are the cross-sections for forward events (with $\cos\theta > 0$) and backward events (with $\cos\theta < 0$) respectively. In the framework of the Standard Model the asymmetry is directly related to the vector, $v$, and axial-vector, $a$, couplings of the electron and fermion, $f$, to the $Z^0$ and therefore to the weak mixing angle $\sin^2\theta_W$. At the $Z^0$ resonance it has the approximate form $[1]$:

$$A_{FB} \approx \frac{3}{4} \frac{2v_e a_e - 2v_f a_f}{v_e^2 + a_e^2}$$  \hspace{1cm} (2)

The forward-backward asymmetry on the resonance is, neglecting mass effects, the same for all fermions with the same charge. The asymmetry for down-like $(d,s,b)$ quarks has a higher sensitivity to the weak mixing angle than that for up-like $(u,c)$ quarks and charged leptons.

The OPAL experiment has already published a measurement of $A_{FB}^b$ based on the identification of prompt leptons originated from heavy flavour decay $[2]$. Recent results from other LEP experiments are summarised in $[3]$.

For this measurement, the relatively long lifetime of weakly-decaying hadrons containing a $b$ quark is exploited; significantly displaced secondary vertices are sought to enrich the sample of $b\bar{b}$ events. A jet-charge method, based on the charge distribution of the final state particles, has been used to distinguish between the direction of the primary quark produced in the decay of the $Z^0$ and that of the primary antiquark. This method, which will be described in more detail in sections 3 and 4, relies very little on Monte Carlo modelling of the $b$ jet charge, as the quantities most relevant for the analysis were measured directly from the data. It makes a statistical determination of the number of forward and backwards events. An analysis, described in section 6, was also performed using a different method, based on a study of the jet charges on an event-by-event basis, which provided a check of the analysis. Analyses using the jet-charge to determine $A_{FB}^b$ have been reported previously by the ALEPH $[4]$ and DELPHI $[5]$ collaborations.

In general, $A_{FB}^b$ depends on the centre-of-mass energy, $\sqrt{s}$; this dependence has a well defined functional form in the Standard Model. In this analysis the $b$-asymmetry was measured for events collected on, and approximately 2GeV above and below the $Z^0$ peak.

2 Event selection

The analysis described here is based on data recorded with the OPAL detector $[6]$ in the years 1991 to 1994 inclusive. Multihadronic decays of the $Z^0$ were selected using the criteria described in $[7]$, and were also required to contain at least seven charged tracks passing certain minimum track quality requirements. In addition, the silicon microvertex detector, the central tracking chambers and the electromagnetic calorimeters were required to have been correctly operating when the data were recorded.

For the purposes of $b$-tagging, charged particle tracks and electromagnetic calorimeter energy clusters not associated to charged tracks were combined into jets using the JADE algorithm $[8]$, with the E0 recombination scheme $[9]$. An invariant mass-squared cut-off of $x_{\text{min}} = (7 \text{ GeV}/c^2)^2$ was used. According to Monte Carlo simulation, the momentum vectors of the jets found in this manner closely follow the $b$-hadron direction. The thrust axis was also determined using both tracks
and unassociated electromagnetic calorimeter energy clusters and was used as an estimator of the direction of the initial quark-antiquark pair. The analysis was restricted to events largely contained within the silicon microvertex detector acceptance by applying a cut $|\cos \theta_T| < 0.8$ on the direction of the thrust axis. A sample of approximately 2.15 million events passed these requirements.

The JETSET 7.3 Monte Carlo program [10] was used to generate event samples, which were then processed by a program that simulates the response of the OPAL detector [11]. Smaller samples generated with the HERWIG 5.5 [12] program were also used. Simulated events were processed through the same reconstruction and selection algorithms as data from the detector. The Lund symmetric fragmentation function [10] was used to describe the hadronisation properties of $u$, $d$ and $s$ quarks in this sample whereas for $b\bar{b}$ and $c\bar{c}$ events the fragmentation was described by the fragmentation function of Peterson et al. [13]. The values of the parameters controlling the fragmentation function used for $b\bar{b}$ and $c\bar{c}$ events were $\epsilon_b = 0.0055$ and $\epsilon_c = 0.05$, respectively, corresponding to LEP average values of $\langle x_F \rangle_b = 0.70$ and $\langle x_F \rangle_c = 0.51$ [14].

To increase the fraction of $b\bar{b}$ events in the multihadron sample a lifetime tag was used. This was based on the selection of events with secondary vertices that were well separated from the primary vertex. These vertices are expected to be formed mainly by the tracks resulting from $b$-flavour hadron decays. The primary vertex for each event was reconstructed in the plane transverse to the beam axis using a $\chi^2$ minimization method which also incorporated the average beamspot position as a constraint. The secondary vertex finding algorithm used for this analysis attempts to reconstruct a secondary vertex separately for each reconstructed jet in the event, and is described in detail in [15]. In a first iteration, all charged tracks in a given jet are fitted to a common vertex point in the plane transverse to the beam axis. If one or more tracks contribute $\Delta \chi^2 > 4$ to the overall $\chi^2$ for the secondary vertex fit, then the track with the largest $\Delta \chi^2$ is removed and the fit repeated. The process is continued until all tracks contribute $\Delta \chi^2 < 4$ or until fewer than four tracks remain, in which case the secondary vertex reconstruction fails for this particular jet.

Additional cuts were applied to the charged tracks which are used by the secondary vertex finding algorithm, aimed mainly at removing poorly measured tracks, or tracks from $K^0$ or $\Lambda$ decay for example. The point-of-closest-approach of each track to the primary vertex in the plane transverse to the beam axis, $d_0$, was required to satisfy $|d_0| < 0.3$ cm, while the error on this quantity, $\sigma(d_0)$ was required to satisfy $\sigma(d_0) < 0.1$ cm.

For each reconstructed secondary vertex the vertex decay length $L$ was calculated. $L$ was defined as the distance of the secondary vertex from the primary vertex, constrained by the direction given by the total momentum vector (in the plane transverse to the beam direction) of the jet containing the tracks assigned to the secondary vertex. The total vertex momentum vector was also used to determine the sign of the decay length: $L > 0$ if the secondary vertex was displaced from the primary vertex in the same direction as the total momentum, and $L < 0$ in the other case. Vertices were required to have a reconstructed decay length $|L| < 2$ cm. We call the quantity $L/\sigma$ the decay length significance, where $\sigma$ is the error on the determination of the decay length $L$, taking into account the uncertainties in the primary and secondary vertex positions. The track parameters were degraded in the Monte Carlo as in [15] to improve the agreement to the data in the region of negative decay length significance, which is dominated by resolution effects. The effects of changing the degradation factors are included in the systematic uncertainties.

Figure 1 shows the decay length significance, $L/\sigma$, distribution for secondary vertices in the data and Monte Carlo samples. Vertices with large positive values of $L/\sigma$ are dominantly produced in $b\bar{b}$ events. The Monte Carlo disagrees with the data in this region. This difference may be due to assumptions in the Monte Carlo about the underlying $b$ quark physics, such as the average $b$ lifetime and decay multiplicity. However, as will be described in detail later, such differences between data and Monte Carlo in this region do not affect the results of the analysis, since the $b$ tagging efficiency
is determined directly from data with very little reliance on Monte Carlo modelling.

The hadronic events were divided into two hemispheres by the plane perpendicular to the thrust axis and containing the interaction point. The forward thrust hemisphere is defined to be the one that contains the positive z axis\(^1\) and the other hemisphere is called the backward thrust hemisphere. To ensure a good charge reconstruction, only events having more than three good charged tracks per hemisphere were used. Each hemisphere was deemed to give a lifetime tag if it contained at least one reconstructed secondary vertex which satisfied the requirements described above and had a decay length significance \(L/\sigma > 8\). In total 165 771 events passed the lifetime tag. Within these events we found 180 499 tagged hemispheres and 14 728 events which were tagged in both hemispheres.

The peak events were defined as those with \(\sqrt{s}\) between 91.05 and 91.40 GeV. Those below the peak had energies between 88.4 and 90.4 GeV and those above the peak had energies ranging from 92.0 to 94.0 GeV. The corresponding event-weighted mean centre-of-mass energies of the three classes were, 89.52, 91.25 and 92.94 GeV, respectively.

### 3 The Jet Charge Method

To measure the forward-backward asymmetry it is necessary to determine the direction of the primary fermion produced in the final state. In the jet charge method the charge of the primary quark is estimated by using the charges and momenta of the tracks produced in the final state.

For each hemisphere, defined by the direction of the thrust axis, the jet charge \(Q_{jet}\) \(^1\) is computed as:

\[
Q_{jet} = \frac{\sum_i^n |p_{i||}|^{\kappa} q_i}{\sum_i^n |p_{i||}|^{\kappa}}
\]  

(3)

where the sum runs over the \(N\) charged tracks of the hemisphere, \(p_{i||}\) is the momentum component of the track \(i\) along the thrust axis, \(q_i\) is the charge of track \(i\) and \(\kappa\) is a parameter which controls the momentum weighting of each particle’s charge. The weighting used \(\kappa = 0.5\) in order to optimise the charge determination. Only tracks with transverse momentum with respect to the beam direction greater than 0.15 GeV/c were used to compute the jet charge. The jet charges in the forward and backward thrust hemispheres, defined in section 2, are labelled \(Q_F\) and \(Q_B\) respectively.

