Test of CP-invariance in $e^+e^- \rightarrow Z^0 \rightarrow \tau^+\tau^-$
and a limit on the weak dipole moment of the $\tau$ lepton

OPAL Collaboration

Using a sample of 5558 $Z^0 \to \tau^+ \tau^-$ decays produced at LEP a direct test of $CP$-invariance in the neutral current reaction $e^+e^- \to \tau^+\tau^-$ is performed. Samples of events where each $\tau$ decays into a single charged particle have been isolated for the construction of $CP$-odd observables. Three different event classes are considered: lepton-lepton, lepton-hadron, and hadron-hadron. No evidence for a non-zero expectation value of the considered $CP$-observables and hence for $CP$-violation is observed. Quantitatively, we deduce from this null result an estimate on the weak dipole...
1. Introduction

The origin of non-conservation of the discrete symmetry \( CP \), where \( C \) stands for charge conjugation and \( P \) for parity, is one of the fundamental questions of particle physics. So far, violation of \( CP \)-invariance has been observed [1] only in the neutral kaon system in \( \Delta S = 2 \) transitions between \( K^0 \) and \( \bar{K}^0 \) and perhaps also in \( \Delta S = 1 \) transitions ("direct \( CP \)-violation") [2]. In the standard model of electroweak interactions, \( CP \)-violation is described by a phase in the quark mixing matrix [3], which enters in the weak charged current couplings among quarks. In neutral current reactions, violation of \( CP \)-symmetry has not been observed and the standard model does not predict any observable effect. Nevertheless, interesting possibilities exist in theories beyond the standard model.

It was pointed out [4-10] that the large number of \( Z^0 \) decays obtained at LEP are well suited to search for \( CP \)-violation in weak neutral current interactions by studying \( CP \)-odd observables. Any non-zero expectation value of such an observable would be direct evidence for \( CP \)-violation. \( CP \)-violation can be introduced in neutral current processes if the participating elementary particles possess electric or weak dipole moments [11,12].

The experimental limit on the electric dipole moment of the muon is \((-3.7 \pm 3.4) \times 10^{-19} \text{ e cm} \) [13] and on the electron electric dipole moment \((-2.7 \pm 8.3) \times 10^{-27} \text{ e cm} \) [14]. In many models the magnitude of the lepton dipole moments depends on the mass of the lepton to the third power [5,7]. This, and the fact that signs of new physics are commonly believed to manifest themselves in interactions involving the heavier quark and lepton families, renders the reaction \( e^+e^- \rightarrow \tau^+\tau^- \) a particularly interesting process to search for \( CP \)-violating effects. Since \((m_{\tau}/m_e)^3 \sim 5 \times 10^{10}\) a sensitivity to the electric dipole moment of the \( \tau \) on the order of \( 10^{-17} \text{ e cm} \) is well in the regime to test new physics. There is, however, no a priori connection between the magnitudes of the weak and electric dipole moments. In this paper we present the study of a \( CP \)-odd tensor observable using data on \( \tau \) pair production measured by the OPAL detector. Apart from a direct test of \( CP \)-violation we use the data to set a limit on the weak dipole moment of the \( \tau \).

2. Study of \( CP \)-odd tensor observables

The authors of ref. [7] use the following effective Lagrangian which corresponds to the only \( CP \)-violating form factor at the \( Z\tau\tau \) vertex to model new \( CP \)-violating effects in \( \tau \) pair production:

\[
\mathcal{L}_{CP} = -\frac{1}{2} i \bar{\tau} \sigma^{\mu\nu} \gamma_5 \tau [d_\tau(q^2) F_{\mu\nu} + \hat{d}_\tau(q^2) Z_{\mu\nu}].
\]

Here \( F_{\mu\nu} \) and \( Z_{\mu\nu} \) are the electromagnetic and weak field tensors. The form factors \( d_\tau(q^2) \) and \( \hat{d}_\tau(q^2) \) are called electric and weak dipole moment, respectively. They determine the strength of the \( CP \)-violating amplitude and are assumed to be real throughout this analysis [8]. In our case the momentum transfer is given by \( q^2 = m_\tau^2 \). In the following we abbreviate \( \hat{d}_\tau(m_\tau^2) = \hat{d}_\tau \) and neglect the term \( d_\tau(q^2) F_{\mu\nu} \). The \( CP \)-violating amplitude \( A_{CP} \) with coupling strength \( \hat{d}_\tau \) adds coherently to the standard model amplitude. For large \( \hat{d}_\tau \) the \( CP \)-even contribution \( |A_{CP}|^2 \) to the cross section dominates, giving rise to a large partial width.
$\Gamma (Z^0 \to \tau^+\tau^-)$. As discussed later in this letter this possibility is excluded by experiment. For small $\bar{d}_t$, the $CP$-odd interference term becomes important.

