Doubly-periodic instability pattern in a smectic A liquid crystal

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We report the observation of a doubly-periodic surface defect-pattern in the liquid crystal 8CB, formed during the nematic–smectic A phase transition. The pattern results from the antagonistic alignment of the 8CB molecules, which is homeotropic at the surface and planar in the bulk of the sample cell. Within the continuum Landau-deGennes theory of smectic liquid crystals, we find that the long period (≈10 µm) of the pattern is given by the balance between the surface anchoring and the elastic energy of curvature wall defects. The short period (≈1 µm) we attribute to a saddle-splay distortion, leading to a non-zero Gaussian curvature and causing the curvature walls to break up.

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The richness of thermotropic liquid crystal (LC) phases 1, their susceptibility to external fields and their unique optical properties make LCs ideally suited to study symmetry breaking phase transitions. These transitions often involve the formation of isolated topological defects or complex ordered spatial structures (topological defect patterns) 2, with analogues in magnetism (grain boundaries, domain walls) 3, superconductivity (Abrikosov lattice, stripes) 4 and cosmology (cosmic strings, monopoles) 5. Contrary to cosmological or quantum systems, LC patterns can be studied at room temperature using polarization microscopy, whence the formation, organization and kinetics of the defect structures can be fully explored. Patterns in the LC nematic phase, with long range orientational order but no positional order, are mostly well understood and readily explained within a continuum elastic theory of LC 1. In contrast, patterns in smectic LC phases are more difficult to describe due to the additional one-dimensional positional order. Many, sometimes rather complex, smectic patterns have been observed, such as undulations of the smectic layers in an applied magnetic field (Helfrich–Hurault instability) 1, 3 or other periodic structures, like stripes 6, 7, squares 8, or hexagons 9. Usually, those structures are explained by the formation of focal conic domains or curvature walls, characterized by one typical length scale 3, 10, 11. In this Letter we report the observation of a novel doubly-periodic defect pattern, which is formed during the nematic-smectic A (N–SmA) phase transition of a liquid crystal in an applied magnetic field. The field imposes an orientation of the LC molecules in the bulk that is orthogonal to the preferred orientation at the surface of the sample cell. Most strikingly, the pattern has two distinct periods: a long one (≈10 µm) along the field direction and a short one (≈1 µm) perpendicular to the field. Interestingly, a quite similar texture develops in LC colloidal shells on cooling towards the N–SmA phase transition 12. We present a model describing the pattern using a geometric construction of a space-filling, energy minimizing, structure of equidistant (smectic) layers. Within this model we identify the driving mechanism as an elastic saddle-splay contribution 14, 15 that breaks the symmetry in such a way that it naturally explains both distinct periodicities of the experiment and the orientation of the pattern with respect to the magnetic field direction.

For our experiments we have used the liquid crystal 8CB (4-n-octyl-4’-cyanobiphenyl) which exhibits both a N and a SmA phase (SmA33.5°C N 41.5°C I). The sample is contained in a cell consisting of two 0.4 mm thick borosilicate glass plates, spaced by a teflon ring with 4.5 mm inner diameter and 1.6 mm thickness (Fig. 1a). A 7 T static magnetic field B was applied in the plane of the cell, along the x-direction. In-situ polarized microscopy was used to visualize the LC phase as a function of time. The sample was positioned in the xy-plane in between two crossed polarizers at ±45° relative to B. In this geometry the transmitted light intensity is maximal when the LC phase is aligned along B and minimal when the LC molecules are randomly aligned (isotropic phase), aligned along the viewing direction (z-axis) or aligned along one of the polarizer axes.

Several cell glass coatings were used to vary the orientation of the 8CB molecules at the surface and the strength of the surface anchoring. To obtain homeotropic alignment (parallel to the normal of the glass) with varying surface anchoring strength we used hexamethyldisilazane (HMDS) coatings or spin-coated polydimethylsiloxane (PDMS) layers, or the untreated glass. Alternatively, coating the cell with polyvinyl alcohol (PVA) induces a planar molecular alignment. The following standard protocol was used: the sample was heated to the
The magnetic field induces a secondary structure: between the surface-anchored lines small elongated defects are visible with a periodicity of about 1–2 µm, perpendicular to the main pattern. This secondary structure develops where the primary defect structure is more dense and regular, i.e. in between straight defect lines. In contrast no secondary structure is formed in the vicinity of an interrupted line pattern. The pattern is absent.

