Multiplicty and Transverse Momentum Correlations in Multihadronic Final States in $e^+e^-$ Interactions at $\sqrt{s} = 91.2$ GeV

The OPAL Collaboration

Abstract

We report a study of forward-backward multiplicity correlations and a measurement of the dependence on charged multiplicity of the mean transverse momentum of charged hadrons, measured with respect to the thrust axis. The study was performed on a high statistics sample of $Z^0$ decays to multihadronic final states collected by the OPAL Collaboration at LEP. The positive forward-backward multiplicity correlation observed in our inclusive sample can be understood in terms of a superposition of distinct event topologies characterized by a different amount of hard gluon radiation (2-, 3- and 4-jet events) and with different mean multiplicities. The residual positive correlation that we see in a clean 2-jet sample can be interpreted in terms of fragmentation properties of different quark flavours and of the production and decay of resonances. We have compared the observed effects with the predictions of QCD-based parton shower models. The data are well described by the Jetset 7.3 Monte Carlo, while Herwig 5.5 does not satisfactorily reproduce the measured correlations. Hard gluon radiation is also shown to be responsible for the observed increase of about 40% in the mean transverse momentum of produced charged hadrons in the multiplicity range from 10 to 30. The comparison with the results obtained in an analysis of a sample enriched in $Z^0 \rightarrow b\bar{b}$ events, shows that the presence of heavy flavours does not contribute significantly to the observed effect.

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1 Introduction

The study of multiplicity distributions provides information concerning the dynamics of the production of multihadronic final states in high energy $e^+e^-$ collisions. Both the shape of the distribution and the centre-of-mass energy dependence of its moments have been compared with the predictions of several models, including analytical formulae inspired by perturbative QCD [1]. It has been experimentally observed at different energies that a simple Poisson form is not able to describe the multiplicity distribution and that, therefore, correlations must be present among the particles produced in the final state.

Several experiments have studied long range, “forward-backward”, correlations between the number of charged particles produced in two opposite event hemispheres. The “forward” multiplicity, $n_F$, is defined as the number of charged particles contained in the “forward” event hemisphere, chosen randomly between the two defined by the plane perpendicular to the thrust axis [2]. The “backward” multiplicity, $n_B$, is the particle multiplicity in the opposite hemisphere. These forward-backward multiplicity correlations, i.e. the dependence on $n_B$ of the average number of charged particles produced in the forward hemisphere, $\langle n_F \rangle$, are expected to be sensitive to the details of the fragmentation process. In the study of a sample of 2-jet events selected by cutting on the sphericity and aplanarity variables, the HRS Collaboration [3] observed no such correlations in $e^+e^-$ interactions at $\sqrt{s} = 29$ GeV. These results were interpreted [3] as giving support to the idea of independent jet fragmentation. The TASSO Collaboration [4], analysing inclusive samples of events collected at energies varying between 14 and 43.6 GeV, observed a weak, almost linear rise of $\langle n_F \rangle$ with increasing $n_B$, and practically no dependence of the correlation strength $^1$ on the centre-of-mass energy. Based on the analysis of events simulated according to the Jetset model [5], it was suggested [4] that the different fragmentation properties of the produced quark flavours and the formation and decay of resonances, could be at the origin of this effect. According to that analysis, the contribution of these sources are expected to decrease at higher energies. The DELPHI Collaboration at LEP, however, has recently reported the observation of a positive forward-backward multiplicity correlation [6] of a strength comparable to that observed at PETRA energies. Several theoretical interpretations of the effect have been published [7], some of which are based on investigations of similar, but stronger, correlations observed in hadron-hadron collisions.

The dependence on the charged multiplicity, $n_{ch}$, of the mean transverse momentum, $\langle p_t \rangle$, of the produced charged particles of the event has been extensively studied in hadronic interactions. Several experiments at hadron colliders at different centre-of-mass energies have measured this effect, observing a strengthening of the correlation with increasing energy [8]. A number of dynamical models have been proposed to explain the origin of this correlation [9, 10, 11]. In a recent paper [12] the multiplicity dependence of the $\langle p_t \rangle$ of charged particles, produced in $e^+e^-$ collisions, computed with respect to the event sphericity axis [13], has been investigated in samples of Monte Carlo events generated with the Jetset model. The observation, in this model, of an increase of $\langle p_t \rangle$ with charged multiplicity was explained by the increase with multiplicity of the relative abundance of multi-jet events, which yield particles with larger $\langle p_t \rangle$. The DELPHI Collaboration at LEP has recently presented results [14] showing that the $\langle p_t \rangle$ of charged particles, computed with respect to the event sphericity axis, is indeed

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$^1$The correlation strength is defined as the slope obtained from a linear fit to the curve of $\langle n_F \rangle$ as a function of $n_B$. 

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an increasing function of the charged multiplicity and their results support the interpretation
given in [12]. There are no papers on the subject published at lower energies.