We consider the reaction $e^+e^- \to Z^0 \to \tau^+\tau^-$, where each $\tau$ decays into one charged particle plus neutrals:

$$e^+(p^+) + e^-(p^-) \to a(q_-) + \bar{b}(q_+) + X,$$

(2)

where $a$ and $\bar{b}$ are the charged $\tau$ decay products and $X$ symbolizes all neutral particles in the final state. We assume that the $e^+e^-$ initial state has no net longitudinal polarization. Transverse polarization would not change the results presented here as long as cuts on the momenta are done in a $CP$-invariant way.

Many observables can be constructed which are sensitive to $CP$-violation [4,8]. Since no information on the $\tau$-spin direction is experimentally accessible on an event by event basis the following symmetric and traceless tensor has been suggested [8] using the momenta $q_{\pm}$ of the final state particles:

$$T_{ij} = (q_- - q_+)_i(q_- \times q_+)_j + (i \leftrightarrow j),$$

(3)

where $1 \leq i, j \leq 3$ are the Cartesian vector indices with the $z$ coordinate along the incoming electron direction. These quantities transform odd under $CP$.

Using the coupling given in eq. (1), integrating over phase space, and respecting rotational invariance, one obtains for $q^2 = m_{Z^0}^2$ and for small $\bar{d}_t$ [8]

$$\langle T_{ij} \rangle_{ab} = \bar{d}_t c_{ab} \frac{m_{Z^0}^2}{e} \text{diag}(-\frac{1}{6}, -\frac{1}{6}, \frac{1}{2}).$$

(4)

The symbol $\text{diag}$ designates a diagonal matrix with the diagonal elements given in parentheses. The constant $c_{ab}$ depends on the decay channels. The sensitivity to $\bar{d}_t$ of each $\tau$ pair decay channel depends on the spin analyzing power of the momenta of the decay particles because the tensor observables can be traced back to observables which contain the $\tau$ spin [8]. For $\tau$ decays with a relatively large branching ratio $c_{ab}$ has been calculated in ref. [8] and is listed in table 1.

A direct test on $CP$-invariance can be made by studying the distributions of $T_{ij}$. $CP$-violation would introduce an asymmetric form of these distributions. A significant deviation of the mean value of $T_{ij}$ from zero would be direct evidence for $CP$-violation. In order to obtain a limit on $\bar{d}_t$ we use a Monte Carlo event generator [8] which includes the tree level standard model and $CP$-violating amplitudes. We have searched for the most sensitive method to observe any possible deviation from $CP$-invariance. The mean value of the tensor element $T_{33}$ is the most sensitive observable to use and is also the one least influenced by systematic effects. Its sensitivity is twice as large as that for the other two diagonal elements. Note that the trace of $T_{ij}$ is zero and the individual tensor elements are strongly correlated. The inclusion of the quantities $\langle T_{11} \rangle$ or $\langle T_{22} \rangle$ in the analysis does not increase the sensitivity.

Table 1 suggests that maximum sensitivity is obtained by isolating the different $\tau$ decay channels because $c_{ab}$, which is a measure of the expected sensitivity, varies strongly for the different decay modes. Since this procedure would lower the statistics considerably we have instead chosen to select three different decay modes of $\tau$ pairs, the lepton–lepton ($\ell^-\ell^-$), lepton–hadron ($\ell^-h$) and hadron–hadron (h–h) decay mode classes, where “lepton” means $\mu$ or $e$ candidate and “hadron” means a single charged track which is not a lepton candidate. In this way one only has to discriminate a lepton from a charged hadron. The average $\langle c \rangle$ for the lepton–hadron and hadron–hadron case is calculated according to the formula

$$\langle c \rangle = \frac{\sum B_a B_b c_{ab}}{\sum B_a B_b},$$

(5)

