Measurement of the Time Dependence of $B^0_d \leftrightarrow \overline{B}^0_d$ Mixing Using a Jet Charge Technique

The OPAL Collaboration

Abstract

The observation and measurement of the time-dependence of $B^0_d \leftrightarrow \overline{B}^0_d$ mixing are described. The $B^0_d$ meson is reconstructed in final states that contain a $D^*\pm$ and an $\ell^+$, where the $b$ flavour of the $B^0_d$ at decay time is tagged by the electric charge of the lepton. A new and efficient method, using a jet charge technique, is developed for identifying the $b$ flavour of the produced $B^0_d$. From a sample of 556 $D^*\pm\ell^+$ candidates reconstructed in the OPAL data collected during 1990-1993, the $B^0_d \leftrightarrow \overline{B}^0_d$ oscillation frequency is measured to be $\Delta m_d = 0.508 \pm 0.075$ (stat) $\pm 0.025$ (syst) ps$^{-1}$ giving an oscillation parameter of $x_d = 0.73 \pm 0.11$ (stat) $\pm 0.08$ (syst), where 0.076 of the systematic error on $x_d$ arises from the uncertainty on the $B^0_d$ lifetime.

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1 Introduction

As in the $K^0\bar{K}^0$ system, particle-antiparticle oscillations are expected in neutral mesons that contain a bottom quark \([1]\). For $B_d^0\leftrightarrow\bar{B}_d^0$ mixing the main mechanism is a second order weak interaction as described by the box diagrams. The mass eigenstates are linear combinations of the flavour eigenstates $B_d^0 (\bar{b}d)$ and $\bar{B}_d^0 (b\bar{d})$. In $Z^0$ decays into $b\bar{b}$, the $B_d^0$ and $\bar{B}_d^0$ states are produced incoherently in the fragmentation process. Oscillations occur between particle and antiparticle states, with frequency proportional to $\Delta m_d$, the difference between the two mass eigenvalues\(^1\), until they decay weakly into lighter flavours.

The probability to find a $B_d^0$ at proper time $t$ after a $B_d^0$ is produced is expected to follow \(^2\)

$$P(t) = \frac{\overline{B}_d^0(t)}{B_d^0(t) + \overline{B}_d^0(t)} = \sin^2\left(\frac{\Delta m_d \cdot t}{2}\right). \quad (1)$$

The oscillation parameter, $x_d$, is introduced as $x_d = \Delta m_d/\Gamma$, where $\Gamma$ is the total width of the $B_d^0$. $B_d^0(t)$ is the probability for finding a $B_d^0$ meson at some later time $t$.

The UA1 Collaboration first observed $B^0\bar{B}^0$ mixing in 1986 \([2]\), where both $B_d^0$ and $\bar{B}_d^0$ contributed. The first experimental evidence for $B_d^0\bar{B}_d^0$ mixing came in 1987 from the ARGUS experiment \([3]\) at the $\Upsilon(4S)$ energy, where the $B_d^0$ is the only neutral $b$ meson produced. The observed $B_d^0\bar{B}_d^0$ mixing is the effect of time-integrated $B_d^0\leftrightarrow\bar{B}_d^0$ oscillation. The time-integrated mixing parameter is $\chi_d = \frac{1}{2} x_d^2 + 1$. The recent and more precise measurements \([4]\) of mixing at the $\Upsilon(4S)$ by the ARGUS and CLEO experiments give $\chi_d = 0.158 \pm 0.026$, corresponding to $x_d = 0.68 \pm 0.08$. Two measurements of time-dependent $B_d^0\leftrightarrow\bar{B}_d^0$ mixing have been recently reported by the ALEPH experiment at LEP \([5]\) which yield $\Delta m_d = 0.52^{+0.10+0.04}_{-0.11-0.03}$ ps\(^{-1}\) and $\Delta m_d = 0.50^{+0.07+0.11}_{-0.06-0.10}$ ps\(^{-1}\).

We use a new approach to measure the $B^0_d\leftrightarrow\bar{B}_d^0$ oscillation. In our analysis the $B_d^0$ meson is detected in $B_d^0\rightarrow D^{*-}\ell^+\nu X$ decays and the $b$ flavour of the $B_d^0$ at decay time is tagged by the

\(^1\)The effect of CP violation is expected to be smaller than that of the $B_d^0\rightarrow\bar{B}_d^0$ oscillation and is neglected here.

\(^2\)Throughout this paper the charge conjugate processes are also implied.
charge of the lepton. To identify the b flavour of this neutral b meson at production time (i.e. whether it is b or \( \overline{b} \)), we have exploited the charge difference between quark and antiquark jets in \( Z^0 \to b \overline{b} \) events. Other jet-charge techniques have already been used at LEP to measure the average B mixing [6].

The following sections describe the OPAL detector and event selection, the technique for distinguishing \( B^0_d \) from \( \overline{B}^0_d \) at production and decay, the \( B^0_d \) selection, \( B^0_d \) decay proper time reconstruction and the measurement of the \( B^0_d \leftrightarrow \overline{B}^0_d \) oscillation.