In this letter we report a detailed study of forward-backward multiplicity correlations and
an analysis of the correlation between the $\langle p_t \rangle$ and $n_{ch}$, as observed in a high statistics sample
of multihadronic events from $Z^0$ decays recorded by the OPAL detector at LEP. Our main
purpose is an attempt to understand the extent to which these correlations originate from
genuine dynamical sources, taking advantage of the clean initial state environment provided by
e$^+$e$^-$ annihilations. We also compare our results with those obtained by other experiments at
LEP and at lower energies, as well as with the predictions of Monte Carlo programs based on
perturbative QCD, implementing different hadronization models.

2 Experimental Procedure

The present analysis is based on 284,149 multihadronic events recorded by the OPAL
detector [15] at LEP during 1991 at a centre-of-mass energy of 91.2 GeV, corresponding to
the $Z^0$ peak energy. For this work we used mainly the information from the central tracking
detector, which comprises a high precision drift chamber for vertex reconstruction, a large
diameter jet chamber which provides up to 159 space points per track, and an outer drift
chamber that measures with high precision the $z$ coordinates of tracks.

We imposed quality requirements on the observed charged particles in order to eliminate
possible spurious or poorly measured tracks. For each charged track, the extrapolated closest
distance of approach to the interaction point was required to be less than 5 cm in the plane
perpendicular to the beam axis and less than 30 cm in the direction along the beam axis.
Only tracks with at least 40 hits in the jet chamber were accepted. The transverse momentum
relative to the beam axis was required to be larger than 0.15 GeV/c.

Candidate hadronic events were required to have at least five good tracks with a total
momentum greater than 15 GeV/c, and an angle between the beam axis and the thrust axis
greater than 30$^\circ$. Two further requirements were made to reject $\tau\tau$ events decaying to hadrons:
the principal thrust value had to be smaller than 0.9975 and the invariant mass of the particles
in at least one event hemisphere was required to be greater than 2 GeV/c$^2$.

After these requirements 217,697 events were available for the analysis. Events were
subdivided according to their observed multiplicity, defined as the number of charged tracks
that survived the above selections.

We have studied forward-backward multiplicity correlations, defined to be the dependence
of $\langle n_F \rangle$ on $n_B$; the meaning of “forward” and “backward” multiplicities has been explained
in the previous section. In figure 1 is shown the dependence of $\langle n_F \rangle$ on $n_B$ as observed in
our uncorrected data. The predictions of two major Monte Carlo models, Jetset 7.3 [5] and
Herwig 5.5 [16], are also shown. These models have been tuned to reproduce the inclusive event
shape distributions measured by OPAL [17]. The Monte Carlo samples consisted, respectively,

$^2$Our coordinate system is defined so that $z$ is the coordinate parallel to the e$^+$ and e$^-$ beam axis, with positive
direction along the e$^-$ beam; $r$ is the coordinate normal to the beam axis, $\phi$ is the azimuthal angle and $\theta$ is the
polar angle with respect to $+z$. 

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of 408,344 and 293,089 events. They were generated including the effect of QED initial state radiation, passed through the OPAL detector simulation program [18] and processed by the same reconstruction and analysis programs as the data. It can be seen that the Jetset model provides a satisfactory description of the data, while Herwig predicts a correlation significantly stronger than the one observed experimentally.

Corrections for detector effects proceeded by means of an iterative matrix unfolding technique [19] based on fully simulated events. As will be demonstrated in section 3, this unfolding method provides a correction to the data with negligible model dependence. We used the sample of Jetset events described in the previous paragraph, from which we built a matrix $P$ whose generic element is:

$$P(n_i^F, n_i^B, n_o^F, n_o^B) = \frac{N_{\text{events}}(n_i^F, n_i^B, n_o^F, n_o^B)}{N_{\text{events}}(n_i^B, n_i^F)}$$

(1)

where $N_{\text{events}}(n_i^F, n_i^B, n_o^F, n_o^B)$ is the number of events with $(n_i^F, n_i^B)$ true multiplicities if $(n_o^F, n_o^B)$ multiplicities are observed and $N_{\text{events}}(n_i^B, n_i^F)$ is the total number of events with $(n_i^B, n_i^F)$ true multiplicities. In the Monte Carlo we define the “true” charged multiplicity of an event to be the total number of stable charged particles produced in the primary fragmentation or in the decays of particles with lifetimes shorter than $3 \cdot 10^{-13}$ s. Then, the probability that an event found with $(n_o^F, n_o^B)$ observed multiplicities originates from an event with $(n_i^F, n_i^B)$ true multiplicities is:

$$M(n_i^F, n_i^B, n_o^F, n_o^B) = \frac{P(n_i^F, n_i^B, n_o^F, n_o^B) \cdot N_{\text{events}}(n_i^B, n_i^F)}{\sum_{n_i^F, n_i^B} P(n_i^F, n_i^B, n_o^F, n_o^B) \cdot N_{\text{events}}(n_i^B, n_i^F)}.$$  

