The binary progenitor evolution of supernovae

Proefschrift

der verkrijging van de graad van doctor
aan de Radboud Universiteit Nijmegen
op gezag van de rector magnificus prof. mr. S.C.J.J. Kortmann,
volgens besluit van het college van decanen
in het openbaar te verdedigen op maandag 18 november 2013
om 10.30 uur precies

door

Joke Simonne Walter Alex Claeys

tegen op 7 april 1986
te Gent, België
Promotor: Prof. dr. F.W.M. Verbunt

Copromotor: Dr. O.R. Pols

Manuscriptcommissie: Prof. dr. A. Fasolino
Prof. dr. D. Maoz Tel-Aviv University, IL
Prof. dr. G. Nelemans
Dr. C.A. Tout Cambridge University, UK
Dr. J. Vink Universiteit van Amsterdam

© 2013, Joke Claeys
The binary progenitor evolution of supernovae
Thesis, Radboud University Nijmegen
Illustrated by Sofie Cammaert; with bibliographic information and Dutch summary

ISBN: 978-94-6191-939-7
# Contents

1 Introduction  
1.1 Single stars .................................. 1  
1.1.1 Massive stars ................................ 3  
1.1.2 Low- and Intermediate-mass stars ............ 3  
1.2 Binary stars .................................. 4  
1.3 Supernovae .................................... 5  
1.3.1 Core collapse supernovae ...................... 7  
1.3.2 Supernovae type Ia ............................ 9  
1.4 Methods ....................................... 13  
1.4.1 Detailed binary stellar evolution codes ....... 13  
1.4.2 Binary population synthesis codes ............ 14  
1.5 Chapters of this thesis ......................... 14  

2 Binary progenitor models of type IIb supernova  
2.1 Introduction .................................. 18  
2.2 Observed type IIb Supernovae .................... 19  
2.2.1 Individual type IIb supernovae ............... 20  
2.3 Stellar evolution calculations .................... 22  
2.4 Single star progenitors .......................... 24  
2.5 Binary star progenitors ......................... 25  
2.5.1 Properties of the companion ................. 27  
2.5.2 Parameter space ............................. 29  
2.5.3 Non-conservative mass transfer ............... 31  
2.6 Predicted rates ................................ 35  
2.6.1 The rate of type IIb supernovae ............... 36  
2.6.2 The relative rates of different cases for the companion ............ 37  
2.7 Discussion & Conclusion ....................... 38  
2.7.1 A brief comparison with previous work ....... 39  
2.7.2 What type IIb SNe teach us about stellar and binary physics ............ 39
3 Spin-up/spin-down models for type Ia supernovae 47
  3.1 Introduction .......................................................... 48
  3.2 Spin-Up and Spin-Down ................................................ 48
  3.3 Observational Signatures ............................................. 50
    3.3.1 Background ...................................................... 50
    3.3.2 Progenitor signatures ........................................... 50
    3.3.3 Detecting the Remnants of the Donors .......................... 52
    3.3.4 Other Connections .............................................. 54
  3.4 Spin-Up/Spin-Down: A new paradigm ................................ 54

4 Theoretical uncertainties of the type Ia supernova rate 57
  4.1 Introduction .......................................................... 58
  4.2 Binary population synthesis ................................ .......... 59
    4.2.1 Single star evolution ........................................... 59
    4.2.2 Binary star evolution ............................................ 62
    4.2.3 Binary population synthesis ..................................... 67
  4.3 Binary progenitor evolution ......................................... 69
    4.3.1 Double degenerate channel ...................................... 69
    4.3.2 Single degenerate channel ....................................... 71
  4.4 Comparison with observations ....................................... 78
    4.4.1 Double degenerate rate ......................................... 80
    4.4.2 Single degenerate rate .......................................... 81
    4.4.3 Overall SN Ia rate ............................................... 81
  4.5 Uncertainties in binary evolution ................................ 82
    4.5.1 Common envelope evolution ..................................... 82
    4.5.2 Stability criterion of Roche-lobe overflowing stars ......... 86
    4.5.3 Accretion efficiency ............................................. 88
    4.5.4 Angular momentum loss ......................................... 90
    4.5.5 Wind prescription ............................................... 91
    4.5.6 Combining two binary parameters ................................. 92
  4.6 Influence of initial binary distributions .......................... 92
    4.6.1 Initial mass ratio distribution .................................. 93
    4.6.2 Initial mass function ........................................... 93
    4.6.3 Combining two distribution functions ........................... 94
  4.7 Discussion & conclusion ............................................. 95
    4.7.1 Progenitor evolution ............................................ 95
    4.7.2 General characteristics of theoretical rate .................. 96
    4.7.3 Uncertainties in binary evolution ............................... 96
    4.7.4 Comparison with other work ..................................... 97
## Contents

4.7.5 Other uncertainties in the results .............................................. 97  
4.7.6 Outlook ................................................................. 98  
4.8 Acknowledgements .......................................................... 103  
4.A Mass distribution of the envelope ............................................. 103  
4.B Accretion efficiency of WDs .................................................. 104

5 PopCORN ................................................................. 107
  5.1 Introduction ............................................................... 108  
  5.2 Binary evolution ........................................................... 109  
    5.2.1 Roche lobe overflow ................................................ 109  
  5.3 Binary population synthesis codes ......................................... 113  
    5.3.1 binary_c/nucsyn ..................................................... 113  
    5.3.2 The Brussels code ................................................... 114  
    5.3.3 SeBa ................................................................. 114  
    5.3.4 StarTrack ............................................................ 115  
  5.4 Method ................................................................. 115  
    5.4.1 Assumptions for this project ..................................... 116  
    5.4.2 Normalisation ...................................................... 118  
  5.5 Results ................................................................. 118  
    5.5.1 Single white dwarf systems ...................................... 118  
    5.5.2 Double white dwarfs ............................................... 151  
  5.6 Summary of critical assumptions in BPS studies ......................... 163  
  5.7 Conclusion .............................................................. 166  
  5.A Backbones of the BPS codes ............................................... 167  
    5.A.1 Single star prescriptions ......................................... 169  
    5.A.2 Stability of mass transfer ....................................... 169  
    5.A.3 Stable mass transfer ................................................ 170  
    5.A.4 Wind mass loss .................................................... 172  
    5.A.5 Angular momentum loss from winds ................................ 172  
    5.A.6 Evolution of helium stars ........................................ 172  
    5.A.7 Generating the initial stellar population ......................... 173  
  5.B Typical variable assumptions in BPS codes ................................ 173  
    5.B.1 Accretion efficiency ............................................... 176  
    5.B.2 Angular momentum loss during RLOF .............................. 176  
    5.B.3 Common envelope evolution ........................................ 177  
    5.B.4 Wind accretion ..................................................... 177  
    5.B.5 Tides ............................................................... 178  
    5.B.6 Magnetic braking ................................................... 178  
    5.B.7 Initial population .................................................. 178

6 Summary and prospects .......................................................... 181
  6.1 Type IIb supernovae .................................................... 181  
  6.2 Type Ia supernovae ..................................................... 182
6.3 Uncertainties in stellar and binary physics .............................................. 183

Bibliography ................................................................. 185

(Vlaamse) Samenvatting .................................................. 195
De levensloop van een enkele ster ........................................... 195
De invloed van een begeleider .............................................. 196
De explosies van sterren ...................................................... 197
  Core collapse supernovae ................................................. 197
  Supernovae type Ia ......................................................... 198
Hoofdstukken van deze thesis ............................................ 199

My publications ............................................................. 203
  Refereed journals ......................................................... 203
  Conference proceedings ................................................ 203

Curriculum vitae .......................................................... 205

Acknowledgments ........................................................ 207
INTRODUCTION

On the first of May, 1006 a very bright light appeared in the sky which was also visible during the day and remained so for several months\(^1\). It was recorded by people all over the world. They observed the explosion of a star, a supernova, in the constellation Lupus. It may have been the brightest supernova ever observed on earth, although the explosion site is about 500 million times further away from us than the Sun\(^2\). The origin of these energetic events, such as SN 1006, are studied in this thesis.

Stars come in different sizes over a wide mass range, from a tenth of the mass of our Sun to one hundred times the mass of our Sun. When observing these stars we see them as unchanging, however they evolve and change significantly over the course of their life: they can heat up, cool down, pulsate, shed layers, explode, and so on. Additionally, stars often have a companion star. The more massive the star, the more likely it is that it has a companion star (Raghavan et al., 2010; Duchêne & Kraus, 2013). These companions can for example spin up the star, remove mass or the two stars can even merge.

However, how is a star destined to end? As a triumphant explosion, or a shy petering out? And can a companion star alter this destiny? This is the main subject of this thesis: the explosive deaths of stars and the effect of the binary companion. In this introduction we first describe the basics of single star evolution and continue with explaining the behaviour of stars in binary systems. Subsequently we identify the different types of explosive deaths. We describe the methods used for our research and we end with a brief description of the different chapters of this thesis.

1.1 SINGLE STARS

A star’s evolution is a constant fight between an inward gravitational force and an outward force, originating from a pressure gradient, and which of these eventually wins determines its death. The source of pressure could be e.g. thermal motions of the gas particles, or radiation. As long as the inward and outward forces balance each other the star is in \textit{hydrostatic equilibrium}.

Stars often form in large groups, called stellar clusters, from the collapse of giant molecular clouds with masses of about $10^5$ to $10^6 M_\odot$ (Williams et al., 2000). As soon as the gas pressure is high enough

\(^1\)http://www.noao.edu/outreach/press/pr03/pr0304.html

\(^2\)Distance to earth of supernova remnant of SN 1006 = 2.18 ± 0.08kpc (Winkler et al., 2003), with 1 pc = 3.1 ×10\(^6\) m
to counteract gravity, a proto-star is formed which is in hydrostatic equilibrium. A proto-star contracts further until the central temperature and density are high enough to start nuclear fusion of hydrogen into helium, a process generally referred to as hydrogen burning. At that point a star is formed and the energy produced in the centre balances the energy losses from the surface, the total energy budget of the star is conserved and the star is in thermal equilibrium. For about 90% of its stellar life the star burns its entire central hydrogen supply into helium in its centre. This stellar phase is called the main sequence. The time the star spends on the main sequence is referred to as the nuclear timescale of hydrogen burning,

\[ \tau_{\text{nuc,H}} \approx 10^{10} \left( \frac{M}{M_{\odot}} \right)^{-2.8} \text{yr}, \]  

(1.1)

were \( M \) is the mass of the star and \( M_{\odot} \) the mass of our Sun. As the central temperature and density are higher in more massive stars, the burning process is faster, and the nuclear timescale is shorter. This timescale also exists for the other burning cycles beyond hydrogen in the stars. However, these nuclear timescales are a factor ten or more shorter than the main-sequence timescale, and therefore the nuclear timescale of hydrogen burning is a good approximation to the lifetime of a star.

If the energy source disappears, the star contracts to restore thermal equilibrium. This happens on the thermal or Kelvin-Helmholtz timescale of the star,

\[ \tau_{\text{KH}} = 1.5 \cdot 10^7 \left( \frac{M}{M_{\odot}} \right)^2 \left( \frac{R}{R_{\odot}} \right) \left( \frac{L}{L_{\odot}} \right) \text{yr}. \]  

(1.2)

where \( R \) is the radius of the star, \( L \) the luminosity of the star which is the total amount of energy radiated away at the surface of the star per unit time, and \( L_{\odot}, R_{\odot} \) the solar luminosity and radius, respectively. The thermal timescale is generally much shorter than the nuclear timescale of a star.

Generally, energy is transported from the centre to the surface of the star by thermal radiation. However, under certain conditions, such as a too high energy flux, this process is not efficient and convection occurs, in which energy is transported by the upward and downward movement of material inside the star, mixing all the material inside the convective region efficiently. This phenomenon is also found on Earth, for example heat circulation in the Earth’s atmosphere is by the upward and downward movement of air. As energy is transported in a different manner, the structure of a convective star is different to the structure of a radiative star.

To determine the boundaries of the convective regions inside a star, two criteria are commonly used in astrophysics, the Schwarzschild and Ledoux criteria, which both consider the temperature gradient inside a star, while the Ledoux criterion additionally considers the chemical composition gradient. As convection displaces material inside a star, it is not expected that the velocity of the material is exactly zero at the boundary indicated by the Schwarzschild or Ledoux criterion, and the material overshoots this boundary, increasing the size of the mixed regions of the star. The amount of overshooting is uncertain and in our work it is expressed by \( \delta_{\text{ov}} \), the overshooting parameter (Schroder et al., 1997).

Another important timescale of a star is the time it takes to react when hydrostatic equilibrium is perturbed, the dynamical timescale,

\[ \tau_{\text{dyn}} \approx 0.02 \left( \frac{M}{M_{\odot}} \right)^{0.5} \left( \frac{R_{\odot}}{R} \right)^{1.5} \text{days}. \]  

(1.3)

During most of the star’s life the dynamical timescale is much shorter than the thermal and nuclear timescale, therefore a star generally recovers quickly from small deviations of hydrostatic equilibrium.
If the star cannot regain hydrostatic equilibrium the star collapses or explodes, depending whether either the outward or the inward force is the biggest, which means the end of the star’s life.

The entire life time of a single star depends on its initial mass, but also on its rotational velocity and on its metallicity, the initial mass fraction of elements heavier than helium in the star. As I do not discuss the effects of rotation and metallicity in this thesis, I will describe the evolution of non-rotating stars having the same initial metallicity as our Sun and only distinguish between massive stars and low- and intermediate-mass stars, as they result in different types of supernovae.

1.1.1 Massive stars

Massive stars have initial masses greater than about 8 $M_\odot$ and can have masses up to 100 $M_\odot$ or more. During the main sequence, a massive star has a convective central region, referred to as the core of the star, because radiation is insufficient to transport the high energy flux. The outer layers, the envelope of the star, are radiative.

When the core is hydrogen-depleted, burning stops and the star is out of thermal equilibrium which results in the contraction of the star. Hydrogen burning moves to a shell around the core. The core keeps contracting and the envelope expands until helium can be fused in the centre. As both changes occur on the thermal timescale of the star, which is much shorter than the star’s time on the main sequence, few stars are observed in this stellar phase, which is therefore called the Hertzsprung gap. After this relatively short stellar phase, the entire star is much larger than on the main sequence and the outer layers are cool. As these stars appear reddish in visible light, they are called red giants. Radiation transport is not efficient in these cool envelopes with a high opacity, and therefore red giants have convective envelopes. After central helium burning, the star burns carbon in its centre, while helium and hydrogen burning occur in separate burning shells around the core. This process of contraction and shell burning continues until an iron core is formed. At this stage fusion is energetically disadvantageous. An onion-like structure is formed with the lighter elements in successively more outward shells in the star.

Another important feature of massive stars is that they suffer from strong stellar winds over their entire lifetime. In hot, luminous stars, such as massive main-sequence stars, the driving mechanism is radiation pressure caused by line scattering, with the lines from heavy elements in the stars (for an overview see Owocki, 2004). In cool luminous stars, such as red giants, the driving mechanism is still uncertain. It is probably caused by pulsations of the cool outer layers resulting in the formation of dust, which can drive a wind through radiation pressure (Bennett, 2010). As the structure of a star depends on its metallicity, the strength of this type of wind mass loss indirectly also depends on metallicity. For both types of wind the rate at which mass is lost remains the subject of debate.

1.1.2 Low- and Intermediate-mass stars

These stars have a mass ranging from approximately 0.1 $M_\odot$, the minimum mass necessary for a star to be able to burn hydrogen, to about 8 $M_\odot$. The boundary between low- and intermediate-mass stars is around 2.2 $M_\odot$, which will be explained below.

Stars greater than 1.2 $M_\odot$ have convective cores and radiative envelopes. In lower mass stars, the energy flux is lower and the cores are radiative, however the high opacity of these cooler stars causes the outer layers of the envelope to be convective. Stars with masses lower than about 0.7 $M_\odot$ have deep convective envelopes and stars with a mass lower than 0.35 $M_\odot$ are completely convective.
Similarly to a massive star, after hydrogen is exhausted in the centre, the core contracts and hydrogen burning continues in a shell around the core increasing the mass of the hydrogen-depleted core. The star first moves across the Hertzsprung gap and afterwards –when the envelope is completely convective– along the giant branch, until central helium burning is possible. However, in low-mass stars ($\lesssim 2.2 \, M_\odot$) the core is degenerate at that point, a state where the pressure is independent of the temperature. In these stars helium burning always starts at the same central density which corresponds to a core mass of approximately $0.5 \, M_\odot$. The burning process starts in an unstable manner and results in a runaway process, which is called a helium flash. During this flash the core expands and the degeneracy is lifted. Afterwards helium burning continues in a stable manner. It should be noted that the lowest-mass stars never reach the point where helium burning is possible ($\lesssim 0.95 \, M_\odot$ for solar metallicity). The nuclear timescale of these stars (Eq.1.1) is very long, longer than the age of our universe, the Hubble time, which is approximately 13.7 Gyr, and therefore these stars are still on the main sequence regardless of their age.

Stars more massive than $0.95 \, M_\odot$ form a carbon-oxygen core, which contracts after helium is centrally exhausted, and hydrogen and helium burning continue in separate shells around the core. This stellar phase of low- and intermediate-mass stars is referred to as the asymptotic giant branch, and the stars have cool outer layers and a compact, degenerate core. These stars also have strong stellar winds, driven by radiation pressure on dust around the stars which is produced by pulsations of the outer layers, similar to massive red giants. The contraction of the core stops when it reaches the point of carbon ignition. In stars with masses lower than about $6.2 \, M_\odot$ this ignition point is not reached before the entire envelope is removed. The core of these stars is degenerate at that point and they evolve into carbon-oxygen white dwarfs (CO WDs). Stars with masses between about $6.2$ and $8 \, M_\odot$ continue burning carbon, but because they become degenerate soon afterwards they are not able to burn beyond this point and form oxygen-neon-magnesium white dwarfs (ONeMg WDs).

Thus low- and intermediate-mass stars form different types of WDs at the end of their lives, which do not collapse because the degeneracy pressure gradient can sustain gravity regardless of their temperature. Without any further interaction, these stellar objects cool down and crystallize.

### 1.2 Binary stars

There are several uncertainties in the life of a single star, such as uncertainties in convection and other mixing processes, rotation and the wind mass loss rates. However, if the star has a companion the star’s life as discussed above can change completely. Binary systems come in different configurations, from wide systems were the two stars take a few thousand years to orbit each other, to close systems were it only takes a few minutes. The mass of the companion can also vary from an almost equal mass to a completely different mass. Observations show that more than 70% of massive and intermediate-mass stars are part of a binary system (of which some are even part of a higher-order multiple system, Kouwenhoven et al., 2007; Sana et al., 2012). Therefore it is of great importance to study the effect of duplicity.

Binary evolution can have a large impact on a star’s life. Stars in close binary systems can distort each other’s shape of by tidal interaction (similar to the tidal effects of the Moon on the Earth). More importantly, stars can transfer material to their companion. One possibility is that stellar wind is accreted by the companion, in which case it is only a small effect. However, during a stellar life the star expands and at one point it can fill a critical equipotential surface, the Roche lobe. Particles on this surface are
gravitationally bound to both stars and when the star fills its Roche lobe a stream of material from the surface of one star flows to the other star. This process is called Roche-lobe overflow and generally continues until the entire envelope is depleted. It alters the life of both stars significantly. It not only affects the masses of both stars, but the mass gainer also gains angular momentum which changes its rotational velocity.

The Roche-lobe overflow process can become dynamically unstable because of the reaction of the donor star and the orbit upon this mass loss, e.g. when the radius of the donor star expands more quickly than its Roche lobe. The donor keeps overfilling its Roche lobe which results in a runaway process. Such a binary system evolves into a situation where the envelope of the donor star engulfs the other star, which is called a common envelope. Interaction with the common envelope drags energy and angular momentum out of the orbit, resulting in shrinkage of the orbit. The core of the donor star and the companion star can merge inside the common envelope, resulting in a single stellar object, or the core and companion can remain in a shorter orbit after the complete envelope is expelled.

If mass transfer is dynamically stable the further evolution also depends on the reaction of the companion star which accretes material. The companion star gains both mass and angular momentum, which can result in the expansion of the star and/or the re-ejection of part or all of the gained material. If the companion star expands it can at one point also fill its Roche lobe, resulting in a contact system, which potentially evolves into a common envelope.

Depending on the stellar evolutionary stage when the initially more massive star fills its Roche lobe, three main cases of mass transfer are distinguished, which determine which type of stellar object remains after mass transfer:

**Case A** mass transfer occurs during the main sequence, when the donor star burns hydrogen in its centre (Kippenhahn & Weigert, 1967). This type of mass transfer is generally dynamically stable because a donor star with a radiative envelope does not expand as a reaction to mass loss, unless the star is of very low mass and has a convective envelope.

When the star fills its Roche lobe after central hydrogen exhaustion and before central helium ignition, **Case B** mass transfer occurs (Kippenhahn & Weigert, 1967). The stability of mass transfer depends on the structure of the outer layers of the star as well as on the mass ratio. If the outer layers are convective, the star expands as a reaction to mass loss. In binary systems in which the conservatively\(^3\) accreting star has a lower mass than the donor star, the Roche lobe radius decreases.

If the Roche lobe is filled after central helium exhaustion, **Case C** mass transfer occurs (Lauterborn, 1970). Mass transfer from these evolved stars is generally unstable because of the radial expansion of their convective envelopes as a reaction to mass loss.

### 1.3 Supernovae

A supernova (SN) is the explosion of a star at the end of its life. Since the 1940’s it has become clear that there is a large variety of SN types (for an overview see Da Silva, 1993). Because of historical reasons the classification system is based on the early spectrum, which shows the elements in the outermost layers of

---

\(^3\)Conservative mass transfer means that all material lost by the donor star is accreted by its companion.
the exploding star, and on the light curve shape of the SN (Fig. 1.1). The light curve consists of two parts, where the first part arises because of the heating of the outer layers during the explosion and the second part is caused by energetic photons from the decay of radioactive iron-group isotopes.

The first distinction between SN types is based on the occurrence of hydrogen in the spectrum, which distinguishes between type I and type II SNe. In type I SNe, further distinction is based on the occurrence of silicon and the occurrence of helium, leading to the classification into type Ia, type Ib and type Ic SNe (Fig. 1.1). Later it became clear that there is a difference between the origin of type Ia SNe and the other SN types. Type Ia SNe originate from the thermonuclear explosion of WDs, while type II, Ib and Ic SNe arise because of the core collapse of massive stars, leading to their explosion, core collapse SNe.
1.3.1 Core collapse supernovae

At the end of a massive star’s life an iron core is formed, which is stable because of thermal and electron degeneracy pressure. However, when the iron core gets too hot iron disintegrates, and when the density becomes high enough protons capture electrons and form neutrons and finally a neutron-rich core is formed. As the disintegration of iron is an endothermic process thermal pressure decreases, in addition to the decrease of electron degeneracy pressure, and the core collapses. The collapse stops when the neutron degeneracy pressure and the nuclear density are high enough to sustain the pressure from the infalling material, and the rest of the material bounces back on the dense core causing a shock to propagate outwards. Hydrodynamical simulations indicate that in most cases this shock is stalled, but it may be revived by neutrinos emitted by the central hot region. If all of the infalling envelope material is expelled, a luminous SN arises leaving a neutron star behind. If part of the material falls back onto the neutron-rich core while the other part is expelled, the core can collapse further and form a delayed black hole, giving rise to a weak SN. In case the stalled outward shock cannot be revived and the infalling material drives an inward shock, the neutron star collapses directly into a black hole together with the rest of the star and no SN occurs.

Figure 1.2: Overview from Heger et al. (2003) of the dependence of the different types of core collapse SN and the formation of a (direct or delayed) black hole (BH) or neutron star (NS) on metallicity and initial mass for single non-rotating stars.

1.3 Supernovae
There remain uncertainties in the details of this mechanism, the resulting masses of the black holes (Fryer, 2003), and the initial masses corresponding to the boundaries between the formation of a neutron star or a black hole (Heger et al., 2003, Fig. 1.2). In addition to core collapse SNe, there are more rare explosive events from very massive stars, such as pair-instability SNe (Heger & Woosley, 2002), but because they are so uncommon and they are not part of this thesis I do not discuss these further.

