Measurement of the Average B Hadron Lifetime in $Z^0$ Decays

OPAL Collaboration

Abstract

A sample of 689 muon candidates and 665 electron candidates identified in multihadronic $Z^0$ decays has been used to measure the average B hadron lifetime. These data were recorded with the OPAL detector during 1990. Maximum likelihood fits to the distributions of the lepton impact parameters yield an average B hadron lifetime of

$$\tau_b = 1.37 \pm 0.07 \pm 0.06 \text{ ps},$$

where the first error is statistical and the second systematic. This result is a weighted average over the semileptonic branching fractions and production rates of the B hadrons produced in $Z^0$ decays.

(Submitted to Physics Letters B)
The OPAL Collaboration

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1 Introduction

The measurement of the lifetime of B hadrons is used to determine the Cabibbo-Kobayashi-Maskawa matrix elements governing the coupling of bottom quarks to charm and up quarks. The first measurements of the lifetime of B hadrons were made at PEP [1] and PETRA [2], at center-of-mass energies between 29 and 35 GeV. More recent measurements have been made at LEP [3]. A measurement of the average B hadron lifetime using muon and electron candidates identified in multihadronic Z⁰ decays is presented in this Letter. The data were collected with the OPAL detector during the 1990 LEP run in which an integrated luminosity of 6.6 pb⁻¹ was recorded.

2 The OPAL detector

The OPAL detector is described in detail elsewhere [4]. Only a brief overview of the components of OPAL that are relevant to this analysis is included here. Of particular importance to this analysis is a precision vertex chamber. The vertex chamber is a cylindrical detector with inner and outer radii of 9 and 24 cm, respectively, surrounding an 8 cm radius, 1.3 mm thick carbon fiber beam pipe with a 0.1 mm aluminum inner lining. The volume of the chamber between radii of 9 and 17 cm is divided into 36 axial sectors with 12 wires each, and the remaining volume is divided into 36 stereo sectors with 6 wires each. The drift distance resolution is about 50 μm over most of the drift space, with a degradation near the anode plane. Surrounding the vertex chamber is a large volume jet chamber. The chamber has a 1.85 m outer radius, is 4 m long, and is divided into 24 azimuthal sectors, each sense wire plane consisting of 159 wires. Each of the wires provides three-dimensional coordinates, from the wire position, from a drift time measurement in the x-y plane and from a charge division measurement in the z direction.¹ The total charge on each wire is recorded for use in determining the mean ionization energy loss, dE/dx. The barrel region of the jet chamber is surrounded by a set of thin drift chambers, called z chambers, which provide 6 precision measurements of the z coordinate. The vertex chamber and z chambers are mounted onto the support structure of the jet chamber and the whole assembly is positioned inside a pressure vessel, which is filled with an argon-methane-isobutane mixture at 4 bar. The pressure vessel is surrounded by a solenoidal coil that produces a uniform magnetic field of 0.435 Tesla. The pressure vessel and the coil have a combined thickness of about two radiation lengths for particles at normal incidence. The components described above form the central detector of OPAL.

Positioned outside the coil is a barrel time-of-flight counter array consisting of 160 scintillator bars with phototube readout at both ends. A lead-glass electromagnetic calorimeter with a presampler, corresponding to 24.6 radiation lengths and about two hadronic interaction lengths, measures the positions and energies of showering particles. The magnet return yoke serves as a hadron calorimeter and is instrumented with 9 layers of streamer tubes, read out via charge induction onto pads and onto 4 mm wide aluminum strips. These detectors are surrounded by four layers of drift chambers for the detection of muons emerging from the hadron calorimeter. In the end cap region, electromagnetic and hadronic calorimeters provide energy measurements down to |cos θ| = 0.98.

¹The coordinate system is defined so that the z axis is in the direction of the electron beam, the x axis is horizontal and points approximately towards the center of the LEP ring, and the y axis is nearly vertical. The polar and azimuthal angles, θ and φ, are defined with respect to the z and x axes, respectively.
extending coverage to 98% of the full solid angle. Four layers of planar muon chambers, consisting of streamer tubes, track muons down to $|\cos \theta| = 0.985$ and extend the muon chamber coverage to 93% of the full solid angle.