This analysis makes use of the mean jet charge separation between forward and backward hemispheres to measure the forward-backward asymmetry. For a given sample of events we define the charge separation as:

\[
\delta = \langle Q_- - Q_+ \rangle
\]  

(4)

where \(Q_-\) and \(Q_+\) are the jet charges measured in the thrust hemispheres with the negatively charged quark and positively charged quark respectively, and the average is over all events in the sample.

From the data one can measure the quantity \(\langle Q_F - Q_B \rangle\) where \(Q_F\) is the jet charge measured in the forward hemisphere and \(Q_B\) that in the backward hemisphere. For a sample consisting of a single type of down-like quark, where there are \(N_F\) events with the negative quark in the forward thrust hemisphere and \(N_B\) events with the negative quark in the backward thrust hemisphere, then:

\[
\langle Q_F - Q_B \rangle = \frac{N_F < Q_- - Q_+ > + N_B < Q_+ - Q_- >}{N_F + N_B}
\]

\[
= \frac{N_F - N_B}{N_F + N_B} \cdot \delta
\]

\[
= A \cdot \delta
\]  

(5)

\(^1\)The OPAL coordinate system is defined with positive \(z\) along the electron beam direction with \(\theta\) and \(\phi\) being the polar and azimuthal angles, respectively. The origin is in the centre of the detector, which is the nominal interaction point.
where $A$ is the forward-backward charge asymmetry in the sample.

For a sample consisting of a mixture of flavours, as in the lifetime tagged sample, the value of $\langle Q_F - Q_B \rangle$ of the whole sample can be related to the individual asymmetries and charge separations using:

$$
\langle Q_F - Q_B \rangle = \sum_i s_i F_i C_i \delta_i A_{\text{FB}}
$$

where the suffix $i$ denotes the quark flavour and the sum is over all flavours. The value of $s_i$ is defined as +1 for the down-like quarks and -1 for the up-like quarks. The fractions of each quark flavour in the sample are denoted by $F_i$ and the factors $C_i$ take into account the angular acceptance. These factors are described in section 3.1.

The charge separation, $\delta$, can be measured almost entirely from the data sample. In the case of no bias in the charge identification between positive and negative primary quarks, and with no correlation between $Q_-$ and $Q_+$, it can be seen that:

$$
\left( \frac{\delta}{2} \right)^2 = \langle Q_- \rangle \cdot \langle -Q_+ \rangle = -\langle Q_- \cdot Q_+ \rangle = -\langle Q_F \cdot Q_B \rangle.
$$

Taking into account the effects of possible charge bias and correlations between $Q_-$ and $Q_+$, one obtains:

$$
\delta^2 = 4 \cdot \frac{-\langle Q_F \cdot Q_B \rangle + \rho(Q_-, Q_+) \sigma^2(Q) + \mu^2(Q)}{1 + \rho(Q_-, Q_+)}
$$

where $\mu(Q)$ and $\sigma^2(Q)$ are the mean and variance of the charge of all hemispheres and $\rho(Q_-, Q_+)$ is the charge correlation between $Q_-$ and $Q_+$. This correlation is due to overall charge conservation in the event, and to migration of particles originating from the quark into the antiquark hemisphere (and vice versa). The derivation of this equation is given in appendix A. Only the correlation coefficient $\rho(Q_-, Q_+)$ has to be estimated from Monte Carlo, with all other quantities taken directly from data. Equation 7 takes into account small differences between the jet charge for positive and negative quarks introduced by the detector (via the $\mu^2(Q)$ term) but does assume that $\sigma(Q_-) = \sigma(Q_+)$. This assumption has been checked using Monte Carlo and found to be a very good approximation.

Equation 7 can be applied to a mixed sample of events or to a sample consisting of just one flavour. In the case of the lifetime tagged events, the charge separation of the sample, $\delta_{\text{tagged}}$, can be related to the individual charge separations for each quark flavour, $i$, using:

$$
\delta_{\text{tagged}} = \sum_i F_i \delta_i.
$$

Hence, from $\delta_{\text{tagged}}$ it is possible to extract $\delta_i$. This $\delta_i$ already includes the effect of $B^0 \overline{B}^0$ mixing and biases due to lifetime tagging, and so the asymmetry measurement is insensitive to these effects. This value of $\delta_i$ can then be used in Equation 6 to calculate the $b \overline{b}$ forward-backward asymmetry. As a mean quantity is used to measure the asymmetry, it is in effect a counting method; this is made explicit in equation 5. However, in the case of the measurement of the asymmetry at the $Z^0$ peak, the high statistics available allow the measurement to be performed separately in several regions of $|\cos \theta_T|$. These separate measurements may be combined, and the additional information from the variation in the measured asymmetry with angle adds precision to the overall asymmetry measurement. The fraction of each flavour in the tagged sample and the charge separation and asymmetry for charm and light quarks must be taken from elsewhere in order to make this measurement. It should be emphasised that the lifetime tagged sample is comprised of approximately 90% $b \overline{b}$ events and therefore only small systematic errors are introduced by the assumptions that are made about the non-$b$ component of the sample.
3.1 Acceptance Correction Factors

The analysis uses the direction of the experimentally determined thrust axis to estimate the direction of the primary quark and to define the event hemispheres. Detector efficiencies and acceptances mean that the thrust axis calculated is a smeared estimator of the true thrust axis of all final state particles. The correction needed to give the asymmetry that would be measured with an ideal detector (which would observe all of the final state particles) with no cut in $\cos \theta_T$ may be determined in a relatively model-independent manner. Such a correction must also account for the reduction in the observed charge separation for events near the edge of the acceptance, due to tracks failing the cuts. The smearing of the experimental thrust axis about the true thrust direction means that some events with $|\cos \theta_T| > 0.8$ satisfy the selection requirement $|\cos \theta_T| < 0.8$, while others with $|\cos \theta_T| < 0.8$ fail it. Monte Carlo predictions based on JETSET 7.3 and HERWIG 5.5 are used to correct for these effects. As this correction is essentially for detector effects, the different generators lead to very similar correction factors.

The correction factor to correct from the experimental to true thrust direction, and for the $\cos \theta_T$ cut, is calculated using the relation:

$$C_i = \frac{8}{3} \int_0^1 a_i(y) \delta_i(y) y dy$$

where $y = |\cos \theta_T^{\text{true}}|$, $a_i(y)$ is the combined event acceptance and lifetime tagging efficiency for flavour $i$ as a function of $y$ and $\delta_i$ is the charge separation as defined previously, for events of flavour $i$ passing the acceptance and tagging requirements. The tagging efficiency as a function of $|\cos \theta_T|$ for $b$-events was measured from the data in bins of $|\cos \theta_T|$. To convert this to a distribution of efficiencies as a function of $|\cos \theta_T^{\text{true}}|$, the events were reweighted (on a bin-by-bin basis) according to the ratio of the efficiency distributions in terms of $|\cos \theta_T|$ and $|\cos \theta_T^{\text{true}}|$ obtained from the Monte Carlo. The shape of efficiency for $c$-events was found to be very similar to that for $b$-events in the Monte Carlo, and also when determined from the data (to within the poor statistical accuracy); the shape determined from the data for $b$-events was therefore also used for the charm events. For the light quarks the Monte Carlo was used to estimate the efficiency as a function of $|\cos \theta_T^{\text{true}}|$ directly. The event acceptances were taken from the Monte Carlo for all quark flavours The $\delta_i$ are also taken from the Monte Carlo; however, the factors $C_i$ are only sensitive to the shape of the distributions $\delta_i(y)$, and this sensitivity is low. Example values for the factors appropriate to correct to the true thrust axis for the on-peak data are given in Table 1. The values obtained are very similar to those calculated assuming a uniform tagging efficiency for all quarks, no charge dilution and perfect estimation of the quark direction, i.e. just correcting for the limited geometrical acceptance.

<table>
<thead>
<tr>
<th></th>
<th>$C_{i}$</th>
<th>$C_{i}^{\text{quark}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>0.855 ± 0.003</td>
<td>0.846 ± 0.003</td>
</tr>
<tr>
<td>c</td>
<td>0.855 ± 0.010</td>
<td>0.846 ± 0.010</td>
</tr>
<tr>
<td>u, d, s</td>
<td>0.897 ± 0.008</td>
<td>0.878 ± 0.008</td>
</tr>
</tbody>
</table>

Table 1: The on-peak event-weighted average asymmetry correction factors appropriate for the thrust axis asymmetry ($C_i$) and the quark direction asymmetry ($C_{i}^{\text{quark}}$) when measured for events with a $|\cos \theta_T| < 0.8$. The errors quoted for u, d and s are purely statistical.