Adjusting the focus of the microscope revealed that the line patterns are formed on both the top and bottom surfaces of the cell, whereas the bulk is homogeneously aligned. Most strikingly, the microscopy images at higher magnifications show that the line pattern contains an additional fine structure: between the surface-lines smaller elongated defects are visible with a periodicity of about 1–2 µm, perpendicular to the main pattern. This secondary structure develops where the primary defect structure is more dense and regular, i.e. in between straight defect lines. In contrast no secondary structure is formed in the vicinity of an interrupted line pattern. This means that the secondary structure is most clearly seen for the cell with strong surface anchoring (PDMS, Fig. 2b), where a regular undulation is observed in the inner structure of the line pattern, giving rise to a zig-zag pattern.

We start our theoretical description in the nematic phase (above the transition temperature $T > T_c$ ($T_c \equiv$...
The director $\mathbf{n}$ reorients by bending in order to minimize the sum of elastic and magnetic free energy:

$$F_{\text{nem}} = \frac{1}{2} \int_V \{ K |\nabla \mathbf{n}|^2 - \chi_a B^2 (\mathbf{n} \cdot \mathbf{e}_z)^2 \}. \quad (1)$$

We are far above the Fredericksz threshold $B_{cr} = \pi \sqrt{\chi_a} \approx 10^{-2} \text{T} \quad (1)$, where $K \approx 7 \cdot 10^{-12} \text{N}$ is the elastic modulus, $\chi_a \approx 10^{-7}$ is the diamagnetic anisotropy in cgs units \(1\) and $2H = 1.6 \text{ mm}$ is the thickness of the cell. The angle $\theta$ between the $z$-axis and the director $\mathbf{n} = \sin \theta \mathbf{e}_x + \cos \theta \mathbf{e}_z$ varies along the thickness of the sample as \(\theta(\zeta) = \arcsin \left( \frac{A e^{2\zeta} - 1}{A e^{2\zeta} + 1} \right), \quad A = 1 + \frac{\mu}{1 - \mu}. \quad (2)\)

Here $\mu = \frac{\sqrt{K} e^{-Bz}}{W_a}$, $W_a$ is the anchoring strength associated with the energy cost $\frac{1}{2} W_a \sin^2 \theta|_{\zeta=0}$ (Rapini–Papoular form) for the deviation of $\mathbf{n}$ from its preferred orientation along the normal to the surface. The dimensionless coordinate $\xi = \frac{z - \zeta}{\zeta_0}$ measures the distance from the glass surface in the $z$-direction in units of the coherence length $\zeta_0 H = \frac{K}{\chi_a B} \approx 0.45 \mu \text{m}$.

The question we pose here is what happens when the SmA order, characterized by equally spaced layers, is imposed on this bent nematic structure. The smectic order is characterized by the complex order parameter $\rho e^{i\omega}$, where $\rho$ is the amplitude of the smectic density modulation, $\phi$ is the scalar function parametrizing the smectic layers, so that $\nabla \phi$ is parallel to the layer's normal. For SmA the director coincides with the normal to the layers, such as $\mathbf{n} = \nabla \phi/|\nabla \phi|$. The associated Landau–deGennes free energy is \(1\):

$$F_{\text{sm}} = \int dV \left\{ \frac{C}{2} (\lambda^2 |\nabla \rho|^2 + \rho^2 |\nabla \phi - \mathbf{n}|^2) + \frac{r}{2} \rho^2 + \frac{q}{2} \rho^4 \right\}. \quad (3)$$

For 8CB the compression modulus $C \approx 10^6 \text{ J/m}^3$ and the interlayer spacing $\lambda \approx 3.2 \text{ nm} \quad (3)$. The coefficients $r$ and $q$ are regular Landau coefficients, where only $r$ depends on temperature, while $q > 0$. We assume that the spatial variations of $\rho$ are negligible, because they occur within the characteristic lengthscale $\lambda \ll \zeta_0 H$ and prior to the change of orientation due to the cooling. Then $\rho = \sqrt{\frac{r}{q}} \propto \sqrt{T_c - T}$ and the second term in \(3\) is minimized if

$$\nabla \phi = \mathbf{n} \rightarrow \frac{dx}{dz} = \cot \theta, \quad (4)$$

yielding the formation of equidistant smectic layers ($(\nabla \phi) = 1$) on top of the distorted nematic phase. Substituting $\theta$ from \(2\) and integrating \(4\) we find:

$$\zeta_0(\xi) = \frac{1}{2} \log \left( \frac{\coth(\xi/2)}{A} \right), \quad (5)$$

which parametrizes the smectic layer in terms of the dimensionless coordinate $\xi = x/(\epsilon_0 H)$ along the $x$-axis as shown in Fig.\(3\) (red curve, $A \approx 1.1$). This important result for a closed form of the generating curve is derived, based on the competition between anchoring and magnetic energy, mediated by elasticity \(1\) under the global geometric constraint of equidistant layers \(1\), rather than assumed a priori as in refs. \(2\) \(\cite{1} \) \(\cite{18}\). To obtain a space-filling two dimensional structure we perform a parallel transport of the generating curve $\zeta_0(\xi)$ along its normal $\nu = \nu_x e_x + \nu_z e_z$ (see Fig.\(3\)), yielding a set of equidistant layers with the $j$th layer given by:

$$x_j = (\xi + j \alpha \omega \nu_x) e_x + (\zeta_0(\xi) + j \alpha \nu_z) e_z, \quad j = 0, \pm 1, \pm 2 \ldots \quad (6)$$