where $B_a$ is the branching ratio of $\tau \to a$ and the sums run over the decay channels in the respective class. The $\tau^+\tau^- \to (\pi\nu)(\pi\nu)$ decay mode is the most sensitive channel for measuring $\bar{d}_t$ but suffers from the relatively low branching ratio. Table 1 shows that the values of $c_{ab}$ have the same sign for the h–h class and even for the $\ell$–h class cancellations are not large. Because the decays $\tau \to e\nu\bar{\nu}$ and $\tau \to \mu\nu\bar{\nu}$ have the same characteristics, the $\ell$–$\ell$ class has only one decay channel to be considered with $c_{ab}$ being relatively large and positive. For the $\ell$–h class the values of $c_{ab}$ are predominantly negative or near zero. In principle one could combine the $\ell$–h and h–h classes but then one would loose in sensitivity because the $\ell$–h channel has a large branching ratio but only a weak sensitivity to $CP$-violating effects. These two classes suffer from the necessary assumption that the decay modes not considered do not contribute, i.e., $\langle c \rangle = 0$. For the
Table 1
Sensitivity of $\tau$ pair decay modes to $d_{\ell}$. $\ell$ denotes electron or muon. $a_1\nu$ denotes the one-prong $a_1$ decays. $B_{\pi}\beta_{\pi}$ is the product branching ratio into the specified channel [13]. $u$ is the sensitivity to $d_{\ell}$ as explained in the text. The modes labeled "other" are assumed to have $u = 0$.

<table>
<thead>
<tr>
<th>$\tau^+\tau^- \rightarrow ab$</th>
<th>$B_{\pi}\beta_{\pi}$ (%)</th>
<th>$c$ (GeV$^3$)</th>
<th>$u$ (GeV$^4$/10$^{-16}$ $e$ cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^+\tau^- u\nu u\nu$</td>
<td>$12.6 \pm 0.2$</td>
<td>$253 \pm 3$</td>
<td>$2.4 \pm 0.2$</td>
</tr>
<tr>
<td>$\pi\nu \pi\nu$</td>
<td>$5 \pm 0.2$</td>
<td>$-490 \pm 7$</td>
<td>$-5.1 \pm 0.3$</td>
</tr>
<tr>
<td>$\rho\nu \rho\nu$</td>
<td>$1.2 \pm 0.1$</td>
<td>$-1397 \pm 11$</td>
<td>$-13.5 \pm 0.4$</td>
</tr>
<tr>
<td>$a_1\nu \pi\nu$</td>
<td>$5.2 \pm 0.2$</td>
<td>$-146 \pm 3$</td>
<td>$-1.74 \pm 0.2$</td>
</tr>
<tr>
<td>$a_1\nu \rho\nu$</td>
<td>$1.7 \pm 0.2$</td>
<td>$-315 \pm 5$</td>
<td>$-3.5 \pm 0.2$</td>
</tr>
<tr>
<td>$a_1\nu a_1\nu$</td>
<td>$3.4 \pm 0.3$</td>
<td>$-75 \pm 2$</td>
<td>$-0.8 \pm 0.1$</td>
</tr>
<tr>
<td>$a_1\nu a_1\nu$</td>
<td>$1.1 \pm 0.2$</td>
<td>$-22 \pm 2$</td>
<td>$-0.2 \pm 0.1$</td>
</tr>
<tr>
<td>other $h-h$</td>
<td>$8.0 \pm 0.3$</td>
<td>$\approx 0$</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td>total $h-h$</td>
<td>$25.6 \pm 0.6$</td>
<td>$-239 \pm 10$</td>
<td>$-2.34 \pm 0.1$</td>
</tr>
<tr>
<td>$\nu\nu \ell\nu u\nu$</td>
<td>$16.2 \pm 0.5$</td>
<td>$9 \pm 3$</td>
<td>$0.3 \pm 0.2$</td>
</tr>
<tr>
<td>$\pi\nu \ell\nu u\nu$</td>
<td>$7.8 \pm 0.3$</td>
<td>$-240 \pm 5$</td>
<td>$-2.5 \pm 0.3$</td>
</tr>
<tr>
<td>$a_1\nu \ell\nu u\nu$</td>
<td>$5.4 \pm 0.5$</td>
<td>$64 \pm 2$</td>
<td>$0.7 \pm 0.1$</td>
</tr>
<tr>
<td>other $\ell-h$</td>
<td>$6.6 \pm 0.3$</td>
<td>$\approx 0$</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td>total $\ell-h$</td>
<td>$36 \pm 0.9$</td>
<td>$-39 \pm 4$</td>
<td>$-0.3 \pm 0.1$</td>
</tr>
</tbody>
</table>