2 The OPAL detector and Data Sample

A complete description of the OPAL detector can be found elsewhere [7, 8]. We describe briefly the aspects of the detector pertinent to this analysis. Tracking of charged particles is performed by the central detector, which consists of a large volume jet chamber, a precision vertex drift chamber and chambers measuring the \( z \) coordinate\(^3\) of tracks as they leave the jet chamber. The central detector is positioned inside a solenoidal coil that provides a uniform magnetic field of 0.435 T. The momentum resolution obtained is approximately \( \left( \sigma_{p_{xy}}/p_{xy} \right)^2 = (0.02)^2 + (0.0015p_{xy})^2 \), where \( p_{xy} \) is the momentum in the plane transverse to the beam axis in GeV. The jet chamber also provides measurements of the ionization loss of charged particles, which are used for particle identification. The coil is surrounded by a time-of-flight counter array and an electromagnetic calorimeter. The electromagnetic calorimeter is composed of lead-glass blocks and instrumented with a presampler. The blocks are approximately \( 10 \times 10 \text{ cm}^2 \) in cross section, and the calorimeter is typically 24 radiation lengths deep. This is followed by a hadron calorimeter consisting of the instrumented return yoke of the magnet, and several layers of muon chambers. For the 1991 run a high precision silicon microvertex detector [8] surrounding a 1.1 mm thick, 5.3 cm radius beryllium-composite beam pipe was placed inside a new 2.2 mm thick, 8.0 cm radius carbon fibre pipe. The silicon detector was operational for 73% of the data collected in 1991, and all of the data collected in 1992. It provided two layers of silicon strip readout in the \( x-y \) plane. The polar angle acceptance is \( |\cos \theta| \leq 0.82 \) (0.76) for the inner (outer) layer, and the impact parameter resolution in the \( x-y \) plane achieved for 45 GeV muon pairs is 18\( \mu \)m for tracks with associated hits in both layers of the silicon microvertex detector. For the 1993 OPAL run the silicon detector was upgraded to provide a \( z \) coordinate measurement in addition to the \( \phi \) measurement [9]. Only the \( \phi \) measurement from that detector is used for the analysis presented here.

The data sample used for this analysis consists of about 1.9 million hadronic \( Z^0 \) decays collected during the period 1990–1993. The selection of hadronic events is described in [10].

The JETSET 7.3 Monte Carlo program [11] was used to generate event samples, together with a program to simulate the response of the OPAL detector [12]. Simulated events were processed through the same reconstruction and selection algorithms as data from the detector.

\(^3\)The OPAL coordinate system is defined with positive \( z \) being along the electron beam direction, \( \theta \) and \( \phi \) being the polar and azimuthal angles respectively.
3 Distinguishing $B^0_d$ from $\bar{B}^0_d$ at Production and Decay

The presence of a $D^{*-}\ell^+$ pair in the same jet is a clean signature for $B^0_d$ hadrons decaying in the mode $B^0_d \rightarrow D^{*-}\ell^+\nu$.

The $b$ flavour at decay time of a $B^0_d$ meson selected this way is automatically identified from the electric charge of the lepton. Since the $D^*$ is at least partially reconstructed, there are almost no leptons from secondary decays of charmed hadrons in this sample (only the $B$ decay into $D^- D_s^+$ gives such leptons in this sample).

The presence of an energetic $e$ or $\mu$ with large transverse momentum, $p_T$, with respect to the associated jet has been widely used at $e^+e^-$ and hadron colliders for tagging $b$ decays as well as the $b$ flavour of a $b$ hadron at its decay time [13, 14]. In principle, the electric charge of such leptons found in the jet opposite to the $B^0_d$ jet can also be used to infer the $b$ flavour at the production time ($t=0$) of the $B^0_d$, i.e. if the decaying $B^0_d$ originated from a produced $B^0_d$ or $B^0_s$. However, the efficiency is limited by the $b$ hadron semileptonic branching ratio, efficiencies for identifying $e$'s and $\mu$'s, and kinematic cuts applied in selecting the leptons. Typically the $b$ flavour at $t=0$ of only about 5-7% of the $B^0_d \rightarrow D^{*-}\ell^+X$ events can be identified, and in 75-80% of these cases the flavour assignment is correct.

In this study, where a semi-exclusive tag is used to identify the $B^0_d$ decays, we have explored a jet charge technique as a more efficient method of tagging the $b$ flavour at production time ($t=0$) of neutral $b$ mesons ($B^0_d$, $B^0_s$). The jet charge is defined as

$$Q_{jet} = \sum_{i=1}^{n} q_i \cdot \left( \frac{p_i^T}{E_{beam}} \right)^\kappa,$$

where $E_{beam}$ is the beam energy, $q_i$ and $p_i^T$ are the charge and the momentum component along the jet direction of track $i$, and $\kappa$ is a weighting factor. The sum runs over all charged tracks associated with the same jet. Jet finding is done using the JADE [15] algorithm with the E0 recombination scheme [16] and a scaled invariant mass cutoff of $y_{cut} = 0.04$.

The jet charges of two jets are used: that of the jet containing the $B^0_d$ candidate, and that of the most energetic other jet (opposite jet). For the jet containing the $B^0_d$ candidate, it is desired to measure the $b$ flavour ($t=0$) rather than at decay time. To this end, the value of $\kappa$ is chosen to be zero. In this case $Q_{jet}$ is simply the sum of particle charges. Since the reconstructed $B^0_d$ is neutral, the resulting jet charge is independent of whether a $B^0_d$ or a $\bar{B}^0_d$ is the decaying meson. However, some sensitivity to the produced $b$ flavour is provided through the fragmentation tracks (generally low momentum) in the jet.