It should be noted that the asymmetry based on the thrust axis as measured by the jet-charge technique corrects for some QCD radiative effects by construction. This is because when hard gluon radiation forces both the $b$ and the $\bar{b}$ into the same event hemisphere, the result is a near-zero charge
being measured in both hemispheres of the event. The charge separation in these events is therefore small and they contribute little to the \( \langle Q_F - Q_H \rangle \) and to \( \delta \), and consequently contribute little to the measured asymmetry.

The asymmetry based on the quark direction is needed to extract \( \sin^2 \theta_W^{\text{eff}} \). This asymmetry includes corrections for the effects of decay and fragmentation, and also the residual QCD corrections not already accounted for by the charge-dilution effect described in the previous paragraph. As in the \( \cos \theta_W^{\text{true}} \) case, the acceptance extends beyond \( |\cos \theta_{\text{quark}}| = 0.8 \). The difference between \( |\cos \theta_T| \) and \( |\cos \theta_{\text{quark}}| \) can lead to a forward-going quark (\( \cos \theta_{\text{quark}} > 0 \)) being assigned to the backward thrust hemisphere, and vice versa, particularly for \( |\cos \theta_{\text{quark}}| \approx 0 \); this effect will be referred to as 'flipping'.

To obtain an asymmetry based on the original quark direction, without gluon radiation, the appropriate factors are:

\[
C_{i}^{\text{quark}} = \frac{8}{3} \int_0^1 a_i(z)((1 - f_i(z))\delta_i^{\text{unflipped}}(z) - f_i(z)\delta_i^{\text{flipped}}(z))dz
\]

where \( z = \cos \theta_{\text{quark}} \). The functions \( f_i \) describe the probability that the event is flipped, and are obtained from the Monte Carlo. Separate \( \delta_i \) functions are used for flipped and unflipped events. The correction factors to give the quark asymmetry are sensitive to the precise nature of the gluon radiation assumed in the generator models. The final corrections differ only slightly from those used to correct to the true thrust axis.

The acceptance functions \( a_i(z) \) were also taken from the Monte Carlo. In a similar way to the true thrust axis case, the light quark efficiencies as a function of \( \cos \theta_{\text{quark}} \) were taken directly from the Monte Carlo, while for b-events the measured tagging efficiency as a function of \( |\cos \theta_T| \) was used after unfolding the data into \( \cos \theta_{\text{quark}} \) using the Monte Carlo. The shape of the b-event efficiency was also assumed for c-events.

4 Analysis and Results

4.1 Assumed Standard Model Values

The small level of contamination from non-b events in the tagged sample must be corrected for, requiring a knowledge of their partial hadronic widths in \( Z^0 \) decay and their asymmetries. Some of this information is constrained by experimental data, but in other cases the Standard Model must be assumed.

For our standard working-point, the Standard Model predictions for the partial hadronic widths of the \( Z^0 \) into \( u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c} \) and \( b\bar{b} \) were predicted using the program ZFITTER (version 4.8) [17], with \( M_Z = 91.187 \text{ GeV}/c^2 \), \( M_{\text{top}} = 169 \text{ GeV}/c^2 \), \( M_{\text{Higgs}} = 300 \text{ GeV}/c^2 \), and \( \alpha_s = 0.12 \) [18]. These predicted values were then varied and the resulting changes in the measured asymmetries determined. The uncertainty \( \Gamma_{b\bar{b}}/\Gamma_{\text{had}} \) was taken as the difference between the world average experimental measured value and the resulting Standard Model fitted value in [18], which corresponds to 2.5%. The predicted and experimental average values coincide for \( \Gamma_{c\bar{c}}/\Gamma_{\text{had}} \) and the 11.7% error on the experimental average was taken as the uncertainty. A 15% error was assumed for the \( \Gamma_{s\bar{s}}/\Gamma_{\text{had}} \) values of the light quark events. For all variations in the partial widths, account was taken of the fact that the quark partial widths must add to give the total hadronic width. The predictions of ZFITTER were also used for the \( u, d, s \) and \( c \) forward-backward asymmetries. The uncertainty on the charm asymmetries was taken as 18% of the predicted value, corresponding to the difference between fitted and average charm pole asymmetries in [18]. The uncertainties in the light quark asymmetries were taken as 20%. Table 2 summarises the values of the Standard Model parameters at the \( Z^0 \) peak which were used.
<table>
<thead>
<tr>
<th>Event type</th>
<th>$\Gamma_i/\Gamma_{\text{had}}$</th>
<th>$A_{\text{FB}}(\sqrt{s}=89.52\text{GeV})$</th>
<th>$A_{\text{FB}}(\sqrt{s}=91.25\text{GeV})$</th>
<th>$A_{\text{FB}}(\sqrt{s}=92.94\text{GeV})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b\bar{b}$</td>
<td>0.216 ± 0.005</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$c\bar{c}$</td>
<td>0.173 ± 0.020</td>
<td>0.0316 ± 0.0057</td>
<td>0.061 ± 0.013</td>
<td>0.120 ± 0.026</td>
</tr>
<tr>
<td>$u\bar{u}$</td>
<td>0.173 ± 0.026</td>
<td>0.0315 ± 0.0063</td>
<td>0.061 ± 0.011</td>
<td>0.120 ± 0.022</td>
</tr>
<tr>
<td>$d\bar{d}$</td>
<td>0.220 ± 0.033</td>
<td>0.057 ± 0.011</td>
<td>0.093 ± 0.019</td>
<td>0.116 ± 0.023</td>
</tr>
<tr>
<td>$s\bar{s}$</td>
<td>0.220 ± 0.033</td>
<td>0.057 ± 0.011</td>
<td>0.093 ± 0.019</td>
<td>0.116 ± 0.023</td>
</tr>
</tbody>
</table>

Table 2: Standard Model parameters at the $Z^0$ peak used in the analysis. All values are as predicted by ZFITTER version 4.8 using input parameters as determined in [18].

4.2 The Composition of the Tagged Sample

The method presented here requires the composition of the tagged sample to be known to correct for the small contamination of non-$b$ events. To extract the fractions of each flavour, $F_i$, a double tag technique similar to that described in [15] was used. The number of tagged hemispheres, $N_t$, and the number of events in which both hemispheres are tagged, $N_{2t}$, can be expressed as:

$$N_t = 2N_{\text{had}}\left(\frac{\Gamma_{b\bar{b}}}{\Gamma_{\text{had}}}\eta_b + \frac{\Gamma_{c\bar{c}}}{\Gamma_{\text{had}}}\eta_c + \frac{\Gamma_{d\bar{d}}}{\Gamma_{\text{had}}}\eta_d + \frac{\Gamma_{s\bar{s}}}{\Gamma_{\text{had}}}\eta_s\right)$$

(11)

$$N_{2t} = N_{\text{had}}\left(\frac{\Gamma_{b\bar{b}}}{\Gamma_{\text{had}}}\eta_b^2\rho_b + \frac{\Gamma_{c\bar{c}}}{\Gamma_{\text{had}}}\eta_c^2\rho_c + \frac{\Gamma_{d\bar{d}}}{\Gamma_{\text{had}}}\eta_d^2\rho_d + \frac{\Gamma_{s\bar{s}}}{\Gamma_{\text{had}}}\eta_s^2\rho_s\right)$$

(12)

where $N_{\text{had}}$ is the number of multihadronic events and $\eta_i$ are the hemisphere tagging efficiencies for the different flavours. The $\rho_i$ are correlation factors which describe the correlation between the probabilities of tagging each of the hemispheres in a given event. The correlation factors for each flavour $i$ can be expressed in terms of single hemisphere tagging efficiency $\eta_i$ and of the efficiency of double tagging an event, $\eta_i^{dt}$, as:

$$\rho_i = \frac{\eta_i^{dt}}{\eta_i^{2t}}$$

(13)

For the cuts applied, the correlation factors are close to unity. As the fraction of the $b$ sample is approximately 90%, even large deviations from unity of $\rho_b$, $\rho_c$, $\rho_d$, and $\rho_s$ do not affect the measurement of $A_{\text{FB}}^b$. Hence, for the central value, only the effect of hemisphere correlations for $b$ quark events were included. For small deviations from unity, the correlation factor can be expressed as $\rho_b = 1 + \Delta \rho_b$, where $\Delta \rho_b = \Delta \rho_b^{\text{geom}} + \Delta \rho_b^{\text{phys}}$, where $\Delta \rho_b^{\text{geom}}$ comes from purely from the geometry of the detector and $\Delta \rho_b^{\text{phys}}$ comes from the underlying physics processes. The correlation $\Delta \rho_b^{\text{geom}}$ due to non-uniform tagging efficiency as a function of the $\cos \theta_T$ and $\phi$ was calculated from the data. This was done separately for the data taken with the two different versions of the silicon microvertex detector and for each year’s data, for which the overall efficiency was different. The two bottom hadrons in a $b\bar{b}$ event are likely to be produced back-to-back. Their decay products are therefore likely to strike geometrically opposite parts of the detector. This introduces an efficiency correlation if the efficiency of the detector is not completely uniform. This correlation can be estimated by measuring the hemisphere tagging probability in the real data as a function of the thrust axis direction as:

$$\Delta \rho_b^{\text{geom}} = 1 - \frac{A(f^+(\theta, \phi) f^-(\theta, \phi))}{(f^+(\theta, \phi) + f^-(\theta, \phi))^2}$$

(14)

where $f^+$ and $f^-$ are the fraction of tagged hemispheres in the $+z$ and $-z$ directions respectively, and the average is taken over the full solid angle acceptance. The actual estimation was carried out in small bins of $|\cos \theta|$ and $\phi$ and the effect of statistical fluctuation of the measurement of $f$ was assessed by a Monte Carlo technique. The correlation $\Delta \rho_b^{\text{geom}}$ was estimated to be $0.0131 \pm 0.0012$ for
the data taken in 1991 and 1992, $0.0268 \pm 0.0019$ for the data taken in 1993, and $0.0215 \pm 0.0011$ for that taken in 1994. The errors are statistical. The values differ due to known changes in the tracking system efficiencies.