![Figure 1.3: Two possibilities for how a star can lose its envelope, through (a) a stellar wind or through (b) Roche-lobe overflow (credits: Selma de Mink).](image)

Additionally, differences are observed in the early spectrum and light curve shape between SNe from collapsing stars. If hydrogen is observed in the spectrum the SN is classified as a type II. However, the light curve shapes of these SNe vary between a plateau shape (SN type IIP), a linearly declining light curve (SN type IIL) and a double-peaked light curve (SN type IIb), depending on the amount of hydrogen remaining in the envelope. If no hydrogen is detected the SN is classified as a type Ib and if both hydrogen and helium lack in the spectrum it is a type Ic SN. The canonical explanation for the difference in the origin of these SN types is the amount by which the star is stripped of its outer layers at the moment it explodes, because of its onion-like structure before explosion. Below we list the SN types in order of increasing depletion of the envelope,

\[ \text{SNIp} \rightarrow \text{SNIIL} \rightarrow \text{SNI Ib} \rightarrow \text{SNIb} \rightarrow \text{SN Ic}. \]

The envelope can be lost by a stellar wind or by Roche-lobe overflow (Fig. 1.3). Massive stars have strong stellar winds over their entire life time (Sect. 1.1.1). The rate at which mass is lost through a stellar wind depends directly or indirectly on both the mass of the star and the metallicity, and both a lower mass and lower metallicity of a star results in a lower mass loss rate.

Fig. 1.2 presents an overview of the dependence of the strength of the SN and the type of SN on the mass and metallicity of a single star. This figure shows that at approximately solar metallicity, type Ib/Ic SNe originate only from massive stars and produce weak SN, which is not observed. This and other arguments against single stars to fully explain SNe Ib/Ic are discussed in Langer (2012). Another explanation for the diversity is necessary, such as binary evolution, which explains better the observed rate of the different core-collapse types (Smith et al., 2011).
1.3.2 **Supernovae Type Ia**

Type Ia SNe generally show no hydrogen and helium, but a strong Si II absorption line in the spectrum, and are observed in both old and young stellar populations, which indicates that they originate from low- and intermediate-mass stars. Their light curves are very homogeneous. Phillips (1993) found a correlation between the peak of the light curve in the B-band and its decline after 15 days, known as the Phillips relation, making SNe Ia standarizable candles. Hence these stellar explosions are commonly used as cosmological distance indicators to determine the expansion of the universe, which led to the Nobel prize of physics for the researchers who derived the accelerated expansion of the universe \(^4\).

Theoretical and observational arguments indicate that type Ia SNe are thermonuclear explosions of CO WDs (Hoyle & Fowler, 1960). In the explosion the entire WD is disrupted and both intermediate-mass elements, material partly burned beyond carbon, such as silicon and sulphur, and iron-peak elements are produced, making SNe Ia the main contributors to the production of iron in the universe. The homogeneity of the light curves requires that the WD explodes under similar conditions in all SNe Ia. A natural way to explode these objects is when they reach a critical density, which implies a mass very close to the Chandrasekhar mass, i.e \(1.4 M_\odot\) in detailed models for non-rotating stars. However, observations and theoretical models show that CO WDs have masses at formation between 0.6 and 1.2 \(M_\odot\) (Weidemann, 2000). Therefore, to reach \(1.4 M_\odot\) accretion from a companion star is necessary.

Different scenarios are proposed in order for the WD mass to reach \(1.4 M_\odot\):


**Figure 1.4:** Illustration of the reaction of a CO WD to accretion of hydrogen-rich material, depicted as the mass accretion rate versus the mass of the WD. It shows the steady hydrogen-burning regime at a rate of approximately \(10^{-7} M_\odot\) yr\(^{-1}\). If material is accreted at a lower rate nova explosions occur at the surface of the WD. If material is accreted at a higher than the steady hydrogen-burning regime, it is unclear what happens with this material. The possibilities discussed by Nomoto (1982) and Hachisu et al. (1996) are depicted.

In the **double degenerate scenario** it is assumed that two CO WDs are formed in a short orbit (orbital period less than 14 hours) and merge because of gravitational wave radiation within a Hubble time. This channel was suggested by Webbink (1984) and Iben & Tutukov (1984). When the orbit of the two CO WDs becomes smaller, the largest and least massive WD fills its Roche lobe and transfers material to the companion WD. Theoretical models show that under these conditions the CO WD ignites carbon off-centre before reaching the Chandrasekhar mass and forms an ONeMg WD. When the ONeMg WD approaches $1.4\, M_\odot$, magnesium starts capturing electrons which decreases the electron degeneracy pressure of the WD and the WD collapses into a neutron star (Nomoto & Kondo, 1991) instead of exploding as a SN Ia. This process is called an accretion-induced collapse. Multi-dimensional hydrodynamic models of the merger process show that the formation of an ONeMg WD and the resulting collapse can be prevented under certain conditions. This is the case, for example, when two CO WDs with comparable masses merge (Pakmor et al., 2010, 2011), or when a thin helium layer on top of the CO WD, a structure which is supported by observations of WDs, ignites first when it accretes material from the other CO WD. The detonation of the helium layer triggers an explosion inside the WD (Pakmor et al., 2013). Radiation transport calculations show that both models, however, need an accreting WD with a high enough density, which correspond to a mass greater than about $1.0\, M_\odot$, for the explosion to resemble a normal type Ia SN (Sim et al., 2010).

Additionally, Badenes & Maoz (2012) determine the WD merger rate in our galaxy, the Milky Way. They find that the merger rate is compatible with the observed SN Ia rate in galaxies similar to our own galaxy (Li et al., 2011b). However, this is the rate of all WD mergers, even mergers with a combined mass lower than $1.4\, M_\odot$. The merger rate of double WDs with a combined mass greater than $1.4\, M_\odot$ is about a factor of ten lower than the rate determined by Li et al. (2011b). This indicates that either another formation channel is necessary to reproduce the observed rate, or that WD mergers with a combined mass below $1.4\, M_\odot$ produce SNe type Ia.

The **single degenerate scenario** was suggested by Whelan & Iben (1973) and Nomoto (1982). In this scenario a CO WD accretes from a non-degenerate companion star. This companion can be a hydrogen-rich star, such as a main-sequence star or a red giant; or a helium-rich star, a star which has lost its entire hydrogen envelope because of previous binary interactions. However, because the WD is degenerate and very compact mass accretion is generally unstable. Nomoto (1982) calculated that a WD burns hydrogen stably into helium when the material is accreted at a rate of approximately $10^{-7}\, M_\odot\, \text{yr}^{-1}$ (Fig. 1.4). When material is accreted at a lower rate a layer of unburned material is built up at the surface which becomes degenerate. When a thick enough layer is accumulated a runaway burning process starts which results in the accreted layer being expelled from the WD. This type of explosion is called a nova (Fig. 1.4, Nariai et al., 1980). When material is accreted at a higher rate, it burns the material at the rate of stable burning. However it is not clear what happens with the rest of the accreted material (Fig. 1.4). According to Nomoto (1982) the material surrounding the WD forms a giant-like structure. Eventually the giant envelope fills its Roche potential and a contact systems is formed which leads to an unstable binary system. On the other hand, Hachisu et al. (1996) propose that a optically thick wind originates from the surface of the burning WD, driven by radiation pressure. This wind blows away the rest of the accreted material which also stabilizes...
mass transfer. Hachisu et al. (2008) take this idea one step further and argue that the ejected material possibly removes some material from the donor star.

The energy that is released during steady burning of the accreted hydrogen-rich material is in the form of soft X-rays (with a photon energy peak around 30 to 50 electronvolts, Van den Heuvel et al., 1992). However, the flux expected to be emitted by these sources in an entire galaxy if all SNe Ia originate from the single degenerate scenario has not been observed (Gilfanov & Bogdán, 2010). However, Hachisu et al. (2010) argue that Gilfanov & Bogdán (2010) overestimate the expected flux as no soft x-rays are emitted in the phase when the WD has an optically thick wind.

Another aspect of the single degenerate scenario is that after the WD explodes the companion star remains and is expected to have a high space velocity from the break-up of the binary system and to have lost some extra material by the impact of the supernova shell (Marietta et al., 2000). This remaining companion star should be observed when the luminosity of the SN explosion diminishes. Different groups tried to identify these surviving companions in several SN remnants, but so without success (Ruiz-Lapuente et al., 2004; Kerzendorf et al., 2009; Schaefer & Pagnotta, 2012).

Different groups have studied the theoretically expected SN Ia rate with binary population synthesis codes (Sect. 1.4.2) but have so far failed to reproduce all the observed characteristics of SNe Ia with either the double degenerate channel or the single degenerate channel, separately or combined (see Chapter 4). Additionally, some groups find that the single degenerate channel is the dominant formation channel, while others find that the double degenerate channel is dominant. Nelemans et al. (2013) compare the results of several groups performing binary population synthesis to investigate the SN Ia rate and find that the number of SNe predicted differs by a factor three between the results for the double degenerate channel and by more than a factor 5000 between the results for the single degenerate channel. Bours et al. (2013) show that differences between the adopted models to describe the retention efficiency of accreting WDs (Fig.1.4) partly explains the findings of Nelemans et al. (2013). However, it cannot fully explain the large range of model results obtained by different groups (Chapters 4 and 5).

There also is no consensus about the explosion mechanism. Below we discuss the main explosion models, for more information an overview is given in Hillebrandt & Niemeyer (2000).

- **In a prompt detonation** the combustion occurs supersonically. In this case the WD cannot expand during the flame propagation and mainly iron-peak elements and barely any intermediate-mass elements are produced, which disagrees with the observed SN Ia sample.

- **A turbulent deflagration** assumes that the combustion is subsonic. The WD expands during the burning phase, which result in the formation of a larger amount of intermediate-mass elements. However, the amount of iron-peak elements is smaller and some simulations show that not enough energy is released to completely unbind the WD and a remnant remains after the explosion, resulting in rather a subluminous SN Ia (Kromer et al., 2013).

- **A delayed detonation** starts as a deflagration and changes into a detonation under certain conditions. The advantage of this model is that the WD expands during the deflagration and therefore produces the right amount of iron-peak and intermediate-mass elements and also reproduces other characteristics of the SN (e.g. Blondin et al., 2013). However it is still under debate under which conditions the transition occurs.
Besides different theoretical aspects also the SN Ia rate remains uncertain. Fig. 1.5 displays the observationally determined delay time distribution, which is the number of SN Ia per unit mass of stars formed as a function of time, assuming that all the stars are formed at $t = 0$, and therefore shows the delay between the formation of the stars and the SN. Generally, stars in a galaxy are not all formed simultaneously, star formation can continue for a few Myr or even Gyr, or there can be multiple starbursts. Therefore to recover the delay time distribution the star formation history has to be determined. Additionally, during the evolution of a galaxy some material is removed by galactic outflows, e.g. because of energetic events such as SNe. As the delay time distribution is normalized to the initial mass of the galaxy, assumptions have to be made for this mass loss. Both give rise to uncertainties in the rate. The various observationally determined delay time distributions are also based on different type of galaxies, e.g. the rate of Maoz et al. (2011) is based on a sample of galaxies in cluster environments, while the Maoz et al. (2012) rate at long delay times ($>2.4$ Gyr) is based on a sample dominated by galaxies in the field, which are not gravitationally bound with other galaxies in a cluster. This might explain the SN Ia rate of Maoz et al. (2011) and Maoz et al. (2012) at long delay times ($>2.4$ Gyr) which differ more than $2\sigma$.

The main characteristics of the observed sample of SNe Ia are that they occur both in young ($\lesssim 300$ Myr) and old ($\gtrsim 300$ Myr) stellar populations and that the delay time distribution is approximately a continuous function. Most observationally determined delay time distributions agree well with a $t^{-1}$-relation (Totani et al., 2008; Maoz et al., 2011, 2012; Graur & Maoz, 2013), although some groups find a different distribution, such as $t^{-0.5}$ (Pritchet et al., 2008). Totani et al. (2008) argue that a $t^{-1}$ distribution corresponds to the delay time distribution expected from SNe Ia originating from the double degenerate channel. The time for two WDs to merge because of gravitational wave radiation depends on the separation $a$ between the two WDs, i.e. $\tau_{\text{GW}} \approx \tau_{\text{GW}} \propto a^4$. Assuming an initial distribution of separations of the binary systems $\psi(a) \propto a^{-1}$ (Abt, 1983; Kouwenhoven et al., 2007), and that double CO WDs follow the
same distribution function at moment of formation, this leads to a delay time distribution

\[
\text{DTD}(t) = \psi(a) \cdot \frac{\text{d}a}{\text{d}\tau_{\text{Ia}}} \propto \tau_{\text{Ia}}^{-1/4} \cdot \tau_{\text{Ia}}^{-3/4} = \tau_{\text{Ia}}^{-1}.
\]

(1.4)

Finally, the time-integrated rate of SNe Ia formed per unit mass found by different groups varies between \(0.39 \cdot 10^{-3}\) and \(2.9 \cdot 10^{-3}\) per solar mass of formed stars. Although the large range of time-integrated rates is mainly the result from differences between the observed rates in certain time bins, it also originates from the uncertainty in the slope of the delay time distribution, which has to be assumed to extrapolate the data points to estimate the time-integrated rate, which according to Graur et al. (2011) results in a uncertainty of the time-integrated rate of a factor three.

The last few years the Chandrasekhar-mass model has been under debate. What if the trigger necessary to explode the WD has a different origin (Fink et al., 2010; Kromer et al., 2010)? What happens if the WD spins up because of accretion and the density is not high enough to trigger the explosion at the moment the WD reaches \(1.4 M_\odot\) (Yoon & Langer, 2005, Chapter 3)? Even the possibility of super-symmetric particles which annihilate in the WD and trigger the explosion has been considered (Biermann & Clavelli, 2011), which would make a companion star unnecessary.

There remain many open question in the study of SNe Ia. In the last few years many (perhaps too many) solutions have been put forward. I would like to end with a quote from Hillebrandt & Niemeyer (2000): “It appears a miracle that all complexity seems to average out in a mysterious way to make the class (of SNe) so homogeneous.”

### 1.4 Methods

For the work that led to this thesis we used two different types of code, a detailed binary stellar evolution code and a binary population synthesis code. While a detailed stellar evolution code computes stellar models by solving the full set of stellar evolution equations, a binary population synthesis code uses recipes and fits to pre-computed stellar models. Below we discuss the advantages and disadvantages of both codes.

#### 1.4.1 Detailed Binary Stellar Evolution Codes

This type of code solves the full set of stellar evolution equations simultaneously, including the equations for the structure and the composition of the stars. We used the code STARS, which was developed by Eggleton (1971) and further updated by Pols et al. (1995), Eggleton (2006) and Glebbeek & Pols (2008). This code calculates the entire structure and evolution of a single or binary star from the zero-age main-sequence until carbon ignition, and follows the elements inside the star which are relevant for energy production by fusion up to this stellar phase. The star is thus followed over its entire life, as in massive stars the explosion follows shortly after carbon ignition \(^5\). The code has the resolution to correctly calculate deviations of thermal equilibrium, but cannot resolve processes that occur on dynamical or shorter timescales, e.g. the helium flash. Another limitation is that code is one dimensional, while to calculate some processes, such as convection or a common envelope phase, a three-dimensional code is necessary. Therefore prescriptions have to be considered to consider these processes.

\(^5\)For a \(15 M_\odot\) star the time spent until carbon burning is about \(10^7\) yrs, while the time between carbon burning and the explosion is only about \(10^3\) yrs.
Chapter 1: Introduction

This code calculates the evolution of one binary system in approximately a few minutes to a half hour of CPU time. Therefore it is not only possible to study the evolution of a binary system in great detail, but also to calculate a reasonably large grid of binary systems and their characteristics. However, it is not advantageous to calculate a full population of stars or to check the effect of many uncertainties in single and binary evolution, while possible, both are in practise too time consuming.

1.4.2 Binary population synthesis codes

To solve the issues of time-consuming simulations and limited flexibility, binary population synthesis codes have been developed over the last three decades. This type of code follows the entire evolution of a star without calculating the full set of stellar equations, instead it uses fits to results of pre-computed models calculated with detailed stellar evolution codes. The code calculates one binary system in a ms to one second of CPU time and, consequently, a million binaries on only a few hours, which makes this type of code ideal to study large stellar populations and the importance and influence of uncertain stellar physics on stellar populations. A binary population synthesis code makes a compromise between accuracy, simulation speed and flexibility.

In addition to these simulation-speed enhancing simplifications, a binary population synthesis code also makes other assumptions, for example about the initial distribution of single and binary stars or the retention efficiency of accreting WDs. This type of code is a very powerful tool, as it can make predictions for large stellar populations, however it should never be treated as a black box.

1.5 Chapters of this thesis

Chapter 2: Binary progenitor models of type IIb supernovae

Supernovae type IIb are classified as a type II SN during the first days of their observation, while at a later stage they are observed as a type Ib SN. This transitional type was first observed in SN 1993J, and regained scientific interest when Maund & Smartt (2009) observed the probable companion of this SN to be a blue supergiant, an uncommon type of SN progenitor. Also with the re-classification of the well-known supernova remnant Cassiopeia A as a type IIb SNe (Krause et al., 2008), the progenitor evolution towards this supernova type and the evolution of the possible remaining companion became more intriguing.

In this chapter we use a detailed stellar evolution code to investigate the binary evolution scenario towards SNe type IIb and to study the evolution of the companion star. We estimate the overall theoretical rate of this SN type and of the sub-paths which lead to different companion stars. Finally, we link the different paths to a set of observed supernovae of this type.

Chapter 3: Spin-up/Spin-down models for Type Ia supernovae

One of the major arguments against the single degenerate channel as the dominant progenitor channel of SN Ia is that a companion star has scarcely been observed, either directly or indirectly, even though it is difficult to hide the companion star during or after the explosion.

However, an accreting WD not only gains material but also angular momentum during the process, which spins up the white dwarf. In this chapter we discuss the effects of this spin-up process, as the density of a rotating white dwarf deviates from the density of a non-rotating white dwarf of the same mass. This not only affects the moment when the WD explodes, but it also implies that the WD possibly has
to spin down before it explodes. This results in a delay between the accretion process and the explosion, and therefore diminishes the probability of observing the companion star at the moment of explosion. We discuss both the observational and the evolutionary consequences of this spin-up/spin-down process.

Chapter 4: Theoretical uncertainties of the type Ia supernova rate
Both SN Ia progenitor channels, the double degenerate and single degenerate channel, have problems reproducing the observed SN Ia rate. Different groups studying the rate with theoretical models cannot reproduce the rate (Sect. 1.3.2).

However, various binary evolution aspects are ill-constrained, as are the initial distributions functions of binary systems. This chapter studies these aspects and their effect on the progenitor evolution and the SN Ia rate. We investigate the progenitor evolution towards SN Ia and the theoretical SN Ia rate with a binary population synthesis code. Subsequently we analyse the influence of uncertain stellar physics, binary physics and distribution functions on the progenitor evolution and the rate. We distinguish the main characteristics of SNe Ia and examine whether the rate and delay time distribution can be reproduced within the uncertainties.

Chapter 5: PopCORN: hunting down the differences between binary population synthesis codes
Different binary population synthesis codes often give conflicting results. In this chapter we compare the results of four different binary population synthesis codes. To make the comparison as simple as possible and to study the inherent differences between the codes, we equalize the assumptions and free parameters in the four codes as far as possible. For two different binary populations, with either one or two white dwarfs at the moment of formation, we compare their progenitor evolution and their formation rate. We identify the similarities between the results of the four codes and if differences arise we discuss whether they are due to differences in the numerical treatments or differences in the input physics.
Binary progenitor models of type IIb supernovae

J.S.W. Claeys, S.E. de Mink, O.R. Pols, J.J. Eldridge, M. Baes

A&A 2011, 528, A131

Abstract

Massive stars that lose their hydrogen-rich envelope down to a few tenths of a solar mass explode as extended type IIb supernovae, an intriguing subtype that links the hydrogen-rich type II supernovae with the hydrogen-poor type Ib and Ic. The progenitors may be very massive single stars that lose their envelope due to their stellar wind, but mass stripping due to interaction with a companion star in a binary system is currently considered to be the dominant formation channel.

Anticipating the upcoming automated transient surveys, we computed an extensive grid of binary models with the Eggleton binary evolution code. We identify the limited range of initial orbital periods and mass ratios required to produce type IIb binary progenitors. The rate we predict from our standard models, which assume conservative mass transfer, is about six times smaller than the current rate indicated by observations. It is larger but still comparable to the rate expected from massive single stars. We evaluate extensively the effect of various assumptions such as the adopted accretion efficiency, the binary fraction and distributions for the initial binary parameters. To recover the observed rate we must generously allow for uncertainties and consider low accretion efficiencies in combination with limited angular momentum loss from the system.

Motivated by the claims of detection and non-detection of companions for a few IIb supernovae, we investigate the properties of the secondary star at the moment of explosion. We identify three cases: (1) the companion is predicted to appear as a hot O star in about 90% of the cases, as a result of mass accretion during its main sequence evolution, (2) the companion
becomes an over-luminous B star in about 3% of the cases, if mass accretion occurred while crossing the Hertzsprung gap or (3) in systems with very similar initial masses the companion will appear as a K supergiant. The second case, which applies to the well-studied case of SN 1993J and possibly to SN 2001ig, is the least common case and requires that the companion very efficiently accretes the transferred material – in contrast to what is required to recover the overall IIb rate. We note that relative rates quoted above depend on the assumed efficiency of semi-convective mixing: for inefficient semi-convection the presence of blue supergiant companions is expected to be more common, occurring in up to about 40% of the cases.

Our study demonstrates that type IIb supernovae have the potential to teach us about the physics of binary interaction and about stellar processes such as internal mixing and possibly stellar-wind mass loss. The fast increasing number of type IIb detections from automated surveys may lead to more solid constraints on these model uncertainties in the near future.

2.1 Introduction

Core collapse supernovae are the bright explosions marking the end of the lives of massive stars. Their light curves and spectral signatures come in a large variety of types and yield information about the structure and chemical composition of the progenitor star and its surroundings. Type II supernovae – characterised by strong hydrogen lines – are associated with massive stars that are still surrounded by their hydrogen-rich envelope at the time of explosion, whereas type Ib and Ic supernovae – in which no signature of hydrogen is found – are thought to result from massive stars that have lost their entire hydrogen-rich envelope.

Type IIb supernovae constitute an intriguing intermediate case. Initially they show clear evidence for hydrogen, but later the hydrogen lines become weak or absent in the spectra. Two famous examples of this subtype are SN 1993J (Ripero et al., 1993) and Cassiopeia A, which was recently classified using the scattered light echo (Krause et al., 2008). The typical light curve of a type IIb supernova, such as SN 1993J, is characterized by two peaks. The first maximum is associated with shock heating of the hydrogen-rich envelope, resembling a type II supernova. The second maximum is caused by the radioactive decay of nickel (Benson et al., 1994). These characteristics can be explained assuming that the progenitor star had an extended low-mass hydrogen envelope at the time of explosion, with between 0.1 and 0.5 $M_\odot$ of hydrogen (Podsiadlowski et al., 1993; Woosley et al., 1994; Elmhamdi et al., 2006). Progenitors with smaller hydrogen envelope masses are compact, but can be classified as type IIb supernovae if hydrogen lines are detected shortly after the explosion, e.g. Chevalier & Soderberg (2010). In this paper we focus on the progenitors of extended type IIb supernovae.

Massive single stars with masses of about 30 $M_\odot$ or higher can lose their envelope as a result of a stellar wind. Alternatively, massive stars can be stripped of their envelope due to interaction with a companion star in a binary system. Quickly after the discovery of SN 1993J many authors realized that the single star scenario requires precise fine tuning of the initial mass of the star in order to have a very low-mass envelope left at the time of the explosion (Podsiadlowski et al., 1993; Woosley et al., 1994, and references therein). According to these authors, mass stripping from an evolved red supergiant in a binary system naturally leads to a hydrogen envelope containing a few times 0.1 $M_\odot$ at the moment of explosion. More compact binaries, where the primary star is stripped in a less evolved stage, result in smaller hydrogen
envelopes and may lead to type IIb supernovae of the compact category (e.g. Yoon et al., 2010) or type Ib supernovae.