For the opposite jets a value of $\kappa = 1$ is used. This choice of $\kappa$ enhances the correlation between the jet charge and the $b$ flavour of the decaying $b$ hadron jet opposite to the $B^0_d$. The $b$-hadron on the other side can be any species: $B^0_d$, $B^0_s$ which is expected to exhibit a larger mixing effect and $B^+$ or $\Lambda_b$ which do not mix. An average mixing of 12% was measured at LEP
Using the jet charges described above, a combined charge measure is defined:

\[ Q_{2\text{jet}} = Q_{\text{jet}}^{c=0}(B^0_d) - 10 \cdot Q_{\text{jet}}^{c=1}(\text{opp}), \]  

where \( Q_{\text{jet}}^{c=0}(B^0_d) \) and \( Q_{\text{jet}}^{c=1}(\text{opp}) \) are the jet charges of the \( B^0_d \) and of the most energetic other jet, respectively. The scaling factor of 10 gives the two jet charges similar numeric ranges. This measure combines the jet charge information from both the \( B^0_d \) jet and the jet containing the other b hadron to improve the b flavour (\( t=0 \)) discrimination. The sign of \( Q_{2\text{jet}} \) is used as an indicator of the sign of the b charge at production. Figure 1 shows the distributions of the jet charge for the \( B^0_d \) jet, the opposite jet and \( Q_{2\text{jet}} \) for simulated events.

In addition to giving a better overall b flavour identification, combining both charges also serves to reduce the effect of events where mixing occurred in the jet opposite the \( B^0_d \). In such a case the two sides tend to give conflicting jet charge information. In order to reject such events, and others where the charge determination is poor, we place a cut on the combined charge measure of:

\[ |Q_{2\text{jet}}| > 1.0. \]  

About 70% of reconstructed simulated \( B^0_d \) events satisfy equation 4. The jet charge performance in simulated events was tested using the JADE [15] and Durham [18] jet-finding algorithms and was seen to have little dependence on the jet finding scheme or the choice of invariant mass cutoff. Changing the scaling factor for \( Q_{\text{jet}}^{c=1}(\text{opp}) \) and the cut on \( Q_{2\text{jet}} \), does not change dramatically the jet charge performance. In about 72% of the events passing the requirement in equation 4, the b flavour (\( t=0 \)) is found to be correctly identified. The effect of mixing in the opposite jet decreases the correct identification of the flavour by only 2% (from 74% to the 72%). If the effect of mixing were not reduced by the use of both jets, the mixing in the opposite jet would have caused a decrease in the fraction correctly identified by 6%.

Had only the opposite jet charge been used, for the same efficiency (70%) only 65% of the events would be correctly tagged. A correct tagging of 72% can be achieved with a single jet only at an efficiency of 26%.

The value found in simulated events for the fraction of events correctly identified by the jet charge method was not used in the analysis. Instead, this fraction was obtained directly from the data as a free parameter in the fit for \( \Delta m_d \). The fitted value is compared to the Monte Carlo prediction as a consistency check.
4 Reconstruction of the $B_d^0$ Meson

The $D^{*+}$ mesons are identified via their decay $D^{*+} \rightarrow D^0\pi^+$ followed by the decay $D^0 \rightarrow K^-\pi^+$ or $D^0 \rightarrow K^-\pi^+\pi^0$, where the $\pi^0$ is not reconstructed (called the satellite channel). Tracks forming the $D^*$ are required to be contained in the same jet and are required to pass the following quality cuts:

- $|d_0| < 5\text{ mm}$, where $d_0$ is the distance of closest approach in the $r-\phi$ plane, between the track and the event vertex;
- $|z_0| < 20\text{ cm}$, with $z_0$ being the $z$ coordinate at the point of closest approach in the $r-\phi$ plane;
- $p_{xy} > 250\text{ MeV}$.

For the two channels the selection of $D^*$ candidates is performed in a very similar way. Pairs of oppositely charged tracks are combined, with the assumption that one is a kaon and the other a pion. The kaon candidate is required to have a $dE/dx$ measurement consistent with the kaon hypothesis, as follows:

- $P(dE/dx, K) > 0.01$, and
- for $D^0$ in the satellite mode $P(dE/dx, K) > 0.05$ if $dE/dx > \langle dE/dx \rangle_K$,

where $\langle dE/dx \rangle_K$ is the expected $dE/dx$ value for a kaon of the observed momentum and $P(dE/dx, K)$ is the probability that a kaon would have a $dE/dx$ measurement at least as far from the expected mean as the measured $dE/dx$. The cut is harder for the satellite mode because there is more background to it and a better $\pi$ rejection is needed.

The invariant mass, $M^{\text{cand}}_{D^0}$, of each combination is calculated. If it lies within one of the intervals specified below, the combination is retained as a $D^0$ candidate. Another track, the slow pion candidate, with a charge opposite to the charge of the kaon candidate track, is combined with the $D^0$ candidate, and the $K^-\pi^+\pi^+$ invariant mass is calculated. The combination is considered a $D^*$ candidate if the difference between this mass and the $D^0$ candidate mass is within certain limits.

The $M^{\text{cand}}_{D^0}$ mass window used for the $D^0 \rightarrow K^-\pi^+$ mode is:

- $1790\text{ MeV} < M^{\text{cand}}_{D^0} < 1940\text{ MeV}$.