Other sources of correlation, represented by the term $\Delta \rho_b^{\text{phys}}$, were investigated using Monte Carlo simulations. Those considered were the momentum-momentum correlation between the b-hadrons produced from the $b$ and $\bar{b}$ quarks, and the negative correlations due to hard gluon radiation (which tends to reduce the back-to-back nature of the event). The correlation factor $\Delta \rho_b^{\text{phys}}$ due to these effects is $\Delta \rho_b^{\text{phys}} = -0.0008 \pm 0.0060$. The overall correlation factor is $\rho_b = 1.0123 \pm 0.0061$ for the 1991 and 1992 data, $1.0260 \pm 0.0062$ for the 1993 data, and $1.0207 \pm 0.0061$ for the 1994 data.

The analysis procedure was:

- $N_t$, $N_{2t}$ and $N_{\text{had}}$ were measured from data
- $\rho_b$ was estimated from data and Monte Carlo.
- $\eta_t$, $\eta_c$ and $\eta_b$ were estimated from Monte Carlo simulation
- the hadronic partial widths were taken from the Standard Model prediction; there is a small flavour bias in the event selection, which is taken from the Monte Carlo predictions.
- the two equations, 11 and 12, were solved for $\eta_b$ and $\eta_c$
- The hemisphere tagging efficiencies $\eta_i$ were converted into the event tagging efficiencies $\overline{\eta}_i$ using the relation $\overline{\eta}_i = 2\eta_i(1 - \eta_i \cdot \rho_i) + \rho_i \cdot \eta_i^2$.
- the fraction of flavour $i$ present in the sample, $F_i$, was extracted using the relation:

$$F_i = \frac{\Gamma_i/\Gamma_{\text{had}} \overline{\eta}_i}{\sum_k \Gamma_k/\Gamma_{\text{had}} \overline{\eta}_k} \quad (15)$$

where $k$ runs over all the five flavours.

Table 3 summarises the tagging efficiencies and the corresponding fractions for the various flavours present in the sample of tagged events.

<table>
<thead>
<tr>
<th>Event type</th>
<th>Tagging efficiency $\eta_i$</th>
<th>Fraction $F_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b\bar{b}$</td>
<td>$0.3192 \pm 0.0013$</td>
<td>$0.9015 \pm 0.0036$</td>
</tr>
<tr>
<td>$c\bar{c}$</td>
<td>$0.0267 \pm 0.0016$</td>
<td>$0.0601 \pm 0.0036$</td>
</tr>
<tr>
<td>$u\bar{u}$</td>
<td>$0.0047 \pm 0.0001$</td>
<td>$0.0106 \pm 0.0003$</td>
</tr>
<tr>
<td>$d\bar{d}$</td>
<td>$0.0047 \pm 0.0001$</td>
<td>$0.0134 \pm 0.0004$</td>
</tr>
<tr>
<td>$s\bar{s}$</td>
<td>$0.0050 \pm 0.0001$</td>
<td>$0.0141 \pm 0.0004$</td>
</tr>
</tbody>
</table>

Table 3: Event tagging efficiencies and fractions in the data sample (all beam energies are included). The errors are from Monte Carlo statistics for $u, d$ and $s$. For $b\bar{b}$ and $c\bar{c}$ events the tagging efficiencies were estimated from the data and the errors include only data and Monte Carlo statistical effects.

4.3 Results

Monte Carlo studies indicate that the $\delta$ and $\langle Q_F \cdot Q_B \rangle$ are independent of $\sqrt{s}$ over the small range in our data. Thus, we measure them from combined on- and off-peak data. The mean forward-backward charge product was found to be $\langle Q_F \cdot Q_B \rangle = -0.00569 \pm 0.00010$, and $\mu(Q) = 0.00526 \pm 0.00035$, while
<table>
<thead>
<tr>
<th>Event Type</th>
<th>Charge Separation $\delta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b\bar{b}$</td>
<td>$-0.132 \pm 0.001$</td>
</tr>
<tr>
<td>$c\bar{c}$</td>
<td>$-0.141 \pm 0.004$</td>
</tr>
<tr>
<td>$u\bar{u}$</td>
<td>$-0.212 \pm 0.009$</td>
</tr>
<tr>
<td>$d\bar{d}$</td>
<td>$-0.110 \pm 0.008$</td>
</tr>
<tr>
<td>$s\bar{s}$</td>
<td>$-0.137 \pm 0.007$</td>
</tr>
</tbody>
</table>

Table 4: Charge separations for the various flavours. The values quoted for $b\bar{b}$ were determined from the data, whereas for the other flavours they were obtained from Monte Carlo. The errors are statistical only.

$\sigma^2(Q) = 0.0418 \pm 0.0004$, where the errors are statistical only. The mean charge separation for the tagged sample was obtained using equation 7 and the Monte Carlo prediction that $\rho(Q_-, Q_+) = -0.035$, and found to be:

$$\delta_{\text{tagged}} = -0.1332 \pm 0.0013$$  \hspace{1cm} (16)

where again the error is statistical only. The negative solution to the equation is taken to correspond to Monte Carlo prediction and naive expectations. It has been confirmed by using lepton from $b$-decays in one hemisphere to infer the sign of the charge of the parton in the opposite hemisphere; the mean jet charge in the opposite hemisphere is found to have the same sign as the expected parton charge, implying that $\delta$ indeed should have a negative sign. This value is then corrected using the known fractions of the different quark flavours, and the $\delta$ values for the non-$b$ events (obtained from JETSET and given in Table 4) to obtain the value of $\delta_b$ for the sample (also given in Table 4).

For the three different centre-of-mass interval samples we obtained:

\[
\begin{align*}
\langle Q_F - Q_B \rangle(\sqrt{s} = 89.52\text{GeV}) & = -0.0069 \pm 0.0038, \\
\langle Q_F - Q_B \rangle(\sqrt{s} = 91.25\text{GeV}) & = -0.0092 \pm 0.0008, \\
\langle Q_F - Q_B \rangle(\sqrt{s} = 92.94\text{GeV}) & = -0.0172 \pm 0.0032.
\end{align*}
\]

By applying Equations 6 and 8, the asymmetries based on the true thrust axis, were determined:

\[
\begin{align*}
A_{FB}^b(\sqrt{s} = 89.52\text{GeV}) & = 0.062 \pm 0.034, \\
A_{FB}^b(\sqrt{s} = 91.25\text{GeV}) & = 0.0935 \pm 0.0074, \\
A_{FB}^b(\sqrt{s} = 92.94\text{GeV}) & = 0.172 \pm 0.028,
\end{align*}
\]

where the errors are statistical only. The statistical error includes the statistical uncertainties on the fractions of the different flavours, $F_i$. The prescription for calculating the statistical error is taken from [19], with a small ($\sim 2\%$) correction to take into account the correlation between $\langle Q_F - Q_B \rangle$ and $\delta$.

The analysis of the peak asymmetry has been repeated in four bins of $|\cos \theta_T|$. The resulting asymmetries are presented in Fig. 2. The errors shown are purely statistical. Each bin provides a result statistically compatible with the result using all events. Additional precision may be obtained by determining the peak asymmetry by forming the weighted average of these results. The additional precision is gained by utilising the additional information on the variation in the measured asymmetry as a function of $|\cos \theta_T|$. In this way, we obtain:

\[
A_{FB}^b(\sqrt{s} = 91.25\text{GeV}) = 0.0963 \pm 0.0067,
\]

where the error is statistical only.
The measured b quark asymmetries, corrected using the \( C_i^{\text{asym}} \) values derived from JETSET 7.3, were:

\[
\begin{align*}
A_{FB}^b(\sqrt{s} = 89.52\,\text{GeV}) & = 0.063 \pm 0.034, \\
A_{FB}^b(\sqrt{s} = 91.25\,\text{GeV}) & = 0.0973 \pm 0.0067, \\
A_{FB}^b(\sqrt{s} = 92.94\,\text{GeV}) & = 0.173 \pm 0.029,
\end{align*}
\]

where the errors are again statistical only.