(2)

The data, therefore, may be corrected by:

$$N_{\text{corr}}^{\text{exp}}(n_F, n_B) = \sum_{n_i^F, n_i^B} \{M(n_F, n_B, n_o^F, n_o^B) \cdot N_{\text{obs}}^{\text{exp}}(n_o^F, n_o^B)\},$$

(3)

where $N_{\text{obs}}^{\text{exp}}$ is the observed distribution and $N_{\text{corr}}^{\text{exp}}$ is the corrected one. Initially the matrix $M$ is computed using pure Monte Carlo information. After one has obtained a first estimate of the corrected distribution $N_{\text{corr}}^{\text{exp}}$, it is possible to iterate the matrix correction procedure putting the information of the experimental distribution $N_{\text{corr}}^{\text{exp}}$ in (2) in place of $N_{\text{events}}(n_i^F, n_i^B)$. A few iterations are usually sufficient to obtain stable $N_{\text{corr}}^{\text{exp}}$ distributions.

A second step in the correction procedure concerns the effects of initial state radiation and of the selection cuts: to take them into account we estimated a correction matrix $c(n_i^F, n_i^B)$ as the ratio between the normalized forward-backward multiplicity distribution computed in events generated without simulation of initial state photon radiation and the same distribution computed using events generated with initial state radiation simulated and satisfying all selection requirements at the detector level. The final corrected forward-backward multiplicity distribution is then:

$$N_{\text{corr}}^{\text{exp}}(n_F, n_B) = c(n_i^F, n_i^B) \cdot N_{\text{corr}}^{\text{exp}}(n_F, n_B).$$

(4)

One should notice that this step of the correction procedure is model-dependent since the result depends on how well the observed data are reproduced by the Monte Carlo simulation. However, in our case, the elements of the matrix $c$ were generally found to be close to unity and the resulting corrections to the data points were small.

The $\langle p_t \rangle$, computed for each multiplicity from the accepted tracks, has been measured with respect to the thrust axis. We have measured separately the average of the two components “$\langle m \rangle$”,
The unfolding of the observed \( \langle p_t \rangle \) as a function of the observed \( n_{ch} \) proceeded as follows. If \( N_i^j \) is the number of Monte Carlo events with true multiplicity \( i \), seen at the detector level with a multiplicity \( j \), and \( N_i \) is the total number of Monte Carlo events having true multiplicity \( i \), then the link between the two vectors \( \langle p_t \rangle_{MC}^{\text{true}} \) and \( \langle p_t \rangle_{MC}^{\text{true}} \) can be written as follows:

\[
\sum_i \left( \frac{N_i^j}{N_i} \frac{\langle p_t \rangle_{MC}^{\text{true}}(i)}{\langle p_t \rangle_{MC}^{\text{obs}}(j)} \right) \langle p_t \rangle_{MC}^{\text{obs}}(j) = \langle p_t \rangle_{MC}^{\text{true}}(i). \tag{5}
\]

The matrix elements \( A(i,j) \), inside the curly brackets, were obtained using the sample of Jetset Monte Carlo events mentioned before. The experimental distributions have been corrected for detector effects (such as geometrical acceptance, resolution and particle interactions in the material) and track finding inefficiencies of the reconstruction program, by multiplying the matrix \( A \) with the vector of the \( \langle p_t \rangle \) observed in the data. As for the case of forward-backward correlations, possible biases introduced by data selection and initial state radiation are taken into account by correction factors defined as the ratio \( c_i = \langle p_t \rangle_{MC,\text{true}}^{\text{true}}(i) / \langle p_t \rangle_{MC,\text{true}}^{\text{true}}(i) \) between \( \langle p_t \rangle \) of Monte Carlo events generated without initial state radiation \( (MC_{\text{true}}) \) and \( \langle p_t \rangle \) computed only for those Monte Carlo events that satisfy, at detector level, the event selection and had initial state radiation simulated \( (MC_{\text{true}}) \). The final, corrected \( \langle p_t \rangle \) distribution is the product of the \( \langle p_t \rangle \) as obtained from the detector acceptance correction procedure and the corresponding factors \( c_i \):

\[
\langle p_t \rangle_{DATA}^{\text{corr}}(i) = c_i \sum_j \{ A(i,j) \cdot \langle p_t \rangle_{DATA}^{\text{true}}(j) \}. \tag{6}
\]

### 3 Forward-backward multiplicity correlations

In figure 2(a) our measured forward-backward multiplicity correlation, corrected as described in section 2, is compared with the predictions of three QCD parton shower Monte Carlo programs based on different hadronization models: Jetset 7.3 [5] with string fragmentation, Herwig 5.5 [16] with cluster fragmentation and Cojets 6.23 [20] with independent fragmentation. Only statistical errors are plotted. From the figure, it is apparent that only Jetset reproduces the data in a satisfactory way above \( n_B \approx 15 \).