At the time of this analysis the momentum resolution in the $x$-$y$ plane for charged particles in the region $|\cos \theta| < 0.7$ was $(\sigma_p/p)^2_{xy} = 0.025^2 + (0.0018 \cdot p_{xy})^2$, where $p_{xy}$ is the momentum in the $x$-$y$ plane in GeV/c. The resolution in $dE/dx$ for tracks leaving the full 159 samples was 3.8%. The ratio of electromagnetic energy to track momentum for 10 GeV/c electrons had an r.m.s. of about 8%. The resolution in the $x$-$y$ plane of the distance of closest approach of the tracks to the beam spot, defined below, was 45 μm for 20 GeV/c tracks, and degraded to 85 μm for 5 GeV/c tracks due to multiple Coulomb scattering.

The analysis presented here relies on the determination of the average intersection point of the LEP beams in the $x$-$y$ plane. This average intersection point, or beam spot, was determined separately for each LEP fill and, statistics allowing, several times within a fill. The procedure for determining the beam spot [5] used tracks from both multihadronic and leptonic decays of the $Z^0$, and resulted in an average precision of 15 μm and 10 μm in the $x$ and $y$ coordinates of the beam spot, respectively. The r.m.s. widths$^2$ of the beam intersection ellipsoid were 160 μm in $x$ and 10 μm in $y$.

Several Monte Carlo datasets were used in this analysis. Multihadronic events were generated using the JETSET program [7, 8] and the HERWIG program [8, 9]. Samples of 164,133 JETSET events and 73,255 HERWIG events were passed through a detailed simulation [10] of the detector. The data resulting from this simulation were subjected to the same reconstruction software as was used on real data. Unless stated otherwise, when the use of Monte Carlo data is discussed the JETSET sample passed through the detailed detector simulation was used.

Additional JETSET events were passed through a fast simulation of the detector. In this fast simulation, the number of hits assigned to each generated track was estimated, taking into account the 2-hit separation performance of each component of the central detector. The covariance matrix of the track parameters was then calculated, taking into account the estimated number of hits, their resolutions, and multiple Coulomb scattering. This covariance matrix was used to smear the track parameters about their generated values. The performance of the central detector was well described by the fast simulation.

3 The event and lepton selection

Multihadronic events were selected using criteria described elsewhere [11]. In addition, the thrust axis of the event, calculated using only charged tracks, had to satisfy $|\cos \theta_T| < 0.7$, and good detector operation was required for the central tracking chambers, the barrel muon chambers, the hadron calorimeter strips, and the barrel presampler and electromagnetic calorimeter. These requirements were satisfied by 86,899 events. In each event, charged tracks were grouped into jets using the scaled invariant mass algorithm described in Reference [12]. Tracks used in the jet finding were required to pass within 1 cm of the beam spot in the $x$-$y$ plane, and to have at least half of the expected number of jet chamber hits with a minimum of 40, a measured momentum between

$^2$The width of the beam in $y$ was estimated from the LEP optics [6]; in $x$ it was measured from the data.
0.15 and 55\,GeV/c and a measured \( z \) coordinate at the point of closest approach to the beam axis within 15\,cm of the nominal collision point. Monte Carlo studies indicated that the r.m.s. difference in the \( x-y \) plane between the reconstructed jet axis and the \( B \) hadron direction was 60\,mrad.

In addition to the requirements listed above, tracks considered as lepton candidates satisfied the following conditions:

- The momentum, \( p \), was at least 4.5\,GeV/c.
- The transverse momentum with respect to the nearest jet, \( p_T \), was at least 1.5\,GeV/c. The \( p_T \) was determined with the candidate track excluded from the jet.
- The absolute value of the cosine of the polar angle was less than 0.7.
- At least 3 hits in the \( z \) chambers were associated with the track.
- At least 6 hits in an axial sector of the vertex chamber were associated with the track.
- Less than half of the axial vertex chamber hits were registered on the ionization tail of earlier hits. This criterion was applied because the drift distance resolution of these hits was worse by a factor of two compared to the resolution of the first hits registered on the wires. Approximately 23\% of the lepton candidates were rejected by this cut after all other cuts were applied.