5 Systematic Errors

The systematic errors on \( A_{FB}^b \) are summarised in Table 5. They are discussed in more detail in the following sections.

<table>
<thead>
<tr>
<th>Source of Error</th>
<th>( \sqrt{s} = 89.52 , \text{GeV} )</th>
<th>( \sqrt{s} = 91.25 , \text{GeV} )</th>
<th>( \sqrt{s} = 92.94 , \text{GeV} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fragmentation modelling</td>
<td>( \pm 0.0011 )</td>
<td>( \pm 0.0024 )</td>
<td>( \pm 0.0044 )</td>
</tr>
<tr>
<td>( b ) decay multiplicity</td>
<td>( \pm 0.0003 )</td>
<td>( \pm 0.0005 )</td>
<td>( \pm 0.0006 )</td>
</tr>
<tr>
<td>Jet Charge identification</td>
<td>( \pm 0.0011 )</td>
<td>( \pm 0.0025 )</td>
<td>( \pm 0.0044 )</td>
</tr>
<tr>
<td>Acceptance model</td>
<td>( \pm 0.0005 )</td>
<td>( \pm 0.0008 )</td>
<td>( \pm 0.0014 )</td>
</tr>
<tr>
<td>Material asymmetry</td>
<td>( \pm 0.0005 )</td>
<td>( \pm 0.0007 )</td>
<td>( \pm 0.0014 )</td>
</tr>
<tr>
<td>( u, d ) and ( s ) efficiency</td>
<td>( \pm 0.0003 )</td>
<td>( \pm 0.0004 )</td>
<td>( \pm 0.0012 )</td>
</tr>
<tr>
<td>Efficiency ( \cos \theta_T ) dependence</td>
<td>( \pm 0.0003 )</td>
<td>( \pm 0.0004 )</td>
<td>( \pm 0.0008 )</td>
</tr>
<tr>
<td>Hemisphere correlation</td>
<td>( \pm 0.0001 )</td>
<td>( \pm 0.0005 )</td>
<td>( \pm 0.0011 )</td>
</tr>
<tr>
<td>( \tau-\phi ) detector resolution</td>
<td>( \pm 0.0010 )</td>
<td>( \pm 0.0014 )</td>
<td>( \pm 0.0023 )</td>
</tr>
<tr>
<td>( \cot \theta ) detector resolution</td>
<td>( \pm 0.0010 )</td>
<td>( \pm 0.0015 )</td>
<td>( \pm 0.0028 )</td>
</tr>
<tr>
<td>Thrust direction resolution</td>
<td>( \pm 0.0007 )</td>
<td>( \pm 0.0010 )</td>
<td>( \pm 0.0018 )</td>
</tr>
<tr>
<td>Detector effects</td>
<td>( \pm 0.0018 )</td>
<td>( \pm 0.0026 )</td>
<td>( \pm 0.0049 )</td>
</tr>
<tr>
<td>Monte Carlo Statistics</td>
<td>( \pm 0.0003 )</td>
<td>( \pm 0.0013 )</td>
<td>( \pm 0.0024 )</td>
</tr>
<tr>
<td>( \Gamma_{b\tau}/\Gamma_{\text{had}} )</td>
<td>( \pm 0.0004 )</td>
<td>( \pm 0.0023 )</td>
<td>( \pm 0.0043 )</td>
</tr>
<tr>
<td>( \Gamma_{\tau}/\Gamma_{\text{had}} )</td>
<td>( &lt; 0.0001 )</td>
<td>( \pm 0.0001 )</td>
<td>( \pm 0.0002 )</td>
</tr>
<tr>
<td>( \Gamma_{\tau}/\Gamma_{\text{had}} )</td>
<td>( \pm 0.0001 )</td>
<td>( \pm 0.0002 )</td>
<td>( \pm 0.0004 )</td>
</tr>
<tr>
<td>( \Gamma_{\tau}/\Gamma_{\text{had}} )</td>
<td>( \pm 0.0001 )</td>
<td>( \pm 0.0004 )</td>
<td>( \pm 0.0006 )</td>
</tr>
<tr>
<td>( \Gamma_{\text{d}/\Gamma_{\text{had}}} )</td>
<td>( &lt; 0.0001 )</td>
<td>( \pm 0.0001 )</td>
<td>( \pm 0.0003 )</td>
</tr>
<tr>
<td>( A_{FB}^c )</td>
<td>( \pm 0.0003 )</td>
<td>( \pm 0.0008 )</td>
<td>( \pm 0.0013 )</td>
</tr>
<tr>
<td>( A_{FB}^s )</td>
<td>( \pm 0.0001 )</td>
<td>( \pm 0.0002 )</td>
<td>( \pm 0.0005 )</td>
</tr>
<tr>
<td>( A_{FB}^d )</td>
<td>( \pm 0.0001 )</td>
<td>( \pm 0.0003 )</td>
<td>( \pm 0.0003 )</td>
</tr>
<tr>
<td>( A_{FB}^s )</td>
<td>( \pm 0.0002 )</td>
<td>( \pm 0.0003 )</td>
<td>( \pm 0.0004 )</td>
</tr>
<tr>
<td>Standard Model parameters</td>
<td>( \pm 0.0006 )</td>
<td>( \pm 0.0025 )</td>
<td>( \pm 0.0046 )</td>
</tr>
<tr>
<td>Total systematic error</td>
<td>( \pm 0.0022 )</td>
<td>( \pm 0.0046 )</td>
<td>( \pm 0.0084 )</td>
</tr>
<tr>
<td>Statistical error</td>
<td>( \pm 0.0337 )</td>
<td>( \pm 0.0067 )</td>
<td>( \pm 0.0282 )</td>
</tr>
</tbody>
</table>

Table 5: Uncertainties on the determination of \( A_{FB}^b \) below, on and above the peak when using the statistical method. Where appropriate, the boldface items represent the sum of the items in the previous section.

5.1 Jet-charge Identification

The uncertainties due to modelling of fragmentation on the \( u, d, s \) and \( c \) jet charge properties were estimated using Monte Carlo events generated with different fragmentation parameters. The parameter variations \([10]\) are given in Table 6. Most are similar to those in our previous publication on the
forward-backward charge asymmetry of hadronic $Z^0$ decays [20]. In addition to these, the JETSET baryon production parameter and the popcorn parameter was also varied. The effect of turning off $B^0\bar{B}^0$ mixing were also considered. (The standard mixing parameters for the Monte Carlo generations were $x_d = 0.7$ and $x_s = 999$, where $x_d = |m_1 - m_2| \tau_{B_d^0}$, $m_1$ and $m_2$ are the two mass eigenstates of the $B_d^0$ and $\tau_{B_d^0}$ its lifetime; $x_s$ is the corresponding parameter for the mixing in the $B_s^0$ system.)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nominal Value</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda_{QCD}$</td>
<td>0.29</td>
<td>0.28 — 0.31</td>
</tr>
<tr>
<td>$Q_0$</td>
<td>1.0</td>
<td>0.7 — 1.8</td>
</tr>
<tr>
<td>$\sigma_q$</td>
<td>0.37</td>
<td>0.32 — 0.40</td>
</tr>
<tr>
<td>$\gamma_s$</td>
<td>0.285</td>
<td>0.250 — 0.320</td>
</tr>
<tr>
<td>$[V/(V + S)]_{u,d}$</td>
<td>0.50</td>
<td>0.30 — 0.75</td>
</tr>
<tr>
<td>$[V/(V + S)]_s$</td>
<td>0.60</td>
<td>0.50 — 0.75</td>
</tr>
<tr>
<td>$[V/(V + S)]_{c,b}$</td>
<td>0.75</td>
<td>0.65 — 0.80</td>
</tr>
<tr>
<td>$\epsilon_b$</td>
<td>0.0055</td>
<td>0.0025 — 0.0095</td>
</tr>
<tr>
<td>$\epsilon_c$</td>
<td>0.05</td>
<td>0.03 — 0.07</td>
</tr>
<tr>
<td>direct baryon rate</td>
<td>0.1</td>
<td>0.08 — 0.12</td>
</tr>
<tr>
<td>popcorn parameter</td>
<td>1</td>
<td>0 — 2</td>
</tr>
<tr>
<td>$x_d$</td>
<td>0.7</td>
<td>0 — 0.7</td>
</tr>
<tr>
<td>$x_s$</td>
<td>999</td>
<td>0 — 999</td>
</tr>
</tbody>
</table>

Table 6: The ranges of parameters assumed in the fragmentation and $B^0\bar{B}^0$ mixing modelling systematic error study.