In order to quantify the strength of the observed correlation and make comparisons with the same effect measured at different energies, we fit the data shown in figure 2(a) to the linear form \( \langle n_F \rangle = a + b \ n_B \). The fit, performed in the range \( 5 \leq n_B \leq 34 \), yields a slope \( b = 0.103 \pm 0.002 \) with \( \chi^2/dof = 3.0 \). The main contributions to the \( \chi^2 \) value come from the range \( n_B \geq 20 \). The choice of a lower bound in the fit range is motivated by the fact that the requirement in our experimental selection of having events with at least five good tracks removes events with low values of both \( n_B \) and \( n_F \). The region \( n_B \leq 4 \) is thus necessarily affected and its correction is rather sensitive to the model prediction for low multiplicity events. The effect on the slope produced by including the region \( 1 \leq n_B \leq 4 \) in the fit and an estimate of the systematic uncertainty associated to a possible model dependence of this effect will be discussed in the next paragraph.

Several tests have been performed in order to evaluate the systematic uncertainties of the measurement. The stability of our results using data taken at different times during 1991 was
checked by performing linear fits separately for 13 different data taking periods. A check was made by using the data collected in 1990. To estimate the magnitude of the systematics due to track and event selection we have repeated our analysis varying the selection requirements within reasonable ranges and then performing the full correction procedure. We have found a negligible effect on the correlation in all cases. The model dependence of the correction procedure has been evaluated in the following way. A sample of events, generated according to the Herwig Monte Carlo, was fully simulated in the OPAL detector and then reconstructed and analyzed with the programs used for the data. The same matrix \( M \) used to correct the data, obtained from Jetset as described in the previous section, was then applied to unfold the Herwig events. The comparison between the correlation observed in this case and the one computed from the "true" Herwig multiplicity distribution, \( N_{\text{events}}(n_F^P, n_B^P) \), provides a direct estimate of the systematic effect produced by the matrix unfolding procedure. Despite the large differences observed between the correlations predicted by Herwig and Jetset, using the iterative method described in section 3 we obtain a forward-backward multiplicity correlation curve that reproduces within statistical errors, with the exception of a few points, the curve obtained from the "true" Herwig distribution, as shown in figures 3(a) and (b). This test demonstrates the negligible model dependence of matrix unfolding and the stability of the correlation strength after a few iterations, see figure 3(c). The systematic uncertainty associated with this step is therefore estimated to be negligible. It was already mentioned, however, that the correction of the region \( n_B \leq 4 \) is sensitive to the model used to compute the matrix \( c \) of equation (4). The forward-backward correlation obtained from a sample of Herwig events corrected as described before together with unfolding by means of the \( c \) matrix, has been compared to the correlation computed from another sample of Herwig events generated without detector and initial state radiation simulation. The discrepancies between the two distributions at low \( n_B \) were significant. In terms of correlation strength, fits made using the full range \( 1 \leq n_B \leq 34 \) differ by 7\%, while they agree within statistical errors if the range used is \( 5 \leq n_B \leq 34 \). We therefore assume a systematic uncertainty of that amount and quote a final value \( b = 0.103 \pm 0.007 \) for our measured correlation strength, where the uncertainty contains both statistical and systematic contributions added in quadrature.

Our value may be compared with the one published by the DELPHI Collaboration [6], \( b = 0.118 \pm 0.009 \), measured at the same energy. The values measured by TASSO [4] at energies of 14, 22, 34.8 and 43.6 GeV are \( b = 0.085 \pm 0.014 \), \( b = 0.084 \pm 0.016 \), \( b = 0.089 \pm 0.003 \) and \( b = 0.111 \pm 0.009 \), respectively. Contrary to the case of hadron-hadron interactions, the correlation strength in \( e^+e^- \) reactions has only a weak energy dependence, if any.

Another way to emphasize forward-backward multiplicity correlations is the comparison of the single hemisphere multiplicity distribution at fixed total charged multiplicity, \( n_{\text{tot}} \), with the binomial distribution:

\[
P_{n_{\text{tot}}}(n_F) = \frac{n_{\text{tot}}!}{n_F!(n_{\text{tot}}-n_F)!} \left( \frac{1}{2} \right)^n_F \left( \frac{1}{2} \right)^{n_B} = \frac{n_{\text{tot}}!}{n_F!(n_{\text{tot}}-n_F)!} \left( \frac{1}{2} \right)^{n_{\text{tot}}}. \tag{7}
\]

Agreement between data and the distribution (7) would indicate a lack of correlation between the multiplicities in the two hemispheres. We find that our \( n_F \) experimental distributions are not well reproduced by the binomial; however we observed better agreement as \( n_{\text{tot}} \) decreases: the ratios between the observed and the binomial dispersions are 1.44 at \( n_{\text{tot}} = 30 \), 1.18 at \( n_{\text{tot}} = 20 \) and 1.03 at \( n_{\text{tot}} = 10 \), with errors of order 0.2\%.