Tracks satisfying these criteria were subjected to lepton identification procedures.

The muon selection was based on the work presented in Reference [13]. The muon candidates were selected from the subset of central detector tracks associated to track segments in both the hadron calorimeter, where the strips were used to track muons, and in the muon chambers. In addition, the track segment in the muon chambers had to satisfy the more restrictive quality cuts described in Reference [13]. The efficiency of this muon selection is 0.61 for muons satisfying all of the kinematic and geometric criteria described above. It was determined from the Monte Carlo simulation and is nearly independent of both \( p \) and \( p_T \). The background in the muon candidate sample was also determined from the Monte Carlo simulation, and is due to decays in flight of charged pions and kaons, and hadrons that either passed through the calorimeters without interacting strongly or showered in such a way that one or more particles reached the muon chambers to fake a muon signal. The probability that a hadron was misidentified as a muon due to any of the aforementioned effects was 0.31\% in the kinematic region of interest, and did not depend strongly on the kinematic variables. The efficiency was cross-checked using a data sample of \( Z^0 \rightarrow \mu^+\mu^- \) decays, and the background estimate was cross-checked using a data sample of \( K_s^0 \rightarrow \pi^+\pi^- \) decays. These comparisons formed the basis for estimating the systematic uncertainties in the muon identification.

A sample of electron candidates was extracted using the procedures described in Reference [14]. Electron candidates were required to be consistent with the expected signature for electrons in their measured \( dE/dx \), in the pulse height registered in the electromagnetic presampler, and in the lateral spread of the associated electromagnetic shower. In addition, the measured electromagnetic energy divided by the reconstructed track momentum, \( E/p \), had to lie between 0.7 and 1.4. The background in the electron candidate sample came principally from misidentified hadron tracks, photon conversions, and the Dalitz decay of neutral mesons. Comparisons between the efficiencies
obtained from the Monte Carlo and from the data were used in assessing systematic errors, and will be explained later.

The final data samples contained 689 muon candidates and 665 electron candidates. The compositions of these samples are given in Table 1 and will be explained in the next section. These numbers have been estimated from the Monte Carlo, with the exception of the electron hadronic background fraction which was fixed to the value measured from the data [14]. Figure 1 shows the momentum and transverse momentum distributions of the lepton samples. There is good agreement between the predicted and observed shapes of these distributions.

4 The lifetime fit

The average B hadron lifetime is determined from maximum likelihood fits to the impact parameter distributions of the two lepton samples. In this analysis, the term impact parameter refers to the smallest distance in the plane transverse to the beam axis between the track in question and the beam spot. The sign of the impact parameter is positive if the angle in the x-y plane between the vector from the beam spot to the point of closest approach of the track and the axis of the jet containing the lepton candidate is less than 90°, otherwise it is negative. The impact parameter distributions for the muon and electron candidate samples are shown in Figure 2.

The method used here is based on the procedure described in Reference [15]. The observed impact parameter distribution is determined by particle lifetimes and decay kinematics, and by the detector resolution. In this analysis, the impact parameter distribution is described in terms of a convolution of the distributions expected from the underlying physics with a resolution function determined using the data themselves. The impact parameter distributions are divided into five components for which separate probability density functions are constructed. They are

1. Leptons arising from the semileptonic decay of B hadrons.
2. Leptons arising from the semileptonic decay of c-flavored hadrons or from the decay of τ leptons which themselves arise from B hadron decay.