For the satellite channel the $\pi^0$ is not reconstructed and therefore is not included in the invariant mass calculation. This yields a second peak around $M^{\text{cand}}_{D^0} \approx 1600\text{ MeV}$ in the $M^{\text{cand}}_{D^0}$
distribution. The width of this peak is about twice as large as that in the $D^0 \to K^-\pi^+$ channel. Since only a $K^-\pi^+$ pair is reconstructed, additional particles other than a single $\pi^0$ could be present if the kinematics are similar to those of the $K^-\pi^+\pi^0$ decay. Candidates in this satellite channel are selected by requiring

- $1410 \text{ MeV} < M_{D^0}^{\text{cand}} < 1770 \text{ MeV}$.

At low values of $x_{D^*} = E_{D^*}/E_{\text{beam}}$, where $E_{D^*}$ is the energy of the $D^*$ meson, the combinatorial background increases dramatically. In addition, in this study the decay time of the $D^*$ is required, and for small $x_{D^*}$ the smaller reconstructed momentum makes the boost and decay proper time estimates less accurate. To improve the lifetime measurement and reduce background, only $D^*$ candidates with $x_{D^*} > 0.15$ for the $D^0 \to K^-\pi^+$ channel and with $x_{K^-\pi^+} > 0.20$ for the satellite channel were used.

The $D^*$ candidates are combined with an $e$ or $\mu$ identified in the same jet. Unlike previous measurements by OPAL [13] an artificial neural network is used here to identify electrons. This selection was designed to provide high efficiency over an enlarged geometric acceptance of the detector. The network was trained on simulated events to identify electrons in the OPAL detector on the basis of 12 measured quantities coming from the electromagnetic calorimeter and the central tracking detector. The input variables were chosen for their ability to discriminate between electrons and hadrons and for the reliability of their simulation. The use of those variables in electron identification has been fully described in [19]. Some details about this method are provided in appendix 1. Electrons identified as originating from photon conversions are rejected as in [13]. Electrons are required to have $p > 2 \text{ GeV}$. Muons are selected as in [13] and are required to have momentum $p > 3 \text{ GeV}$. Both types of lepton are required to have $p_T > 0.6 \text{ GeV}$ with respect to the jet axis calculated including the lepton momentum. We require the lepton candidate to have at least one silicon vertex detector hit, or that a majority of its hits are “first hits” in the vertex drift chamber.  

The invariant mass of the $D^*\ell$ system is required to satisfy $2.8 \text{ GeV} < M_{D^*\ell} < 5.3 \text{ GeV}$ to suppress combinatorial background.

The event was rejected if the estimated $D^0$ vertex position was inconsistent (in direction or distance from the primary vertex) with the decay of a $D^0$ originating from the B decay vertex. This rejects a small number of badly reconstructed vertices as well as reducing combinatorial background containing tracks from the primary vertex. The vertex reconstruction is explained in the next section.

Figure 2 shows the distributions of $\delta_M = M(D^*) - M(D^0)$ for $D^*\ell$ candidates in the $D^0 \to K^-\pi^+$ and satellite channels. Overlaid are the distributions where the lepton has the same charge as the slow pion (wrong sign). The distribution of $\delta_M$ for the wrong sign candidates is used to estimate the shape of the combinatorial background. The absolute number of the wrong sign candidates underestimates the combinatorial background in the right sign

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4The “first hit” on a particular wire is that with the shortest measured drift time.
sample because the probability for the slow pion and lepton candidates to have a total charge of 2 is smaller than that for the zero charge combination.

The wrong sign background is fit to a function of the form:

\[ B(\delta_M) = (\delta_M - 0.1386)^r(a + b \cdot \delta_M + c \cdot \delta_M^2), \]

where \( a, b, c \) and \( r \) are free parameters in the fit.

To estimate the amount of combinatorial background in the signal region, the distribution of the right sign candidates for the \( K\pi \) channel is fit to the function:

\[ F(\delta_M) = d \cdot B(\delta_M) + A \cdot e^{-\frac{1}{2}\left(\frac{\delta_M - m_0}{\sigma}\right)^2}, \]

where \( m_0 \), the mean mass difference between the \( D^* \) and the \( D^0 \), \( \sigma \), the experimental resolution on this mass difference, \( d \) and \( A \) are free parameters in the fit.

For the satellite channel, the shape of the signal is determined by the kinematics rather than by the experimental resolution. In this case the signal is fit to a gaussian with different widths above and below the mean. Those widths were determined from simulated events.

\[ F(\delta_M) = \begin{cases} 
 d \cdot B(\delta_M) + A \cdot e^{-\frac{1}{2}\left(\frac{\delta_M - m_0}{\sigma_1}\right)^2} & \text{for } \delta_M < m_0 \\
 d \cdot B(\delta_M) + A \cdot e^{-\frac{1}{2}\left(\frac{\delta_M - m_0}{\sigma_2}\right)^2} & \text{for } \delta_M > m_0 
\end{cases} \]

The signal region was defined as \( \delta_M < 0.15 \) GeV for the \( K\pi \) channel and \( \delta_M < 0.16 \) GeV for the satellite channel. The signal and background estimates for the two channels are given in Table 1. These events passed the requirement of equation 4, which is 67% efficient for the data. Of the 556 \( D^{*-}\ell^+ \) candidates, 278 contain muons and 278 contain electrons. The number of signal events is 426 ± 15.