It is usual to analyze multiplicity correlations separately for particles produced in different
ranges of rapidity; the rapidity variable is defined as:

$$Y = \frac{1}{2} \ln \left( \frac{E + p_L}{E - p_L} \right)$$  \hspace{1cm} (8)$$

where $E$ is the particle’s energy and $p_L$ its momentum component along the thrust axis. Figures 2(b) and 2(c) show the corrected forward-backward correlations for particles having rapidity $|Y| \leq 1$ and $|Y| > 1$, respectively. In this analysis all particles have been assigned the pion mass. Particles produced in the central rapidity region show a strong linear positive correlation in the range $n_B \leq 10$; for larger backward multiplicities the value of $\langle n_F \rangle$ remains constant at about five. This is different from the linear increase reported, with lower statistics, by the DELPHI Collaboration [6] for the same range of rapidity. All the models considered are able to reproduce the data reasonably well in this range. For $|Y| > 1$ the data show almost no correlation and are reproduced by Jetset; in contrast Herwig predicts a clear positive correlation and Cojets a weak anticorrelation.

In the attempt to understand better the origin of the observed correlation we have analyzed our data in terms of different event topologies. For this purpose, subsamples of events with a fixed number of jets were selected with the JADE jet finding algorithm [21] with the E0 recombination scheme. In figure 4(a) we plot $\langle n_F \rangle$ as a function of $n_B$ separately for 2-, 3- and 4-jet events. The results shown are corrected for detector effects as well as for selection criteria and initial state radiation, and were obtained with a value of the jet resolution parameter $y_{cut} = 0.015$. Their relative abundance is, respectively, 39%, 48% and 12%. We have checked that the shape of the correlations remains practically unchanged if one varies the resolution parameter up to a value of 0.08, for which the relative abundance of 2-, 3- and 4-jet events becomes 84%, 16% and negligible, respectively. Of course $\langle n_F \rangle$ moves toward higher values for the 2-jet sample. One can understand the positive correlation observed in the inclusive sample, also shown in the figure, as a consequence of the well established phenomenon of hard gluon radiation. The inclusive sample, indeed, can be thought to be a superposition of classes of events with fixed number of resolved jets, each class having a different mean multiplicity and its own correlated behaviour. In this light it is likely that also the correlations observed in hadron-hadron interactions, despite the more complicated initial state involved, have the same origin. In figures 4(b) and 4(c) we show the results obtained by restricting the analysis to limited regions of rapidity. It can be seen that the effect just described is almost completely concentrated in the central region, $|Y| \leq 1$, where the difference in mean multiplicity between the 2-jet and the multi-jet \(^3\) event samples is pronounced. In the region $|Y| > 1$ the difference between the two samples is much smaller, resulting in a very weak correlation for the inclusive sample. The anticorrelation for the 3- and 4-jet samples shown in figure 4(a), follows from the fact that the low $n_B$ region is preferably populated by events with only one jet in the backward hemisphere while the tendency becomes the opposite when approaching higher $n_B$ values. The consequence is a decrease of $\langle n_F \rangle$ with increasing $n_B$.

It is interesting to remark, at this point, that the simplest class (2-jets) still shows a positive correlation in both rapidity regions. As a check, we repeated the analysis by using a cone based jet finding algorithm of the type commonly used in $\bar{p}p$ experiments. We found the same qualitative behaviour as the one seen by using the JADE algorithm. In a recent OPAL paper [22] it was shown that the mean charged multiplicity is higher for $b\bar{b}$ events than for events originating from light quarks. To investigate further the origin of the correlation we have studied, within

\(^3\)By multi-jet event we mean an event with at least three resolved jets.
the 2-jet sample, a possible effect produced by the superposition of events originating from different primary quark flavours. For this study we have analyzed Jetset events generated with fragmentation parameters adjusted so as to reproduce several event shape variables measured by OPAL and the measured mean multiplicities of events corresponding to different primary quark flavours [23]; in particular the Peterson fragmentation form [24] was used to hadronize c and b quarks.

In figure 5(a) we show the forward-backward correlation for 2-jet events, separately for $b\bar{b}$, light flavours and all flavours. It is clear that the positive correlation observed in the 2-jet sample with all flavours is due to the superposition of event classes with different multiplicity, namely those originating from light flavours and those from $b$ quarks. Although less pronounced, a positive correlation is still present in both the light quark and $b$ quark 2-jet samples. Restricting the analysis to the particles produced in limited regions of rapidity one can see that the difference in multiplicity is concentrated in the region $|Y| > 1$, figure 5(b), where the two separate samples show practically no correlation. Particles produced with low rapidity, instead, still show a clear forward-backward multiplicity correlation, with an $\langle n_F \rangle$ at fixed $n_B$ independent from the originally produced quark flavour.