modes not calculated only those where one $\tau$ decays into the $\pi\nu$ channel are expected to be sensitive [15]. These have, however, a small branching ratio. Nevertheless, it should be pointed out that only the $\ell-\ell$ class is without bias in this respect.

3. The OPAL detector

The OPAL detector is a large general-purpose detector [16] covering almost the entire solid angle. Here we give only a brief description. A cylindrical coordinate system $(r, \phi, z)$ is defined such that the $z$ axis is along the $e^-$ beam direction. The polar angle $\theta$ is the angle with respect to the $z$ axis.

The central detector consists of three sets of drift chambers: a high precision vertex chamber, a large-volume jet chamber and "$z$-chambers" which give a precise $z$ measurement in the barrel region. The jet chamber is divided into 24 azimuthal sectors each containing 159 sense wires. A uniform magnetic field of 0.435 T is provided by a solenoidal coil.

Outside the coil is a time-of-flight counter array which covers the region $|\cos \theta| < 0.82$, which contains 9440 lead-glass blocks pointing towards the interaction region, and endcaps covering the region $0.81 < |\cos \theta| < 0.98$, consisting of 2264 lead-glass blocks parallel to the beam direction.

The magnet return yoke is instrumented with nine layers of streamer tubes and serves as a hadron calorimeter and muon tracker. On the outside of the detector, four layers of drift chambers are used for muon detection.

The momentum resolution of the tracking chambers is measured to be $\Delta p/p \approx 9\%$ for $p \approx 45$ GeV/c from $e^+e^- \rightarrow \mu^+\mu^-$ events. In the barrel region the electromagnetic calorimeter has an energy resolution of $\Delta E/E \approx 3\%$ for $E \approx 45$ GeV as determined from $e^+e^- \rightarrow e^+e^-$ events. For Monte Carlo studies the OPAL detector response is simulated by a program [17] which treats in detail the detector geometry and material, as well as effects of detector resolutions and efficiencies.

4. Selection of $\tau$ pair events

The cuts described here to select $\tau$ pair events are the same as those of a previous OPAL publication [18]. The detector acceptance is restricted to the bar-
The distinctive feature of $\tau$ pair events as compared to the other event types at LEP is given by two nearly back-to-back jets consisting of only one or a few charged particles possibly accompanied by neutrals. Due to neutrino production in $\tau$ decays, the center-of-mass energy of the incoming electron–positron pair is not fully visible in the final state, in contrast to $\mu$ pair production, for example.

The background to $\tau$ pair production is due to three different types of events. First there are the leptonic events $e^+e^- \rightarrow e^+e^- (\gamma)$ and $e^+e^- \rightarrow \mu^+\mu^- (\gamma)$. If there is no $\gamma$ produced these events are characterized by two back-to-back particles whose momenta sum up to the total center-of-mass energy. For the electron case the total energy is also detected in the electromagnetic calorimeter. For the $\mu$ case there is only very little calorimetric energy detected. Radiative events can also be identified due to the hermeticity of the detector. A second type of background arises from the low multiplicity tail of multihadron production for which high particle multiplicity is characteristic. A third type of background comes from two-photon processes $e^+e^- \rightarrow (e^+e^-)X$, where the $e^+e^-$ pair escapes undetected and the $X$ system is (mis)identified as a $\tau$ pair with low visible energy. This two-photon background has significantly lower visible energy than the signal process and its cross section is not enhanced on the $Z^0$ resonance. The higher energy $X$ system consists mainly of lepton pairs with balanced transverse momenta. Less important backgrounds from cosmic rays and single beam interaction can be suppressed by time-of-flight requirements, by the location of the event vertex and the event topology. For a more detailed discussion of the background suppression see ref. [18].