<table>
<thead>
<tr>
<th>Decay modes</th>
<th>Number of ( D^{*-}\ell^+ ) candidates</th>
<th>background</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D^0 \to K^-\pi^+ )</td>
<td>187</td>
<td>34±6</td>
</tr>
<tr>
<td>( D^0 \to K^-\pi^+\pi^0 )</td>
<td>369</td>
<td>96±14</td>
</tr>
<tr>
<td>Sum</td>
<td>556</td>
<td>130±15</td>
</tr>
</tbody>
</table>

Table 1: List of reconstructed \( D^{*-}\ell^+ \) events and background estimates

In the absence of \( D^{**} \)'s (or non resonant \( D^*\pi^- \)) in B decays, the \( D^*\ell \) events would arise almost completely from \( B^+_d \) decays. However, \( B^+ \) can also decay into this combination via \( D^{**} \) states from the decay \( B^+ \to D^{*0}\ell\nu \) where the \( D^{*0} \) decays into a \( D^* \) and a charged pion. The fraction
of D* and D in inclusive semileptonic decays of b hadrons was measured by CLEO [20] and ARGUS [21]. Using these measurements and theoretical considerations, the B⁺ contamination in D*ℓ events was calculated in [22]. That calculation gave the relative fraction of B⁺ to B_d⁰ in the selected D*ℓ sample as (16 ± 9)%. Rather than subtract this background, its influence on the Δm_d measurement was introduced in the fit as described below.

Other background arises from hadron tracks misidentified as leptons and then combined with a D*. This background is estimated by fitting a D* peak shape to the wrong sign events. It is estimated to contribute to the D*ℓ sample at no more than the 1% level and is neglected.

5 Reconstruction of B_d⁰ Decay Proper Time

The B_d⁰ decay proper time, t, can be expressed as

\[ t = \frac{L}{c(\beta\gamma)} \]  

where L is the 3-d decay length of the B_d⁰ and \( \beta\gamma \) is the Lorentz boost of the B_d⁰. Measurements of L and \( \beta\gamma \) are required in order to reconstruct t. The 2-d decay length of the B_d⁰ is measured in the plane transverse to the beam direction following the scheme described in [23]. The slow pion momentum direction, which follows closely that of the D*, is used to constrain the D⁰ vertex. The D* momentum is extrapolated back to an intersection point with the lepton. This 2-d decay length is converted into L using the direction of the reconstructed D*ℓ.

The estimate of \( \beta\gamma \) is obtained by parametrising \( \beta\gamma \) as a function of the momentum and the invariant mass of the D*ℓ pair. For the fully reconstructed D⁺ → D⁰π⁺ events where D⁰ → K⁻π⁺, the boost of the B_d⁰ is estimated following

\[ \beta\gamma = \frac{p_{D\ell}}{m_B} \cdot s(p_{D\ell}, m_{D\ell}), \]  

where \( p_{D\ell} \) is the momentum of the D*ℓ pair, \( m_B=5278.7 \) MeV is the mass of B_d⁰ [24], and s is a scaling factor that corrects for the missing energy carried by the undetected \( \nu \) in the semileptonic B_d decay. Monte Carlo simulation has shown that s depends on the kinematic properties of the D*ℓ, namely \( p_{D\ell} \) and \( m_{D\ell} \). We have extracted values of s in eleven \( p_{D\ell} \) and \( m_{D\ell} \) bins from the Monte Carlo study, as summarised in Table 2.

For the satellite D⁰ events correction factors were calculated separately in a similar way and are shown in Table 3. Although there is a \( \pi^0 \) missing in addition to the \( \nu \), the correction factors are not very different, largely due to the binning in mass and momentum.

The relative uncertainty on the \( \beta\gamma \) estimate, \( \sigma_s/s \), ranges from 24% at low mass and low momentum to 5% at high mass and high momentum. For events passing the selection criteria,
Table 2: $p_{B_d}/p_{D^0}$ boost correction factors for the $K\pi$ channel. The second number for each entry is the r.m.s. of the distribution.

<table>
<thead>
<tr>
<th>$s = p_{B_d}/p_{D^0}$</th>
<th>$m_{D^0}$ (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{D^0}$ (GeV)</td>
<td>2.8 - 3.7</td>
</tr>
<tr>
<td>$&lt; 20$</td>
<td>1.82±0.46</td>
</tr>
<tr>
<td>20 - 30</td>
<td>1.41±0.25</td>
</tr>
<tr>
<td>$&gt; 30$</td>
<td>1.13±0.10</td>
</tr>
</tbody>
</table>

Table 3: $p_{B_d}/p_{K\pi\ell}$ boost correction factors for the satellite channel. The second number for each entry is the r.m.s. of the distribution.

<table>
<thead>
<tr>
<th>$s = p_{B_d}/p_{K\pi\ell}$</th>
<th>$m_{K\pi\ell}$ (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{K\pi\ell}$ (GeV)</td>
<td>2.8 - 3.7</td>
</tr>
<tr>
<td>$&lt; 20$</td>
<td>1.83±0.42</td>
</tr>
<tr>
<td>20 - 30</td>
<td>1.46±0.23</td>
</tr>
<tr>
<td>$&gt; 30$</td>
<td>1.19±0.09</td>
</tr>
</tbody>
</table>

The average uncertainty is about 13%.

The reconstructed $B^0_d$ decay proper time is compared to the true time in Figure 3 for simulated $B^0_d \rightarrow D^{*-}\ell^+\nu X$ events. Depending on the kinematics of a $D^*\ell$ pair, the average values of $\sigma_t/t$ ranges from 16% to 20%. These uncertainties are not used in the fit; rather they are used in the estimation of the systematic errors due to the decay time resolution.