Nomoto et al. (1993) proposes a third scenario which involves a common envelope phase and finally a merger of the core of the primary star and its companion. Energy to eject the envelope is extracted from the orbital energy, but a small layer of hydrogen may remain on the surface of the merger product until the moment of explosion as a supernova type IIb. This scenario was recently considered by Young et al. (2006) for Cassiopeia A, in which no evidence for a companion star was found, despite numerous attempts.

A decade after the detection of 1993J, the interest for type IIb supernova revived when Maund et al. (2004) found evidence for the presence of a blue supergiant companion at the location where 1993J had just faded away. These findings nicely fitted the predictions for the companion star in the models by Podsiadlowski et al. (1992, 1993).Stancliffe & Eldridge (2009) performed the first systematic study of binary progenitor models that matched the properties of the progenitor of 1993J and its companion. They computed a grid of binary models varying the initial primary masses and initial orbital periods. Even though they find a suitable progenitor model, they emphasize that it proves to be extremely difficult to explain the properties of the blue supergiant companion star.

At the time of writing about 69 supernovae have been classified as IIb supernova\(^1\). This number will increase quickly thanks to upcoming systematic wide-field surveys of transient events, such as the Palomar Transient Factory (e.g. Arcavi et al., 2010), Pan-STARRS (Kaiser et al., 2002) and the Large Synoptic Survey Telescope (Ivezic et al., 2008). About 9 have been studied well enough such that constraints can be derived on the progenitors and in some cases on possible companion stars.

These observational developments and the questions raised by Stancliffe & Eldridge (2009) motivated us to further investigate binary progenitor models for type IIb supernova. In contrast with previous work we will not just discuss the case of SN 1993J. Instead we discuss the properties of the progenitor stars and their companions in general. We extend the work of Stancliffe & Eldridge (2009) by exploring the parameter space of binary progenitor models in detail, in particular the dependence on the initial mass ratio. Furthermore we discuss the properties of the companion star at the time of explosion.

In section 2.2 we review observed type IIb supernovae and the observed rates. In section 2.3 we outline the assumptions in our binary evolution code. In section 2.4 we briefly discuss single-star progenitor models. Section 2.5 we discuss our progenitor models and in section 2.6 the rate of IIb supernovae predicted from our models. Section 2.7 contains a discussion, conclusion and outlook.

### 2.2 Observed type IIb Supernovae

In recent years about 69 supernovae of type IIb have been detected\(^1\). These observations enabled the first estimates for the rate of type IIb supernovae. The determination of the rate is not straightforward due to observational biases and selection effects. For example, if the supernova is observed too late it will be classified as a type Ib supernova instead of type IIb (see also Maurer et al., 2010). Van den Bergh et al. (2005) and Li et al. (2007) derive that 3.2 ± 1.0% and 1.5 ± 1.5% of all core collapse SNe (CCSNe) are of type IIb. They based their estimates on the discoveries by the Lick Observatory Supernova Search (LOSS), respectively using a 140 and 30 Mpc distance-limited sample. Van den Bergh et al. (2005) also specified that the distribution of SNe IIb and SNe II does not depend on the morphological type of the

---

\(^1\)http://www.cfa.harvard.edu/iau/lists/Supernovae.html
Chapter 2: Binary progenitor models of type IIb supernova

host galaxy. Smartt et al. (2009); Smartt (2009) evaluated different determinations of the rate and the effect of observational biases. They estimate the rate to be $5.4 \pm 2.7\%$ based on observations covering 10.5 years within 28 Mpc. However, there are only 5 IIb supernovae in their sample, of which at least two are compact IIb supernovae. Preliminary results of the Palomar Transient Factory survey point towards a rate of $3.6 \pm 2.5\%$ in giant hosts and a larger fraction in dwarf galaxies, $20 \pm 11\%$ (Arcavi et al., 2010).

While this study was near completion Smith et al. (2011) presented the results of a homogeneous volume limited survey within 60 Mpc which includes 80 CCSNe. They find a significantly higher fraction of type IIb supernova, $10.6^{+3.6}_{-3.1}\%$, with respect to previous studies. They attribute the difference to their more complete photometric and spectroscopic follow-up observations used to classify type II supernovae into different subtypes. It is unclear how many of the type IIb supernovae quoted in these studies are extended instead of compact.

For the purpose of comparison with our model predictions we will use the rate quoted by Smartt et al. (2009) after taking out the two compact Type IIb, resulting in a fraction of extended type IIb of about $3\%$ with respect to all core collapse supernovae. We emphasize that this number is still very uncertain. More reliable determinations of the rate of extended type IIb supernovae are expected in the near future from the current and upcoming automated transient surveys."

In the follow paragraphs we discuss some observed supernovae type IIb and discuss their observed characteristics.

2.2.1 Individual type IIb supernovae

2.2.1.1 SN 1987K

SN 1987K was the first supernova observed with the characteristics of a type IIb supernova, namely the transition from a supernova type II to type Ib (Filippenko, 1988), but it was only later that it was defined a SN of type IIb. Filippenko (1988) already suggested that these characteristics could be due to mass loss of a massive star, with an initial mass between 20 and 25 $M_\odot$ or a combination of mass transfer and winds of a less massive star, between 8 and 20 $M_\odot$.

2.2.1.2 SN 1993J

SN 1993J was the first supernova classified as type IIb. This supernova has been well studied because the progenitor was detected and recognized as a star with a spectral class K0Ia (Filippenko et al., 1993). Its binary companion has been observed recently as an early B-supergiant (with best estimate a B2Ia star, Maund & Smartt, 2009). The light curve of the supernova can be explained if the star had an amount of hydrogen in the envelope of about 0.1–0.5 $M_\odot$ at time of explosion (Woosley et al., 1994; Filippenko et al., 1993). Maund et al. (2004) determined the luminosity and effective temperature of the progenitor of the supernova and its companion, namely $\log L/L_\odot = 5.1 \pm 0.3$ and $\log T_{\text{eff}}/K = 3.63 \pm 0.05$ for the progenitor and $\log L/L_\odot = 5 \pm 0.3$ and $\log T_{\text{eff}}/K = 4.3 \pm 0.1$ for the companion. The ejecta mass was determined, using the width of the second peak of the light curve, to be about 4 $M_\odot$ (Shigeyama et al., 1994). Radio and X-ray observations showed evidence for a mass loss rate of about $4 \cdot 10^{-5} M_\odot \text{yr}^{-1}$ at the time of explosion (Fransson et al., 1996).
2.2 Observed type IIb Supernovae

2.2.1.3 SN 1996cb

Supernova 1996cb was also determined to be of type IIb (Qiu et al., 1999). The light curve of this supernova was not observed until several days after the explosion. Instead of the two peaks that are typical for type IIb light curves, the light curve consists of a short-term plateau phase similar to a type II supernova. These differences with SN 1993J arise from a more massive hydrogen envelope of SN 1996cb (Qiu et al., 1999).

2.2.1.4 SN 2000H

Benetti et al. (2000) classified SN 2000H a type IIb supernova because of the hydrogen lines observed in the spectrum. However, these hydrogen lines were not obvious and the light curve showed more resemblance with a SN Ib (Branch et al., 2002). Therefore, this supernova has been classified by other authors as a type Ib supernova (Branch et al., 2002; Elmhamdi et al., 2006). The amount of hydrogen was estimated to be 0.08 $M_\odot$ (Elmhamdi et al., 2006).

2.2.1.5 SN 2001gd

The observations of radio light curves give an estimate of the mass loss rate of the progenitor system of the supernova. The radio light curve of the type IIb SN 2001gd indicates a mass loss rate of about $2 \sim 12 \cdot 10^{-5} M_\odot \text{yr}^{-1}$ (Stockdale et al., 2003; Pérez-Torres et al., 2005), which is in between the rates for typical type II and type Ib SNe.

2.2.1.6 SN 2001ig

Another supernova of type IIb, SN 2001ig, was observed in 2001 in NGC 7424 and shows a spectral evolution similar to that of SN 1993J (Ryder et al., 2006). Evidence was found for a star of spectral type late-B through late-F at the location of SN 2001ig, a possible companion of the progenitor of SN 2001ig (Ryder et al., 2006). Kotak & Vink (2006) suggested that observed modulations in the radio light curve indicate the star was not a red giant but a luminous blue variable (LBV) at time of explosion.

2.2.1.7 SN 2003bg

SN 2003bg evolved from a type Ic supernova to a hydrogen-rich type IIb, to a hydrogen-poor type Ibc (Soderberg et al., 2006). It was observed as a broad-lined type IIb supernova and proclaimed to be 'the first type IIb hypernova' (Mazzali et al., 2009; Hamuy et al., 2009). The broadness of the lines indicates a high progenitor mass (Hamuy et al., 2009). The light curve and spectral evolution indicate the presence of a thin layer of hydrogen at time of explosion, $\approx 0.05 M_\odot$ (Mazzali et al., 2009). The velocity of the ejecta resembles more closely the velocity of SNe type Ib than type II (Soderberg et al., 2006). This implies a compact Wolf-rayet progenitor, with a progenitor mass between 20 and 25 $M_\odot$. Soderberg et al. (2006) conclude that this event is an intermediate case between SNe type IIb and type Ib. The progenitor of this supernova was also suggested to be a LBV from its radio light curve by Kotak & Vink (2006).
2.2.8 SN 2008ax

The light curve of SN 2008ax shows some differences with the light curve of SN 1993J, namely the lack of the first peak and it has slightly bluer colors (Pastorello et al., 2008). These features can be explained by a less massive hydrogen envelope at time of explosion, less than a few × 0.1 \( M_\odot \), in comparison with the progenitor of SN 1993J (Crockett et al., 2008).

2.2.9 Cas A

Cas A is the supernova remnant of a star that exploded about 350 years ago (Thorstensen et al., 2001). A light echo from this explosion (Krause et al., 2008) shows evidence that it was a supernova of type IIb. Direct methods to determine the mass of the progenitor star are difficult, but the ejecta mass was calculated to be 2–4 \( M_\odot \) and the remnant would be expected to be a neutron star with a mass between 1.5 and 2.2 \( M_\odot \) (Young et al., 2006). This sets the mass of the star at time of its explosion at about 4–6 \( M_\odot \). There is no direct evidence as to whether this supernova was of the compact or extended type IIb, but the possibility that the progenitor was a red supergiant is left open. For this supernova single and binary progenitor models were calculated (Young et al., 2006). The single star models indicated fine-tuning of the stellar wind is necessary to evolve to the specific characteristics of the supernova remnant Cas A (Young et al., 2006). Besides, there is evidence that the progenitor could only have had a very short-lived Wolf-rayet phase, which is difficult to explain with single stars (Schure et al., 2008; Van Velzen et al., 2009). There is no evidence for a companion star. Therefore a common envelope scenario was proposed, in which the two stars merge into a single star before explosion. Observations show tentative evidence for this scenario (Krause et al., 2008), such as the asymmetric distribution of the quasi-stationary flocculi near Cas A, which could arise from the loss of a common envelope.

The observations put constraints on the general properties of a SN type IIb: the explosion of a supergiant with a hydrogen envelope mass between 0.1 and 0.5 \( M_\odot \). We consider the lower limit of the mass of the hydrogen envelope to be 0.1 \( M_\odot \) rather than 0.01 \( M_\odot \), the lower limit proposed by Chevalier & Soderberg (2010). The explosion of a star with a hydrogen envelope mass smaller than 0.1 \( M_\odot \) will exhibit hydrogen lines in its spectrum, but only in the early phases of the supernova. In addition the light curve resembles the typical light curve of a SN type Ib. Therefore such a supernova will more likely be defined as a supernova type Ib or a transitional type between IIb and Ib (see examples above, e.g. SN 2003bg and SN 2000H). Elmhamdi et al. (2006) places the upper limit of the mass of the hydrogen envelope of the progenitor of a type Ib supernova at 0.1 \( M_\odot \).

A binary progenitor is confirmed or considered likely in some cases, and the secondary has been detected as a blue supergiant in possibly two cases. However, no general constraints on the secondary can be set.

2.3 Stellar evolution calculations

We use a version of the binary evolution code STARS originally developed by Eggleton (1971) and later updated and described by various authors (e.g. Pols et al., 1995; Eggleton, 2006; Glebbeek & Pols, 2008). The code is fully implicit and solves the equations for the structure and composition of the star simultane-
2.3 Stellar evolution calculations

It employs an adaptive non-Lagrangian mesh that places mesh points in regions of the star where higher resolution is required. This allows us to evolve stars with a reasonable accuracy using as few as 200 mesh points. The code therefore is fast and suitable to compute the large numbers of models needed to investigate wide initial parameter space of binary systems (e.g. De Mink et al., 2007).


Convection is implemented using a diffusion approximation (Eggleton, 1972) of the mixing-length theory (Böhm-Vitense, 1958), assuming a mixing length of 2.0 pressure scale heights. We use the Schwarzschild criterion to determine the boundaries of the convective regions. Convective overshooting is taken into account using the prescription of Schroder et al. (1997) with an overshooting parameter of $\delta_{ov} = 0.12$, which was calibrated against accurate stellar data eclipsing binaries (Pols et al., 1997). In terms of the pressure scale height, as the overshooting parameter is commonly defined in other stellar evolution codes, this value approximately compares to $\alpha_{ov} \approx 0.25$.

Mass loss in the form of a stellar wind is taken into account adopting the prescription by De Jager et al. (1988). Although for O and B stars this prescription has been superseded by more recent mass-loss determinations, this is not the case for the red supergiant region of the H-R diagram where the only significant mass loss in our binary models occurs. For computational reasons we ignore stellar-wind mass loss from the less massive companion star and we ignore any possible accretion from the stellar wind of the primary.

To compute the evolution of interacting binaries we evolve the two stars quasi-simultaneously. First we follow the primary for several steps and store the changes of the masses of both stars and of the orbit. Afterwards the secondary is evolved applying these mass changes until its age reaches that of the primary. When the primary star expands beyond its Roche lobe we compute the mass-transfer rate as a function of the difference in potential between the Roche-lobe surface and the stellar surface. The mass flux of each mesh point beyond the Roche lobe is given by

$$\frac{dM}{dm} = -C \cdot \frac{\sqrt{2\phi_s}}{r},$$

as in De Mink et al. (2007), where $m$ and $r$ denote the mass and radius coordinate of the mesh point and $\phi_s$ the difference in potential with respect to the Roche-lobe surface. $C$ is a proportionality constant which we set to $10^{-2}$ for numerical convenience. The mass transfer rate is then given by the integral of Eq. (2.1) over all mesh points outside the Roche radius. As a result of the implementation of mass transfer in our code we implicitly assume that the entropy and composition of the accreted material is equal to the entropy and composition of material at the surface of the accreting star (e.g. Pols, 1994). Tidal effects are not taken into account in this work.

To investigate the effects of non-conservative mass transfer, i.e. mass and angular momentum loss from the system during Roche-lobe overflow, we assume that a constant fraction $\beta$ of the transferred mass is accreted by the companion. We adopt different values: $\beta = 1$ (conservative mass transfer) and $\beta = 0.75, 0.5, 0.25$ (non-conservative mass transfer). The specific angular momentum of the mass lost from the system is assumed to be the specific angular momentum of the orbit of the accreting star.

Using this code, we calculated a grid of binary systems for an initial primary mass of $15 M_\odot$. We varied the initial orbital period between 800 and 2100 days in steps of 100 days and we varied the initial
secondary mass between about 10 and 15 $M_\odot$ in steps of 0.1 $M_\odot$. For systems with similar masses we increased the resolution by varying the secondary mass in steps of 0.01 $M_\odot$. In addition we computed several systems with different primary masses.

The stars are evolved until the onset of carbon burning. The star will explode shortly after this point, about $2.6 \cdot 10^3$ years for a star of 15 $M_\odot$ (El Eid et al., 2004). To determine the masses of the stars and in particular of their envelopes at the time of explosion we extrapolate the mass-loss rate by stellar winds and Roche-lobe overflow to determine the masses at time of explosion. When both stars fill their Roche lobe at the same time and a contact binary is formed, we end our simulation. In this case the binary system will probably evolve into a common envelope.

We note that whether or not a contact system forms is sensitive to some of our assumptions regarding mass transfer. The chosen value of $C$ in eq. (2.1) leads to a maximum mass-transfer rate that is lower than the self-regulated rate on the thermal timescale of the donor star. If we choose a larger value of $C$ the mass-transfer rate increases, which affects the response of the secondary star. Since mass transfer is faster than the thermal timescale of the secondary, more rapid accretion results in a stronger radius expansion of the secondary and a higher likelihood of forming a contact system. On the other hand, the transferred material comes from the surface of a red supergiant which has a much smaller specific entropy than the hot surface of the secondary. Although the gas may undergo additional heating during accretion, this is probably not sufficient to make its entropy equal to the surface entropy of the secondary as we implicitly assume. Taking this into account properly, which is very difficult, would likely result in less radius expansion of the secondary. These two simplifying assumptions thus have opposite effects, making the true boundary between systems that do and do not come into contact hard to predict from our models (see Section 2.5.2 and Section 2.7).

<table>
<thead>
<tr>
<th>$M_i$  ($M_\odot$)</th>
<th>$M_f$  ($M_\odot$)</th>
<th>$M_H$  ($M_\odot$)</th>
<th>$\log \left( \frac{L}{L_\odot} \right)$</th>
<th>$T_{\text{eff}}$ ($10^3$ K)</th>
<th>SN type</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>13.94</td>
<td>1.014</td>
<td>5.56</td>
<td>3.6</td>
<td>IIL</td>
</tr>
<tr>
<td>32.5</td>
<td>12.89</td>
<td>0.505</td>
<td>5.57</td>
<td>3.8</td>
<td>Ib</td>
</tr>
<tr>
<td>33</td>
<td>11.81</td>
<td>0.080</td>
<td>5.58</td>
<td>4.3</td>
<td>Ib</td>
</tr>
</tbody>
</table>

Notes: The final mass $M_f$ and the amount of hydrogen in the envelope $M_H$ are given at the time of explosion, together with the luminosity $\log(L/L_\odot)$ and effective temperature $T_{\text{eff}}$ at the onset of central carbon burning.

### 2.4 Single star progenitors

At the time of explosion the progenitor of a type IIb supernova is surrounded by a low-mass hydrogen envelope. In massive single stars the envelope can be removed by the stellar wind. Heger et al. (2003) find that at solar metallicity the range for type IIb and IIL supernovae combined ranges from roughly 26–34 $M_\odot$. Type IIL supernovae show a linear decline in the light curve, but without transition to a type Ib, which requires a hydrogen-rich envelope of less than about 2 $M_\odot$ at the time of explosion. Eldridge & Tout (2004) map single-star progenitors of different types of supernovae as a function of metallicity. At solar
metallcity they find that IIL and IIb supernova progenitors combined should have initial masses between approximately 25 and 30 \(M_\odot\). Georgy et al. (2009) discuss single star SN progenitors from rotating models at different metallicities. Although they do not discuss type IIb or IIL in particular, they find that the transition from type II to Ib should occur around 25 \(M_\odot\). Pérez-Rendón et al. (2009) investigate possible single star progenitors for Cas A and need a progenitor of approximately 30 \(M_\odot\). In particular, a single star scenario has been proposed for hypernova SN 2003bg to explain the high mass. On the other hand a single star scenario cannot explain the characteristics of SN 1993J (see Sec. 2.2.1.2). Consequently, the single star scenario cannot explain all observed type IIb supernovae.

The determination of the rate from single stars is sensitive to uncertainties in the stellar wind mass loss rates. Especially with respect to stellar winds from massive red giants our understanding is limited. Yoon & Cantiello (2010) speculate that a superwind driven by pulsational instabilities may drive a strong mass loss, bringing the minimum mass for type IIb supernovae down to about 20 \(M_\odot\).

Single stars with a mass above \(\sim 25 M_\odot\) are believed to produce only faint supernovae (Fryer, 1999). Consequently, these type IIb SNe will appear different than type IIb SNe formed by binary stars. Nevertheless, in the correct mass range, single stars can explode as type IIb SNe and therefore it is reasonable to compare the expected rate from binary and single-star progenitors. To be able to do this comparison we computed single stellar models with the same input physics as the binary models we discuss in the next section. We find that the initial mass range for single stars resulting in type IIb progenitors should be within 32.5–33 \(M_\odot\). In Table 2.1 we list the final mass and the amount of hydrogen left in the envelope near this range of initial masses. Assuming a Kroupa initial mass function (Kroupa, 2001) we estimate that about 0.3% of all single stars more massive than 8 solar masses are within this mass range and will end their lives as a type IIb supernova.

### 2.5 Binary star progenitors

Table 2.2: Properties of binary stellar models for an initial primary mass of 15 \(M_\odot\), conservative mass transfer and with variation of the initial secondary mass \(M_{2i}\) and initial orbital period \(P_{\text{orb}}\).

<table>
<thead>
<tr>
<th>#</th>
<th>(M_{2i} (M_\odot))</th>
<th>(M_{1f} (M_\odot))</th>
<th>(M_{2f} (M_\odot))</th>
<th>(P_{\text{orb}}) (days)</th>
<th>(M_{\text{H}<em>2} (M</em>\odot))</th>
<th>(\log\left(\frac{L_1}{L_\odot}\right))</th>
<th>(T_{\text{eff,1}} (10^3 \text{K}))</th>
<th>(\log\left(\frac{L_2}{L_\odot}\right))</th>
<th>(T_{\text{eff,2}} (10^3 \text{K}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>14</td>
<td>5.93</td>
<td>20.92</td>
<td>1500</td>
<td>0.354</td>
<td>5.07</td>
<td>3.48</td>
<td>4.95</td>
<td>31.4</td>
</tr>
<tr>
<td>#2</td>
<td>14.35</td>
<td>5.98</td>
<td>21.22</td>
<td>1500</td>
<td>0.370</td>
<td>5.07</td>
<td>3.47</td>
<td>5.06</td>
<td>17.9</td>
</tr>
<tr>
<td>#3</td>
<td>14.55</td>
<td>6.01</td>
<td>21.40</td>
<td>1500</td>
<td>0.390</td>
<td>5.07</td>
<td>3.45</td>
<td>4.85</td>
<td>3.82</td>
</tr>
<tr>
<td>#4</td>
<td>14.95</td>
<td>6.02</td>
<td>21.79</td>
<td>1400</td>
<td>0.401</td>
<td>5.07</td>
<td>3.51</td>
<td>5.07</td>
<td>3.63</td>
</tr>
</tbody>
</table>

**Notes:** The final mass of the primary and secondary, \(M_{1f}\) and \(M_{2f}\), and the amount of hydrogen in the envelope \(M_{\text{H}_2}\) are given at the time of explosion, together with the luminosity \(\log(L/L_\odot)\) and effective temperature \(T_{\text{eff}}\) of the primary and secondary at the onset of central carbon burning in the primary (except for model #4, where \(L_2\) and \(T_{\text{eff,2}}\) refer to the last computed model of the secondary).

In this section we discuss binary progenitor models for type IIb supernova. We compute the evolution of binary models with different initial orbital periods and different initial mass ratios. We adopt an initial primary mass of 15 \(M_\odot\) in agreement with the progenitor model proposed for 1993J (Maund et al., 2004).
We assume that type IIb supernovae result from massive stars that undergo core collapse with an envelope which contains between 0.1 and 0.5 $M_\odot$ of hydrogen. This criterion is based on observations, previous models (Podsiadlowski et al., 1993; Woosley et al., 1994; Elmhamdi et al., 2006) and some test models which proved that a hydrogen mass less than 0.1 $M_\odot$ gives rise to a compact rather than an extended progenitor.