The fragmentation model influences the result in two ways: through the non-b quark $\delta$ values; and through the charge correlation coefficient, $\rho[Q_-, Q_+]$. The overall relative uncertainty on the $\delta$ values is about 10%, with the $\epsilon$, popcorn and $[V/(V + S)]_{u,d}$ contributions largest, while the relative uncertainty on the correlation is about 8%, with the mixing and $[V/(V + S)]_{u,d}$ contributions largest. The effects of incorrect modelling of the b-decay multiplicities were investigated. The b-decay multiplicity per thrust hemisphere was varied by the OPAL measured uncertainty of 0.51 [21], which corresponds to approximately 10%, to which an additional multiplicity variation of 0.25 has been added to account for the observed multiplicity bias in tagged hemispheres. The effects on the final result are small. This is because the correlation coefficient, $\rho[Q_-, Q_+]$, is insensitive to such modelling effects.

The effect of varying the charm decay multiplicity by $\pm 0.14$, as suggested in [21], is negligible.

### 5.2 Detector Effects

The acceptances for $Z^0$ decays to the different quark flavours are predicted to differ by small amounts in the Monte Carlo simulations. The central value for the asymmetry is obtained using the predictions for JETSET 7.3. The resulting change in the measured $A_{FB}^\gamma$ when the predictions of HERWIG 4.5 are used are taken as an estimate of the systematic uncertainty in these acceptances.

The analysis assumes that within our acceptance the material in front of the tracking detectors is symmetric in $\cos \theta_T$, which is true within the statistical precision in the Monte Carlo samples. Any material asymmetry can lead to an apparent forward-backward charge asymmetry in the sample, as the charge bias will differ in the forward and backward hemispheres. The maximum extent of such an effect may be determined using the asymmetry in the rate of $\gamma$ conversions as a function of $|\cos \theta_T|$. Within the acceptance, this conversion asymmetry is both independent of $\cos \theta$ and consistent with
zero to within the 0.7% statistical precision. This uncertainty is combined with the observed charge bias to obtain a relative uncertainty on the final measured $A_{FB}^b$ of 0.7%.

The determination of the $b$ asymmetry is sensitive to the average tagging efficiencies of the various quark flavours. These have to be known to determine the fraction of the different flavours in the sample. The charm and bottom tagging efficiencies are determined directly from the data, which makes the measurement insensitive to the modelling of the heavy quark fragmentation and decay. The $u$, $d$ and $s$ tagging efficiencies were determined from Monte Carlo. The efficiency for tagging a $b$-event, $\mathcal{E}_b$, is found to be 13% higher in the data than the Monte Carlo. This could be due to the incorrect $b$-lifetimes and mean $b$-decay multiplicity in the Monte Carlo, or else due to smearing effects in the data that are not modelled in the Monte Carlo. To account for such a possible effect, the tagging efficiencies for the light quarks were increased by 13%, and the resulting change in the measured asymmetry is taken as a systematic uncertainty. Effects due to the fragmentation modelling of $u,d$ and $s$ quarks were found to have a negligible effect on the tagging efficiencies.

The shapes of the quark tagging efficiencies as a function of $|\cos \theta_T|$ are required for the calculation of the acceptance factors $C_i'$. These are taken from the Monte Carlo for the light quarks. In the case of the $b$-quarks, the efficiency as a function of $|\cos \theta_T|$ was inferred from the single and double tagging rates. The statistical precision of the efficiencies obtained in a similar fashion for the $c$-quark is poor, and so the $b$-quark form is assumed, and the difference between the factor calculated in this way and that when the Monte Carlo curve was used is negligible. The shape of the $b$-quark efficiency curve is parameterised and the change caused by varying the shape according to the fit errors was used to estimate the systematic uncertainty. The various efficiency curves are shown in Fig. 3.

The factors $\rho$ describing the correlation between the tagging efficiencies in the two hemispheres of an event where determined in part from the data and in part from Monte Carlo. For the central value, the non-$b$ events were assumed to have no such correlation. This is supported by the Monte Carlo in the case of the $u$, $d$ and $s$ quark events. In the case of charm, the correlations were seen to be similar to those for $b$-events. However, the effect of the small deviations from unity in the charm case were not seen to change the $b$- and $c$-efficiencies determined in the Monte Carlo sample to within the statistical precision for the test. As an estimate of the systematic uncertainty in setting the charm correlation factor to unity, the data efficiencies were re-estimated with the charm correlation factor set equal to that determined for $b$-events. The change in the resulting $b$ asymmetry was taken as a systematic error. The limited Monte Carlo and data statistics also contribute to the systematic error.

The effects of incorrect modelling of the track resolutions in $r$-$\phi$ and $\cos \theta$ in the Monte Carlo on the corrections to our observed quantities were investigated by smearing the Monte Carlo. The factor rescaling the difference between the true and reconstructed values of the $r$-$\phi$ track parameters was varied by 20%, while that of the $\cot \theta$ was increased by 100%. As the effect of these resolution changes on the light quark tagging efficiency is smaller than the 13% variation mentioned earlier, it has not been included in this systematic uncertainty to avoid double-counting.

The correction to unfold the quark asymmetry from the asymmetry obtained using the thrust axis is performed by calculating the factors $C_i$ over $\cos \theta_{\text{quark}}$ using Equation 10. Thus, the Monte Carlo is used to describe the detector smearing, the hadronisation effects and the effects of final-state photon and of gluon radiation. This full correction leads to a larger asymmetry than that measured using the experimental thrust axis. Similar corrections may be performed to correct only for the smearing of the true thrust axis by detector resolutions and reconstruction efficiencies, which lead to a peak asymmetry that is 0.001 smaller than that calculated using the quark axis.

To investigate any possible sensitivity to the $b$-tagging cut used, the analysis has been repeated in
five bins of $L/\sigma$, beginning at four, each with an approximately equal number of tagged events. The results are statistically consistent with the central value quoted, and no significant trend is observed.

The analysis was also repeated for different values of $\kappa$ between 0.3 and 2.0, and after allowing for the correlation between the results at different $\kappa$ values, no statistically significant differences in the $A_{FB}^b$ obtained were observed.

5.3 Monte Carlo Statistics
Monte Carlo events were used to estimate the $u$, $d$ and $s$ tagging efficiency as well as the properties of the jet charge of $u\tau$, $d\bar{d}$, $s\bar{s}$ and $c\bar{c}$ events. The uncertainties due to Monte Carlo statistics on these parameters are reported in Table 3 and 4.

5.4 Dependence on Standard Model Parameters
The extracted value of $A_{FB}^b$ depends on the assumed hadronic partial widths and the forward-backward asymmetries of the non-$b$ events. The partial widths and asymmetries were varied according to the uncertainties described in section 4.1. The uncertainty on $\Gamma_{b\tau}/\Gamma_{had}$ has the largest effect on the overall systematic error, since the selected events are $b$-dominated. When the assumed value of $\Gamma_{b\tau}/\Gamma_{had}$ is increased the measured asymmetry is reduced. Although the fraction of charm in the sample is larger than $u$, $d$ or $s$, the measurement of $A_{FB}^b$ is almost completely insensitive to the uncertainty on $\Gamma_{c\tau}/\Gamma_{had}$. This is because the charm partial width enters in the determination of $A_{FB}^b$ always as the product $\Gamma_{c\tau}/\Gamma_{had} \cdot \eta_c$, and therefore a variation of $\Gamma_{c\tau}/\Gamma_{had}$ is compensated by an opposite variation of the charm tagging efficiency, which is determined from the data, thus keeping the product about constant. This does not happen in the case of $u$, $d$ and $s$, for which the tagging efficiencies are taken from Monte Carlo.

The measured value of $A_{FB}^b$ is increased when the assumed value of $A_{FB}^{c}$ is increased, and vice versa. The contributions to the overall systematic error from the forward-backward asymmetries of $u$, $d$ and $s$ are very small.

6 The Event-by-Event Method
As an independent check the analysis was performed using a different method, still based on the jet charge determination, which will be referred to as the event-by-event method. In this method the angular distribution of Equation 1 was constructed by estimating event-by-event the direction of the quark emitted in the final state. Within the sample of lifetime tagged events, the jet charge $Q_{jet}$ was computed for each hemisphere using Equation 3 with $\kappa = 0.4$, which optimizes the precision of this measurement. In addition only events having hemispheres with jet charges of opposite sign were accepted. The sign of the jet charge was then used to indicate the charge of the primary quark in a given hemisphere. The requirement of oppositely charged hemispheres rejects about 45% of the tagged events, but enhances the probability of correct identification of the direction of the primary quark.

The observed angular distribution of the outgoing quark can be expressed as:

$$\frac{d\sigma^{obs}}{dx} = C(1 + x^2 + \frac{8}{3} A_{FB}^{obs} x ) \eta(x)$$

(17)

where $x = -Q_{jet} \cdot |\cos \theta_T|/|Q_{jet}|$ where $Q_{jet}$ is measured in the forward hemisphere. The constant $C$ is for normalization, and $\eta(x)$ is the tagging efficiency as a function of angle for an event. It is assumed that the efficiencies of events for each primary flavour all have the same shape; the systematic error introduced by this assumption is addressed later. It is also assumed that the efficiencies are even functions of $x$. The observed asymmetry is defined as:

$$A_{FB}^{obs} = \sum_i s_i F_i(2P_i - 1) A_{FB}^{i}$$

(18)
where $s_i$ is +1 for the down-like quarks and −1 for the up-like quarks, and $F_i$ is the fraction of the flavour type $i$ present in the data sample, defined in Equation 15. The term $P_i$ is the probability of correctly identifying the direction of the outgoing quark with flavour $i$.