The data have been recorded in 1990 and 1991 at center-of-mass energies between 88.28 and 94.28 GeV. About 85% of the $\tau$ pairs were recorded on the peak of the $Z^0$ resonance. The integrated luminosity is 10.5 pb$^{-1}$. A total of 5558 $\tau$ pair candidates were found. The residual backgrounds have been estimated from Monte Carlo studies of $e^+e^- \rightarrow \mu^+\mu^-$ [19], $e^+e^- \rightarrow e^+e^- (\gamma)$ [20], $e^+e^- \rightarrow q\bar{q}$ [21,22] and $e^+e^- \rightarrow (e^+e^-)X$ [23]. The total background is found to be (1.9±1.0)% in the barrel part of the detector.

5. Selection of $\tau$ decays

To select the desired $\tau$ pair events we classify a $\tau$ jet as a leptonic decay or a decay into a single charged hadron. For leptons also two tracks in the $\tau$ jet are allowed to account for photon radiation of the lepton with successive conversion. These events are usually detected as two-track $\tau$ jets. Each $\tau$ jet must be in the range ($|\cos\theta| \leq 0.7$) and its momentum must be at least five percent of the beam energy. A $\tau$ jet is a hadron candidate if it consists of a single charged track and is not classified as an electron or muon by the cuts below.

The $\tau$ jet is an electron candidate if it satisfies the following cuts:

- the ratio of the electromagnetic cluster energy $E_{\text{cl}}$ associated with the track and the track momentum $0.7 \leq E_{\text{cl}}/p_{\text{track}} \leq 2.0$,
- the number of blocks in the clusters containing more than 90% energy $N_{\text{cl}}^{90} \leq 3$.

This cut is introduced because the lateral spread of electromagnetic showers limits the shower deposit to a few lead-glass blocks. In order to reject $\tau$ decays with $\pi^0$'s, additional cuts are used:
- We require that $\delta\phi_{\text{max}}$, the largest angle in $(r\phi)$ projection between a presampler cluster defining the shower position and an associated track from the $\tau$ decay, satisfies $\delta\phi_{\text{max}} \leq 5^\circ$,
- and that $x_{\text{cl}} = E_{\text{cl}}/E_{\text{beam}}$, the fractional electromagnetic energy not associated with the track, satisfies $x_{\text{cl}} \leq 0.04$.

In order to reject muons,
- no hits in the muon chambers or in the outer layers of the hadron calorimeter are allowed.

The total background to the $\tau \rightarrow e\nu\bar{\nu}$ candidate sample is found to be $(7.7 \pm 0.7)$%.

The $\tau$ jet is a $\mu$ candidate if it satisfies the following cuts:
Table 2
Background to $\tau^+\tau^- \rightarrow$ hadron-hadron and lepton-lepton.

<table>
<thead>
<tr>
<th>Background</th>
<th>$\ell-\ell$ (%)</th>
<th>h-h (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^+\tau^- \rightarrow \ell-\ell$</td>
<td>$1.3 \pm 0.2$</td>
<td></td>
</tr>
<tr>
<td>$\tau^+\tau^- \rightarrow \ell-h$</td>
<td>$9.0 \pm 0.6$</td>
<td>$19.0 \pm 0.5$</td>
</tr>
<tr>
<td>$\tau^+\tau^- \rightarrow h-h$</td>
<td>$0.7 \pm 0.2$</td>
<td></td>
</tr>
<tr>
<td>$e^+e^- \rightarrow e^+e^- e^+e^-$</td>
<td>$0.5 \pm 0.1$</td>
<td>$\leq 0.06$</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow \mu^+\mu^-$</td>
<td>$\leq 0.03$</td>
<td>$0.4 \pm 0.1$</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>$10.2 \pm 0.7$</td>
<td>$20.7 \pm 0.6$</td>
</tr>
</tbody>
</table>

- the number of hadron calorimeter strips hit per layer $H_{strl} \leq 3$,
- the electromagnetic energy associated with the jet $E_{\text{ecal}} \leq 0.10 E_{\text{beam}}$.

The last cut is introduced to suppress hadrons with or without accompanying photons. In addition, a $\mu$ candidate must fulfill at least one of the following criteria:
- the number of hits in the muon barrel or endcap detectors in the cone must be $N_{\text{barrel}} \geq 2$ or $N_{\text{endcap}} \geq 3$,
- one of the four outer layers of the hadron barrel subdetector is hit.