In figure 5(d) we show, for one single flavour, the correlation predicted by Jetset for the partons produced in the shower as well as the correlation predicted for primary hadrons before they decay into the final charged particles, whose correlation is also shown in the same figure. There is no clear correlation at the parton level, while one can see a complicated behaviour of alternating positive and negative correlations after hadronization. Finally, a smoother clear positive correlation appears after primary hadrons have decayed. We have checked that for light flavours the behaviour is qualitatively the same. Following Jetset, then, one can conclude that the correlation seen in the rapidity interval $|Y| \leq 1$ is dominated by the primary hadron decay process, and is presumably due to the fact that decay products of a centrally produced hadron are shared between the two event hemispheres. For completeness, we extended this analysis also to the rapidity region $|Y| > 1$ where no correlation was seen at the final charged particle level, figure 5(b). No correlation is observed also for partons and primary hadrons.

We end this section by commenting on possible sources of the strong correlations predicted by Herwig. We have checked that the effect is absent at the parton level. However, a correlation appears already at the cluster level, the step of the fragmentation process where quark and antiquark pairs of low virtuality (at the end of the parton shower) and opposite color charges find themselves close in space and momentum and are associated in colorless objects, called clusters, whose decay products are the primary hadrons. This correlation is probably introduced by the non-perturbative gluon splitting mechanism [16], that forces all gluons present at the end of the perturbative shower to split into $q\bar{q}$ pairs. The simulation of particle decays is identical in our versions of Herwig and Jetset; consequently differences in the forward-backward correlations predicted by the two models cannot be due to the modelling of the decays. The absolute yields of various unstable particles and resonances are, however, not necessarily the same in the two models and differences in the forward-backward correlations could arise from this. The absolute particle yields depend upon the details of the fragmentation process in the two models. In Herwig, with default parameters, the simulation of heavy quark fragmentation is such that the average number of charged particles in $b\bar{b}$ events is appreciably higher than the measured value [22]. This would increase the forward-backward multiplicity correlation.
4 Dependence of the mean transverse momentum on the charged multiplicity

Since the first observation of a strong correlation between the mean transverse momentum of the produced particles and the charged multiplicity in hadron-hadron collisions [8], a number of models have been developed to explain this effect. Among them, some [9] deal with collective effects in the hadronic matter, expected to be absent in $e^+e^-$ interactions, while others [11] attribute the correlation to the superposition of two components (one soft, non perturbative, and the second hard, treated perturbatively, that gives rise to the so called “mini-jets”) in the hadron-hadron collision process. A measure of this correlation in $e^+e^-$ annihilations could therefore be useful in testing such models.

In our analysis we have followed the line of reference [14] and checked how the radiation of hard gluons can be the cause of an increase of the mean transverse momentum with multiplicity. Furthermore, we have investigated in the data possible contributions to the effect due to the different fragmentation properties of light and heavy quarks.

In figure 6(a) the values of $\langle p_T^{\text{in}} \rangle$ and $\langle p_T^{\text{out}} \rangle$ computed with respect to the event thrust axis, corrected for detector effects, initial state radiation and event selection as described in section 2 are plotted as a function of the charged multiplicity, $n_{ch}$, and compared with the predictions of Jetset 7.3. In the range of multiplicities between 10 and 30, where the event sample is largest, one observes an increase of the order of 40 % of both $\langle p_T^{\text{in}} \rangle$ and $\langle p_T^{\text{out}} \rangle$. For $n_{ch}$ larger than 30 there seems to be flattening in the $\langle p_T^{\text{in}} \rangle$ correlation curve. There is a good agreement between data and the Jetset Monte Carlo for both $\langle p_T^{\text{in}} \rangle$ and $\langle p_T^{\text{out}} \rangle$. Our results agree with those of the DELPHI Collaboration [14].

In order to estimate the magnitude of the systematic errors introduced by our selection criteria, we varied within reasonable limits the cuts listed in section 2 and repeated the analysis. The differences are small at low multiplicities and of the order of 6 - 7% at $n_{ch} \approx 30 - 40$. Systematic effects due to the finite momentum resolution have been estimated by smearing the momenta of the generated stable, charged hadrons according to the resolution of the detector and have been found to be negligible.