Using Equation 17 the observed asymmetry $AFB^{obs}$ was obtained by maximising the log likelihood:

$$\ln L = \sum_j \ln[C\eta(x_j)] + \sum_j \ln[1 + x_j^2 + \frac{8}{3} AFB^{obs} x_j^2]$$

(19)

where the sum is over all the selected events and $AFB^{obs}$ is the only free parameter in the fit. The first term is a constant for a given set of events, so that the efficiency as a function of $x$ does not need to be known.

For a given flavour $i$ the fraction of events with opposite jet charges $f_{opp}^{i}$, assuming no correlation between hemispheres, is given by:

$$f_{opp}^{i} = P_i^2 + (1 - P_i)^2$$

(20)

where $P_i$ is the probability of correctly identifying the sign of the charge of the outgoing quark with flavour $i$ in a given hemisphere. The probability $P_i$ of Equation 18 is related to $P_i$ by the relation:

$$P_i = P_i^2 / (P_i^2 + (1 - P_i)^2).$$

(21)

The observed fraction of opposite charged events over the total number of tagged events, $f^{obs}_{opp}$, is given by:

$$f^{obs}_{opp} = \sum_i f_{opp}^{i} \cdot P_i.$$  

(22)

The small correlation between the jet charges of opposite hemispheres was estimated from Monte Carlo, and was taken into account when computing the charge identification probabilities $P_i$. The values of the probabilities $P_u$, $P_d$, $P_s$ and $P_c$ were derived from the Monte Carlo. The charge identification probability for b̅b̅ events was then obtained from the data using Equations 20, 21 and 22. Due to the high b-fraction of the data sample the uncertainties in the modelling of the light flavour fragmentation are expected to have a small effect on the determination of $AFB^{b\bar{b}}$. Table 7 summarises the values of $P_i$ used in the analysis with their statistical errors.

<table>
<thead>
<tr>
<th>Event Type</th>
<th>Charge id. probability $P_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>b̅b̅</td>
<td>0.756 ± 0.005</td>
</tr>
<tr>
<td>c̅c̅</td>
<td>0.777 ± 0.001</td>
</tr>
<tr>
<td>u̅u̅</td>
<td>0.868 ± 0.001</td>
</tr>
<tr>
<td>d̅d̅</td>
<td>0.763 ± 0.001</td>
</tr>
<tr>
<td>s̅s̅</td>
<td>0.794 ± 0.001</td>
</tr>
</tbody>
</table>

Table 7: Charge identification probabilities. The values quoted for b̅b̅ were determined from the data, whereas for the other flavours they were obtained from Monte Carlo. The errors are statistical only.

Using the likelihood function given in Equation 19, the data were fitted to obtain the observed asymmetry, which was then corrected to extract $AFB^{b\bar{b}}$ according to Equation 18. The predictions from ZFITTER were used for the u, d, s and c forward-backward asymmetries. For the three different centre-of-mass interval samples, the following asymmetries were obtained:

$AFB^{b\bar{b}}(\sqrt{s} = 89.52\text{GeV}) = 0.051 \pm 0.038 \pm 0.002,$

$AFB^{b\bar{b}}(\sqrt{s} = 91.25\text{GeV}) = 0.1030 \pm 0.0082 \pm 0.0051,$

$AFB^{b\bar{b}}(\sqrt{s} = 92.94\text{GeV}) = 0.173 \pm 0.032 \pm 0.009,$

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where the first error is statistical and the second systematic. The statistical error includes the statistical uncertainty on the determination of $P_b$ from Equation 21 as well as the statistical uncertainty on the $b$ and $c$ tagging efficiencies from data.

The systematic errors were estimated as for the statistical method, and are summarised in Table 8. In calculating the $b$ asymmetry using the likelihood fit, it was assumed that the tagging efficiencies for the various quark flavours have the same $|\cos \theta_T|$ dependence. This assumption was tested on both Monte Carlo and data and found to be valid within the statistical uncertainties for $c\bar{c}$ and $b\bar{b}$ which represent almost the entire data sample. The tagging efficiencies for $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ events seems to have a slightly different shape in the range $0.6 < |\cos \theta_T| < 0.8$. To determine the sensitivity of the measurement to possible different $|\cos \theta_T|$ dependence of the efficiency for different event types, we divided the $|\cos \theta_T|$ range in two different bins corresponding to $|\cos \theta_T| < 0.6$ and $0.6 < |\cos \theta_T| < 0.8$. In each bin the analysis was repeated independently, using its relative tagging efficiencies, and determining the relative $b$ asymmetry. Then a weighted average of the $b$ asymmetries extracted in this way was computed, which differed by 0.0011 from the asymmetry determined assuming the same efficiency over the whole $|\cos \theta_T|$ range. This difference was quoted as a systematic error.

The results are in good agreement with the main method.

<table>
<thead>
<tr>
<th></th>
<th>$\sqrt{s} = 89.52$ GeV</th>
<th>$\sqrt{s} = 91.25$ GeV</th>
<th>$\sqrt{s} = 92.94$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>fragmentation modelling</strong></td>
<td>$\pm 0.0020$</td>
<td>$\pm 0.0034$</td>
<td>$\pm 0.0057$</td>
</tr>
<tr>
<td><strong>b decay multiplicity</strong></td>
<td>$\pm 0.0003$</td>
<td>$\pm 0.0005$</td>
<td>$\pm 0.0009$</td>
</tr>
<tr>
<td><strong>Jet Charge identification</strong></td>
<td>$\pm 0.0020$</td>
<td>$\pm 0.0035$</td>
<td>$\pm 0.0058$</td>
</tr>
<tr>
<td>acceptance model</td>
<td>$\pm 0.0005$</td>
<td>$\pm 0.0010$</td>
<td>$\pm 0.0017$</td>
</tr>
<tr>
<td>$u$, $d$ and $s$ efficiency</td>
<td>$\pm 0.0001$</td>
<td>$\pm 0.0005$</td>
<td>$\pm 0.0008$</td>
</tr>
<tr>
<td>Efficiency $\cos \theta_T$ dependence</td>
<td>$\pm 0.0005$</td>
<td>$\pm 0.0011$</td>
<td>$\pm 0.0018$</td>
</tr>
<tr>
<td>Hemisphere correlation</td>
<td>$\pm 0.0001$</td>
<td>$\pm 0.0007$</td>
<td>$\pm 0.0016$</td>
</tr>
<tr>
<td>$r$-$\phi$ detector resolution</td>
<td>$\pm 0.0005$</td>
<td>$\pm 0.0011$</td>
<td>$\pm 0.0019$</td>
</tr>
<tr>
<td>$\cot \theta$ detector resolution</td>
<td>$\pm 0.0005$</td>
<td>$\pm 0.0010$</td>
<td>$\pm 0.0016$</td>
</tr>
<tr>
<td>Thrust direction resolution</td>
<td>$\pm 0.0002$</td>
<td>$\pm 0.0005$</td>
<td>$\pm 0.0008$</td>
</tr>
<tr>
<td><strong>Detector effects</strong></td>
<td>$\pm 0.0010$</td>
<td>$\pm 0.0023$</td>
<td>$\pm 0.0040$</td>
</tr>
<tr>
<td><strong>Monte Carlo Statistics</strong></td>
<td>$\pm 0.0001$</td>
<td>$\pm 0.0001$</td>
<td>$\pm 0.0002$</td>
</tr>
<tr>
<td>$\Gamma_{\bar{b}b}/\Gamma_{\text{had}}$</td>
<td>$\pm 0.0003$</td>
<td>$\pm 0.0026$</td>
<td>$\pm 0.0047$</td>
</tr>
<tr>
<td>$\Gamma_{c\bar{c}}/\Gamma_{\text{had}}$</td>
<td>$\pm 0.0011$</td>
<td>$\pm 0.0003$</td>
<td>$\pm 0.0006$</td>
</tr>
<tr>
<td>$\Gamma_{h}\bar{}^{'}/\Gamma_{\text{had}}$</td>
<td>$&lt; 0.0001$</td>
<td>$\pm 0.0002$</td>
<td>$\pm 0.0003$</td>
</tr>
<tr>
<td>$T_{\bar{c}\tau}/T_{\text{had}}$</td>
<td>$\pm 0.0001$</td>
<td>$\pm 0.0006$</td>
<td>$\pm 0.0011$</td>
</tr>
<tr>
<td>$T_{d\tau}/T_{\text{had}}$</td>
<td>$&lt; 0.0001$</td>
<td>$\pm 0.0002$</td>
<td>$\pm 0.0003$</td>
</tr>
<tr>
<td>$A_{bFB}$</td>
<td>$\pm 0.0005$</td>
<td>$\pm 0.0008$</td>
<td>$\pm 0.0016$</td>
</tr>
<tr>
<td>$A_{cFB}$</td>
<td>$\pm 0.0001$</td>
<td>$\pm 0.0002$</td>
<td>$\pm 0.0004$</td>
</tr>
<tr>
<td>$A_{dFB}$</td>
<td>$\pm 0.0002$</td>
<td>$\pm 0.0003$</td>
<td>$\pm 0.0004$</td>
</tr>
<tr>
<td>$A_{tFB}$</td>
<td>$\pm 0.0002$</td>
<td>$\pm 0.0003$</td>
<td>$\pm 0.0004$</td>
</tr>
<tr>
<td><strong>Standard Model parameters</strong></td>
<td>$\pm 0.0007$</td>
<td>$\pm 0.0029$</td>
<td>$\pm 0.0051$</td>
</tr>
<tr>
<td><strong>Total systematic error</strong></td>
<td>$\pm 0.0024$</td>
<td>$\pm 0.0051$</td>
<td>$\pm 0.0088$</td>
</tr>
<tr>
<td><strong>Statistical error</strong></td>
<td>$\pm 0.0384$</td>
<td>$\pm 0.0082$</td>
<td>$\pm 0.0321$</td>
</tr>
</tbody>
</table>