6 Measurement of the Time-Dependence of $B^0_d\bar{B}^0_d$ Mixing

The sample of events that pass the $D^{*-}\ell^+$ selection and that satisfy equation 4 were divided into two samples. A sample of unmixed events, where the calculated jet charge, $Q_{2\text{ jet}}$, has the same sign as the lepton charge, and a sample of mixed events, where the jet and lepton charges are of opposite sign.

The decay time distributions for the combinatorial background were estimated from 1559 events with $\delta_M$ outside the signal region (0.16-0.25 GeV for the $K^-\pi^+$ mode and 0.17-0.25 GeV for the satellite mode). Those distributions were scaled to the number of estimated background events in the signal region and subtracted from the respective distributions. The Monte Carlo indicates that the proper time distribution of events outside the signal region is a good estimator for the background in the signal region. The decay time distributions are shown for all selected $D^{*-}\ell^+$ events in Figure 4a and for the mixed $D^{*-}\ell^+$ events in Figure 4b. The decay time
distribution of these events is consistent with the $B_d^0$ lifetime of 1.44 ps used in this paper.

The time dependence of $B_d^0\overline{B}_d^0$ mixing is measured from the decay time distribution of the ratio

$$R(t) = \frac{N_{\text{mix}}(t) - N_{\text{mix}}^{\text{back}}(t)}{N_{\text{tot}}(t) - N_{\text{tot}}^{\text{back}}(t)},$$

(7)

where $N_{\text{tot}}(t)$ and $N_{\text{tot}}^{\text{back}}(t)$ are the total number of candidates and estimated background decaying at proper time $t$, and $N_{\text{mix}}(t)$ and $N_{\text{mix}}^{\text{back}}(t)$ are the corresponding number for the mixed events.

For a time dependence of the mixing as in equation 1 and a flavours mistag probability $f$, this ratio corresponds to

$$R(t) = f + (1 - 2f) \cdot \sin^2(\Delta m_d \cdot t/2).$$

(8)

Furthermore, if a fraction of the events come from $B^+$ decays, equation 8 becomes

$$R(t) = f + \frac{(1 - 2f)}{1 + N_+(t)/N_0(t)} \cdot \sin^2(\Delta m_d \cdot t/2),$$

(9)

where $N_+(t)$ and $N_0(t)$ are the number of $B^+$ and $B_d^0$ events which decay at time $t$.

$$\frac{N_+(t)}{N_0(t)} = \frac{N_+(0)}{N_0(0)} \cdot \alpha^{(t/\tau_0) + (\delta \tau/\tau_+)}$$

(10)

where $\tau_0$ is the $B_d^0$ lifetime, $\tau_+$ is the $B^+$ lifetime, $\delta \tau = (\tau_+ - \tau_0)$ and $N_+(N_0)$ are the number of $B^+(B_d^0)$ in the sample.

The distribution of $R$ is shown in Figure 5 and is fitted to the functional form of equations 9 and 10 using the values of $\tau_0 = 1.44 \pm 0.15$ ps, $\delta \tau = -0.02 \pm 0.23$ ps [22, 25] and $\frac{N_+(0)}{N_0(0)} = 0.16 \pm 0.09$ [22]. The mistag fraction, $f$, is a free parameter in the fit. For negative estimated proper times the ratio was considered to be the same as for $t=0$ since our simulation indicates those events come from small positive decay times. The fit gave a $\chi^2$ of 2.41 with 7 degrees of freedom. The resulting fit parameters are:

$$\Delta m_d = 0.508 \pm 0.075(\text{stat}) \, \text{ps}^{-1}$$

and

$$f = 0.263 \pm 0.033(\text{stat}),$$
in good agreement with the mistag fraction of 0.28 predicted by the Monte Carlo simulation. The amplitude of this oscillation is approximately 0.41±0.06. Repeating the fit with \(\Delta m_d = 0\), the \(\chi^2\) rises to 22.1, which indicates that \(\Delta m_d = 0\) is disfavoured by 4.44\(\sigma\). The probability for a \(\chi^2\) of at least 22.1 with 8 degrees of freedom is 0.5%.

7 Systematic Errors

We have studied the following sources of systematic uncertainty on the measurement of \(\Delta m_d\):

- the decay proper time resolution,
- \(D^*\) background subtraction,
- the \(B^+\) fraction in \(D^*\ell\) events,
- the \(B^0_d\), \(B^+\) lifetime difference,
- the error on \(B^0_d\) lifetime,
- fake \(e\) and \(\mu\),
- the background process \(B_{ud} \rightarrow D^-D_s^+, D_s^+ \rightarrow \ell^+X\),
- \(b\) fragmentation.

The systematic error due to proper time resolution was estimated from a sample of Monte Carlo events generated with \(\tau_0 = 1.48\) ps and \(x_d = 0.7\) without detector simulation. The time was smeared by the resolution estimated in Section 5. The difference between the generated \(\Delta m_d\) and the fitted value for \(\Delta m_d\) was 0.010±0.008 ps\(^{-1}\). The fully simulated events shown in Figure 3 were used to check for a possible bias in the time reconstruction. The resulting bias was -0.002 ± 0.017 ps. The uncertainty of 0.017 ps gave a systematic error of 0.003 ps\(^{-1}\) on \(\Delta m_d\). The effect of alignment and calibration uncertainties is assessed using the decay length uncertainty of 43 \(\mu\)m found in a detailed study of 3-prong \(\tau\) decays [26]. Adding this extra possible bias to the decay length results in a systematic error of 0.005 ps\(^{-1}\) on \(\Delta m_d\). The above three errors are added in quadrature as the estimate of the error due to decay time resolution.