Finally, to remove residual $\mu^+\mu^-$ events which account for an excess in the $\mu$ momentum spectrum we veto the event if both $\tau$’s decay into a $\mu$ candidate and one candidate satisfies $p \geq 0.08 E_{\text{beam}}$. The total background to the $\tau \rightarrow \mu\nu\nu$ candidate sample is found to be $(3.5 \pm 1.0)\%$.

6. Selection of event topologies

Now we describe the selection of the three types of $\tau$ pair decays: $\ell-\ell$, $\ell-h$ and h-h. To select $\tau^+\tau^-$ where both $\tau$’s decay leptonically it is required that two leptons in opposite hemispheres are identified by the criteria given above. We obtain 447 events of this type. The background is listed in table 2. The dominant background consists of lepton-hadron events with the hadron misidentified as a lepton. To isolate $\tau^+\tau^- \rightarrow \ell-h$ decays, we require that one $\tau$ decays leptonically and the other decays into one charged particle which is a hadron candidate. For this signature we obtain 1421 candidate events. The background consists primarily of $\tau$ decays where a lepton is not identified or a hadron is misidentified as a lepton. We will not use this channel to estimate $\delta_t$ because of its low sensitivity. For the h-h class we require that both $\tau$'s decay into a single charged hadron candidate. We obtain 1180 candidate events. There is a relatively large background from $\tau^+\tau^- \rightarrow \ell-h$. Despite this background, the h-h channel is the most sensitive one to $\delta_t$. Its theoretical cleanliness suffers, however, from the assumption that the channels not included in the CP-violating Monte Carlo do not contribute to $\delta_t$, as explained above.

The measured distributions of the observable $T_{33}$ are shown in fig. 1 for the selected event classes together with Monte Carlo expectations for $\delta_t = 0$ (solid line) and also for $\delta_t = 10^{-16} e$ cm (dashed line) for the lepton-lepton and hadron-hadron classes. The mean values of the distributions are given
Table 3

<table>
<thead>
<tr>
<th>Signature</th>
<th>\langle T_{33} \rangle (\text{GeV}^3)</th>
<th>\hat{d}_t (10^{-17} \text{ ecm})</th>
</tr>
</thead>
<tbody>
<tr>
<td>\ell-\ell</td>
<td>-11.4 \pm 13.5 \pm 3.3</td>
<td>-4.5 \pm 5.3 \pm 1.4</td>
</tr>
<tr>
<td>\ell-\h</td>
<td>-1.2 \pm 8.6 \pm 2.7</td>
<td>1.4 \pm 3.7 \pm 1.3</td>
</tr>
<tr>
<td>h-h</td>
<td>-3.2 \pm 8.6 \pm 2.7</td>
<td>1.4 \pm 3.7 \pm 1.3</td>
</tr>
<tr>
<td>Combined</td>
<td>-0.5 \pm 3.0 \pm 1.4</td>
<td></td>
</tr>
</tbody>
</table>

in table 3. All values are compatible with zero within the measurement errors. Hence no CP-violation in the neutral current reaction \( e^+e^- \rightarrow Z^0 \rightarrow \tau^+\tau^- \) is observed. More specifically we have \(|\langle T_{33} \rangle| \leq 38.8 \text{ GeV}^3\) and \(|\langle T_{33} \rangle| \leq 21.1 \text{ GeV}^3\) for the lepton–lepton and hadron–hadron signature, respectively, at 95% confidence level.