3. Leptons arising from the semileptonic decay of c-flavored hadrons that do not come from B hadron decay.

4. Hadrons that are misidentified as leptons.

5. Decays in flight of charged pions and kaons to muons, photon conversions and Dalitz decays of neutral mesons.

The first and second components are functions of the average B hadron lifetime, \( \tau_b \), whereas the other components have little or no dependence on \( \tau_b \). The likelihood function to be maximized is

\[
\mathcal{L} = \prod_{j=1}^{N} \left[ f_1 P_1(\tau_b, d_0^j, \sigma^j) + f_2 P_2(\tau_b, d_0^j, \sigma^j) + f_3 P_3(d_0^j, \sigma^j) + f_4 P_4(d_0^j, \sigma^j) + f_5 P_5(d_0^j, \sigma^j) \right],
\]

where \( d_0^j \) is the measured impact parameter of candidate \( j \), \( \sigma^j \) is the assigned uncertainty in \( d_0^j \) as described in section 6, \( P_i \) is the probability density function for component \( i \), \( f_i \) is the fraction of the lepton sample due to component \( i \), as given in Table 1, and \( N \) is the total number of lepton candidates. The only free parameter in the fit is \( \tau_b \).

5 Determination of the probability density functions

The probability density functions for components 1, 2, and 3 were constructed by convoluting the underlying physics distributions with the resolution function described in the next section. This convolution was performed track-by-track since the resolution depended on, amongst other variables, the track momentum and the track direction.

The underlying physics distributions for components 1 to 3 were determined using two million \( Z^0 \to bb \) and one million \( Z^0 \to cc \) events generated with JETSET. These events were processed using the fast simulation of the OPAL detector. The impact parameters were taken directly from the generated information before the detector simulation, but only particles whose reconstructed tracks passed all kinematic and lepton identification criteria were entered into these distributions. The impact parameters were calculated relative to the \( Z^0 \) production point instead of the beam spot, and their signs were determined using the jet axes reconstructed from the charged tracks after detector simulation. This procedure was necessary because the signing of the impact parameters is determined by the other tracks in the jet whereas the resolution function applies only to the lepton candidate track, and therefore cannot account for signing mistakes due to imprecise estimates of the B hadron direction. These distributions were parametrized using a sum of exponential functions in order to facilitate an analytic convolution with the resolution function. The underlying physics distribution for electrons from B hadron decays, together with the parametrized curve, is shown in Figure 3. The distributions were expressed in terms of the impact parameter divided by the average generated decay length of the B hadrons for components 1 and 2, and of the c-flavored hadrons for component 3. The generated lifetime for B hadrons was 1.30 ps and the average generated lifetime for c-flavored hadrons was 0.70 ps. The Peterson scheme was used for b and c fragmentation with
parameter $c_b$ set to 0.0035 [16], and with parameter $c_e$ set to 0.024 [17]. These correspond to $\langle x_b \rangle = 0.72$ and $\langle x_e \rangle = 0.55$, where $x$ is the energy of the hadron divided by the beam energy.

The probability density functions for component 4, the misidentified hadrons, were determined directly from the data. The impact parameter distribution for the muons was obtained using very loose muon identification requirements, rejecting candidates that passed the normal, more stringent criteria. Monte Carlo studies indicated that the resulting sample was composed of 95.4% hadrons. This distribution was then normalized and fitted to yield an analytic probability density function. Similarly for the electrons, the corresponding probability density function was derived from the set of tracks that passed all electron identification criteria except the $dE/dx$ requirement, and which had $E/p$, the ratio of calorimeter shower energy to track momentum, in the range 0.2 to 0.7. Monte Carlo studies indicated that 99.9% of the tracks in this sample were hadrons.

The probability density function for component 5 of the electron sample was chosen to be identical to the electron misidentification probability density function. The broadening of the impact parameter distribution due to extrapolating tracks from converted photons back to the origin was less than the width resulting from the tracking resolution and the width of the beam spot. This consideration, coupled with the small fraction (0.3%) of the data ascribed to this component, justify the use of the misidentification distribution. The probability density function for component 5 of the muon sample was determined from a large sample of pion and kaon decays in multihadronic events simulated by the Monte Carlo.

6 Determination of the resolution function

The impact parameter resolution depends on the kinematic variables used to select the lepton candidates. Therefore the detector resolution function was measured with the set of tracks that passed all kinematic criteria but failed the lepton identification. In addition, tracks were selectively discarded to bring the distributions of $p$, $p_T$, and the number of jet chamber hits into agreement with the distributions observed from the lepton samples for these quantities. The remaining tracks are called resolution function tracks.