The statistical error on the \(D^*\) background estimate from the fits to the \(\delta_M\) spectrum was 3\% and a systematic error on the background, estimated by fitting the spectrum to a Gaussian and a polynomial without the constraints from the wrong sign fit, gave a maximal variation of 11\%. The systematic error on \(\Delta m_d\) when varying the background level by 12\% is 0.004 ps\(^{-1}\).

Some of the \(D^*\ell\) candidates have a correctly reconstructed lepton and slow pion but one of the tracks forming the \(D^0\) is misidentified. These events have the right charge assignment. They constitute approximately 10\% of the reconstructed \(D^*\ell\) events. Their reconstructed lifetime
distribution was checked for a possible bias coming from vertex or boost estimation. No bias was found, and the lifetime resolution was found to be consistent with that of correctly reconstructed events. The excess seen in Figure 2b around $\delta \chi = 0.16$ is due to this kind of events, and is included in the systematic error on the background.

An additional check on the background subtraction was made by calculating the mixed fraction, $R$, assuming the fraction of background events in the mixed sample does not depend on the proper time. Since the side band mixed fraction is consistent with being independent of $t$, this measures the sensitivity of the results to statistical fluctuations in the background subtraction. The fitted $\Delta m_d$ is changed by 0.001 ps$^{-1}$. This small deviation should be fully accounted for in the statistical error.

The error due to the uncertainty in the fraction of $B^+$ in the $D^+\ell$ sample was evaluated by changing this fraction in the fit in the range $(16\pm 9)\%$[22]. The variation in $\Delta m_d$ is $\pm 0.019$ ps$^{-1}$.

The lifetime difference between $B^+$ and $B^0_d$ introduces a time dependence in the oscillation amplitude. This effect was evaluated by changing the lifetime difference in the fit by $\pm 0.23$ ps [22, 25]. This changed the fitted value of $\Delta m_d$ by $\pm 0.012$ ps$^{-1}$.

The $B^0_d$ lifetime was varied by $\pm 0.15$ ps [22, 25] while keeping $\tau_+ - \tau_0$ constant. This changed $x_d$ by $\pm 0.076$. The corresponding change in $\Delta m_d$ is less than 0.001 since no scaling with $\tau_0$ is necessary for $\Delta m_d$.

As described in Section 4, the background due to fake $e'$s or $\mu'$s is at the 1% level. The contribution to the $\Delta m_d$ measurement is smaller than 1% and is neglected. The possible bias on $\Delta m_d$ as a result of the background process $B \rightarrow D^{\ast}D^+_s$, $D^+_s \rightarrow \ell^+X$, was also evaluated using available measurements [22, 24]. The fraction of events from this process in the selected $D^{\ast\to\ell^+}$ sample is estimated to be about 1%. This will affect the amplitude of the oscillation but not its frequency. We neglect this background in this measurement.

In this analysis a large fraction of the $B^0_d$ momentum is reconstructed and the corrections to the boost are binned in the mass and momentum of the reconstructed $D^+\ell$ system. Therefore, the effect of the uncertainty of the $b$ fragmentation on this measurement is expected to be insignificant. This has been checked by reweighting simulated events according to fragmentation distributions corresponding to the fragmentation parameters measured by OPAL [14]. The effect was found to be negligible.

The bias in the estimated boost for $D^+$ decays from higher mass states was found to be less than 1%. About 24% of the events are expected to come from such decays. The error resulting from this is negligible compared with the possible bias in decay time already included in the systematic error on the time resolution.

We have also repeated the analysis with a cut on the leptons of $p_T > 0.8$ GeV, rather than $p_T > 0.6$ GeV. The result obtained was $\Delta m_d = 0.526 \pm 0.080$ (stat) ps$^{-1}$, which is consistent with the fit result from section 6, taking into account the correlation between the samples.
Table 4 lists the systematic errors evaluated. The error on $x_d$ due to uncertainty on $\tau_d$ is 0.076. The remaining errors combined in quadrature add up to 0.037.

<table>
<thead>
<tr>
<th>Source of error</th>
<th>Variation</th>
<th>Error on $f$</th>
<th>Error on $\Delta m_d$ (ps$^{-1}$)</th>
<th>Error on $x_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decay time resolution</td>
<td></td>
<td></td>
<td>0.012</td>
<td>0.017</td>
</tr>
<tr>
<td>Background subtraction</td>
<td>$\pm 12%$</td>
<td>0.008</td>
<td>0.004</td>
<td>0.006</td>
</tr>
<tr>
<td>$B^+$ fraction</td>
<td>$\pm 0.09$</td>
<td>0.002</td>
<td>0.019</td>
<td>0.028</td>
</tr>
<tr>
<td>$\tau_+ - \tau_0$</td>
<td>$\pm 0.23\text{ ps}$</td>
<td>0.001</td>
<td>0.012</td>
<td>0.017</td>
</tr>
<tr>
<td>$\tau_0$</td>
<td>$\pm 0.15\text{ ps}$</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>0.008</td>
<td>0.025</td>
<td>0.085</td>
</tr>
</tbody>
</table>

Table 4: List of systematic errors

The error due to the uncertainty on the $B^0_d$ lifetime constitutes a very small part of the systematic error on $\Delta m_d$ but is the dominant error on $x_d$.