Here $\alpha = \frac{\epsilon_0 H}{\chi_a B^2}$ is the dimensionless interlayer distance, the components of the normal $\nu_x = \frac{\cosh \xi}{\sqrt{4 + \cosh^2 \xi} \xi}$ and $\nu_z = \frac{2}{\sqrt{4 + \cosh^2 \xi}}$. In the bulk $(\xi \rightarrow \infty)$ we obtain a set of flat parallel layers perpendicular to the $x$-axis. The curved smectic layers (green lines in Fig.\(3\)) cannot fill the space without defects such as curvature walls shown by the grey and dashed-blue regions. Note that Fig.\(3\) illustrates only part of the whole structure, while the actual size of the grey region is determined by the energy balance between the surface and the bulk contributions (see below). Within our two dimensional model the size of the blue region is characterized by the intersection of the two red curves (at the $z$-axis) at $|\xi| = \pm 2 \coth^{-1}(A)$ and not by the energy minimizing structure of this region. This latter approach gives different space-filling structures as described previously \(2\) \(\cite{18}\).

The resulting structure consists of stripes parallel to the $y$-axis with period $L_x$ in the $x$-direction, along the magnetic field, like in the experiment. $L_x$ is determined by two contributions (see supplemental material for derivation \(19\)):

$$L_x \approx 2 \frac{W_a}{\lambda a B^2} - 2 \rho \sqrt{\frac{K}{\chi_a}} (1 - \frac{1}{\epsilon_0 H}) + 2 \frac{\epsilon_0 H}{\max |\kappa|} \quad (7)$$

The first term corresponds to the competition between the anchoring energy (green region in Fig.\(3\)) and the energy of the curvature wall (grey region), given by $f_\omega = \rho \sqrt{K/C} (\tan \omega - \omega) \cos \omega \quad (10)$ $(\omega = \pi/4)$ per unit area. The second term originates from a non-zero curvature of the generating curve \(\alpha\), given by $\kappa = \partial \xi \zeta_0/((1 + (\partial \zeta_0)^2)^{3/2})$. The first term in \(7\) is non-negative only if $W_a \geq \rho \cdot 10^{-3} \text{ J/m}^2$, yielding $L_x \approx (9 \pm 4) \mu \text{m}$ for a strong surface anchoring $(W_a \approx$
anism is the growth of the elastic constants in the vicinity to explain it is a truly challenging task. A plausible mechanism is our most important experimental finding and planar and small homeotropic surface anchoring.

In the vicinity of the N–SmA transition, which is compatible with our experimental observations. Indeed, ‘breaking’ of the smectic layers implies that the linear correction to the level set function \( \phi + \varepsilon \hat{\phi} \) should satisfy \( \nabla \hat{\phi} = \nabla \hat{n} \), leading to:

\[
\frac{dy}{dz} = -\frac{\theta}{\varphi} \to \sin(\eta y) = (\cosh \zeta + \mu \sinh \zeta)^{-\sigma}.
\]

Here \( \sigma = \frac{\omega(1 + \tau \omega)}{\mu^2 (1 + \tau \omega)} \), \( \omega^2 = 1 + q_y^2 \varepsilon_0^2 H^2 \) and \( \tau = \frac{K_{24}}{K_{06}} \). The correction to the level set function \( \zeta_0(\xi) + \varepsilon \zeta(y/(\varepsilon_0 H)) \) at the onset of instability acquires a simple analytic form \( \zeta \approx \cosh^{-1} \left[ \sin(q_y y) \right]^{1/\sigma} \) (\( \mu \approx 0.06 \) for strong anchoring). In Fig. 3 we plot \( \zeta_0(\xi) + \varepsilon \zeta(y/(\varepsilon_0 H)) \) for a periodically distorted generating layer (red curve in Fig. 3) with non-zero Gaussian curvature. This simplified analysis allows to identify the intrinsic periodicity of modulated stripes \( L_y \approx (3 \pm 1) \times 10^{-4} \text{ J/m}^2 \) and a smectic order parameter \( \rho \approx 0.1 \) in the vicinity of the N–SmA transition, which is compatible with our experimental observations. Within this model indeed no striped defect-patterns are expected for planar and small homeotropic surface anchoring.