As an example we discuss a system with an initial orbital period of 1500 days and initial masses of 15 and 14.35 $M_\odot$ for the primary and secondary star respectively. The initially most massive star evolves faster and experiences significant mass loss in the form of a stellar wind when it ascends the giant branch and during central helium burning. After about 13.1 Myr, when the helium mass fraction has dropped below 0.5 in the center, it fills its Roche lobe (late case B mass transfer, Kippenhahn & Weigert, 1967). At this moment it has already lost more than 1 $M_\odot$ and has become less massive than its companion. The

<table>
<thead>
<tr>
<th>$N^0$</th>
<th>$M_{2i}$ ($M_\odot$)</th>
<th>M(U)</th>
<th>M(B)</th>
<th>M(V)</th>
<th>M(R)</th>
<th>M(I)</th>
<th>M(J)</th>
<th>M(H)</th>
<th>M(K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>14</td>
<td>-5.89</td>
<td>-4.99</td>
<td>-4.65</td>
<td>-4.66</td>
<td>-4.37</td>
<td>-4.02</td>
<td>-3.91</td>
<td>-3.80</td>
</tr>
<tr>
<td>#2</td>
<td>14.35</td>
<td>-7.12</td>
<td>-6.49</td>
<td>-6.22</td>
<td>-6.27</td>
<td>-6.03</td>
<td>-5.77</td>
<td>-5.68</td>
<td>-5.65</td>
</tr>
</tbody>
</table>
reversal of the mass ratio before the onset of Roche-lobe overflow helps to stabilize the mass transfer.

In Figure 2.1 we depict the mass-transfer rate as a function of the remaining envelope mass. We find that the mass transfer initially takes place on a timescale equal to the thermal timescale of the primary star. The maximum mass-transfer rate during this phase is \( 6 \cdot 10^{-4} \, M_{\odot} \, \text{yr}^{-1} \). Although this phase lasts only about 0.05 Myr, about \( 4.5 \, M_{\odot} \) is transferred. After this phase the star keeps filling its Roche lobe and mass transfer continues on the nuclear timescale, at a rate comparable to the mass-loss rate in the form of a stellar wind, about \( 3 \cdot 10^{-6} \, M_{\odot} \, \text{yr}^{-1} \). During this phase the primary expands on its nuclear timescale while it is burning helium in its center. This phase lasts about 0.8 Myr and about \( 2.5 \, M_{\odot} \) is transferred. After central helium exhaustion the star expands again on its thermal timescale and a second maximum in the mass-transfer rate occurs. Finally, at the onset of carbon burning the star expands again resulting in a third peak in the mass-transfer rate, see Fig. 2.1. We follow the evolution of the system up to this point, when the mass of hydrogen in the envelope has decreased to 0.46 \( M_{\odot} \). Extrapolating the mass-loss rate we find that the amount of hydrogen in the envelope at the time of explosion will be about 0.37 \( M_{\odot} \). Therefore we expect that the primary star explodes as a type IIb supernova.

In wider systems mass transfer starts in a later phase of the evolution of the primary star. Because the primary stars in these systems are more evolved, there is less time available to reduce the mass of the envelope before the explosion. In addition, stellar winds had more time to reduce the mass of the primary star before the onset of Roche-lobe overflow. Reversal of the mass ratio stabilizes the process of mass transfer. Vice versa we find that stars in binary systems with lower initial orbital periods remain with smaller envelope masses at the time of explosion. We give details of all our computed models in Tables 2.7–2.10 in the Appendix.

### 2.5.1 Properties of the companion

In general the companion star is relatively unevolved, i.e. still on the main sequence, at the onset of mass transfer. However, when the initial mass ratio is close to one, the evolutionary timescales of the primary and secondary star are comparable and the companion can be more evolved. The response of the companion to mass accretion depends on its evolutionary stage. We distinguish three different cases: (1) accretion starts while the companion is on the main sequence, (2) accretion starts while the companion is crossing the Hertzsprung gap and (3) accretion starts while the companion is a giant. The evolution of the stars in these three cases is illustrated in Figure 2.2, where we give the evolutionary tracks of both stars in the Hertzsprung-Russell diagram and a Kippenhahn diagram illustrating the changes in the internal structure as a result of mass transfer. Table 2.2 list several properties of the binary models described here.

The first case, accretion during the main sequence, is depicted in the left panels of Figure 2.2. The accreting star responds to the increase in mass by adapting its internal structure. The size of its convective core increases and fresh hydrogen is mixed towards the center, effectively rejuvenating the star. After accretion the properties of the star are similar to the properties of a younger single star of the same mass. The star becomes brighter but remains hot, appearing as an O star, see also Table 2.2 (model #1).
Figure 2.2: Response of the accreting star in three different cases (1) left panels: accretion during main sequence, (2) central panels: accretion during Hertzsprung gap and (3) right panels: accretion during giant branch. In the top row we show the evolutionary tracks of both stars in the Hertzsprung-Russell diagram. In the bottom row we illustrate the evolution of the internal structure of the accreting star as a function of time around the moment of accretion. The total mass (black line), the core mass (black line) and the mass coordinates of the region in which nuclear burning takes place (red line) are plotted versus time. Grey areas indicate convective regions. In all models we assumed a primary mass of 15 $M_\odot$, an initial orbital period of 1500 days and a secondary masses of 14, 14.35 and 14.55 $M_\odot$ to illustrate the three different cases. These diagrams correspond to models #1, #2 and #3 in Table 2.2.
2.5 Binary Star Progenitors

The central panels of Figure 2.2 show an example of the second case, accretion during the Hertzsprung gap (model #2 in Table 2.2). In this phase nuclear burning takes place in a shell around the core. This prevents the star from adapting its internal structure to that of a single star of its new increased total mass. Having a core mass which is too small compared to the core mass of a normal single star, the star appears as an over-luminous B supergiant. This type of progenitor model has been proposed to explain the properties of the blue companion of SN 1993J.

For systems with very similar initial masses, accretion takes place while the secondary resides on the giant branch, see the right panels of Fig 2.2. In this third case we find that the secondary will appear as a K supergiant at the moment the primary explodes (model #3 in Table 2.2). If the initial mass ratio is even closer to one we find that the evolution of the secondary is accelerated enough to catch up with and overtake the evolution of the primary. In this case the secondary explodes first, as a normal type II supernova, while the primary explodes afterwards as a type IIb supernova. The time difference between the explosions is 8000 years for model #4 in Table 2.2, but can be up to $10^5$ years for very close mass ratios (see Table 2.9 in the Appendix).

While the three companions of the SN IIb progenitors in these examples are of similar luminosity, it is important to consider how easily they might be observed in pre- and post-explosion images. For example the O star has a high surface temperature and so most of its emission is in the ultraviolet. In Table 2.3 we list the expected broad-band magnitudes of our example progenitors using the methods outlined in Eldridge et al. (2007) and Eldridge & Stanway (2009) to calculate the colours. We see that while all three progenitors are of a similar bolometric luminosity, the B and K supergiants output more of their light in the optical bands, such as V and I which are typical of those most commonly available in pre-explosion imaging (Smartt et al., 2009), making them easier to identify in pre- and post-explosion images.

2.5.2 Parameter Space

Figure 2.3 depicts the range of initial mass ratios and initial orbital periods of binary systems in which the primary star is expected to explode as a type IIb supernova. In systems with initial orbital periods larger than about 1600–1800 days, depending on the mass ratio, the primary star has more than $0.5 M_\odot$ of hydrogen left in its envelope at the time of explosion. We assume that the supernova would be classified as type II. We therefore do not find any SNe type IIb progenitors which have undergone Case C mass transfer (Lauterborn, 1970), since these all end up with hydrogen masses greater than $0.5 M_\odot$. In systems with initial orbital periods smaller than about 1000–1300 days the envelope mass left at the time of explosion is less than $0.1 M_\odot$ and we assume that the primary explodes as a type Ib supernova. In systems with $M_2/M_1 \lesssim 0.7$–0.8, depending on the orbital period, the mass-transfer rate is so high that the stars come into contact. These systems are expected to experience a common envelope phase. As we have discussed in section 2.3, whether or not a binary evolves into contact during this phase is sensitive to some of our model assumptions. Therefore the critical mass ratio separating contact from non-contact systems is uncertain and we regard the location of this boundary in Fig. 2.3 as indicative only.

The borders between type IIb / II and between type Ib / IIb run diagonally across this diagram, i.e. the critical orbital period dividing different supernova types increases with mass ratio. This is caused by the fact that systems with more extreme initial mass ratios exhibit a higher mass-transfer rate in the initial phase of mass transfer. For a given initial orbital period this results in lower envelope masses at the time of explosion in systems with more extreme mass ratios.
Figure 2.3: Ranges of initial mass ratios and orbital periods of binaries for which the primary star is expected to explode as a type IIb of the extended type supernova. We assumed a primary mass $M_1 = 15\, M_\odot$, a metallicity $Z = 0.02$ and conservative mass transfer. We assume that type Ib supernovae result from stars with less than $0.1\, M_\odot$ of hydrogen in their envelope at the time of explosion, type IIb supernovae from stars with $0.1–0.5\, M_\odot$ of hydrogen and type II with more than $0.5\, M_\odot$ of hydrogen. Furthermore we distinguish different cases based on the response of the companion star. Case 1: The secondary evolves to an O star at time of explosion of the primary. Case 2: the secondary evolves to a B supergiant. Case 3: the secondary evolves to a K supergiant. See also Fig. 2.4 for an enlargement.

The border between type Ib and type IIb shows a horizontal step near mass ratios of about 0.89. This feature is related to the evolutionary stage of the primary star at the onset of mass transfer. For systems with orbital periods smaller than about 1100–1200 days, the primary star fills its Roche lobe relatively early, while it is still in the Hertzsprung gap just before ascending the giant branch. The effects of stellar winds, which enlarge or even reverse the mass ratio and stabilize the mass transfer, are still limited. In addition, the rapid expansion of the primary on its thermal timescale results in a high mass-transfer rate. We find that within this period range only in systems with very similar initial masses, $M_2/M_1 \gtrsim 0.89$, enough hydrogen can be retained on the surface of the primary star until the time of explosion to result in a type IIb supernova.

In Figure 2.3 we also indicate the three different cases distinguishing the properties of the secondary star at the moment of explosion, discussed in Sect. 2.5.1. The information from our model grid on which this figure is based can be found in Tables 2.7–2.9 in the Appendix. In the most common case 1, the
2.5 Binary star progenitors

companion is still on the main sequence at the onset of accretion and it will appear as an O star at the time of explosion. Case 2 is the region where the companion accretes while it is on the Hertzsprung gap and will appear as a B supergiant. The range of mass ratios for this case is very limited. This is a direct result of the short time spent by the star in the Hertzsprung gap, about 0.5% of time that it spends on the main sequence. Case 3 indicates the region where the companion will evolve to a K supergiant. This case is more likely to occur for wider systems in which the primary star fills its Roche lobe in a later stage of helium burning. In Figure 2.4 we show an enlarged region of the parameter space for initial mass ratios near 1. Here we indicate case 3b in which the secondary explodes before the primary. This only occurs if the stars are initially very close in mass, to within 0.5%.

![Figure 2.4: Zoom-in of Fig. 2.3 for initial mass ratios close to one. The region (case 3) where the secondary evolves to a K supergiant is subdivided into case 3a, where the primary explodes before the secondary, and case 3b, where the secondary explodes before the primary. The location of the four models in Table 2.2) is also indicated.](image)

2.5.3 Non-conservative mass transfer

So far we have assumed that the mass transferred during Roche-lobe overflow is efficiently accreted by the secondary. This assumption may not be valid, given the high mass-transfer rates reached and the fact that the secondary star may quickly spin up to break-up rotation after accreting only a fraction of the transferred mass (e.g. Packet, 1981). In this section we investigate the effect of variations in the accretion efficiency $\beta$, i.e. the fraction of material accreted by the secondary with respect to the amount of material lost by the primary star as a result of Roche-lobe overflow.
Table 2.4: Properties of binary stellar models for an initial primary mass of $15 \, M_\odot$, with an initial orbital period of 1500 days and with variation of the initial secondary mass $M_2$ and the accretion efficiency of the companion $\beta$.

<table>
<thead>
<tr>
<th>$#^0$</th>
<th>$M_2$</th>
<th>$M_{1f}$</th>
<th>$M_{2f}$</th>
<th>$\beta$</th>
<th>$M_{1f}$</th>
<th>$T_{eff,1}$</th>
<th>$T_{eff,2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>14</td>
<td>5.93</td>
<td>20.92</td>
<td>1.00</td>
<td>0.354</td>
<td>5.07</td>
<td>4.95</td>
</tr>
<tr>
<td>#1a</td>
<td>&quot;&quot;</td>
<td>6.38</td>
<td>17.23</td>
<td>0.50</td>
<td>0.598</td>
<td>5.07</td>
<td>3.40</td>
</tr>
<tr>
<td>#2</td>
<td>14.35</td>
<td>5.98</td>
<td>21.23</td>
<td>1.00</td>
<td>0.370</td>
<td>5.07</td>
<td>3.45</td>
</tr>
<tr>
<td>#2a</td>
<td>&quot;&quot;</td>
<td>6.06</td>
<td>20.46</td>
<td>0.90</td>
<td>0.416</td>
<td>5.07</td>
<td>3.45</td>
</tr>
<tr>
<td>#2b</td>
<td>&quot;&quot;</td>
<td>6.14</td>
<td>19.72</td>
<td>0.80</td>
<td>0.462</td>
<td>5.07</td>
<td>3.43</td>
</tr>
<tr>
<td>#2c</td>
<td>&quot;&quot;</td>
<td>6.19</td>
<td>19.35</td>
<td>0.75</td>
<td>0.489</td>
<td>5.07</td>
<td>3.43</td>
</tr>
<tr>
<td>#2d</td>
<td>&quot;&quot;</td>
<td>6.24</td>
<td>18.97</td>
<td>0.70</td>
<td>0.518</td>
<td>5.07</td>
<td>3.42</td>
</tr>
<tr>
<td>#2e</td>
<td>&quot;&quot;</td>
<td>6.34</td>
<td>18.25</td>
<td>0.60</td>
<td>0.575</td>
<td>5.07</td>
<td>3.40</td>
</tr>
<tr>
<td>#2f</td>
<td>&quot;&quot;</td>
<td>6.45</td>
<td>17.54</td>
<td>0.50</td>
<td>0.633</td>
<td>5.07</td>
<td>3.39</td>
</tr>
<tr>
<td>#2g</td>
<td>&quot;&quot;</td>
<td>6.74</td>
<td>15.87</td>
<td>0.25</td>
<td>0.806</td>
<td>5.07</td>
<td>3.37</td>
</tr>
<tr>
<td>#3</td>
<td>14.55</td>
<td>6.01</td>
<td>21.40</td>
<td>1.00</td>
<td>0.390</td>
<td>5.07</td>
<td>3.45</td>
</tr>
<tr>
<td>#3a</td>
<td>&quot;&quot;</td>
<td>6.49</td>
<td>17.72</td>
<td>0.50</td>
<td>0.660</td>
<td>5.07</td>
<td>3.39</td>
</tr>
<tr>
<td>#A</td>
<td>8.6</td>
<td>5.41</td>
<td>10.44</td>
<td>0.25</td>
<td>0.091</td>
<td>5.07</td>
<td>3.79</td>
</tr>
<tr>
<td>#B</td>
<td>8.8</td>
<td>5.41</td>
<td>12.49</td>
<td>0.50</td>
<td>0.096</td>
<td>5.07</td>
<td>3.80</td>
</tr>
<tr>
<td>#C</td>
<td>9.7</td>
<td>CONTACT</td>
<td>/</td>
<td>0.75</td>
<td>/</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>#D</td>
<td>10</td>
<td>5.50</td>
<td>15.49</td>
<td>0.75</td>
<td>0.131</td>
<td>5.07</td>
<td>3.69</td>
</tr>
</tbody>
</table>

Notes: The final mass of the primary and secondary, $M_{1f}$ and $M_{2f}$, and the amount of hydrogen in the envelope $M_{H1}$ are given at the time of explosion, together with the luminosity $\log(L/L_\odot)$ and effective temperature $T_{eff}$ of the primary and secondary at the onset of central carbon burning. If the evolution of the binary system ended because of formation of a contact binary, the parameters cannot be determined and are indicated by ‘/’.

The effect on the primary star is rather modest as its evolution has been determined largely by its development before the onset of Roche-lobe overflow. In Table 2.4 we list various properties of models computed with accretion efficiencies varying between 1 and 0.25. The luminosity and temperature of the primary star at the moment of explosion are hardly affected; in all cases we find that the primary star will be a cool supergiant at the time of explosion. Nevertheless its final mass and therefore the amount of hydrogen in the envelope at the moment of explosion $M_{H1}$ increases if we assume a lower accretion efficiency. This can be understood as an effect related to response of the orbit and thus the size of Roche lobe to mass transfer and mass and angular-momentum loss from the system. In our models mass loss from the system widens the orbit and therefore the Roche lobe. This results in smaller mass-transfer rates and therefore larger envelope masses at the time of explosion. This influences the parameter space for which we predict type IIb supernovae, as depicted in Fig. 2.3. For an accretion efficiency of 50% the range of initial orbital periods resulting in SN IIb progenitors shifts by about 200 days to smaller periods in comparison with the conservative case.

The evolution and properties of the secondary are more sensitive to the adopted accretion efficiency. The lower accretion rate reduces the expansion of the companion star. As a result, the formation of contact can be avoided in systems with more extreme initial mass ratios. This effect widens the parameter space...
Figure 2.5: Evolution tracks in the Hertzsprung-Russel diagram of the secondary of binary systems with initial mass $15+14.35\, M_\odot$ and initial orbital period 1500 days. The differences between the evolution tracks of the secondary are caused by variation of the accretion efficiency ($\beta$). (see models #2 - #2g in Table 2.4)

For type IIb supernovae. In table 2.4 we list a few test models that explore the effect of nonconservative mass transfer for extreme mass ratios. For example, we find that the mass ratio leading to contact for a system with an initial orbital period of 1500 days shifts from $M_2/M_1 \approx 0.74$ assuming conservative mass transfer to about 0.65 if we assume a accretion efficiency of 75% (see models #C-#D in Table 2.4). For lower accretion efficiencies we find that formation of contact is no longer the effect that limits the parameter space for type IIb progenitors. In systems with such extreme mass ratios, that mass stripping from the primary star becomes so efficient that these stars do not have enough hydrogen left at the moment of explosion to become IIb. The most extreme mass ratio resulting in a IIb progenitor assuming the same initial orbital period shifts to 0.59 (0.58) assuming an accretion efficiency of 50% (25%), see model #B, (#A) in Table 2.4. In section 2.6 we discuss the influence of $\beta$ on the rate of type IIb supernova.

The appearance of the companion star at the moment of explosion is also affected. In the first case of accretion during the main sequence the secondary rejuvenates, regardless of the adopted accretion efficiency. However, the effective temperature and luminosity decrease for lower accretion efficiencies as a result of the smaller amount of accreted mass (e.g. compare models #1 and #1a in Table 2.4). The evolution of the secondary in the second case of accretion during the Hertzsprung gap depends most strongly on the accretion efficiency. In Fig. 2.5 we illustrate the evolutionary tracks of the secondary in the Hertzsprung-Russel diagram. For conservative mass transfer the secondary evolves to an overluminous B star. The effective temperature decreases with decreasing accretion efficiency. If the accretion efficiency is 75%, the secondary evolves to an A supergiant, and for accretion efficiencies of 50% or less the secondary evolves to a much cooler K supergiant (e.g. compare models #2 and #2g in Table 2.4). In the third case of mass transfer while the secondary is already on the giant branch, the secondary still evolves to a K supergiant. In
Table 2.5: Predicted rates for type IIb supernovae from stable mass transfer in binaries and its dependence on different input assumptions.

<table>
<thead>
<tr>
<th>Mass ratio distr.</th>
<th>fav. extreme ratios</th>
<th>flat</th>
<th>fav. equal masses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>((x = -1))</td>
<td>((x = 0))</td>
<td>((x = 1))</td>
</tr>
<tr>
<td>Fraction of binaries producing type IIb SNe via stable mass transfer</td>
<td>0.83%</td>
<td>1.33%</td>
<td>1.85%</td>
</tr>
</tbody>
</table>

Number of type IIb relative to the number of all core collapse SNe (assuming that the companion does – does not produce a core collapse SNe)

\[ f_{\text{bin}} = 25\% \quad 0.17 - 0.21\% \quad 0.27 - 0.33\% \quad 0.37 - 0.46\% \]
\[ f_{\text{bin}} = 50\% \quad 0.28 - 0.42\% \quad \textbf{0.44 - 0.67\%} \quad 0.62 - 0.93\% \]
\[ f_{\text{bin}} = 100\% \quad 0.42 - 0.83\% \quad 0.67 - 1.33\% \quad 0.93 - 1.85\% \]

Estimates of the rates assuming lower accretion efficiencies \( \beta^1 \)

\[ \beta = 1.0 \quad \textbf{0.44 - 0.67\%} \]
\[ \beta = 0.75 \quad 0.60 - 0.92\%^2 \]
\[ \beta = 0.5 \quad 0.69 - 1.06\%^2 \]
\[ \beta = 0.25 \quad 0.71 - 1.09\%^2 \]

Assuming that all systems with orbital periods between 1000 and 2000 days produce IIb’s\(^1,3\)

- 2.3–3.5\%

Notes: See Section 2.6 for details. The rate predicted based on our standard assumptions is marked in boldface.

(1) Assuming a binary fraction of \( f_{\text{bin}} = 50\% \).

(2) The estimates for lower accretion efficiencies are based on just a few test models and should only be taken as indicative. A more extended grid of non-conservative models is required for a proper evaluation of these rates.

(3) This estimate does no longer depend on the assumed distribution of initial mass ratios.

In summary we conclude that lower accretion efficiencies shift and widen the parameter space for type IIb supernovae from stable mass transfer. However, it proves to be even more difficult to produce a companion that is a B supergiant at the moment of explosion for non-conservative mass transfer. If the accretion efficiency is smaller than about 60\% our models predict no companions to reside in the middle of the Hertzsprung-Russel diagram. They will either be hot and compact residing in the main sequence band or, if the initial masses were very similar, the secondary will appear as a K supergiant.
Table 2.6: Relative rates for the properties of the companion star at the moment of explosion for different initial mass ratio distributions based on conservative models.

<table>
<thead>
<tr>
<th>Mass ratio distr.</th>
<th>fav. extreme ratios $(x = -1)$</th>
<th>flat $(x = 0)$</th>
<th>fav. equal masses $(x = 1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1 (O star)</td>
<td>91.5%</td>
<td>90.3%</td>
<td>89.1%</td>
</tr>
<tr>
<td>Case 2 (B supergiant)</td>
<td>2.5%</td>
<td>2.8%</td>
<td>3.2%</td>
</tr>
<tr>
<td>Case 3 (K supergiant)</td>
<td>6.0%</td>
<td>6.9%</td>
<td>7.7%</td>
</tr>
</tbody>
</table>

2.6 Predicted rates

The currently most likely observed rate of type IIb supernovae with respect to all core collapse supernovae is about 3% (see Sect. 2.2 for a discussion). The accuracy of the observed rate is expected to go up in the near future thanks to the automated detection and classification of supernova light curves. Computing the rate predicted from our models is not straightforward and requires adopting further uncertain assumptions about for example the binary fraction and the distribution functions of the initial parameters. That said, we still consider it worthwhile to estimate the rate predicted from our models, which we do below adopting commonly made, most reasonable assumptions and assessing the effect of uncertainties in these assumptions on the rates.

In the previous sections we discussed progenitor models assuming an initial mass of $15 M_\odot$ for the primary star. Based on a few test models we conclude that the parameter space does not significantly change for systems with primary masses between roughly 10 and 20 solar masses. This mass range dominates the population of massive binaries in which the primary is massive enough to undergo a core collapse supernova. In the following derivations we will assume that the characteristics of our conservative models with a $15 M_\odot$ primary are representative for the population of massive binaries.