As already mentioned, previous studies [12, 14] indicate that the source of the increase of the mean transverse momentum with multiplicity is the increasing contribution of hard gluon bremsstrahlung. One check is the study of the behaviour of events with different $n$-jet topologies as classified by the JADE algorithm. The jet finder has been used with a resolution parameter $y_{cut} = 0.04$. Figure 6(b) shows the dependence of $\langle p_T^{\text{in}} \rangle$ on $n_{ch}$ separately for two subsamples of events: the first contains only 2-jet events and the second $n$-jet events, with $n > 2$. In the 2-jet sample the correlation is basically absent, while for events with $\geq 3$ jets $\langle p_T^{\text{in}} \rangle$ is larger, by about 300 MeV/$c$, than for the 2-jet sample and it has only a minor dependence on $n_{ch}$. The increase of $\langle p_T \rangle$ with multiplicity in the inclusive sample thus arises from the higher value of $\langle p_T \rangle$ for the sample of $n$-jet events, where the emission of hard gluons has occurred, combined with the increasing relative abundance of this subsample at higher multiplicities.

As pointed out in reference [12], different values of the mean multiplicity and transverse momentum in events originating from the fragmentation of light and heavy primary quarks could also contribute to producing an increase of $\langle p_T \rangle$ with $n_{ch}$. To check this possibility we selected a sample enriched in $Z^0 \rightarrow b \bar{b}$ events by requiring the presence of a muon with
momentum greater than 4.5 GeV/c and transverse momentum with respect to the nearest jet axis greater than 1 GeV/c [25]. The purity of this enriched sample was estimated to be about 66%. In order to reduce possible biases introduced by the b-tagging requirement we measured the $\langle p_t \rangle$ as a function of $n_{ch}$ for the particles in the hemisphere, defined with respect to the thrust axis of the event, opposite to the one containing the muon. Figure 6(c) shows the result, for the $\langle p_t^{in} \rangle$ component, and a comparison with the result for the inclusive sample. In this case the hemisphere was chosen randomly. The sample enriched in beauty events shows a correlation of strength comparable to the one observed in the inclusive sample; however the curve is shifted to lower values of $\langle p_t^{in} \rangle$ by about 60 - 80 MeV/c. Our data, therefore, indicate that the production of heavy quarks does not contribute significantly to the observed increase of mean $p_t$ with multiplicity.

The average over all multiplicities of the mean $p_t$ of the charged tracks in an event, $\langle p_t \rangle$, and the corresponding averages of the components “in” and “out” of the event plane, have been calculated at $\sqrt{s} = 91.2$ GeV. We have integrated the $\langle p_t \rangle$ ($n_{ch}$) distributions, weighted by the appropriate multiplicity distribution [26], in order to compute the mean $\langle p_t^{in} \rangle$ and $\langle p_t^{out} \rangle$. The values obtained, $\langle p_t^{in} \rangle = (0.467 \pm 0.001_{stat} \pm 0.009_{syst})$ GeV/c and $\langle p_t^{out} \rangle = (0.241 \pm 0.001_{stat} \pm 0.005_{syst})$ GeV/c, are little affected by systematic effects due to the residual contamination of $\gamma\gamma$, $\pi\pi$ and beam-gas background at the lowest multiplicities, $n_{ch} < 8$. We have also measured the overall $\langle p_t \rangle$ from the inclusive $p_t$ distributions, by applying bin by bin correction factors obtained by comparing the Jetset $p_t$ distribution at the hadron level to the same distribution observed at the detector level. We obtain $\langle p_t \rangle = (0.586 \pm 0.001_{stat} \pm 0.011_{syst})$ GeV/c. In all cases the examined sources of systematic uncertainties were the model dependence in the correction procedure and the track selection criteria. In figure 7 we compare these results with published values of $\langle p_t \rangle$ and of its mean components [27]. The figure also shows that the data are globally well reproduced by the predictions of Jetset 7.3 with parameters tuned to data at $\sqrt{s} = 91.2$ GeV, [23].

5 Summary and conclusions

In this letter we have presented a study of correlations between the charged particle multiplicities in opposite event hemispheres, defined by the plane normal to the thrust axis, commonly known as forward-backward multiplicity correlations, and of the dependence of the mean transverse momentum on charged multiplicity, performed on a sample of approximately 220,000 multi-hadronic $Z^0$ final states measured with the OPAL detector at LEP.

We observed forward-backward correlations: the measured correlation strength, $b = 0.103 \pm 0.007$, is in good agreement with the value measured by the DELPHI Collaboration, and a comparison with results obtained at lower energies indicates that this quantity does not appreciably vary with centre-of-mass energy.

The comparison with QCD-based Monte Carlo programs shows that Jetset 7.3, implementing a parton shower model with string fragmentation, is able to reproduce quite well the behaviour observed in the data. Herwig 5.5 predicts much stronger multiplicity correlations than are observed, while Cojets 6.23 predicts no correlations.