Table 8: Uncertainties on the determination of the peak $A_{bFB}$ with the event-by-event method. Where appropriate, the boldface items represent the sum of the items in the previous section.
7 Conclusions

The forward-backward asymmetry of the process $e^+e^- \to b\bar{b}$ was measured below, at and above the peak of the $Z^0$ resonance using a statistical method based on the jet charge. The results are:

\[
\begin{align*}
A_{FB}^b(\sqrt{s} = 89.52\text{GeV}) &= 0.062 \pm 0.034 \pm 0.002, \\
A_{FB}^b(\sqrt{s} = 91.25\text{GeV}) &= 0.093 \pm 0.0067 \pm 0.0046, \\
A_{FB}^b(\sqrt{s} = 92.94\text{GeV}) &= 0.172 \pm 0.028 \pm 0.008,
\end{align*}
\]

where the first error is statistical and the second is systematic.

The forward-backward asymmetries of the tagged sample were used to determine the effective weak mixing angle $\sin^2 \theta_W^{\text{eff}}$. In determining the asymmetries quoted above, the values of various quantities that are predicted by the Standard Model had to be assumed, as has been described in subsection 4.1. The analysis was repeated using ZFITTER to predict these Standard Model inputs, with the top mass varied until the $\chi^2$ between the observed and predicted asymmetries on- and off-peak was minimised. The other parameters assumed in ZFITTER were $M_Z = 91.187$ GeV$/c^2$, $M_{Higgs} = 300$ GeV$/c^2$, and $\alpha_s = 0.12$. The asymmetries were calculated with correction factors $C_i^{\text{quark}}$ appropriate to produce the $\cos \theta_W^{\text{quark}}$ asymmetries. The QCD corrections to the $A_{FB}^b$ values were not applied, being largely inherent in the method. The $\sin^2 \theta_W^{\text{eff}}$ of the electron channel which corresponded to the top mass that minimised the $\chi^2$ was evaluated, where $\sin^2 \theta_W^{\text{eff}}$ is defined by:

\[
\frac{\nu_e}{a_e} = 1 - 4\sin^2 \theta_W^{\text{eff}},
\]

(23)

and where $\nu_e/a_e$ is defined in terms of the electron asymmetry at the $Z^0$ pole. The measured asymmetries along with the Standard Model prediction (using the fitted value of $\sin^2 \theta_W^{\text{eff}}$) are shown in Fig. 4. The result for $\sin^2 \theta_W^{\text{eff}}$ is:

\[
\sin^2 \theta_W^{\text{eff}} = 0.2313 \pm 0.0012 \pm 0.0006
\]

with $M_{\text{top}} = 196^{+33}_{-38}^{+16} \text{GeV}/c^2$, where the first error is statistical and the second is systematic. A variation in the assumed mass of the Higgs boson between 60 and 1000 GeV$/c^2$ corresponds to an uncertainty in $\sin^2 \theta_W^{\text{eff}}$ of 0.00006 and on $M_{\text{top}}$ of $^{+20}_{-26}$ GeV$/c^2$.

A Relating $\langle Q_F \cdot Q_B \rangle$ to the Charge Separation $\delta$

We define $Q_F$ and $Q_B$ as the jet charges measured in the forward and backward hemispheres. Using these jet charges we measure the quantities $\langle Q_F - Q_B \rangle$ and $\langle Q_F \cdot Q_B \rangle$. We also measure the mean, $\mu(Q)$, and width, $\sigma(Q)$, of the distribution of jet charge for all forward and backward hemispheres. These distributions have exactly equal contributions from positive and negative quarks. In the Monte Carlo it is also possible to decide which of these charges corresponds to the jet charge of the negatively charged quark and which corresponds to the positively charged quark. These we define as $Q_-$ and $Q_+$ respectively.

Starting from the definition of the covariance of $Q_-$ and $Q_+$,

\[
\text{cov}[Q_-, Q_+] = \langle Q_- Q_+ \rangle - \langle Q_- \rangle \langle Q_+ \rangle
\]

\[
= \langle Q_- Q_+ \rangle + \frac{1}{4}(\delta^2 - \xi^2)
\]

(24)

where

\[
\delta = \langle Q_- \rangle - \langle Q_+ \rangle
\]

\[
\xi = \langle Q_- \rangle + \langle Q_+ \rangle = 2\mu(Q).
\]
The quantity $\delta$ is called the charge separation and $\xi$ is termed the charge offset. If the jet charge is an estimator of the quark charge then $\delta$ will have some non-zero value. If the detector has different response to positively and negatively charged quarks then $\xi$ will also have some non-zero value. Using $\langle Q_x Q_y \rangle = \langle Q_-, Q_+ \rangle$ and $\xi^2 = 4\mu^2(Q)$ and rearranging Equation 24 gives

$$\delta^2 = 4\{ -\langle Q_-, Q_+ \rangle + \text{cov}[Q_-, Q_+] \} + \xi^2$$

$$= 4\{ -\langle Q_x Q_y \rangle + \text{cov}[Q_-, Q_+] + \mu^2(Q) \}. \quad (25)$$

This is the important result; it will now be refined to write it explicitly referring to a correlation coefficient.

If we assume that $\sigma(Q_-) = \sigma(Q_+)$, confirmed by the Monte Carlo, then the covariance term of Equation 25 can be rewritten in the following way:

$$\text{cov}[Q_-, Q_+] = \rho[Q_-, Q_+]\sigma(Q_-)\sigma(Q_+)$$

$$= \rho[Q_-, Q_+][\sigma^2(Q) - \frac{1}{4}\delta^2]$$

where we have used

$$\sigma^2(Q) = \langle Q^2 \rangle - \langle Q \rangle^2$$

$$\quad = \frac{1}{2}\langle Q_-^2 \rangle + \frac{1}{2}\langle Q_+^2 \rangle - (\frac{1}{2}\langle Q_- \rangle + \frac{1}{2}\langle Q_+ \rangle)^2$$

$$\quad = \frac{1}{2}\sigma^2(Q_-) + \frac{1}{2}\sigma^2(Q_+) + \frac{1}{4}\delta^2. \quad (26)$$

The expression for $\delta$ then becomes:

$$\delta^2 = 4 \left( -\langle Q_x Q_y \rangle + \frac{\rho[Q_-, Q_+]\sigma(Q) + \mu^2(Q)}{1 + \rho[Q_-, Q_+]} \right),$$

which is the form used in the analysis.

References


    T. Sjöstrand, CERN-TH.6488/92. OPAL optimised parameters were used, as described in
    DELPHI Collaboration, P. Abreu et al., Z. Phys. C 56 (1992) 47;
    OPAL Collaboration, R. Akers et al., Z. Phys. C 60 (1993) 199;
Figure 1: Decay length significance distribution of the most significant secondary vertex in an event. The dots represent the data, superimposed on the Monte Carlo. Both distributions are normalised to the same number of events. The charm and light flavour expected contributions are shown. The $L/\sigma$ cut is also shown. The Monte Carlo and data disagree for large positive $L/\sigma$. This is not relevant for the analysis, since the $b$ tagging efficiency is determined from the data.
Figure 2: $A_F^b$ in bins of $|\cos \theta_T|$. The line and shaded area indicate the weighted average of the results and its error. All errors are statistical only.
Figure 3: The tagging efficiency for bottom, charm and light quark events as a function of $|\cos \theta_T|$. The b-quark values are taken from the data, the charm and light quark values are taken from the Monte Carlo. The curve is a parameterisation of the points in the region $|\cos \theta_T| < 0.8$. 

Efficiency

Bottom events
Charm events
Light quark events

OPAL Preliminary

$\cos \theta_{\text{thrust}}$
Figure 4: The measured values of $A_{FB}^b$ as a function of the centre of mass energy. The curve is the Standard Model prediction for $A_{FB}^b$ using the fitted value of $\sin^2\theta_W$. 