We assume that the initial orbital periods, $P$, are distributed according to Opik’s (1924a) law (Kouwenhoven et al., 2007),

$$ f(P) \propto P^{-1} \quad \text{for} \quad 0.5 \leq P (d) \leq 10^4. $$

The upper limit is chosen close to the widest orbital period for which we expect binaries to interact via Roche-lobe overflow during their life time. For the initial mass ratio, which we define as the mass of the initially less massive star over the mass of the initially most massive star, $q \equiv M_2/M_1$, we adopt a power-law distribution,

$$ f(q) \propto q^x \quad \text{for} \quad 0.25 \leq q \leq 1. $$

Binaries with extreme mass ratio are hard to detect and even if they have been detected it is still difficult to determine their mass ratio accurately. Therefore we limit ourselves to systems with $M_2/M_1 > 0.25$, following the approach of for example Pols et al. (1991). The parameter $x$ describes whether the distribution of initial mass ratios is flat ($x = 0$), skewed towards systems equal masses ($x > 0$) or favors systems with unequal masses ($x < 0$). Although these distribution functions are uncertain, they are consistent with observed distributions. Kouwenhoven et al. (2007) and references therein find $x = -0.4$ for the nearby association Scorpius OB2 while $x = -1$ corresponds to a study of spectroscopic binaries by Trimble (1990). A flat or uniform mass ratio distribution $x = 0$ has been quoted by Sana et al. (2009) for young open
clusters, whereas e.g. Pinsonneault & Stanek (2006) and Kobulnicky & Fryer (2007) claim that massive binaries like to be twins, i.e. \( x > 0 \). We will adopt \( x = 0 \) as our standard assumption and consider the two extreme cases \( x = \pm 1 \) as well.

We adopt a binary fraction \( f_{\text{bin}} \) of 50\% as our standard assumption, i.e. for every single star with mass \( M_1 \) there is exactly one binary system of which the primary mass is \( M_1 \) and the orbital period and mass ratio are within the ranges specified above. For comparison, Mason et al. (1998) derives a binary fraction of 59\%-75\% for O stars in clusters and associations. García & Mermilliod (2001) find fractions between 14\%-80\%, whereas Kobulnicky & Fryer (2007) infer a fraction higher than 70\%. On the other hand, Pols et al (1991) adopted a fraction of 27.5\% of binaries with B-type primaries, \( P < 10 \text{ yr} \) and \( 0.25 < q < 1 \).

Besides our standard assumption we will consider binary fractions of \( f_{\text{bin}} = 25\% \) and 100\%. We note that exploring the effect of different binary fractions partially covers the uncertainties in the assumed upper limits for the orbital period and mass ratio that we assumed above for the initial parameter distributions.

The observed rate of type IIb supernovae is expressed with respect to the rate of all types of core collapse supernova. To compute this number we must consider that a significant fraction of the companion stars end their lives as core collapse supernovae. The precise fraction will depend on the initial mass ratio distribution, the amount of mass lost from the secondary and the fraction that is actually accreted by the secondary. For clarity, we will just consider the two extreme cases in which all or none of the companion stars result in a core collapse supernova.

### 2.6.1 The rate of type IIb supernovae

Under the standard assumptions described above, we find that the number of type IIb supernovae over the number of core collapse supernovae formed via stable mass transfer as predicted from our models is roughly 0.6 \%, see Table 2.5, about a factor five lower than the observed rate (Sect. 2.2). The rate is larger, but only by a factor two, than the rate predicted from single stars, 0.3\%, under the assumption that all stars are single (see Section 2.4). Our predicted rate is compatible with the rate found by Podsiadlowski et al. (1992) for the ‘stripped’ supernovae. In contrast to what some previous authors have suggested (e.g. Podsiadlowski et al., 1993), we emphasize that not only for the single star channel but also for the binary channel one needs to fine-tune the initial parameters of the system. For single stars the initial mass must be finely-tuned, whereas for binaries the combination of initial orbital period and initial mass ratio must be carefully arranged.

The derived rate depends on the adopted initial mass ratio distribution, the assumed binary fraction and on whether or not the companion star results in a core-collapse supernovae, see Table 2.5. Even when we adopt the assumptions favoring a high SN IIb rate from binaries (100\% binaries, mass ratio distribution skewed to equal-mass systems and no core-collapse supernovae from the companions) the rate derived from our conservative models is still below the observed rate.

Another uncertainty to consider is the efficiency of mass transfer. In our models, lower efficiencies shift and widen the parameter space. In Table 2.5 we show that the number of SNe IIb over the number of core-collapse supernovae may increase to over 1\%, with the warning that for a proper evaluation of the parameter space one needs a more extensive model grid.

A further uncertainty we have to consider is in the range of hydrogen-envelope masses in the progenitor that give rise to a SN IIb. The lower limit of 0.1 \( M_\odot \) is determined by the boundary between compact and extended type IIb. The upper limit of about 0.5 \( M_\odot \) is determined by the lack of a plateau in the light curve,
2.6 Predicted rates

which depends on the chemical structure of the stellar envelope and therefore is less certain. If we change
the boundary from 0.5 to 0.6 $M_\odot$, test models show that the initial orbital period range will only widen by
about 80 days. This means an increase of the overall rate of type IIb SNe of about 10%. The uncertainties
in these boundaries will therefore not have a large effect on the rate.

As an extreme assumption, we consider that all systems with orbital periods between 1000 and 2000
days produce IIb progenitors. This corresponds to 7% of all binary systems with initial mass ratios and
orbital periods in the ranges specified earlier. This would increase the relative rate compared to the rate of
core collapse supernova to 2.3–3.5%, depending on whether the companion star produces a core-collapse
supernova or not and assuming a binary fraction of 50%. This number is consistent with the observed rate.

2.6.2 The relative rates of different cases for the companion

Table 2.6 indicates the relative rates for the different characteristics of the companion at the moment of
explosion, as described in section 2.5.1. The large majority, about 90%, of type IIb supernova resulting
from stable mass transfer is expected to have an O-star companion at the moment of explosion. The rate of
systems with a blue supergiant companion, which applies to SN 1993J and possibly 2001ig, is predicted to
be quite low according to our models, about 3% of the type IIb supernova rate. These relative rates reflect
the large and very small regions in parameter space for O-star and B-supergiant companions, respectively,
as depicted in Fig. 2.3. However, these percentages do not necessarily reflect the probability of detecting
a companion of a certain type. Detecting an O-star companion in post-explosion images of a supernova is
more difficult than detecting a B or K supergiant, as we demonstrated in Section 2.5.1.

The probability for the presence of blue companions to type I Ib SNe we derive is strikingly lower than
predicted by Podsiadlowski et al. (1992). They find that, if the companion starts to accrete after becoming
a giant star, it will contract in a similar way to a feature known as “blue loops” seen in some evolutionary
tracks of single stars. We do not find this behavior with our stellar evolution code. Whether or not stellar
models perform blue loops is very sensitive to the ratio of core mass to total stellar mass and to details in
the chemical profile outside the stellar core (Kippenhahn & Weigert, 1994). If a significant fraction of the
case 3 models would result in blue companions, the relative rate for the presence of blue companions may
increase by a factor of two or three (see Table 2.6).

In the most common case, accretion during the main sequence, the star rejuvenates, i.e. the size of
its convective core increases to adapt to the new stellar mass, see section 2.5.1. In our code, all main
sequence stars that accrete rejuvenate. However, whether stars rejuvenate or not depends on the assumed
efficiency of semi-convection. Stars that have evolved towards the end of the main sequence build up a
chemical gradient between the helium rich core and the hydrogen rich envelope. Braun & Langer (1995)
show that, assuming a low efficiency of semi-convective mixing, the growth of the convective core in an
accreting main-sequence star is prevented. These stars may also appear as blue supergiants at the moment
of explosion.

Based on their models, we estimate that in our case of conservative mass transfer the companion must
have reached a central helium mass fraction of about 0.75 or more to prevent rejuvenation. In our models
this occurs for initial mass ratios larger than about 0.87, and we estimate that this may increase the relative
rate of the presence of blue companions up to about 50%. For lower accretion efficiencies it becomes even
easier to prevent rejuvenation: a central helium mass fraction for the accreting star of just 0.6 and higher
are required if $\beta = 0$. However, the companion must accrete enough to finish its main sequence evolution
before the explosion of the primary star, to be observed as a blue supergiant. More detailed estimates are beyond the scope of this study, but we do emphasize this as an interesting possibility to use the companions of supernovae to gain insight in internal mixing processes.

### 2.7 Discussion & Conclusion

We identified binary progenitor models for extended type IIb supernovae. In these models the most massive star is stripped from its envelope by interaction with its companion, such that only several tenths of solar masses of hydrogen remain at the moment of explosion. We find that the most massive star must fill its Roche lobe during central helium burning in order to achieve this. We derive for which range of initial orbital periods and initial mass ratios binary systems are expected to produce type IIb supernova progenitors.

We have discussed in detail the properties of the companion star at the moment of explosion, motivated by the detection of a companion for SN 1993J, a possible companion for 2001ig and the non-detection of a companion in the supernova remnant of Cas A. We distinguish three cases: (1) the companion appears as a hot O star, when it accreted while still being on the main sequence, (2) the companion becomes an over-luminous B-star if mass transfer started while the companion was crossing the Hertzsprung gap, a scenario which may apply to 1993J and possibly 2001ig and (3) the companion will be a K-supergiant when it accreted after ascending the giant branch. The third case applies to systems with very similar initial masses and can be subdivided in (3a) systems in which the initially most massive star explodes first and (3b) systems in which the companion star explodes first, up to $10^5$ years before the primary. These models may apply to Cassiopeia A -if it was an extended-IIb SN- explaining the lack of evidence for a companion\(^2\). If the progenitor of Cas A was compact, not a red supergiant, at time of explosion another evolutionary scenario then discussed here could explain this specific supernova remnant, e.g. a common envelope scenario.

However, our models predict that the scenarios we propose for 1993J, 2000ig (case 2) and Cas A (case 3b) are very rare and require that the companion efficiently accretes the transferred material. The accretion efficiency, i.e. the fraction of transferred material that is accreted by the companion, is a major uncertainty for binary evolutionary models. A lower accretion efficiency shifts the parameter space to smaller orbital periods (by about 200 days for an efficiency of 50% compared to conservative mass transfer) and widens the parameter space towards more extreme initial mass ratios (assuming an efficiency of 50% we estimate that the IIb rate predicted by our models increases by roughly a factor 1.6 compared to conservative mass transfer). However a B supergiant is identified more easily in comparison with the other possible evolutionary paths of the companion star, as pointed out in section 2.5.1 and Table 2.3, which gives a bias towards a higher fraction of observed B-supergiants as a companion star.

Our conservative models predict $\sim 0.6\%$ of all core collapse supernova to be of type IIb (using our standard assumptions, see Section 2.6), about a factor of five lower than the currently most likely observed rate. This rate is larger, but only by a factor of two than the rate predicted for a pure single star population, $\sim 0.3\%$, in which high mass stars lose their envelope due to stellar winds. We emphasize that both the

\(^2\)Signatures from the first supernova, for example in radio emission or from interaction with the interstellar medium will be hard to detect as Cas A is bright radio source itself and the remnant of the first supernova will be roughly 10 to 100 times larger than the remnant of Cas A.
binary and single stars scenario only produce type IIb in a very limited region of the initial parameter space.

We warn the reader that the rates quoted above are uncertain. The rate from single stars is affected by our limited understanding of mass loss rates from massive red giants. The rate from binary stars is affected by uncertainties in the physics of binary interaction as well as the adopted distributions of initial binary parameters. If we make several assumptions that favor a high rate of IIb supernovae (a high binary fraction, a distribution of initial mass ratios which is skewed towards systems with equal masses, a low mass-transfer efficiency and only moderate angular momentum loss from the system), we derive a rate that is consistent with the lower limit of the currently most reliable observed rate of Smartt et al. (2009).

In addition, the observed rate is based on only a few SNe and it is difficult to make a distinction between an extended type IIb and a compact, the latter of which is more similar to a type Ib SN and is formed by another evolutionary scenario (e.g. Yoon et al., 2010). Therefore, the observed rate is also one of the uncertainties. Nevertheless, the rates predicted from our standard models seem to indicate that there is definitely room for a single star scenario and a common envelope scenario leading to a type IIb.

### 2.7.1 A BRIEF COMPARISON WITH PREVIOUS WORK

Our findings are consistent with earlier studies by Podsiadlowski et al. (1992, 1993), Woosley et al. (1994), Maund et al. (2004) and Stancliffe & Eldridge (2009). Nevertheless we can note two major differences. Contrary to suggestions by Podsiadlowski et al. (1993) we find that it proves to be hard to explain the presence of a blue companion star at the time of explosion, confirming the thought raised by Stancliffe & Eldridge (2009) with our more extended model grid. In addition, we find that the primary star needs to fill its Roche lobe during helium burning (late Case B mass transfer), whereas Podsiadlowski et al. (1993) require mass transfer to start after helium burning (Case C mass transfer). This can be attributed to differences in the adopted physics: the adopted initial composition, the opacity tables and the mixing length parameter which have a not negligible effect on the radii of giants and therefore on the onset of and response to Roche-lobe overflow. Furthermore there are small differences in prescription for the mass-transfer rate, the treatment of non-conservative mass transfer and the mass loss occurring between the last computed model and the explosion. As a result we find that slightly different initial orbital periods are required to produce a IIb progenitor. However, if we allow for uncertainties, our estimate of the IIb rate is consistent with previous studies.

### 2.7.2 WHAT TYPE IIb SNE TEACH US ABOUT STELLAR AND BINARY PHYSICS

The rate of type IIb supernovae and the relative occurrence of different companions have the potential to constrain both stellar physics, such as internal mixing processes and stellar wind mass loss, but also the physics of interacting binaries, when the accuracy of the determined rates increases in the near future.

The discrepancy between the currently most likely observed rate and the rate predicted by our models seems to point towards lower accretion efficiencies. At lower accretion efficiencies, systems with more extreme mass ratios can avoid contact and retain enough hydrogen in their envelope to become progenitors of IIb supernovae. On the other hand, the presence of a blue companion for SN 1993J and possibly SN 2000ig indicates that at least a substantial fraction must be accreted. Therefore, improving statistics about IIb supernovae may in the future help to constrain the efficiency of mass transfer, one of the major uncertainties in binary evolutionary models.
Chapter 2: Binary progenitor models of type IIb supernova

The need to avoid contact and a subsequent spiral in for these binaries with an evolved giant companion also gives information about two other uncertain input assumption related to the physics of mass transfer. It may indicate that the entropy of accreted material must be low, to prevent swelling of the companion, and it may indicate that the mass-transfer rate in such systems remains lower than the self-regulated thermal-timescale rate, see section 2.3.

Finally the detection of one and possibly two blue companions while our standard models predict that this should be a rare event is puzzling. As discussed in section 2.6.2, this may be explained with a low efficiency of semi-convection.

2.7.3 Outlook

We illustrated the potential of type IIb supernovae, being on the border between the hydrogen rich type II and the hydrogen poor type Ib/c supernova, to give insight in stellar physics such as internal mixing processes and stellar winds as well as physics of interacting binaries. Even though the predictions from stellar evolutionary models are still challenged by various uncertainties, it has become feasible to compute large grids of binary evolutionary models and assess the impact of the various assumptions. Especially from the observational side we expect large progress in the near future. Statistics are improving with the large samples resulting from automated surveys. These will allow us to even assess the rate of IIb supernovae in different host galaxies probing different environments and even metallicity regimes (Modjaz et al., 2011). Also detailed multiwavelength studies such as Kotak & Vink (2006) will reveal indirect methods to probe the progenitors of these supernovae.

2.8 Acknowledgements

We thank Philip Podsiadlowski for the useful discussions regarding the comparison with his work about type IIb SNe, Matteo Cantiello and Sung-Chul Yoon for their input about the c-IIb SNe, as well the referee for the useful feedback. SdM is supported by NASA through Hubble Fellowship grant HST-HF-51270.01-A awarded by the Space Telescope Science Institute, which is operated by the Association of Universities for Research in Astronomy, Inc., for NASA, under contract NAS 5-26555.
Table 2.7: Properties of binary stellar models for an initial primary mass of 15 $M_{\odot}$, conservative mass transfer and with variation of the initial secondary mass $M_{2}$, and initial orbital period $P_{\text{orb}}$.

<table>
<thead>
<tr>
<th>No</th>
<th>$M_1$ ($M_{\odot}$)</th>
<th>$M_2$ ($M_{\odot}$)</th>
<th>$P_{\text{orb}}$ (Days)</th>
<th>$M_{\text{H}}$ ($M_{\odot}$)</th>
<th>log ($\frac{L}{L_{\odot}}$)</th>
<th>$T_{\text{eff,1}}$ (10^4K)</th>
<th>log ($\frac{L}{L_{\odot}}$)</th>
<th>$T_{\text{eff,2}}$ (10^4K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0a</td>
<td>14.99</td>
<td>5.20</td>
<td>22.88</td>
<td>1000</td>
<td>0.0997</td>
<td>5.04</td>
<td>3.75</td>
<td>5.17</td>
</tr>
<tr>
<td>0b</td>
<td>14.95</td>
<td>5.19</td>
<td>22.85</td>
<td>1000</td>
<td>0.0992</td>
<td>5.04</td>
<td>3.76</td>
<td>5.17</td>
</tr>
<tr>
<td>0c</td>
<td>14.9</td>
<td>5.19</td>
<td>22.80</td>
<td>1000</td>
<td>0.0982</td>
<td>5.04</td>
<td>3.76</td>
<td>5.16</td>
</tr>
<tr>
<td>0d</td>
<td>14.87</td>
<td>5.19</td>
<td>22.78</td>
<td>1000</td>
<td>0.0982</td>
<td>5.04</td>
<td>3.76</td>
<td>5.16</td>
</tr>
<tr>
<td>0e</td>
<td>14.86</td>
<td>5.19</td>
<td>22.77</td>
<td>1000</td>
<td>0.0981</td>
<td>5.04</td>
<td>3.76</td>
<td>5.15</td>
</tr>
<tr>
<td>1a</td>
<td>14.99</td>
<td>5.32</td>
<td>22.72</td>
<td>1100</td>
<td>0.1283</td>
<td>5.05</td>
<td>3.69</td>
<td>5.16</td>
</tr>
<tr>
<td>1b</td>
<td>14.95</td>
<td>5.32</td>
<td>22.68</td>
<td>1100</td>
<td>0.1276</td>
<td>5.05</td>
<td>3.69</td>
<td>5.16</td>
</tr>
<tr>
<td>1c</td>
<td>14.9</td>
<td>5.31</td>
<td>22.64</td>
<td>1100</td>
<td>0.1267</td>
<td>5.05</td>
<td>3.69</td>
<td>5.15</td>
</tr>
<tr>
<td>1d</td>
<td>14.87</td>
<td>5.31</td>
<td>22.61</td>
<td>1100</td>
<td>0.1262</td>
<td>5.05</td>
<td>3.69</td>
<td>5.15</td>
</tr>
<tr>
<td>1e</td>
<td>14.86</td>
<td>5.31</td>
<td>22.60</td>
<td>1100</td>
<td>0.1260</td>
<td>5.05</td>
<td>3.69</td>
<td>5.15</td>
</tr>
<tr>
<td>2a</td>
<td>14.93</td>
<td>5.46</td>
<td>22.42</td>
<td>1200</td>
<td>0.1570</td>
<td>5.06</td>
<td>3.66</td>
<td>5.15</td>
</tr>
<tr>
<td>2b</td>
<td>14.92</td>
<td>5.46</td>
<td>22.41</td>
<td>1200</td>
<td>0.1470</td>
<td>5.06</td>
<td>3.66</td>
<td>5.15</td>
</tr>
<tr>
<td>2c</td>
<td>14.9</td>
<td>5.46</td>
<td>22.40</td>
<td>1200</td>
<td>0.1556</td>
<td>5.06</td>
<td>3.66</td>
<td>5.15</td>
</tr>
<tr>
<td>2d</td>
<td>14.85</td>
<td>5.45</td>
<td>22.35</td>
<td>1200</td>
<td>0.1555</td>
<td>5.06</td>
<td>3.67</td>
<td>5.14</td>
</tr>
<tr>
<td>2e</td>
<td>14.84</td>
<td>5.45</td>
<td>22.34</td>
<td>1200</td>
<td>0.1553</td>
<td>5.06</td>
<td>3.67</td>
<td>5.14</td>
</tr>
<tr>
<td>3a</td>
<td>14.73</td>
<td>5.62</td>
<td>22.00</td>
<td>1300</td>
<td>0.1949</td>
<td>5.07</td>
<td>3.60</td>
<td>5.12</td>
</tr>
<tr>
<td>3b</td>
<td>14.72</td>
<td>5.62</td>
<td>22.00</td>
<td>1300</td>
<td>0.1958</td>
<td>5.07</td>
<td>3.60</td>
<td>5.12</td>
</tr>
<tr>
<td>3c</td>
<td>14.71</td>
<td>5.61</td>
<td>21.99</td>
<td>1300</td>
<td>0.1911</td>
<td>5.07</td>
<td>3.60</td>
<td>5.12</td>
</tr>
<tr>
<td>3d</td>
<td>14.7</td>
<td>5.61</td>
<td>21.98</td>
<td>1300</td>
<td>0.1911</td>
<td>5.07</td>
<td>3.60</td>
<td>5.12</td>
</tr>
<tr>
<td>3e</td>
<td>14.68</td>
<td>5.61</td>
<td>21.96</td>
<td>1300</td>
<td>0.1912</td>
<td>5.07</td>
<td>3.60</td>
<td>5.11</td>
</tr>
<tr>
<td>3f</td>
<td>14.66</td>
<td>5.61</td>
<td>21.94</td>
<td>1300</td>
<td>0.2028</td>
<td>5.07</td>
<td>3.60</td>
<td>5.10</td>
</tr>
<tr>
<td>4a</td>
<td>14.54</td>
<td>5.79</td>
<td>21.63</td>
<td>1400</td>
<td>0.2774</td>
<td>5.07</td>
<td>3.53</td>
<td>5.09</td>
</tr>
<tr>
<td>4b</td>
<td>14.53</td>
<td>5.79</td>
<td>21.62</td>
<td>1400</td>
<td>0.2787</td>
<td>5.07</td>
<td>3.53</td>
<td>5.09</td>
</tr>
<tr>
<td>4c</td>
<td>14.5</td>
<td>5.79</td>
<td>21.59</td>
<td>1400</td>
<td>0.2711</td>
<td>5.07</td>
<td>3.53</td>
<td>5.08</td>
</tr>
<tr>
<td>5a</td>
<td>14.36</td>
<td>5.98</td>
<td>21.23</td>
<td>1500</td>
<td>0.3677</td>
<td>5.07</td>
<td>3.47</td>
<td>5.06</td>
</tr>
<tr>
<td>5b</td>
<td>14.35</td>
<td>5.98</td>
<td>21.23</td>
<td>1500</td>
<td>0.3699</td>
<td>5.07</td>
<td>3.47</td>
<td>5.06</td>
</tr>
<tr>
<td>5c</td>
<td>14.33</td>
<td>5.98</td>
<td>21.21</td>
<td>1500</td>
<td>0.3625</td>
<td>5.07</td>
<td>3.47</td>
<td>5.06</td>
</tr>
<tr>
<td>5d</td>
<td>14.32</td>
<td>5.97</td>
<td>21.20</td>
<td>1500</td>
<td>0.3765</td>
<td>5.07</td>
<td>3.47</td>
<td>5.04</td>
</tr>
<tr>
<td>6a</td>
<td>14.17</td>
<td>6.20</td>
<td>20.80</td>
<td>1600</td>
<td>0.4772</td>
<td>5.07</td>
<td>3.42</td>
<td>5.03</td>
</tr>
<tr>
<td>6b</td>
<td>14.15</td>
<td>6.19</td>
<td>20.79</td>
<td>1600</td>
<td>0.4749</td>
<td>5.07</td>
<td>3.43</td>
<td>5.02</td>
</tr>
<tr>
<td>7a</td>
<td>14.01</td>
<td>6.44</td>
<td>20.37</td>
<td>1700</td>
<td>0.6257</td>
<td>5.07</td>
<td>3.39</td>
<td>5.00</td>
</tr>
<tr>
<td>7b</td>
<td>14</td>
<td>6.44</td>
<td>20.37</td>
<td>1700</td>
<td>0.6458</td>
<td>5.07</td>
<td>3.39</td>
<td>5.00</td>
</tr>
</tbody>
</table>

Notes: The secondary is, for every model, at the start of mass transfer in its Hertzsprung gap. The final mass of the primary and secondary, $M_{1}$ and $M_{2}$, and the amount of hydrogen in the envelope $M_{\text{H}}$, are given at the time of explosion, together with the luminosity log($L/L_{\odot}$) and effective temperature $T_{\text{eff}}$ of the primary and secondary at the onset of central carbon burning.
Table 2.8: Properties of binary stellar models for an initial primary mass of $15 \, M_\odot$, conservative mass transfer and with variation of the initial secondary mass $M_{2i}$ and initial orbital period $P_{\text{orb}}$.