There are three sources for the systematic error in measuring the mean values \( \langle T_{33} \rangle \). First, the detector will smear the true momenta of the decay particles. Since our CP-violating Monte Carlo contains only tree level amplitudes we use the \( \tau \) pair generator KORALZ [19], which generates \( \tau \) pairs according to the standard model but selectively choose events such that the resulting \( T_{ij} \) distribution becomes asymmetric as if CP were violated. The detector simulation program [17] was then applied to these events. \( \langle T_{33} \rangle \) is shifted by 2.5 GeV\(^3\) as compared to the generated momenta. This value is taken as a systematic error. Second, the measured values of \( \langle T_{33} \rangle \) depend on the particular values of cuts taken for the lepton selection. The magnitude of this effect has been estimated by varying the selection cuts within reasonable bounds. For the \( \ell-\ell \) case an error of 2.1 GeV\(^3\) and for the \( \ell-\h \) case an error of 0.7 GeV\(^3\) is attributed to the selection criteria. Third, the detector itself may introduce fake CP-violating effects. This would be the case if the opening angle between the tracks were systematically shifted away from 180°. Such an effect could be imagined if, for example, the end flanges of the tracking chamber were rotated against each other. To estimate the magnitude of such a possibility we use multihadronic \( Z^0 \) decays. For two-jet events, \( \langle T_{33} \rangle \) is calculated using two individual tracks in either jet. We find that the level at which the detector may fake CP-non-invariance is less than 0.5 GeV\(^3\). Comparing this value with the error on \( \langle T_{33} \rangle \) for \( \tau \) pair decays we conclude that the detector cannot account for any possible CP-non-invariance at the level of the investigation reported here. As an additional check we determined \( \langle T_{33} \rangle \) for the process \( e^+e^- \rightarrow \mu^+\mu^- \). Using 7389 \( \mu \) pairs in the barrel region we find \( \langle T_{33} \rangle = -5.6 \pm 3.6 \text{ GeV}^3 \).

Limit on \( \hat{d}_t (q^2 = m_Z^2) \)

First we derive a limit on \( \hat{d}_t \) from the partial width \( \Gamma (Z^0 \rightarrow \tau^+\tau^-) \) adopting a slightly different procedure from that used in refs. [8,10,24]. We follow the prescription of ref. [13] to take account of the fact that a CP-violating contribution can only increase the width and thus constrains \( \Gamma_{SM} \leq \Gamma_{exp} \). Using \( \Gamma_{SM} = 82.8 \) to 84.6 MeV—the range for this value is due to the unknown masses of the top quark (50–230 GeV/c\(^2\)) and Higgs boson (50–1000 GeV/c\(^2\))—and the OPAL result \( \Gamma_{exp} = 82.7 \pm 1.9 \) MeV [25] we obtain for the difference between standard model and experiment \( (\Gamma_{exp} - \Gamma_{SM}) \leq 1.9 \) MeV at 68% confidence level. The deviation from the standard model width due to the CP-violating interaction is given by \( \Delta \Gamma = \frac{\hat{d}_t^2 m_Z^3}{(24\pi)} \) [6,10], resulting in \( |\hat{d}_t| \leq 2.8 \times 10^{-17} \text{ ecm} \) at 68% confidence level. Note that for this limit no CP-odd observable has been used. It is therefore not a direct test on CP-invariance. The formula used rests on the assumption that only the CP-violating amplitude and not any other new processes contribute to the width. As has been pointed out in ref. [26] \( \Gamma (Z^0 \rightarrow \tau^+\tau^-) \) may even decrease in multi-Higgs models.

For large values of \( \hat{d}_t \), eq. (4) becomes invalid and the dependence of \( \langle T_{33} \rangle \) on \( \hat{d}_t \) is described by [8]

\[
\langle T_{33} \rangle = \frac{u \hat{d}_t}{\Gamma (Z^0 \rightarrow \tau^+\tau^-)} \cdot \Gamma (Z^0 \rightarrow \tau^+\tau^-) = u + w |\hat{d}_t|^2,
\]

where \( u, v \) and \( w \) are constants. For the \( \ell-\ell \) and \( \ell-\h \) cases the functional form is plotted in fig. 2. \( \langle T_{33} \rangle \) attains a maximum for large values of \( \hat{d}_t \) and decreases again for still larger \( \hat{d}_t \). To estimate errors on \( \hat{d}_t \) we prefer to compare \( \langle T_{33} \rangle \cdot \Gamma \) with the theoretical expectation rather than \( \langle T_{33} \rangle \) itself. This way, the dependence on \( \hat{d}_t \) is linear. The error of the width measurement is negligible compared to the error on \( \langle T_{33} \rangle \). We have \( \langle T_{33} \rangle \cdot \Gamma (Z^0 \rightarrow \tau^+\tau^-) = u \hat{d}_t \) and calculate \( u \)
Fig. 2. Dependence of \( <T_{33}> \) (left hand scale) on \( d_e \) for \( \tau \) pair decays to (a) lepton–lepton and (b) hadron–hadron. The dashed line shows \( <T_{33}> \cdot \Gamma(Z^0 \rightarrow \tau^+ \tau^-) \) (right hand scale). The shaded area represents the measured \( <T_{33}> \).

with the \( CP \)-violating Monte Carlo including cuts on the particle momenta and angles. The sensitivity \( u \) is shown in table 1 for the different decay channels.