The distribution of the impact parameters divided by their calculated uncertainties for resolution function tracks from the data is shown in Figure 4a. The uncertainties were calculated according to the formula

$$\sigma^2 = a^2 \cdot V_{dd} + \sin^2 \phi \cdot S_x^2 + \cos^2 \phi \cdot S_y^2,$$

where $V_{dd}$ is the variance of the impact parameter from the track fit, which includes contributions due to multiple Coulomb scattering, $S_x$ and $S_y$ are the widths of the beam spot in $x$ and $y$, respectively, and $a$ is a parameter which would be unity if the estimate of the impact parameter variance was perfect. In practice the true deviations of the measured impact parameters are not perfectly described by $V_{dd}$. This $\sigma$ describes the measurement accuracy on most tracks, but there is a second, broader component to the distribution that comes from pattern recognition mistakes, large angle multiple scattering, imperfect detector calibrations, and the presence of long-lived particles in the data sample. Therefore the resolution function was parametrized as the sum of two Gaussians,

$$R(d_0, V_{dd}) = \frac{(1 - f)}{\sqrt{2\pi}\sigma} \exp \left( -\frac{d_0^2}{2\sigma^2} \right) + \frac{f}{\sqrt{2\pi}\sigma_t} \exp \left( -\frac{d_0^2}{2\sigma_t^2} \right),$$
Table 2: The parameters describing the resolution function. The errors in the column labelled “raw” are statistical. The errors on the corrected parameters include statistical and systematic contributions. The errors quoted for $S_x$ and $S_y$ are conservative; see Reference [5] for more precise estimates.

<table>
<thead>
<tr>
<th>parameter</th>
<th>raw</th>
<th>corrected</th>
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<tr>
<td>$a$</td>
<td>$1.20 \pm 0.03$</td>
<td>$1.11 \pm 0.11$</td>
</tr>
<tr>
<td>$f$</td>
<td>$0.081 \pm 0.005$</td>
<td>$0.063 \pm 0.008$</td>
</tr>
<tr>
<td>$w$</td>
<td>$10.7 \pm 0.4$</td>
<td>$13.4 \pm 2.5$</td>
</tr>
<tr>
<td>$S_x$</td>
<td>$160 \pm 10 \mu m$</td>
<td></td>
</tr>
<tr>
<td>$S_y$</td>
<td>$10 \pm 10 \mu m$</td>
<td></td>
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</table>

where $f$ is the fractional area of the broad component of the distribution and

$$\sigma_i^2 = w^2 \cdot a^2 \cdot V_{di} + \sin^2 \phi \cdot S_x^2 + \cos^2 \phi \cdot S_y^2$$

is the square of the width of the broad component. The parameters $a$, $f$, and $w$ were determined from a maximum likelihood fit to the resolution function tracks. The width of the narrower Gaussian (in $d_y/\sigma$) was constrained to unity in the fit. Only the entries with a negative impact parameter were used in the fit, since tracks from the decays of long-lived particles introduce an asymmetry in the distribution, as seen in Figure 4a. The fitted values of the parameters describing the resolution function appear in Table 2 under the column labelled “raw.”

Shown in Figures 4b and 4c are the distributions of resolution function tracks from the Monte Carlo dataset. Figure 4b shows the distribution obtained using the same criteria as applied to the data. Figure 4c shows the distribution after subtracting the contribution to the impact parameters due to the lifetime of the parent hadron. This was done by calculating the true impact parameters of the tracks relative to the $Z^0$ production point, using only the generated information, and subtracting these from the usual measured impact parameters of the tracks after the detector simulation. Therefore the difference between the measured and true impact parameters contains only contributions from the detector resolution and from the width of the beam spot in the $x$-$y$ plane. By comparing the Monte Carlo distributions before and after this subtraction, correction factors were determined to scale the resolution function parameters in order to remove the lifetime information. These correction factors were applied to the resolution function determined from the data. This procedure was necessary because tracks coming from long-lived hadrons can still be assigned a negative impact parameter, therefore biasing the resolution function. The corrected resolution function parameters for the data are given in Table 2, and defined the resolution function used in the likelihood fits for the lifetime.