<table>
<thead>
<tr>
<th>$N^a$</th>
<th>$M_{2i} , (M_\odot)$</th>
<th>$M_{2f} , (M_\odot)$</th>
<th>$P_{\text{orb}} , (\text{Days})$</th>
<th>$M_{\text{He1}} , (M_\odot)$</th>
<th>$\log \left( \frac{L_{\text{He1}}}{L_\odot} \right)$</th>
<th>$T_{\text{eff},1} , (10^3 , K)$</th>
<th>$\log \left( \frac{L_{\text{He2}}}{L_\odot} \right)$</th>
<th>$T_{\text{eff},2} , (10^3 , K)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>h0a</td>
<td>13.5</td>
<td>5.10</td>
<td>21.52</td>
<td>1000</td>
<td>0.0835</td>
<td>5.03</td>
<td>3.86</td>
<td>4.96</td>
</tr>
<tr>
<td>h0b</td>
<td>14</td>
<td>5.13</td>
<td>21.98</td>
<td>1000</td>
<td>0.0886</td>
<td>5.03</td>
<td>3.82</td>
<td>5.01</td>
</tr>
<tr>
<td>h0c</td>
<td>14.85</td>
<td>5.19</td>
<td>22.76</td>
<td>1000</td>
<td>0.0980</td>
<td>5.04</td>
<td>3.76</td>
<td>5.09</td>
</tr>
<tr>
<td>h1a</td>
<td>13.3</td>
<td>5.16</td>
<td>21.25</td>
<td>1100</td>
<td>0.1014</td>
<td>5.04</td>
<td>3.76</td>
<td>4.94</td>
</tr>
<tr>
<td>h1b</td>
<td>13.4</td>
<td>5.17</td>
<td>21.34</td>
<td>1100</td>
<td>0.1111</td>
<td>5.04</td>
<td>3.73</td>
<td>5.00</td>
</tr>
<tr>
<td>h1c</td>
<td>14</td>
<td>5.23</td>
<td>21.85</td>
<td>1100</td>
<td>0.1258</td>
<td>5.05</td>
<td>3.69</td>
<td>5.08</td>
</tr>
<tr>
<td>h1d</td>
<td>14.85</td>
<td>5.31</td>
<td>22.59</td>
<td>1100</td>
<td>0.1083</td>
<td>5.05</td>
<td>3.78</td>
<td>4.92</td>
</tr>
<tr>
<td>h2a</td>
<td>13.2</td>
<td>5.32</td>
<td>20.86</td>
<td>1200</td>
<td>0.1012</td>
<td>5.04</td>
<td>3.77</td>
<td>4.93</td>
</tr>
<tr>
<td>h2b</td>
<td>13.3</td>
<td>5.32</td>
<td>20.95</td>
<td>1200</td>
<td>0.1195</td>
<td>5.06</td>
<td>3.72</td>
<td>4.99</td>
</tr>
<tr>
<td>h2c</td>
<td>14</td>
<td>5.37</td>
<td>21.60</td>
<td>1200</td>
<td>0.1450</td>
<td>5.06</td>
<td>3.67</td>
<td>5.07</td>
</tr>
<tr>
<td>h2d</td>
<td>14.83</td>
<td>5.45</td>
<td>22.33</td>
<td>1200</td>
<td>0.1450</td>
<td>5.06</td>
<td>3.67</td>
<td>5.07</td>
</tr>
<tr>
<td>h2e</td>
<td>12.1</td>
<td>5.35</td>
<td>19.65</td>
<td>1300</td>
<td>0.0983</td>
<td>5.06</td>
<td>3.78</td>
<td>4.80</td>
</tr>
<tr>
<td>h3b</td>
<td>12.2</td>
<td>5.36</td>
<td>19.75</td>
<td>1300</td>
<td>0.1079</td>
<td>5.06</td>
<td>3.77</td>
<td>4.81</td>
</tr>
<tr>
<td>h3c</td>
<td>14.64</td>
<td>5.61</td>
<td>21.93</td>
<td>1300</td>
<td>0.2009</td>
<td>5.07</td>
<td>3.60</td>
<td>5.04</td>
</tr>
<tr>
<td>h4a</td>
<td>11.4</td>
<td>CONTACT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>h4b</td>
<td>11.5</td>
<td>5.43</td>
<td>18.85</td>
<td>1400</td>
<td>0.1223</td>
<td>5.06</td>
<td>3.72</td>
<td>4.74</td>
</tr>
<tr>
<td>h4c</td>
<td>11.9</td>
<td>5.47</td>
<td>19.29</td>
<td>1400</td>
<td>0.1329</td>
<td>5.06</td>
<td>3.69</td>
<td>4.77</td>
</tr>
<tr>
<td>h4d</td>
<td>12</td>
<td>5.48</td>
<td>19.38</td>
<td>1400</td>
<td>0.1430</td>
<td>5.07</td>
<td>3.68</td>
<td>4.78</td>
</tr>
<tr>
<td>h4e</td>
<td>14.49</td>
<td>5.79</td>
<td>21.58</td>
<td>1400</td>
<td>0.2785</td>
<td>5.07</td>
<td>3.53</td>
<td>5.01</td>
</tr>
<tr>
<td>h5a</td>
<td>11.1</td>
<td>CONTACT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>h5b</td>
<td>11.2</td>
<td>5.56</td>
<td>18.37</td>
<td>1500</td>
<td>0.1615</td>
<td>5.07</td>
<td>3.65</td>
<td>4.70</td>
</tr>
<tr>
<td>h5c</td>
<td>11.6</td>
<td>5.60</td>
<td>18.84</td>
<td>1500</td>
<td>0.1843</td>
<td>5.07</td>
<td>3.62</td>
<td>4.73</td>
</tr>
<tr>
<td>h5d</td>
<td>13</td>
<td>5.79</td>
<td>20.06</td>
<td>1500</td>
<td>0.2711</td>
<td>5.07</td>
<td>3.53</td>
<td>4.86</td>
</tr>
<tr>
<td>h5e</td>
<td>14</td>
<td>5.93</td>
<td>20.92</td>
<td>1500</td>
<td>0.3538</td>
<td>5.07</td>
<td>3.48</td>
<td>4.95</td>
</tr>
<tr>
<td>h5f</td>
<td>14.31</td>
<td>5.97</td>
<td>21.19</td>
<td>1500</td>
<td>0.3604</td>
<td>5.07</td>
<td>3.47</td>
<td>4.98</td>
</tr>
<tr>
<td>h6a</td>
<td>10.7</td>
<td>CONTACT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>h6b</td>
<td>10.8</td>
<td>5.68</td>
<td>17.81</td>
<td>1600</td>
<td>0.2183</td>
<td>5.07</td>
<td>3.58</td>
<td>4.64</td>
</tr>
<tr>
<td>h6c</td>
<td>11.3</td>
<td>5.76</td>
<td>18.36</td>
<td>1600</td>
<td>0.2442</td>
<td>5.07</td>
<td>3.55</td>
<td>4.69</td>
</tr>
<tr>
<td>h6d</td>
<td>13</td>
<td>6.01</td>
<td>19.81</td>
<td>1600</td>
<td>0.3916</td>
<td>5.07</td>
<td>3.46</td>
<td>4.84</td>
</tr>
<tr>
<td>h6e</td>
<td>14.1</td>
<td>6.19</td>
<td>20.73</td>
<td>1600</td>
<td>0.4740</td>
<td>5.07</td>
<td>3.43</td>
<td>4.94</td>
</tr>
<tr>
<td>h6f</td>
<td>14.14</td>
<td>6.19</td>
<td>20.77</td>
<td>1600</td>
<td>0.4862</td>
<td>5.07</td>
<td>3.43</td>
<td>4.95</td>
</tr>
</tbody>
</table>
Table 2.8: continued.

<table>
<thead>
<tr>
<th>( N^0 )</th>
<th>( M_1 ) (( M_\odot ))</th>
<th>( M_2 ) (( M_\odot ))</th>
<th>( P_{\text{orb}} ) (Days)</th>
<th>( M_{\text{H}<em>1} ) (( M</em>\odot ))</th>
<th>( \log \left( \frac{L_{\text{1}}}{L_\odot} \right) )</th>
<th>( T_{\text{eff},1} ) (10^3 K)</th>
<th>( \log \left( \frac{L_{\text{2}}}{L_\odot} \right) )</th>
<th>( T_{\text{eff},2} ) (10^3 K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>h7a</td>
<td>10.4</td>
<td>CONTACT</td>
<td>/</td>
<td>1700</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>h7b</td>
<td>10.5</td>
<td>5.81</td>
<td>17.34</td>
<td>1700</td>
<td>0.2716</td>
<td>5.08</td>
<td>3.53</td>
<td>4.61</td>
</tr>
<tr>
<td>h7c</td>
<td>11.1</td>
<td>5.91</td>
<td>17.98</td>
<td>1700</td>
<td>0.3318</td>
<td>5.08</td>
<td>3.50</td>
<td>4.66</td>
</tr>
<tr>
<td>h7d</td>
<td>11.4</td>
<td>5.96</td>
<td>18.23</td>
<td>1700</td>
<td>0.3468</td>
<td>5.08</td>
<td>3.48</td>
<td>4.68</td>
</tr>
<tr>
<td>h7e</td>
<td>12.7</td>
<td>6.20</td>
<td>19.31</td>
<td>1700</td>
<td>0.4784</td>
<td>5.07</td>
<td>3.42</td>
<td>4.80</td>
</tr>
<tr>
<td>h7f</td>
<td>12.8</td>
<td>6.21</td>
<td>19.38</td>
<td>1700</td>
<td>0.5041</td>
<td>5.07</td>
<td>3.42</td>
<td>4.81</td>
</tr>
<tr>
<td>h8a</td>
<td>10.3</td>
<td>CONTACT</td>
<td>/</td>
<td>1800</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>h8b</td>
<td>10.4</td>
<td>5.95</td>
<td>17.08</td>
<td>1800</td>
<td>0.3467</td>
<td>5.08</td>
<td>3.49</td>
<td>4.58</td>
</tr>
<tr>
<td>h8c</td>
<td>11</td>
<td>6.05</td>
<td>17.70</td>
<td>1800</td>
<td>0.4006</td>
<td>5.08</td>
<td>3.45</td>
<td>4.63</td>
</tr>
<tr>
<td>h8d</td>
<td>11.5</td>
<td>6.16</td>
<td>18.11</td>
<td>1800</td>
<td>0.4668</td>
<td>5.08</td>
<td>3.43</td>
<td>4.68</td>
</tr>
<tr>
<td>h8e</td>
<td>11.8</td>
<td>6.22</td>
<td>18.34</td>
<td>1800</td>
<td>0.4860</td>
<td>5.08</td>
<td>3.42</td>
<td>4.70</td>
</tr>
<tr>
<td>h8f</td>
<td>11.9</td>
<td>6.24</td>
<td>18.41</td>
<td>1800</td>
<td>0.5104</td>
<td>5.08</td>
<td>3.42</td>
<td>4.71</td>
</tr>
<tr>
<td>h9a</td>
<td>10.3</td>
<td>CONTACT</td>
<td>/</td>
<td>1900</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>h9b</td>
<td>10.4</td>
<td>6.12</td>
<td>16.95</td>
<td>1900</td>
<td>0.4531</td>
<td>5.08</td>
<td>3.44</td>
<td>4.57</td>
</tr>
<tr>
<td>h9c</td>
<td>10.8</td>
<td>6.19</td>
<td>17.24</td>
<td>1900</td>
<td>0.4873</td>
<td>5.08</td>
<td>3.42</td>
<td>4.60</td>
</tr>
<tr>
<td>h9d</td>
<td>10.9</td>
<td>6.22</td>
<td>17.31</td>
<td>1900</td>
<td>0.5014</td>
<td>5.08</td>
<td>3.42</td>
<td>4.61</td>
</tr>
</tbody>
</table>

Notes: The secondary is, for every model, at the start of mass transfer a main sequence star. The other symbols have a similar meaning as in Table 2.7. If the evolution of the binary system ended because of formation of a contact binary, the parameters cannot be determined and are indicated by ’/’. 
Table 2.9: Properties of binary stellar models for an initial primary mass of $15 M_\odot$, conservative mass transfer and with variation of the initial secondary mass $M_{2i}$ and initial orbital period $P_{\text{orb}}$.

<table>
<thead>
<tr>
<th>$N^0$</th>
<th>$M_{2i}$ ($M_\odot$)</th>
<th>$M_{1f}$ ($M_\odot$)</th>
<th>$P_{\text{orb}}$ (Days)</th>
<th>$M_{\text{H1}}$ ($M_\odot$)</th>
<th>log ($\frac{L_1}{L_\odot}$)</th>
<th>$T_{\text{eff},1}$ (10$^3$ K)</th>
<th>log ($\frac{L_2}{L_\odot}$)</th>
<th>$T_{\text{eff},2}$ (10$^3$ K)</th>
<th>*1/ *2</th>
<th>$#$ time (yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>g2a</td>
<td>14.94</td>
<td>5.60</td>
<td>22.26</td>
<td>1200</td>
<td>0.2178</td>
<td>5.06</td>
<td>3.66</td>
<td>5.08</td>
<td>3.63</td>
<td>*2 14200</td>
</tr>
<tr>
<td>g2b</td>
<td>14.99</td>
<td>5.61</td>
<td>21.75</td>
<td>1200</td>
<td>0.2211</td>
<td>5.06</td>
<td>3.66</td>
<td>5.08</td>
<td>3.63</td>
<td>*2 70400</td>
</tr>
<tr>
<td>g3a</td>
<td>14.74</td>
<td>5.62</td>
<td>22.01</td>
<td>1300</td>
<td>0.2061</td>
<td>5.07</td>
<td>3.60</td>
<td>4.90</td>
<td>3.79</td>
<td>*1 /</td>
</tr>
<tr>
<td>g3b</td>
<td>14.75</td>
<td>5.62</td>
<td>22.02</td>
<td>1300</td>
<td>0.2058</td>
<td>5.07</td>
<td>3.60</td>
<td>4.90</td>
<td>3.79</td>
<td>*1 /</td>
</tr>
<tr>
<td>g3c</td>
<td>14.93</td>
<td>5.64</td>
<td>22.18</td>
<td>1300</td>
<td>0.2036</td>
<td>5.07</td>
<td>3.59</td>
<td>5.02</td>
<td>3.67</td>
<td>*1 /</td>
</tr>
<tr>
<td>g3d</td>
<td>14.94</td>
<td>5.76</td>
<td>22.18</td>
<td>1300</td>
<td>0.2638</td>
<td>5.07</td>
<td>3.59</td>
<td>5.07</td>
<td>3.64</td>
<td>*2 300</td>
</tr>
<tr>
<td>g3e</td>
<td>14.99</td>
<td>5.80</td>
<td>21.37</td>
<td>1300</td>
<td>0.2894</td>
<td>5.07</td>
<td>3.58</td>
<td>5.08</td>
<td>3.62</td>
<td>*2 63100</td>
</tr>
<tr>
<td>g4a</td>
<td>14.55</td>
<td>5.79</td>
<td>21.64</td>
<td>1400</td>
<td>0.2761</td>
<td>5.07</td>
<td>3.53</td>
<td>4.86</td>
<td>3.83</td>
<td>*1 /</td>
</tr>
<tr>
<td>g4b</td>
<td>14.9</td>
<td>5.84</td>
<td>21.94</td>
<td>1400</td>
<td>0.3081</td>
<td>5.07</td>
<td>3.51</td>
<td>4.95</td>
<td>3.74</td>
<td>*1 /</td>
</tr>
<tr>
<td>g4c</td>
<td>14.94</td>
<td>5.84</td>
<td>21.98</td>
<td>1400</td>
<td>0.3087</td>
<td>5.07</td>
<td>3.51</td>
<td>5.04</td>
<td>3.66</td>
<td>*1 /</td>
</tr>
<tr>
<td>g4d</td>
<td>14.95</td>
<td>6.02</td>
<td>21.79</td>
<td>1400</td>
<td>0.4012</td>
<td>5.07</td>
<td>3.51</td>
<td>5.07</td>
<td>3.63</td>
<td>*2 7900</td>
</tr>
<tr>
<td>g4e</td>
<td>14.99</td>
<td>6.03</td>
<td>21.01</td>
<td>1400</td>
<td>0.4012</td>
<td>5.07</td>
<td>3.51</td>
<td>5.08</td>
<td>3.61</td>
<td>*2 58300</td>
</tr>
<tr>
<td>g5a</td>
<td>14.37</td>
<td>5.98</td>
<td>21.24</td>
<td>1500</td>
<td>0.3656</td>
<td>5.07</td>
<td>3.47</td>
<td>4.81</td>
<td>3.85</td>
<td>*1 /</td>
</tr>
<tr>
<td>g5b</td>
<td>14.4</td>
<td>5.99</td>
<td>21.27</td>
<td>1500</td>
<td>0.3728</td>
<td>5.07</td>
<td>3.47</td>
<td>4.82</td>
<td>3.85</td>
<td>*1 /</td>
</tr>
<tr>
<td>g5c</td>
<td>14.55</td>
<td>6.01</td>
<td>21.40</td>
<td>1500</td>
<td>0.3904</td>
<td>5.07</td>
<td>3.45</td>
<td>4.85</td>
<td>3.82</td>
<td>*1 /</td>
</tr>
<tr>
<td>g5d</td>
<td>14.9</td>
<td>6.06</td>
<td>21.69</td>
<td>1500</td>
<td>0.4240</td>
<td>5.07</td>
<td>3.45</td>
<td>4.95</td>
<td>3.74</td>
<td>*1 /</td>
</tr>
<tr>
<td>g5e</td>
<td>14.94</td>
<td>6.06</td>
<td>21.73</td>
<td>1500</td>
<td>0.4244</td>
<td>5.07</td>
<td>3.45</td>
<td>5.04</td>
<td>3.66</td>
<td>*1 /</td>
</tr>
<tr>
<td>g5f</td>
<td>14.95</td>
<td>6.27</td>
<td>21.56</td>
<td>1500</td>
<td>0.5305</td>
<td>5.07</td>
<td>3.45</td>
<td>5.07</td>
<td>3.63</td>
<td>*2 9100</td>
</tr>
<tr>
<td>g5g</td>
<td>14.99</td>
<td>6.28</td>
<td>20.56</td>
<td>1500</td>
<td>0.5328</td>
<td>5.07</td>
<td>3.45</td>
<td>5.07</td>
<td>3.60</td>
<td>*2 59500</td>
</tr>
<tr>
<td>g6a</td>
<td>14.18</td>
<td>6.20</td>
<td>20.81</td>
<td>1600</td>
<td>0.4796</td>
<td>5.07</td>
<td>3.42</td>
<td>4.78</td>
<td>3.88</td>
<td>*1 /</td>
</tr>
<tr>
<td>g6b</td>
<td>14.2</td>
<td>6.20</td>
<td>20.83</td>
<td>1600</td>
<td>0.4980</td>
<td>5.07</td>
<td>3.42</td>
<td>4.77</td>
<td>3.88</td>
<td>*1 /</td>
</tr>
<tr>
<td>g6c</td>
<td>14.3</td>
<td>6.22</td>
<td>20.91</td>
<td>1600</td>
<td>0.4997</td>
<td>5.07</td>
<td>3.42</td>
<td>4.80</td>
<td>3.86</td>
<td>*1 /</td>
</tr>
<tr>
<td>g6d</td>
<td>14.4</td>
<td>6.23</td>
<td>20.99</td>
<td>1600</td>
<td>0.5236</td>
<td>5.07</td>
<td>3.42</td>
<td>4.82</td>
<td>3.85</td>
<td>*1 /</td>
</tr>
</tbody>
</table>