8 Conclusion

We have measured the time-dependence of $B^0_d \rightarrow \overline{B}^0_d$ mixing in $Z^0$ decays. Using $426 \pm 15$ $D^\pm \ell \pi^\pm$ pairs that are dominated by semileptonic $B^0_d$ decays, we determine the oscillation frequency for the $B^0_d$ to be $\Delta m_d = 0.508 \pm 0.075(\text{stat}) \pm 0.025(\text{syst})$ ps$^{-1}$. Using $\tau_d = 1.44 \pm 0.15$ ps this gives $x_d = 0.73 \pm 0.11(\text{stat}) \pm 0.08(\text{syst})$, where most of the systematic error comes from the uncertainty on the $B^0_d$ lifetime.

Through this measurement a new jet charge technique for identifying the $b$ flavour of neutral $b$ mesons at production time is demonstrated. Employing this technique enables us to use a semi-exclusive event sample because the flavour ($\ell=0$) determination is very efficient. In addition this analysis shows that the mistag fraction predicted by Monte Carlo is consistent with that measured in the data. The result presented here, however, is completely independent of this prediction.

References


The results above were combined by M. Danilov. Presented at the International Europhysics Conference on High Energy Physics, Marseille, July 1993.


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OPAL optimised parameters were used, as described in OPAL Collaboration, Z. Phys. C 47 (1990) 505.
The fragmentation of the b quarks was described by the fragmentation function of C.Peterson et al., Phys. Rev. D 27 (1983) 105.
The value of the fragmentation parameter used corresponds to the LEP average value of \( \langle x_E \rangle_b = 0.70 \).


Appendix 1

The identification of electrons in the OPAL detector relies on the specific ionization loss of a track in the jet chamber, $dE/dx$, and the amount and distribution of energy in the electromagnetic calorimeter around the extrapolated track.

The electron selection used in this paper is based on an artificial neural network of the feed forward type [27]. The network has one hidden layer made of 15 neurons. It was trained on simulated events to identify electrons in the OPAL detector on the basis of the following 12 measured quantities coming from the electromagnetic calorimeter and the central tracking detector:

- $p$, the track momentum.
- $\cos \theta$, the cosine of the track angle to the $e^-$ beam.
- $dE/dx$, the specific ionization loss in the central tracking chamber.
- $\sigma(dE/dx)$, the estimated error on $dE/dx$.
- $E/p$, the energy in the associated electromagnetic calorimeter cluster divided by the track momentum.

- the number of lead-glass blocks in the electromagnetic cluster.

- $E_{\text{cone}}/p$

- The number of lead glass blocks with centres within 30 mrad of the extrapolated track position at the front face of the lead glass.

- $E_{\text{cone}}/(E_{\text{cone}} + \Delta E)$

- $\theta_{\text{track}} - \theta_{\text{cluster}}$, the difference in $\theta$ between the extrapolated track position and the centre of the electromagnetic cluster.

- $\phi_{\text{track}} - \phi_{\text{cluster}}$, the difference in $\phi$ between the extrapolated track position and the centre of the electromagnetic cluster.

- The presampler signal associated with the track.

$E_{\text{cone}}$ is the total energy deposited in the blocks in the electromagnetic cluster associated with the track whose centres are within 30 mrad of the extrapolated track position at the front face of the lead glass. $E_{\text{cone}} + \Delta E$ is the total energy in this cone plus adjacent blocks.

These input variables have been chosen for their ability to discriminate between electrons and hadrons or to indicate variations in the strength of other discriminating variables, and for the reliability of their simulation. A full description of these variables can be found in [19]. In the kinematic range $p > 2$ GeV and $p_T > 0.6$ GeV the efficiency is estimated to be about 85% for electrons coming from semileptonic decays of b hadrons in simulated events. The corresponding hadronic contamination for inclusive electrons is about 9%. It is important to note that the analysis described here does not rely on any knowledge of the electron selection efficiency and that the background estimation is done directly from the wrong sign $D^*$ lepton combinations.

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Figure Captions

Figure 1:
The jet charge distribution for (a) $B^0_d$ jets, (b) opposite jets and (c) the combined jet charge measure. The solid (dashed) lines are the distributions for simulated $B^0_d(\overline{B}^0_d)$ events.

Figure 2:
Distributions of $M(D^*)-M(D^0)$ for a) $D^0 \rightarrow K^-\pi^+$ events and b) satellite events. The histogram represents the wrong sign events.

Figure 3:
Reconstructed proper time versus true Monte Carlo proper time (a) and reconstructed proper time minus generated proper time (b) for simulated $B^0_d \rightarrow D^*-\ell^+X$ events.

Figure 4:
Reconstructed proper time distribution for a) all $D^*-\ell^+$ events, and b) mixed events. The dashed histogram is the estimated background.

Figure 5:
The ratio $R$ of mixed to total events as a function of proper decay time. Overlaid is the result of the fit.
Figure 1

(a) Distribution of $Q_{\text{jet}}^{\Lambda^0(B_d^0)}$

(b) Distribution of $Q_{\text{jet}}^{\Lambda^+}(\text{opp})$

(c) Distribution of $Q_{2\text{jet}}$
Figure 2
Figure 3
Figure 4