7 Results

The results of the one parameter fits to the impact parameter distributions were

$$\tau_b = 1.26 \pm 0.10 \text{ ps} \quad (\text{muons}),$$

$$\tau_b = 1.48 \pm 0.11 \text{ ps} \quad (\text{electrons}).$$
The associated $\chi^2$ values per degree of freedom, determined from the binned histograms in Figure 2, were 15.6/16 (muons) and 14.0/16 (electrons). The curves in Figure 2 display the results of the fits.

8 Consistency checks

Several checks were performed to look for possible detector-based systematic effects. In various tests, the lepton samples were split into positive and negative tracks, into horizontal and vertical quadrants, and into high and low ranges of $p$ and $p_T$. The data were also divided according to the distance of the tracks from the vertex chamber anode planes. The sensitivity of the result to tracks with large impact parameters was investigated by performing the likelihood fit over a reduced range. The results from the subsamples were in each case consistent.

The method outlined above was also applied to several Monte Carlo samples of known input lifetime. These included JETSET samples generated with B hadron lifetimes of 0.0 ps and 2.6 ps, the standard JETSET sample generated with a lifetime of 1.3 ps, and the HERWIG sample, also generated with a lifetime of 1.3 ps. These samples were statistically independent from those used to determine the underlying physics distributions for components 1 to 3. Separate resolution functions were determined for each sample using the prescription described above. The probability density functions for components 4 and 5 were similarly determined from these Monte Carlo samples. The fitted lifetimes were in each case consistent with the generated values.

9 Determination of systematic errors

The systematic errors fall into two categories: errors common to both the muon and the electron samples, and errors that are uncorrelated between the two samples. Table 3 lists the common systematic errors. They were determined as follows:

- The semileptonic branching fraction of B hadrons was varied by $\pm 10\%$ [18].

- The fraction of the muon and electron samples arising from the semileptonic decays of $c$-flavored hadrons or from the decays of $\tau$ leptons, which themselves arise from B hadron decays, was varied by $\pm 20\%$. This variation takes into account uncertainties in the numbers of charmed mesons produced per B decay and in the amount of $D^+$ produced relative to other charmed states.

- The semileptonic branching fraction of $c$-flavored hadrons that do not arise from B hadron decay was varied by $\pm 20\%$.

- The average charm lifetime was varied in the range 0.62 to 0.79 ps. The variations in this average number are due almost entirely to different assumptions about the fraction of charm quarks which result in $D^0, D^+, D_s$, and $\Lambda_c$ hadrons before decaying weakly.
• The fragmentation parameter $c_b$ in the Peterson scheme was varied between 0.0017 and 0.0055, corresponding to $\langle x_b \rangle = 0.74$ and $\langle x_b \rangle = 0.70$, a range consistent with a 2 standard deviation change in the results quoted in Reference [16].

• Each parameter describing the resolution function was varied within the accuracy given in the column of Table 2 labelled “corrected”, and the maximum corresponding variation in the fitted $\tau_0$ is quoted as a systematic error. The dominant component of the parameter errors comes from an estimate of how well the resolution function, obtained from hadrons, represents the resolution function appropriate for the leptons. This estimate was obtained by comparing resolution function fits to the Monte Carlo leptons and to the Monte Carlo resolution function tracks, after removing the lifetime information from the impact parameters as described above. The resolution function parameters obtained from the leptons were consistent with the parameters obtained from the resolution function tracks. This check was only valid within the statistical uncertainty in the parameters obtained from the leptons; therefore these uncertainties were incorporated in the calculation of the corrected resolution function parameter errors for the data, which are given in Table 2. These errors also include contributions from the statistical uncertainty of the resolution function fits to the Monte Carlo and to the data.

• The B hadron lifetime in the Monte Carlo used to determine the correction factors for the resolution function was varied by ±15%.