Notes: The secondary is, for every model, at the start of mass transfer on the giant branch. The column with *1/ *2 indicates which star in the binary system explodes first. The time difference between the two stars reaching carbon burning is indicated if star 2 explodes before star 1. The other symbols have a similar meaning as in Table 2.7.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>8.5</td>
<td>14.9</td>
<td>5.15</td>
<td>18.86</td>
<td>800</td>
<td>0.50</td>
<td>0.0897</td>
<td>5.04</td>
<td>3.84</td>
<td>5.08</td>
<td>3.56</td>
<td>*1</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>9.9</td>
<td>14.9</td>
<td>5.13</td>
<td>22.07</td>
<td>900</td>
<td>0.90</td>
<td>0.0814</td>
<td>5.03</td>
<td>3.84</td>
<td>5.14</td>
<td>12.6</td>
<td>*1</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>9.5</td>
<td>14.9</td>
<td>5.38</td>
<td>18.75</td>
<td>900</td>
<td>0.50</td>
<td>0.1671</td>
<td>5.04</td>
<td>3.71</td>
<td>5.10</td>
<td>3.55</td>
<td>*2</td>
<td>11900</td>
<td></td>
</tr>
<tr>
<td>0c9</td>
<td>14.9</td>
<td>5.23</td>
<td>21.97</td>
<td>1000</td>
<td>0.90</td>
<td>0.1050</td>
<td>5.04</td>
<td>3.73</td>
<td>5.13</td>
<td>11.8</td>
<td>*1</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>1b9</td>
<td>14.95</td>
<td>5.50</td>
<td>21.41</td>
<td>1100</td>
<td>0.90</td>
<td>0.2107</td>
<td>5.05</td>
<td>3.66</td>
<td>5.08</td>
<td>3.62</td>
<td>*2</td>
<td>20200</td>
<td></td>
</tr>
<tr>
<td>1b7</td>
<td>14.95</td>
<td>5.60</td>
<td>19.93</td>
<td>1100</td>
<td>0.70</td>
<td>0.2469</td>
<td>5.05</td>
<td>3.62</td>
<td>5.07</td>
<td>3.59</td>
<td>*2</td>
<td>16700</td>
<td></td>
</tr>
<tr>
<td>1c9</td>
<td>14.9</td>
<td>5.36</td>
<td>21.81</td>
<td>1100</td>
<td>0.90</td>
<td>0.1512</td>
<td>5.05</td>
<td>3.66</td>
<td>5.12</td>
<td>10.7</td>
<td>*1</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>1c5</td>
<td>14.9</td>
<td>5.73</td>
<td>18.54</td>
<td>1100</td>
<td>0.50</td>
<td>0.2878</td>
<td>5.06</td>
<td>3.58</td>
<td>5.09</td>
<td>3.55</td>
<td>*2</td>
<td>12300</td>
<td></td>
</tr>
<tr>
<td>1d75</td>
<td>14.87</td>
<td>5.43</td>
<td>20.56</td>
<td>1100</td>
<td>0.75</td>
<td>0.1712</td>
<td>5.05</td>
<td>3.63</td>
<td>5.08</td>
<td>8.81</td>
<td>*1</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>1d5</td>
<td>14.87</td>
<td>5.57</td>
<td>18.57</td>
<td>1100</td>
<td>0.50</td>
<td>0.2680</td>
<td>5.06</td>
<td>3.58</td>
<td>4.94</td>
<td>3.69</td>
<td>*1</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>1d25</td>
<td>14.87</td>
<td>5.74</td>
<td>16.67</td>
<td>1100</td>
<td>0.25</td>
<td>0.2857</td>
<td>5.06</td>
<td>3.53</td>
<td>4.93</td>
<td>3.64</td>
<td>*1</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>2c5</td>
<td>14.9</td>
<td>5.76</td>
<td>18.47</td>
<td>1200</td>
<td>0.50</td>
<td>0.2661</td>
<td>5.07</td>
<td>3.54</td>
<td>4.95</td>
<td>3.66</td>
<td>*1</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>3d5</td>
<td>14.7</td>
<td>5.96</td>
<td>18.16</td>
<td>1300</td>
<td>0.50</td>
<td>0.3658</td>
<td>5.07</td>
<td>3.48</td>
<td>4.89</td>
<td>3.71</td>
<td>*1</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>4a7</td>
<td>14.54</td>
<td>6.01</td>
<td>19.34</td>
<td>1400</td>
<td>0.70</td>
<td>0.3906</td>
<td>5.07</td>
<td>3.46</td>
<td>4.85</td>
<td>3.77</td>
<td>*1</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>4c9</td>
<td>14.5</td>
<td>5.96</td>
<td>19.68</td>
<td>1400</td>
<td>0.90</td>
<td>0.3671</td>
<td>5.07</td>
<td>3.47</td>
<td>5.02</td>
<td>12.7</td>
<td>*1</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>4c7</td>
<td>14.5</td>
<td>6.00</td>
<td>19.31</td>
<td>1400</td>
<td>0.70</td>
<td>0.3873</td>
<td>5.07</td>
<td>3.46</td>
<td>5.00</td>
<td>9.64</td>
<td>*1</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>4c5</td>
<td>14.5</td>
<td>6.18</td>
<td>17.84</td>
<td>1400</td>
<td>0.50</td>
<td>0.4844</td>
<td>5.07</td>
<td>3.43</td>
<td>4.86</td>
<td>3.74</td>
<td>*1</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>g4b7</td>
<td>14.9</td>
<td>6.06</td>
<td>19.66</td>
<td>1400</td>
<td>0.70</td>
<td>0.4204</td>
<td>5.07</td>
<td>3.45</td>
<td>4.95</td>
<td>3.69</td>
<td>*1</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>g4c7</td>
<td>14.94</td>
<td>6.07</td>
<td>19.70</td>
<td>1400</td>
<td>0.70</td>
<td>0.4191</td>
<td>5.07</td>
<td>3.45</td>
<td>5.04</td>
<td>3.61</td>
<td>*1</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>g4d7</td>
<td>14.95</td>
<td>6.28</td>
<td>19.60</td>
<td>1400</td>
<td>0.70</td>
<td>0.5387</td>
<td>5.07</td>
<td>3.45</td>
<td>5.07</td>
<td>3.58</td>
<td>*2</td>
<td>9700</td>
<td></td>
</tr>
<tr>
<td>h5.0</td>
<td>8.3</td>
<td>5.37</td>
<td>10.15</td>
<td>1500</td>
<td>0.25</td>
<td>0.0795</td>
<td>5.07</td>
<td>3.87</td>
<td>3.88</td>
<td>25.1</td>
<td>*1</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>h5.1</td>
<td>8.6</td>
<td>5.41</td>
<td>10.44</td>
<td>1500</td>
<td>0.25</td>
<td>0.0912</td>
<td>5.07</td>
<td>3.79</td>
<td>3.93</td>
<td>25.4</td>
<td>*1</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>h5.2</td>
<td>8.8</td>
<td>5.44</td>
<td>10.64</td>
<td>1500</td>
<td>0.25</td>
<td>0.1074</td>
<td>5.07</td>
<td>3.75</td>
<td>3.96</td>
<td>25.5</td>
<td>*1</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>h5.3</td>
<td>8.8</td>
<td>5.41</td>
<td>12.49</td>
<td>1500</td>
<td>0.50</td>
<td>0.0961</td>
<td>5.07</td>
<td>3.80</td>
<td>4.17</td>
<td>27.8</td>
<td>*1</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>h5.4</td>
<td>9.1</td>
<td>5.44</td>
<td>12.78</td>
<td>1500</td>
<td>0.50</td>
<td>0.1164</td>
<td>5.07</td>
<td>3.75</td>
<td>4.20</td>
<td>28.0</td>
<td>*1</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>h5.5</td>
<td>9.4</td>
<td>5.55</td>
<td>11.22</td>
<td>1500</td>
<td>0.25</td>
<td>0.1510</td>
<td>5.07</td>
<td>3.65</td>
<td>4.06</td>
<td>26.0</td>
<td>*1</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>h5.6</td>
<td>9.4</td>
<td>5.49</td>
<td>13.07</td>
<td>1500</td>
<td>0.50</td>
<td>0.1239</td>
<td>5.07</td>
<td>3.70</td>
<td>4.24</td>
<td>28.2</td>
<td>*1</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>h5.7</td>
<td>9.4</td>
<td>CONTACT</td>
<td>/</td>
<td>1500</td>
<td>0.75</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>h5.8</td>
<td>9.7</td>
<td>CONTACT</td>
<td>/</td>
<td>1500</td>
<td>0.75</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>h5.9</td>
<td>10</td>
<td>5.50</td>
<td>15.49</td>
<td>1500</td>
<td>0.75</td>
<td>0.1308</td>
<td>5.07</td>
<td>3.69</td>
<td>4.46</td>
<td>30.5</td>
<td>*1</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>h5d5</td>
<td>14</td>
<td>6.38</td>
<td>17.23</td>
<td>1500</td>
<td>0.50</td>
<td>0.5975</td>
<td>5.07</td>
<td>3.40</td>
<td>4.43</td>
<td>27.1</td>
<td>*1</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>5b9</td>
<td>14.35</td>
<td>6.06</td>
<td>20.46</td>
<td>1500</td>
<td>0.90</td>
<td>0.4163</td>
<td>5.07</td>
<td>3.45</td>
<td>5.04</td>
<td>16.6</td>
<td>*1</td>
<td>/</td>
<td></td>
</tr>
</tbody>
</table>
### Table 2.10: continued.

<table>
<thead>
<tr>
<th>$N^0$</th>
<th>$M_{2i}$ ($M_\odot$)</th>
<th>$M_{2f}$ ($M_\odot$)</th>
<th>$P_{orb}$ (Days)</th>
<th>$\beta$</th>
<th>$M_{H1}$ ($M_\odot$)</th>
<th>$\log (\frac{L}{L_\odot})$</th>
<th>$T_{eff,1}$ (10³ K)</th>
<th>$\log (\frac{L}{L_\odot})$</th>
<th>$T_{eff,2}$ (10³ K)</th>
<th>$\pm$ time (yr)</th>
<th>$\pm$1/2</th>
</tr>
</thead>
<tbody>
<tr>
<td>5b8</td>
<td>14.35</td>
<td>6.14</td>
<td>19.72</td>
<td>1500</td>
<td>0.80</td>
<td>0.4624</td>
<td>5.07</td>
<td>3.43</td>
<td>5.01</td>
<td>14.3</td>
<td>*1</td>
</tr>
<tr>
<td>5b75</td>
<td>14.35</td>
<td>6.19</td>
<td>19.35</td>
<td>1500</td>
<td>0.75</td>
<td>0.4886</td>
<td>5.07</td>
<td>3.43</td>
<td>4.97</td>
<td>8.67</td>
<td>*1</td>
</tr>
<tr>
<td>5b7</td>
<td>14.35</td>
<td>6.24</td>
<td>18.97</td>
<td>1500</td>
<td>0.70</td>
<td>0.5176</td>
<td>5.07</td>
<td>3.42</td>
<td>4.84</td>
<td>3.77</td>
<td>*1</td>
</tr>
<tr>
<td>5b6</td>
<td>14.35</td>
<td>6.34</td>
<td>18.25</td>
<td>1500</td>
<td>0.60</td>
<td>0.5750</td>
<td>5.07</td>
<td>3.40</td>
<td>4.81</td>
<td>3.78</td>
<td>*1</td>
</tr>
<tr>
<td>5b5</td>
<td>14.35</td>
<td>6.45</td>
<td>17.54</td>
<td>1500</td>
<td>0.50</td>
<td>0.6325</td>
<td>5.07</td>
<td>3.39</td>
<td>4.81</td>
<td>3.77</td>
<td>*1</td>
</tr>
<tr>
<td>5b25</td>
<td>14.35</td>
<td>6.74</td>
<td>15.87</td>
<td>1500</td>
<td>0.25</td>
<td>0.8062</td>
<td>5.07</td>
<td>3.37</td>
<td>4.80</td>
<td>3.72</td>
<td>*1</td>
</tr>
<tr>
<td>g5c5</td>
<td>14.55</td>
<td>6.49</td>
<td>17.72</td>
<td>1500</td>
<td>0.50</td>
<td>0.6595</td>
<td>5.07</td>
<td>3.39</td>
<td>4.85</td>
<td>3.79</td>
<td>*1</td>
</tr>
<tr>
<td>6b9</td>
<td>14.15</td>
<td>6.29</td>
<td>20.04</td>
<td>1600</td>
<td>0.90</td>
<td>0.5445</td>
<td>5.07</td>
<td>3.41</td>
<td>5.00</td>
<td>17.5</td>
<td>*1</td>
</tr>
</tbody>
</table>

Notes: The column with *1/*2 indicates which star in the binary system explodes first. The time difference between the two stars reaching carbon burning is indicated if star 2 explodes before star 1. If the evolution of the binary system ended because of formation of a contact binary, the parameters cannot be determined and are indicated by '/'. The other symbols have a similar meaning as in Table 2.7.
In the single degenerate scenario for Type Ia supernova (SNe Ia), a white dwarf (WD) must gain a significant amount of matter from a companion star. Because the accreted mass carries angular momentum, the WD is likely to achieve fast spin periods, which can increase the critical mass, $M_{\text{crit}}$, needed for explosion. When $M_{\text{crit}}$ is higher than the maximum mass achieved by the WD, the central regions of the WD must spin down before it can explode. This introduces super-Chandrasekhar single-degenerate explosions, and a delay between the completion of mass gain and the time of the explosion. Matter ejected from the binary during mass transfer therefore has a chance to become diffuse, and the explosion occurs in a medium with a density similar to that of typical regions of the interstellar medium. Also, either by the end of the WD’s mass increase or else by the time of explosion, the donor may exhaust its stellar envelope and become a WD. This alters, generally diminishing, explosion signatures related to the donor star. Nevertheless, the spin-up/spin-down model is highly predictive. Prior to explosion, progenitors can be super-$M_{\text{Ch}}$ WDs in either wide binaries with WD companions or cataclysmic variables. These systems can be discovered and studied through wide-field surveys. Post explosion, the spin-up/spin-down model predicts a population of fast-moving WDs, low-mass stars, and even brown dwarfs. In addition, the spin-up/spin-down model provides a paradigm which may be able to explain both the similarities and the diversity observed among SNe Ia.
Chapter 3: Spin-up/spin-down models for type Ia supernovae

3.1 Introduction

Type Ia supernovae (SNe Ia) are believed to be the explosions of carbon-oxygen white dwarfs (CO WDs). To explode, a CO WD must reach a critical mass ($M_{\text{crit}}$) generally assumed to be the Chandrasekhar mass ($M_{\text{ch}} \sim 1.4 M_\odot$). This can be achieved either through accretion from a companion star (the single-degenerate (SD) scenario) or through the merger of two WDs (the double-degenerate (DD) scenario). Key signatures of the SD scenario include direct detection of progenitors in archival images, direct detection of companions in supernova remnants, and radiation emitted when light and matter from the supernova interact with the companion star or with circumstellar material ejected from the progenitor binary. With the exception of signatures of absorption by circumstellar material in a small number of SNe Ia (Patat et al., 2007b), these strong signatures have not been definitively detected, calling into question the relevance of SD models.

In SD models, the WD must accrete and retain matter. This requires high mass infall rates, with $\dot{M} > 10^{-7} M_\odot \text{yr}^{-1}$ (Iben, 1982; Nomoto, 1982; Prialnik & Kovetz, 1995; Shen & Bildsten, 2007). Because infalling matter carries angular momentum, the angular momentum of the WD must increase. Although spin-up seems certain to occur, its effects are difficult to compute from first principles. One effect is an increase in the value of $M_{\text{crit}}$ (Anand, 1965; Roxburgh, 1965; Ostriker & Bodenheimer, 1968; Hachisu, 1986; Yoon & Langer, 2005). We will show that an increase in $M_{\text{crit}}$ has a profound effect on the progenitor signatures. Some of the oft-expected donor signatures are diminished, possibly explaining why they have either not been detected or have been detected only rarely. Nevertheless, the spin-up/spin-down model is testable because it suggests alternative ways to identify the progenitors and test SD models. In Sect. 3.2 we discuss the model, using four key points to summarize the features relevant to observations of the progenitors and explosions, to which we turn in Sect. 3.3. In Sect. 3.4 we discuss how spin-up/spin-down provides a testable paradigm that can explain both the unity and diversity among SNe Ia.

3.2 Spin-Up and Spin-Down

1. Infalling matter spins up the WD to near-critical rotation. Because infalling matter carries angular momentum, the angular momentum of the WD must increase when the infalling matter is retained. Spin-up is a common process in accreting compact objects. Neutron stars (NSs), for example, can be spun up to periods of a few milliseconds (Lorimer, 2008). Similarly a number of fast-spinning WDs that must have been spun up by accretion are known, for example, WZ Sge, 27.87 s (Patterson 80); AE Aqr, 33.06 s (Patterson, 1979); V842 Cen, 56.82 s (Woudt et al., 2009) and V455 And, 67.2 s (Araujo-Betancor et al., 2005). These periods are much longer than for NSs, due to the much higher moment of inertia of WDs, but similar to the millisecond pulsars, the surface velocity is only a factor of a few lower than the escape speed.

We can measure the spins in these specific systems because the WDs are intermediate polars (IPs) where the accretion is channeled along the field lines of the WD (Warner, 2003). This is possible only for relatively modest accretion rates; higher rates will increase the infalling matter density and probably quench the magnetic fields. The binaries most likely to be progenitors of SNe Ia have rates of mass transfer that are hundreds or thousands of times greater than those inferred for IPs. The
retention of mass should make it possible to spin mass-gaining WDs to even shorter periods than measured for IPs, even though measurements are difficult.

GK Per, which experienced a classical nova in 1901, has a spin period of 351 s, and is spinning up at a rate measured to be $0.00027 \pm 0.0005$ s yr$^{-1}$ (Mauche, 2004), corresponding to a spin-up of $2.7 \cdot 10^5$ s per solar mass accreted in this system. The WDs which evolve toward SNe Ia must accrete at least $0.2 M_\odot$. Although the specific angular momentum carried by infalling matter will vary among binary systems (see, e.g., Popham & Narayan, 1991), the spin-up of GK Per suggests that WDs can gain enough angular momentum to reach critical rotation.

2. The rotation increases the critical mass $M_{\text{crit}}$, needed for the explosion. This implies that accreting WDs can achieve masses in excess of $M_{\text{Ch}}$ without either exploding or imploding. For a rigid rotator, maximal rotation produces an increase in $M_{\text{crit}}$ of roughly 5% (Anand, 1965; Roxburgh, 1965). For more complex radial distributions of the internal angular momentum, Ostriker & Bodenheimer (1968) showed that the critical mass could become very high; they constructed models with $M_{\text{crit}}$ as high as $4 M_\odot$, noting, however, that not all of the configurations they considered were likely to be realized in nature. Hachisu (1986) also found stable equilibrium configurations with $M_{\text{WD}} > 2 M_\odot$.

Yoon & Langer (2005) considered spin-up due to accretion and derived comparably high masses. Piro (2008) included viscous effects and found that, under certain input assumptions, the WD should be able to achieve a state close to uniform rotation during much of the accretion phase, but that differential rotation could be important during a shortlived ($\sim 10^3$ years) “simmering” phase just prior to explosion. The bottom line is that the values of $M_{\text{crit}}$ are difficult to compute from first principles and that the rigid-rotation limit can be taken to give a lower bound.

3. Spin-down can occur when $M$ is low or when mass transfer has ceased. The crucial element for explosion is that angular momentum be lost from the central region of the WD. We therefore use the term “spin-down” to refer to the spindown of the central region, which may not be exactly tracked by the surface spin. Spin-down can be achieved through a combination of angular momentum redistribution (AMR) and angular momentum loss from the WD. As $M$ decreases, AMR may begin; in addition, more angular momentum may be lost per unit time than gained (as is seen in AE Aqr Meintjes, 2002; Ikhsanov et al., 2004). Even isolated WDs can spin down, e.g., through gravitational radiation associated with spin-induced effects (the r-mode instability) (Sedrakian et al., 2006). Spin-down times are uncertain, but almost certainly exhibit a large range, from $< 10^6$ years to $> 10^9$ years (Lindblom, 1999; Yoon & Langer, 2005).

4. Explosion occurs when the central spin period has been reduced to a critical value, $P_{\text{crit}}$. As the spin decreases, so does the value of $M_{\text{crit}}$. When the value of $M_{\text{crit}}$ falls below the current mass of the WD, the WD will explode. This requires that the WD has not yet crystallized (in which case the outcome is an accretion-induced collapse (AIC) into a neutron star, Nomoto & Kondo, 1991). Although AIC is possible, note that the time needed for the WD to cool to low-enough temperatures is several $10^9$ years (Yoon & Langer, 2005). Furthermore, continued accretion and the r-mode instability act as heat sources; crystallization may therefore be avoided.


### 3.3 Observational Signatures

#### 3.3.1 Background

**Population:** Let $f$ denote the fraction of all SNe Ia progenitors in which (a) spin-up produces a significant change in the value of $M_{\text{crit}}$ and (b) the maximum mass achieved by the WD is smaller than $M_{\text{crit}}$, necessitating an interval of spin-down. If the rate of SNe Ia is $R$, then the number, $N_{\text{SD}}$, of spinning-down progenitors in the Galaxy is

$$N_{\text{SD}} = f \cdot (3 \cdot 10^4) \cdot (\tau/10^7 \text{ yr}) \cdot (R/0.003 \text{ yr}^{-1}),$$

(3.1)

where $\tau$ is the spin-down time, the time between the end of genuine mass gain by the WD and the explosion. A few percent of the spinning-down progenitors could lie within a kiloparsec of Earth. There could be an even larger number of Galactic postexplosion systems: $f \cdot (3 \cdot 10^7) \cdot (R/0.003 \text{ yr}^{-1})$, if SNe Ia have been occurring in the Galaxy for $10^{10}$ years.

**Binary Evolution.** SD SNe Ia progenitors must have donor stars whose state of evolution, mass, and orbital separation enable them to contribute mass at high rates. Giant donors can do this if the orbital separation is favorable. Once $\dot{M}$ from a giant donor is high enough to promote nuclear burning by the WD, it is likely to stay high. The binary will be a symbiotic in which the WD gains mass and angular momentum until the giant’s envelope is depleted. The final pre-SNe Ia state is a wide-orbit double WD.

The same evolutionary path can be followed when the donor starts mass transfer as a subgiant if its core is evolved enough. For less evolved subgiant donors and main-sequence (MS) donors the mass ratio, $q$, between the donor and the WD is important. The value of $\dot{M}$ can be high enough for nuclear burning only when $q > 1$. When the mass ratio reverses, the rate of mass transfer decreases dramatically, and the WD can begin to lose angular momentum. Subgiant donors could become WDs, reproducing the signatures described above for giant donors. For MS donors, the binary will become an accretion-powered cataclysmic variable (CV); long spin-down times would transform the donor into a degenerate object of brown-dwarf mass, with an orbital period as low as $\sim 90$ minutes. Figure 3.1 shows the evolution of a subgiant donor whose WD companion gains enough mass to slightly exceed $M_{\text{Ch}}$.

#### 3.3.2 Progenitor Signatures

**Missing Signatures.** Signatures thought to be integral parts of SD models are diminished. For example, even a spin-down time of $10^5$ years allow circumbinary material to dissipate. Furthermore, the donors are likely to be either compact objects at the time of explosion or low-mass stars. Signatures of interaction with the supernova would therefore be diminished relative to the case in which spin is unimportant.\(^1\) In addition, the donors tend to be dim, making them difficult to detect. Nor are the WDs likely to be burning nuclear fuel just prior to explosion. This is consistent with the small numbers of supersoft X-ray sources found in external galaxies (Di Stefano et al., 2003, 2004; Di Stefano, 2010b,a; Di Stefano et al., 2010).\(^2\)

---

\(^1\)Note that signatures related to circumbinary material and/or interactions with a companion can be ambiguous. For example, a DD may take place inside a common envelope if the envelope ejection efficiency is low. Or, if some SNe Ia take place in multiple systems, stars not directly involved in the explosion may produce detectable signatures.

\(^2\)Nuclear burning should take place, however, while the WD gains mass. The lack of SSS-like emission may be due to an extended photosphere or absorption by circumstellar matter that dissipates prior to explosion (Di Stefano, 2010b,a).
3.3 Observational Signatures

![Hertzsprung-Russell diagram](image)

**Figure 3.1:** Hertzsprung-Russell diagram of the donor star in an initial binary system of an $1.1 \, M_\odot$ WD and a $2 \, M_\odot$ MS star and initial orbital period of 2.0 days. The solid line indicates the phase of mass transfer onto the WD. The mass accretion rate is based on Hachisu et al. (1996, 1999b). Mass transfer starts when the donor star is in the Hertzsprung gap and continuous on the GB. After mass transfer the donor evolves into a He WD. The final system is a WD of $\sim 1.5 \, M_\odot$ and a companion of $\sim 0.3 \, M_\odot$. The crosses indicate different times after mass transfer has ceased ($10^6$, $10^7$, and $10^9$ years). After $10^6$ yr: the donor will appear as a low-mass He-star, $10^7$ yr: as a hot He WD, and $10^9$ yr: as a cooler He WD.

Tests of the models. Systematic searches of data from widefield surveys, including SDSS, Pan-STARRS, and LSST, can identify those Galactic progenitors nearest to us [See, e.g., Kleinman et al. (2007); Szkody et al. (2006) for SDSS-based identification of WDs, and CVs]. To test spin-up/spin-down models, we want to measure the mass function of the spinning-down WDs. The maximum mass will tell whether differential rotation occurs. Even if no super-$M_{Ch}$ WDs were found, the mass distribution would provide hitherto unavailable information on the mass gain during binary evolution.

Wide double WDs. Binaries containing a super-$M_{Ch}$ in wide orbit with a compact companion are distinctive: they exhibit the spectra of two hot WDs (Figure 3.2). The lower the mass of the secondary, the cooler it will be, and the larger the spectral contrast will be. Studies which have identified WD/M-dwarf pairs in data from, e.g., SPY (Maxted et al., 2007), demonstrate that it will be possible to either identify or place limits on the existence of the wide double-WD progenitors we predict. The double-WD SNe Ia progenitors with the smallest spectral contrast would be those in which the secondaries are the most
**Figure 3.2:** Logarithm of luminosity vs. logarithm of temperature for cooling WDs in wide double-WD binaries. Each sequence of single-color points with a fixed number of sides corresponds to a given time after the end of mass transfer. The top row (magenta points) corresponds to $10^5$ years, and red, cyan, green-blue, and black points correspond to $10^6$, $10^7$, $10^8$, $10^9$, $10^{10}$ years, respectively. The hottest and brightest system in each sequence corresponds to a Chandrasekhar-mass WD. The super-Chandrasekhar-mass primaries we consider may be somewhat hotter and brighter. Each subsequent point in the same-age sequence corresponds to a WD with a mass 0.2$M_\odot$ lower than the previous point of the sequence. The minimum mass shown is 0.2$M_\odot$. We used the realization of Mestel’s cooling law suggested by Kawaler (1998). The figure illustrates the important feature that the massive WD is likely to be brighter and hotter than its lower-mass companion, and that both WDs are bright and hot compared with the majority of Galactic WDs, which are older.

massive. These would, however, be distinctive in another way: the separation between the two components could be resolvable (the top panel of Figure 3.3). When the spectrum indicates that the secondary mass is also high, follow-up observations to determine if the WDs can be resolved would also be useful.

**CVs and other mass-transfer binaries.** Wide-field surveys, combined with X-ray-source catalogs, can identify CVs and other mass transfer binaries. It is interesting to note that AE Aqr appears to have had an evolution that mirrors what is expected for SNe Ia progenitors. The key difference is that the WD’s mass, while larger than typical of WDs, is smaller than $M_{Ch}$ (see, e.g., Meintjes, 2002).