The origin of the forward-backward multiplicity correlations has been investigated in detail. We have shown that the positive correlation observed in the inclusive sample, at this energy,
is primarily a consequence of the superposition of events belonging to distinct \(n\)-jet topologies which have different mean charged multiplicity. This is particularly evident when restricting the analysis to the limited region of rapidity \( |Y| \leq 1 \), which is more sensitive to the phenomenon of hard gluon radiation. However, even the analysis of a clean 2-jet sample showed the presence of a positive correlation. In the framework of the Jetset model and with the support of recent results published by OPAL on the charged multiplicity of \(Z^0 \rightarrow b\bar{b} \) events [22], we interpreted this effect as the consequence of the superposition of events originating from different primary quark flavours, for particles produced in the rapidity region \( |Y| > 1 \), while for centrally produced particles the effect is presumably dominated by resonance production and decay.

The mean transverse momentum of the produced charged particles has been observed to increase by about 40\% in the multiplicity range between 10 and 30. This is in good agreement with a previous measurement at LEP [14]. The correlation between \( \langle p_t \rangle \) and the charged multiplicity can be explained by the increasing relative abundance of multi-jet events with respect to 2-jet events at increasing multiplicities, since the \( \langle p_t \rangle \) of the first class of events is larger. The analysis of a sample enriched in \(Z^0 \rightarrow b\bar{b} \) events showed that the shape of the correlation does not significantly differ from the one observed for the inclusive sample, supporting the idea that heavy flavours do not contribute much to this effect. These results as well as the energy dependence of the measured \( \langle p_t \rangle \) averaged over all multiplicities, \( \langle p_t \rangle \), and of its components “in” and “out” of the event plane are reasonably well reproduced by the Jetset Monte Carlo.

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Figure Captions

**Figure 1** The uncorrected forward-backward multiplicity correlations compared with the predictions of Jetset 7.3 and Herwig 5.5 Monte Carlos.

**Figure 2** (a) Corrected forward-backward multiplicity correlations compared with the predictions of Jetset 7.3, Herwig 5.5 and Cojets 6.23 Monte Carlos. Also shown is the straight line that best fits the data.
(b) As (a), but only for particles with rapidity $|Y| \leq 1$.
(c) As (a), but only for particles with rapidity $|Y| > 1$.

**Figure 3** Test of the iterative matrix unfolding technique. The multiplicity correlations predicted by Herwig 5.5 (black points) are compared with those obtained by analysing a sample of Herwig events passed through the full detector simulation, reconstruction and analysis programs, and successively corrected by Jetset according to formula (3) of section 2 (open points). Both samples are not corrected for the effects of event selection requirements and initial state photon radiation.
(a) The result when only one iteration was performed.
(b) The result after five iterations.
Also shown, in (c), is the correlation strength $b$ versus the number of iterations. The parameter becomes stable after four iterations.

**Figure 4** (a) Corrected forward-backward multiplicity correlations plotted separately for 2-jet, 3-jet and 4-jet events, compared with the inclusive sample of events.
(b) As (a), but only for the particles with rapidity $|Y| \leq 1$; the samples of 3- and 4-jet events have been merged and analyzed together.
(c) As (b), for particles with rapidity $|Y| > 1$.
The JADE jet finder algorithm has been used with a value of the resolution parameter $y_{cut} = 0.015$.

**Figure 5** (a) The prediction of Jetset 7.3 for the forward-backward multiplicity correlations, in 2-jet events, for $Z^0 \rightarrow u\bar{u}, d\bar{d}, s\bar{s}, Z^0 \rightarrow b\bar{b}$ and $Z^0 \rightarrow$ all flavours.
(b) As (a), but only for particles with rapidity $|Y| > 1$.
(c) As (a), but for particles with rapidity $|Y| \leq 1$.
(d) Jetset 7.3 predictions for multiplicity correlations in $Z^0 \rightarrow u\bar{u}, d\bar{d}, s\bar{s}$ 2-jet events at the parton, primary hadron and charged particle levels.

**Figure 6** (a) The measured corrected $\langle p_t^{in} \rangle$ and $\langle p_t^{out} \rangle$ plotted versus the charged multiplicity, $n_{ch}$. The predictions of the Jetset 7.3 Monte Carlo are also shown.
(b) Dependence of the uncorrected $\langle p_t^{in} \rangle$ on $n_{ch}$ for 2-jet events and for multi-jet events (three or more jets). For comparison, also the dependence of the inclusive sample is plotted. The JADE algorithm has been used with $y_{cut} = 0.04$.
(c) Dependence of the uncorrected $\langle p_t^{in} \rangle$ on the charged multiplicity, computed in a single hemisphere, for the inclusive data sample and for a subsample enriched in $b\bar{b}$ events.

**Figure 7** The measured $\langle p_t \rangle$, $\langle p_t^{in} \rangle$ and $\langle p_t^{out} \rangle$ of charged hadrons produced in $e^+e^-$ interactions, as a function of $\sqrt{s}$; the OPAL results are compared with published data [27]. Also shown are the predictions of the Jetset Monte Carlo with parameters tuned to data at $\sqrt{s} = 91.2$ GeV, [23].
Fig. 1
Fig. 3
Fig. 4
Fig. 5
Fig. 6