• The resolution with which the reconstructed jet axis models the B hadron direction in the $x$-$y$ plane was varied by ±3.5 mrad. This was the difference between the width of the jet-jet acoplanarity distribution in 2-jet events reconstructed in the data with that reconstructed in the fast Monte Carlo, and indicates the scale at which the angular information in reconstructed jets is properly simulated. The systematic error resulting from this variation was dominated by the corresponding changes to the underlying physics distributions; the effects of this variation on the signs of the impact parameters of resolution function tracks, and hence on the corrections to the resolution function, were negligible.

• The assumed size and position of the beam spot were varied by their estimated uncertainties.

The total systematic error common to the muon and electron samples was ±0.047 ps, determined by adding the individual contributions in quadrature.

The systematic errors specific to one lepton species or the other are given in Table 4. They were determined as follows:

• The muon and electron background normalizations were allowed to vary by ±25% and ±15%, respectively.

• The muon identification efficiency was varied by ±10%. This variation only affected the ratio of muons to misidentified hadrons. There is no corresponding variation for the electrons since the background fraction was determined from the data.

• The lepton identification efficiencies as a function of $p$ and $p_T$ were varied, keeping the lepton sample fractions fixed. This variation only altered the underlying physics distributions. The lifetime measurements quoted in this Letter were obtained using the lepton identification efficiencies as a function of $p$ and $p_T$ determined from the Monte Carlo. The lifetimes were
<table>
<thead>
<tr>
<th>source</th>
<th>error on $\tau_b$ (ps)</th>
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<tbody>
<tr>
<td>$\text{BR}(b \rightarrow \ell)$</td>
<td>0.013</td>
</tr>
<tr>
<td>$\text{BR}(b \rightarrow (c \text{ or } \tau) \rightarrow \ell)$</td>
<td>0.008</td>
</tr>
<tr>
<td>$\text{BR}(c \rightarrow \ell)$</td>
<td>0.011</td>
</tr>
<tr>
<td>average charm lifetime</td>
<td>0.006</td>
</tr>
<tr>
<td>b fragmentation</td>
<td>0.007</td>
</tr>
<tr>
<td>resolution function</td>
<td>0.039</td>
</tr>
<tr>
<td>B hadron lifetime in the Monte Carlo</td>
<td>0.006</td>
</tr>
<tr>
<td>uncertainty in B hadron direction</td>
<td>0.012</td>
</tr>
<tr>
<td>beam spot size and position</td>
<td>0.008</td>
</tr>
<tr>
<td>total</td>
<td>0.047</td>
</tr>
</tbody>
</table>

Table 3: Summary of systematic errors common to the muon and electron samples.

Also determined using efficiencies independent of $p$ and $p_T$, and for the electrons the efficiency versus $p$ and $p_T$ as determined from the data [14] was also used. The range of results obtained with these efficiencies was taken as a systematic error.

- The efficiency is expected to depend on the physics process that produced the electron due to the isolation requirements implicit in the electron identification procedure. The Monte Carlo predictions for the efficiencies of components 2 and 3 of the electron sample relative to the efficiency for component 1 were varied by $\pm 40\%$.

- Each parameter of the functions describing the underlying physics distributions was varied by its statistical uncertainty. The maximum variation in the lifetime was taken as a systematic error.

- Each parameter of the functions describing the misidentification probability density functions was varied by its statistical uncertainty. The maximum variation in the lifetime was taken as a systematic error.

- Each parameter of the probability density function describing the charged pion and kaon decays to muons was varied by its statistical uncertainty. The maximum variation in the lifetime was taken as a systematic error. Varying the normalization of this component by $\pm 25\%$ contributed a negligible amount to this error.

- The underlying physics distributions for the electrons were corrected for the effects of electron bremsstrahlung. The corresponding change in the lifetime was taken as a systematic error.

Adding each contribution in quadrature results in the total uncorrelated systematic errors listed in Table 4.