The estimate on \( d_e \) for the two classes, lepton–lepton and hadron–hadron, can be derived from table 1 by calculating the average sensitivity \( u \) to \( d_e \) for the contributing decay channels. We then combine \( d_e \) derived from the \( \ell-\ell \) and \( h-h \) classes using the weighted mean. Although different sets of events are used in the \( \ell-\ell \) and \( h-h \) classes the resulting values of \( d_e \) may not be statistically independent because the division line between the sets is selection dependent. However, the selection cuts predominantly cause migrations between \( \ell-\ell \) and \( h-h \) and between \( \ell-\ell \) and \( \ell-h \) while migrations between \( \ell-\ell \) and \( h-h \) are suppressed.

The systematic error is not independent for the two channels considered. They arise from the error on \( <T_{33}> \), the error on the \( \tau \) branching ratios, the estimate of the background of the selected \( \tau \) decay event types, and the error on the sensitivity \( u \). The systematic error on \( <T_{33}> \) is the most important contribution. When calculating averages of the sensitivity \( u \) over several decay channels, the \( \tau \) branching ratios and the background estimates for the three classes have been used. The errors on these quantities have only a minor influence on \( d_e \). For the error on \( u \) as calculated from the Monte Carlo, one has to take into account that only tree level amplitudes are calculated. To test the influence of radiative corrections we have compared \( T_{ii} \) computed with momenta from the \( CP \)-violating Monte Carlo (with \( d_e = 0 \)) and \( \tau \) decays produced by KORALZ. No significant difference in the \( T_{ii} \) distributions has been seen. We assume that the systematic

Fig. 3. The partial width \( \Gamma(Z^0 \rightarrow \tau^+ \tau^-) \) versus \( <T_{33}> \) for (a) the lepton–lepton and (b) the hadron–hadron cases. \( d_e \) ranges from 0 to \( 6 \times 10^{-17} \text{ e cm} \) in this figure. The points correspond to the indicated values of \( d_e \) in units of \( 10^{-17} \text{ e cm} \). The range for the width is due to the unknown top quark and Higgs boson masses and is indicated by the shaded area. The data point is the measured \( <T_{33}> \) and partial width \( \Gamma(Z^0 \rightarrow \tau^+ \tau^-) \) [25].
errors are fully correlated and the combined systematic error is taken to be the average of the two classes rather than adding them in quadrature. Table 3 includes the estimates of $\Delta_\ell$ from the individual classes and the combined result. The $\ell-h$ channel has not been considered due to its low sensitivity.

Following this procedure we obtain the upper limit $|d_\ell| \leq 3.8 \times 10^{-17} \text{ e cm}$ at 68% confidence level. Note that this number is the result of a true CP-test and much less affected by theoretical bias than the estimate from the width measurement. The value obtained from the $\ell-h$ class alone is: $|d_\ell| \leq 1.0 \times 10^{-16} \text{ e cm}$ at 68% confidence level. For the $\ell-\ell$ and h-h classes fig. 3 shows the connection between $\langle T_{33} \rangle$ and $\Gamma(Z^0 \rightarrow \tau^+\tau^-)$ together with the measured values of $\langle T_{33} \rangle$ and $\Gamma_{\text{exp}}$ [25]. The shaded area indicates the theoretical uncertainty on $\Gamma(Z^0 \rightarrow \tau^+\tau^-)$ due to the unknown masses of the top quark and Higgs boson.

7. Summary

No evidence for a CP-violating contribution to the reaction $e^+e^- \rightarrow Z^0 \rightarrow \tau^+\tau^-$ has been found using 5558 $Z^0 \rightarrow \tau^+\tau^-$ events recorded with the OPAL detector at LEP. From this null result we have placed a limit on the weak dipole moment of the $\tau$ lepton of $|d_\ell| \leq 7.0 \times 10^{-17} \text{ e cm}$ at 95% confidence level. Following theoretical prejudice that CP-violating effects can be proportional to the lepton mass to the third power this result has similar sensitivity as the current limit on the electron electric dipole moment [14].

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