The appearance of the second periodicity within the pattern is our most important experimental finding and to explain it is a truly challenging task. A plausible mechanism is the growth of the elastic constants in the vicinity of the N–SmA phase transition, in particular an enhanced saddle-splay contribution

\[
\mathcal{F}_{K_{24}} = -K_{24} \int dV \left\{ \nabla \cdot (\mathbf{n} \nabla \cdot \mathbf{n}) + \mathbf{n} \times \nabla \times \mathbf{n} \right\},
\]

which should be added to (1). Note that the expression under the integral can be cast into the form \( \mathbf{n} \cdot \nabla \times \mathbf{n} \), where \( \mathbf{\Omega} \) is the spin connection. This is equivalent to the Gaussian curvature of the smectic layers, since \( \mathbf{n} \) coincides with its normal or analogous to the Wen–Zee action, responsible for the curvature coupling in effective field theory of the quantum Hall systems. Thus layers with non-zero Gaussian curvature may become energetically favorable relative to the layers with zero Gaussian curvature considered above.

To quantify the saddle-splay effect we perturb the nematic state by a small variation of polar \( \theta(y, z) \) and azimuthal \( \varphi(y, z) \) angles. The linearised perturbation of the director \( \mathbf{n} \) then reads \( \hat{\mathbf{n}} = \hat{\theta} \mathbf{e}_x \cos \theta \mathbf{e}_y + \hat{\varphi} \mathbf{e}_z - \hat{\theta} \sin \theta \mathbf{e}_z + O(\varepsilon^2) \). Solving the variational problem for \( \hat{\theta} = \varepsilon f(z) \sin(q_y y) \) and \( \hat{\varphi} = \varepsilon g(z) \cos(q_y y) \), we can find the threshold \( (q_y, \mu) \) when the nematic state is linearly unstable with respect to periodic distortions along the \( y \)-direction. The incompressibility of the smectic layers implies that the linear correction to the level set function \( \phi + \varepsilon \hat{\phi} \) should satisfy \( \nabla \hat{\phi} = \nabla \hat{n} \), leading to:

\[
\frac{dy}{dz} = -\frac{\theta}{\varphi} \to \sin(\eta y) = (\cosh \zeta + \mu \sinh \zeta)^{-\sigma}. \tag{9}
\]

Here \( \sigma = \frac{\omega(1 + \tau \omega)}{\mu^2 (1 + \tau \omega)} \), \( \omega^2 = 1 + q_y^2 \varepsilon_0^2 H^2 \) and \( \tau = \frac{K_{24}}{K_{06}} \). The correction to the level set function \( \zeta_0(\xi) + \varepsilon \zeta(y/(\varepsilon_0 H)) \) at the onset of instability acquires a simple analytic form \( \zeta \approx \cosh^{-1} \left[ \sin(q_y y) \right]^{1/\sigma} \) (\( \mu \approx 0.06 \) for strong anchoring). In Fig. 3 we plot \( \zeta_0(\xi) + \varepsilon \zeta(y/(\varepsilon_0 H)) \) for a periodically distorted generating layer (red curve in Fig. 3) with non-zero Gaussian curvature. This simplified analysis allows to identify the intrinsic periodicity of modulated stripes \( L_y \approx (3 \pm 1) \times 10^{-4} \text{ J/m}^2 \) and a smectic order parameter \( \rho \approx 0.1 \) in the vicinity of the N–SmA transition, which is compatible with our experimental observations. Indeed, ‘breaking’ of the smectic layers in the vicinity of the glass surface could relax the structure and prevent the formation of energetically expensive curvature wall defects unavoidable in the planar case (blue region in Fig. 3).

It is well known that curvature walls can become unstable with respect to focal conic domains (FCDs). FCDs are characterised by a semi-major axis \( a \) of the ellipse and its eccentricity \( e \). The latter can be related to the angle of a curvature wall as, \( e = \sin(\omega/2) \) (Fig. 1), yielding \( L_y/L_x = \sqrt{1-e^2} \approx 0.7 (\omega = \pi/2) \), which is observed in experiments (Fig. 2). However, the formation of FCDs, with one characteristic length scale \( L_x \approx L_y \), is different from the appearance of our doubly-periodic pattern.

In conclusion, we have observed a new doubly-periodic defect texture in liquid crystals formed during the N–SmA phase transition. Based on a simple geometric argument and energy minimization including the saddle-splay elastic contribution, we have proposed a plausible scenario for instabilities at two different length scales. Our
findings can give insight into the organization of other lamellar microstructures, ubiquitous in nature \cite{24,25}, and can be of general interest to analyze symmetry breaking phase transitions in condensed matter systems \cite{2,4}.

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\[19\] See Supplemental Material at [URL will be inserted by publisher] for derivation of equation \[13\].
\[23\] The amplitude of the perturbation $\varepsilon$ cannot be identified within a linear stability analysis.