The resulting lifetimes are $1.26 \pm 0.10 \pm 0.07 \text{ps}$ for the muons and $1.48 \pm 0.11 \pm 0.07 \text{ps}$ for the electrons. These results are combined, weighting them by their statistical errors and the systematic errors that were not correlated between the two samples. The result is

$$\tau_b = 1.37 \pm 0.07 \pm 0.06 \text{ps},$$

where the first error is statistical and the second systematic.
<table>
<thead>
<tr>
<th>source</th>
<th>error on $\tau_b$ (ps)</th>
</tr>
</thead>
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<tr>
<td>background to the lepton samples</td>
<td>0.024 0.013</td>
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<td>muon identification efficiency normalization</td>
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<td>lepton identification efficiency shapes</td>
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<td>electron identification relative efficiencies</td>
<td>– 0.025</td>
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<td>misidentification function statistics</td>
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<tr>
<td>$K \rightarrow \mu$ and $\pi \rightarrow \mu$ decays</td>
<td>0.011 –</td>
</tr>
<tr>
<td>electron bremsstrahlung</td>
<td>– 0.017</td>
</tr>
<tr>
<td>total</td>
<td>0.047 0.054</td>
</tr>
</tbody>
</table>

Table 4: Summary of systematic errors specific to each of the two lepton samples.

10 Conclusion

The average B hadron lifetime has been measured using the impact parameters of muon and electron candidates identified in the 1990 OPAL data sample. The result is $\tau_b = 1.37 \pm 0.07 \pm 0.06$ ps. This value is a weighted average over the production fractions and semileptonic branching ratios of the B hadrons produced in multihadronic $Z^0$ decays.

Acknowledgements

It is a pleasure to thank the SL Division for the efficient operation of the LEP accelerator and their continuing close cooperation with our experimental group. In addition to the support staff at our own institutions we are pleased to acknowledge the Department of Energy, USA,
National Science Foundation, USA,
Science and Engineering Research Council, UK,
Natural Sciences and Engineering Research Council, Canada,
Israeli Ministry of Science,
Minerva Gesellschaft,
Japanese Ministry of Education, Science and Culture (the Monbusho) and a grant under the Monbusho International Science Research Program,
American Israeli Bi-national Science Foundation,
Direction des Sciences de la Matière du Commissariat à l’Energie Atomique, France,
Bundesministerium für Forschung und Technologie, FRG,
A.P. Sloan Foundation and Junta Nacional de Investigação Científica e Tecnológica, Portugal.
References


     The E0 recombination scheme was used in the jet finding with the parameter $y_{cut}$ set to 0.04.


[16] OPAL Collab., reported by A. Ball, proceedings of the Third Topical Seminar on Heavy Flavours, San Miniato, Italy (1991), to be published in Nucl. Phys. B.


Figure captions

Figure 1  Momentum and transverse momentum distributions of the muon and electron candidates. The contributions from the 5 components of the candidate lepton samples described in the text are indicated. The sum of the Monte Carlo components was normalized to the observed number of leptons in the data. The momentum distributions contain only tracks with $p_T > 1.5 \text{ GeV}/c$, and the transverse momentum distributions contain only tracks with $p > 4.5 \text{ GeV}/c$. The analysis requires both $p_T > 1.5 \text{ GeV}/c$ and $p > 4.5 \text{ GeV}/c$.

Figure 2  Distributions of the impact parameters for muon and electron candidates. The curves are the result of a maximum likelihood fit for $\tau_b$. The shaded area shows the contribution from all sources other than leptons produced in the semileptonic decay of B hadrons.

Figure 3  Distribution of the true impact parameters for B hadron decays to electrons. The negative entries are due solely to imperfect resolution on the B hadron direction. The impact parameters are scaled by the generated B hadron lifetime to facilitate subsequent fitting. The curve is an analytic description of the distribution.

Figure 4  Distributions of the impact parameter divided by its associated error for resolution function tracks in (a) data and (b,c) Monte Carlo. The curve is the result of a maximum likelihood fit using the function described in the text. Only negative entries are fitted; the function is extrapolated into the positive region. Figures (b) and (c) show the Monte Carlo distributions before and after correction for the non-zero lifetime content. The widths of the central Gaussians have been constrained to unity. The parametrized $\sigma$ which results from this procedure is described in the text.
Figure 1
Figure 4

(a) Data

(b) Monte Carlo

(c) Monte Carlo (no lifetime)