### 3.3.3 Detecting the Remnants of the Donors

The SNe Ia release the donor from orbit. Hansen (2003) and Justham et al. (2009) considered donors that had not yet finished evolving at the time of explosion. In the spin-up/spin-down scenario, many donors
3.3 Observational Signatures

Figure 3.3: **Top panel:** Logarithm of the angular separation (in mas) between the pre-explosion super-$M_{\text{Ch}}$ WD and its WD companion, as a function of the companion’s mass. Top curve: the distance, $D$, to the binary is 100 pc; bottom curve: $D = 1$ kpc. To compute the orbital separation we assumed that the donor is either a subgiant or giant that fills its Roche lobe until its envelope is exhausted. **Bottom panel:** The logarithm of the companion’s vs. its mass. Bottom curve: results for the spin-up/spin-down model, in which the donor has lost its envelope prior to the explosion. Plotted is the orbital speed (presumably close to the ejection speed) vs. donor’s the core mass. Upper curve is computed assuming that the donor has a total mass of 3 $M_\odot$ at the time its WD companion explodes. The dot-dash line represents the speed of brown dwarfs released from close orbits by the explosion. These fast brown dwarfs are new predictions of our model.

will have lost their envelopes prior to explosion; the binary will therefore be lighter and have a lower orbital velocity (bottom panel of Figure 3.3). With the current observational sample (Oppenheimer et al., 2001; Justham et al., 2009) it is not possible to verify that high-speed WDs and isolated low-mass WDs

53
Chapter 3: Spin-up/spin-down models for type Ia supernovae

are remnants of SNe Ia explosions, or to distinguish among models. New surveys, particularly those that allow high-proper-motion remnants to be identified, will provide more data.

A unique feature of our model is that, if the donor started as an MS star and if $\tau$ is large, the donor will be a degenerate brown-dwarf-mass object at the time of explosion. Its speed will be high: for a two-hour period around a $1.6 M_\odot$ WD, $v \sim 570$ km s$^{-1}$. Although they constitute a small fraction of Galactic brown dwarfs (at most $10^{-4} - 10^{-3}$), some of these objects could be discovered through their action as lenses, if complementary data allow radiation from the brown dwarf to be detected (Di Stefano, 2008).

3.3.4 Other Connections

Cosmology. If explosions at different cosmic times have different amounts of local absorption, this would introduce a systematic uncertainty into measurements of the universe’s acceleration. When the WD must spin down before explosion, circumstellar material will disperse and play a smaller role regardless of redshift. If, therefore, spin-up/spin-down is common, the systematic uncertainty is less significant.

Prompt explosions. Some SNe Ia occur within a few $\times 10^8$ years after star formation (see e.g., Maoz & Badenes, 2010). If spin-up occurs in these “prompt” systems, then their spin-down times must be well under $10^9$ years.

Variety. Our model predicts variety in the mass of the exploding WD and in the spin-down times. While some of the standard signatures may be detected in some cases, the trend in spin-up/spin-down predictions is toward diversity among the explosions, with fewer systems exhibiting post-explosion interaction with the donor or circumbinary material. The distribution of explosion properties can provide indirect tests. Direct model tests may be provided by discovering the distinctive pre-explosion (super-$M_{Ch}$ WDs) or post-explosion (high-speed brown dwarfs and WDs) systems in our Galaxy.

3.4 Spin-Up/Spin-Down: A New Paradigm

Conservation of angular momentum plays an important role in astrophysics. It allows NSs and black holes to be spun up to near maximal rotation. It seems almost certain that WDs can be similarly spun up. Indeed, given the variety of donors and accretion geometries exhibited in nature, spin-up can fail only if there is a fundamental physical principle that disallows it. As long as spin-up to near-maximal rotation occurs, the effects we predict will occur. Although theoretical uncertainties make predictions difficult, we have shown that spin-up/spin-down has testable consequences. The measurements we propose can therefore provide input for theoretical work.

The spin-up/spin-down model appears capable of explaining the full range of SNe Ia properties. The mass, $M$, of the WD at the time of explosion is the first parameter that determines the observable characteristics. Without spin-up, SD explosions should occur soon after the WDs reach a critical mass that is very close in value to $M_{Ch}$. With spin-up, the value of $M$ is influenced by the properties of the initial binary. For a rigid rotator, the WD masses should lie in the range $M_{Ch} - 1.05 M_{Ch}$. In other models, the mass can be higher. By identifying the maximum WD mass, we will learn about the angular momentum profile of the pre-explosion WDs. Of course, only a small fraction of donors can provide enough mass to allow the WD to significantly exceed $M_{Ch}$; thus, typical pre-explosion WDs should have masses close to $M_{Ch}$. By measuring the distribution of primary WD masses, we will therefore learn about the binaries whose evolutions produce SNe Ia.
3.4 Spin-Up/Spin-Down: A new paradigm

All other things being equal, each value of $M$ would correspond to a specific value of $P_{\text{crit}}$, the spin at which the value of the critical mass would become equal to $M$. In fact, however, the angular momentum and internal states will differ at the time mass accretion halts, introducing a difference in the values of $\tau$. Furthermore, if there is residual low-level accretion, this also affects the spin-down time. Thus, the value of $P_{\text{crit}}$ may be viewed as a second parameter which influences the explosion characteristics.

Finally, the variety of conditions expected at the time when high-$M$ mass infall ceases, combined with a wide range of possible spin-down evolutions can yield very different pre-explosion conditions. These can in turn produce some truly unusual light curve and spectral evolutions. While we cannot determine whether spin-up effects explain the characteristics of any specific explosion, it is instructive to consider SN 2008ge (Foley et al., 2010), an SN 2002cx-type explosion, showing an unusual light curve and pattern of spectral evolution. The chemical composition, pre-explosion Hubble Space Telescope images, and lack of star formation in the host galaxy make it almost certain that SN 2008ge was the explosion of a WD, yet the explosion itself may have been different from most SNe Ia. A complete deflagration or else incomplete burning have been invoked as possible explanations.

Spin-up/spin-down produces a new paradigm for the progenitors of SNe Ia. Key elements can be tested through observations. While not all SNe Ia progenitors may be SD, and not all SDs may be significantly affected by spin-up, it seems inevitable that angular momentum plays a role in some of the progenitors.

Acknowledgements
We thank Ryan Chornock, Ryan Foley, Boris Gaensicke, Robert P. Kirshner, and Andrew MacFadyen for comments. This work was supported in part by NSF through AST-0908878 and by a development award from the Smithsonian Astrophysical Observatory. The authors thank the Lorentz Center for its hospitality during the workshop “Observational signatures of Type Ia supernova progenitors”, where this work was started.
THEORETICAL UNCERTAINTIES OF THE TYPE Ia SUPERNova RATE

J.S.W. Claeys, O.R. Pols, R.G. Izzard, J.Vink, F.W.M. Verbunt

Submitted to A&A

Abstract

It is thought that type Ia supernovae (SNe Ia) are explosions of carbon-oxygen white dwarfs (CO WDs). Two main evolutionary channels are proposed for the WD to reach the critical density required for a thermonuclear explosion: the single degenerate (SD), in which a CO WD accretes from a non-degenerate companion, and the double degenerate scenario (DD), in which two CO WDs merge. However, it remains difficult to reproduce the observed SN Ia rate with these two scenarios.

With a binary population synthesis code we study the main evolutionary channels that lead to SNe Ia and we calculate the SN Ia rates and the associated delay time distributions. We find that the DD channel is the dominant formation channel for the longest delay times. The SD channel with helium-rich donors is the dominant channel at the shortest delay times. Our standard model rate is a factor five lower than the observed rate in galaxy clusters.

We investigate the influence of ill-constrained aspects of single- and binary-star evolution and uncertain initial binary distributions on the rate of type Ia SNe. These distributions, as well as uncertainties in both helium star evolution and common envelope evolution, have the greatest influence on our calculated rates. Inefficient common envelope evolution increases the relative number of SD explosions such that for $\alpha_{\text{ce}} = 0.2$ they dominate the SN Ia rate. Our highest rate is a factor three less than the galaxy-cluster SN Ia rate, but compatible with the rate determined in a field-galaxy dominated sample. If we assume unlimited accretion onto WDs, to maximize the number of SD explosions, our rate is compatible with the observed galaxy-cluster rate.
4.1 Introduction

Type Ia supernovae (SNe Ia) are important astrophysical phenomena. On the one hand they drive galactic chemical evolution as the primary source of iron, on the other hand they are widely used as cosmological distance indicators (Phillips, 1993; Riess et al., 1996, 1998; Perlmutter et al., 1999) because of their homogeneous light curves. Even so, the exact progenitor evolution remains uncertain. It is generally accepted that SNe Ia are thermonuclear explosions of carbon-oxygen white dwarfs (CO WDs; Nomoto, 1982; Bloom et al., 2012). The explosion can be triggered when the CO WD reaches a critical density, which is reached when the mass approaches the Chandrasekhar mass ($M_{\text{Ch}} = 1.4 M_\odot$, for non-rotating WDs). Single stars form a CO WD with a mass up to about $1.2 M_\odot$ (Weidemann, 2000). To explain how the WD then reaches $M_{\text{Ch}}$ two main channels are proposed: the single degenerate channel (SD; Whelan & Iben, 1973; Nomoto, 1982), in which the WD accretes material from a non-degenerate companion; and the double degenerate channel (DD; Webbink, 1984; Iben & Tutukov, 1984), in which two CO WDs merge.

Both channels cannot fully explain the observed properties of SNe Ia (Howell, 2011). WDs in the SD channel burn accreted material into carbon and oxygen, and therefore these systems should be observed as supersoft X-ray sources (SSXS, Van den Heuvel et al., 1992), not enough of which are observed to explain the number of SNe Ia (Gilfanov & Bogdán, 2010; Di Stefano, 2010a). However, Nielsen et al. (2013) argue that only a small amount of circumstellar mass loss rates is able to obscure these sources, making it difficult or even impossible for observers to detect them as SSXS. Moreover, during the supernova explosion some of the material from the donor star is expected to be mixed with the ejecta, which has never been conclusively observed (Leonard, 2007; García-Senz et al., 2012). In some cases NaD absorption lines have been observed, which can be interpreted as circumstellar material from the donor star (Patat et al., 2007a; Sternberg et al., 2011). Furthermore, the donor star is expected to survive the supernova explosion and to have a high space velocity. In the case of the 400 year old Tycho supernova remnant, we would expect to observe this surviving star. Nevertheless, no such object has been unambiguously identified (Ruiz-Lapuente et al., 2004; Kerzendorf et al., 2009; Schaefer & Pagnotta, 2012). Some SNe and supernova remnants show evidence of interaction with circumstellar material which links them to the SD channel (Hamuy et al., 2003; Chiotellis et al., 2012; Dilday et al., 2012). However, a variation on the DD channel, in which the merger occurs during or shortly after the CE phase (Hamuy et al., 2003; Chevalier, 2012; Soker et al., 2013), cannot be excluded as the progenitor channel for these SNe. Additionally, SN 2011fe, a SN Ia which exploded in a nearby galaxy, showed no evidence of interaction with circumstellar material, in radio, X-ray and optical (e.g. Margutti et al., 2012; Chomiuk et al., 2012; Patat et al., 2013) and pre-explosion images exclude most types of donors stars of the SD channel (Li et al., 2011a).

These difficulties in reconciling the observational evidence with the predictions of the SD channel are avoided in the DD channel, but this channel has its own difficulties. In particular, Nomoto & Kondo (1991) show that the merger product evolves towards an accretion-induced collapse (AIC) rather than towards a normal SN Ia. Only recently, some studies indicate that the merger can lead to a SN Ia explosion under certain circumstances (Pakmor et al., 2010, 2011, 2013; Shen et al., 2013; Dan et al., 2013). Finally, binary population synthesis studies show that neither channel reproduces the observed SN Ia rate (e.g. Ruiter et al., 2009b; Mennekens et al., 2010; Toonen et al., 2012; Bours et al., 2013).

We investigate with a binary population synthesis (BPS) code the progenitor evolution towards SNe Ia through the canonical Chandrasekhar mass channels: the SD and the DD channels. Our code makes it possible to study large stellar populations, to calculate SN Ia rates and to compare these with the observed
rate of SNe Ia. We analyse not only the different evolutionary channels, but also –in the case of the SD channel– determine general properties of the donor star at the moment of the explosion, and –in the case of the DD channel– of the merger product. The SN Ia rate has been studied by several groups through binary population synthesis, however they used only a standard model, without investigating in great detail the uncertainties in their model, or they investigated one uncertainty in binary evolution, such as the common envelope evolution (e.g. Yungelson & Livio, 2000; Ritter et al., 2009b; Wang et al., 2009a, 2010; Mennekens et al., 2010; Toonen et al., 2012). We perform a study of the main parameters that play a role in the evolution towards type Ia SNe which are ill-constrained. We study the effects on the predicted rate and on the progenitor evolution towards SNe Ia.

In Sect. 4.2, the binary population synthesis code and our model are described. In Sect. 4.3, the general progenitor evolution as it follows from our standard model is outlined. The resulting SN Ia rate is discussed in Sect. 4.4, while Sect. 4.5 and 4.6 treat the effects of varying the parameters on the progenitor evolution and the total rate. Finally, Sect. 4.7 discusses the validity of the Chandrasekhar model.

4.2 Binary population synthesis

We employ the rapid binary evolution code binary_c/nucsyn based on the work of Hurley et al. (2000, 2002) with updates described in Izzard et al. (2004, 2006, 2009) and below. We discuss the key assumptions of our model of single and binary star evolution. In Table 4.1 we list the assumptions in our standard model and give an overview of possible other assumptions, and the dependence of the theoretical SN Ia rate on them is discussed in Sects. 4.5 and 4.6. This code serves as the basis of our binary population synthesis (BPS) calculations.

4.2.1 Single star evolution

Our single star models are analytic fits to detailed stellar models, described in Hurley et al. (2000), with updates of the thermally pulsating asymptotic giant branch (TP-AGB) which are fits to the models of Karakas et al. (2002), as described in Izzard et al. (2006). In the present study we adopt a metallicity $Z$ of 0.02. We assume that only a CO WD can explode as a SN Ia based on the work of Nomoto & Kondo (1991) who found that an ONe WD that reaches $M_{Ch}$ undergoes AIC. CO WDs are formed only in low and intermediate mass binary systems, with a primary mass up to $10 M_{\odot}$, therefore we limit our discussion to this mass range.

4.2.1.1 TP-AGB models

The evolution of the core, luminosity and radius of the TP-AGB star versus time are based on the models of Karakas et al. (2002), with the prescriptions described in Izzard et al. (2004, 2006). Because the core masses are determined without taking overshooting during previous evolution phases into account, a fit is made to take overshooting into consideration, based on the core masses calculated by Hurley et al. (2002) during the early-AGB (E-AGB). Up to and including the first thermal pulse, the core mass is calculated by the formulae described by Hurley et al. (2002). A smooth function is built-in to guarantee a continuous transition of the radius between the E-AGB and the TP-AGB. CO WDs form from initial stars with masses up to 6.2 $M_{\odot}$, which corresponds to a maximum CO WD of about $1.15 M_{\odot}$ (Fig. 4.1).
### Table 4.1: Physical parameters of single and binary evolution in our model and distribution functions of initial binary parameters for the standard model and the variations of these (used in Sects. 4.5 and 4.6).

<table>
<thead>
<tr>
<th>Name parameter</th>
<th>Reference</th>
<th>Standard model</th>
<th>Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta \ (R75)$</td>
<td>Eq. 4.1</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>TP-AGB wind</td>
<td>Eq. 4.2 - 4.3</td>
<td>HPT00</td>
<td>KLP02, R75 ($\eta = 1$), B95, VL05</td>
</tr>
<tr>
<td>$\alpha_{BH}$</td>
<td>Eq. 4.6</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>$B_{\text{wind}}$</td>
<td>Eq. 4.7</td>
<td>0.0</td>
<td>10$^3$</td>
</tr>
<tr>
<td>$Q_{\text{crit, HG}}^{(1)}$</td>
<td>Table 4.2</td>
<td>$Q_{\text{crit, He--HG}} = Q_{\text{crit, HG}} = 0.25$</td>
<td>$Q_{\text{crit, He--HG}} = Q_{\text{crit, He--GB}} = 1.28$</td>
</tr>
<tr>
<td>$Q_{\text{crit, He--stars}}$</td>
<td>Table 4.2</td>
<td>variable</td>
<td></td>
</tr>
<tr>
<td>$\alpha_{\text{ce}}$</td>
<td>Eq. 4.8</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>CE accretion$^{(3)}$</td>
<td>Eq. 4.8</td>
<td>0.0</td>
<td>0.5</td>
</tr>
<tr>
<td>$\lambda_{\text{ce}}$</td>
<td>Eq. 4.9 &amp; Appendix 4.A</td>
<td>variable</td>
<td></td>
</tr>
<tr>
<td>$\lambda_{\text{ion}}$</td>
<td>Eq. 4.9 &amp; Appendix 4.A</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Eq. 4.14</td>
<td>10</td>
<td>1,20, $\infty^{(2)}$</td>
</tr>
<tr>
<td>$\gamma_{\text{RLOF}}$</td>
<td>Eq. 4.15</td>
<td>$M_d/M_d$</td>
<td></td>
</tr>
<tr>
<td>$\gamma_{\text{wind}}$</td>
<td>Eq. 4.15</td>
<td>$M_d/M_d$</td>
<td></td>
</tr>
<tr>
<td>$\eta_{\text{H}}, \eta_{\text{He}}$</td>
<td>Eq. 4.38 &amp; 4.40</td>
<td>HKN96</td>
<td></td>
</tr>
<tr>
<td>$\psi(M_{1,i})$</td>
<td>Sect. 4.2.3</td>
<td>KTG93</td>
<td>S98, K01, C03, B03</td>
</tr>
<tr>
<td>$\phi(q_1) \propto q_1^x$</td>
<td>Sect. 4.2.3</td>
<td>$x=0$</td>
<td>$-1 \leq x \leq 1$</td>
</tr>
</tbody>
</table>

**Notes:**
1. Only in the case of mass transfer with a non-degenerate accretor.
2. The symbol $\infty$ implies conservative mass transfer to all types of stars.
3. Only when companion is a MS star.

**References:**
- HPT00 = Hurley et al. (2000), KLP02 = Karakas et al. (2002), R75 = Reimers (1975), B95 = Bloecker (1995), VL05 = Van Loon et al. (2005),
- HPT02 = Hurley et al. (2002), HKN96 = Hachisu et al. (1996), KTG93 = Kroupa et al. (1993), S98 = (Scalo, 1998), K01 = Kroupa (2001), C03 = Chabrier (2003), B03 = Bell et al. (2003).
4.2 Binary population synthesis

Figure 4.1: Initial ($M_i$) versus final ($M_f$) mass of single stars that become CO WDs for different TPAGB models (Sect. 4.2.1.1). The dotted line shows the results of the Hurley et al. (2000)-models, the full line shows the results for the fits to the models of Karakas et al. (2002), and the dashed line shows our model.

4.2.1.2 Helium star evolution

In our model, if a hydrogen-rich star is stripped of its outer layers after the main-sequence (MS) but before the TP-AGB, and the helium core is not degenerate, a helium star is formed. If the exposed core is degenerate a He WD is made. The prescriptions to describe the evolution of naked helium stars are discussed in Hurley et al. (2000). However, because of its importance for the formation of type Ia SNe we emphasize some details.

A distinction is made between three phases of helium star evolution: He-MS, the equivalent of the MS, when helium burns in the centre of the helium star; He-HG, the equivalent of the Hertzsprung gap (HG), when He-burning moves to a shell around the He-depleted core; and He-GB, the equivalent of the giant branch (GB), when the helium star has a deep convective envelope. If a non-degenerate helium core is exposed during the E-AGB, a star on the He-HG forms, otherwise a He-MS star forms.

A He-MS star becomes a He-WD when its mass is less than $0.32 \, M_\odot$, the minimum mass necessary to burn helium. The boundary between a helium star forming a CO WD or a ONe WD is determined by the mass of the star at the onset of the He-HG. The mass of the star on the He-HG should exceed $1.6 \, M_\odot$ in order to form a dense enough core to burn carbon and form an ONe-core, which is based on detailed models (carbon ignites off-centre when $M_{\text{CO,core}} \gtrsim 1.08 \, M_\odot$, Pols et al., 1998). The evolution of a He-HG star with $M < 1.6 \, M_\odot$ leads to the formation of a CO WD with a mass greater than $1.2 \, M_\odot$ unless its envelope is lost by wind or binary mass transfer.
Chapter 4: Theoretical uncertainties of the type Ia supernova rate

4.2.1.3 Wind mass loss

We adopt for both low- and intermediate-mass stars the prescription based on Reimers (1975) during the HG and beyond, multiplied by a factor $\eta$,

$$M_R = \eta \cdot 4.0 \cdot 10^{-13} \frac{R}{R_\odot} \frac{L}{L_\odot} \frac{M_\odot}{M} \text{yr}^{-1}, \quad (4.1)$$

where $M$ is the mass, $R$ the radius and $L$ the luminosity of the star, and $\eta = 0.5$ in our standard model (Table 4.1). During the TP-AGB we use the prescription based on Vassiliadis & Wood (1993),

$$\log \left( \frac{M_{\text{VW}}}{M_\odot \text{yr}^{-1}} \right) = -11.4 + 0.0125 \left( \frac{P_0}{d} - 100 \max \left[ \frac{M}{M_\odot} - 2.5, 0.0 \right] \right), \quad (4.2)$$

where $P_0$ is the Mira pulsation period in days ($d$), given by

$$\log \left( \frac{P_0}{d} \right) = \min \left( 3.3, -2.7 - 0.9 \log \left[ \frac{M}{M_\odot} \right] + 1.94 \log \left[ \frac{R}{R_\odot} \right] \right). \quad (4.3)$$

The wind is limited by the steady superwind rate, $M_{\text{SW}} = 1.36 (L/L_\odot) M_\odot \text{yr}^{-1}$.

In our model the wind of helium stars is either Reimers-like (Eq. 4.1) or Wolf-Rayet-(WR) like (Hamann et al., 1995; Hamann & Koesterke, 1998),

$$M_{\text{WR}} = 10^{-13} (L/L_\odot)^{1.5} M_\odot \text{yr}^{-1}, \quad (4.4)$$

depending on which of the two is stronger,

$$M_{\text{He-star}} = \max(M_{\text{WR}}, M_R). \quad (4.5)$$

4.2.2 Binary star evolution

Binary evolution can significantly impact the evolution of the individual stars in a binary system. Binary evolution is primarily determined by the initial semi-major axis ($a$) and the masses of the two stars. In the widest systems ($a \gtrsim 10^5 R_\odot$) the stars do not interact, but evolve as though they were single. In closer systems ($a \lesssim 3 \cdot 10^3 R_\odot$) Roche-lobe overflow (RLOF) or common envelope (CE) evolution often has dramatic consequences for the further evolution of the two stars (Hurley et al., 2002). These different interactions are discussed below.

We assume the initial eccentricity ($e_i$) is zero, based on the work of Hurley et al. (2002) who show that the evolution of close binary populations is almost independent of the initial eccentricity. We use the subscripts d and a for the donor and accreting star, respectively, and i for the initial characteristics of the star on the zero-age main-sequence (ZAMS). We use the subscripts 1 and 2 for the initially most massive star, the primary, and initially least massive star, the secondary, respectively. The mass ratio $Q$ is $M_a/M_d$, while the initial mass ratio $q_i$ is $M_{2,i}/M_{1,i}$.

4.2.2.1 Interaction with a stellar wind

Mass lost in the form of a stellar wind is accreted by the companion at a rate that depends on the mass loss rate, the velocity of the wind and the distance to the companion star. To describe this process and
4.2 Binary population synthesis

Table 4.2: The critical mass ratio $Q_{\text{crit}}$ for stable RLOF for different types of donor stars, in the case of a non-degenerate and a degenerate accretor.

<table>
<thead>
<tr>
<th>Type accretor $\rightarrow$ Type donor $\downarrow$</th>
<th>Non-degenerate$^{(1,2)}$</th>
<th>Degenerate$^{(1)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MS, $M &gt; 0.7 M_\odot$</td>
<td>0.625$^{(1)}$ (accretor = MS star)</td>
<td>-</td>
</tr>
<tr>
<td>MS$^{(4)}$, $M &lt; 0.7 M_\odot$</td>
<td>1.44</td>
<td>1.0</td>
</tr>
<tr>
<td>HG</td>
<td>0.25</td>
<td>0.21</td>
</tr>
<tr>
<td>(A)GB$^{(5)}$</td>
<td>2.13/ $[1.67 - x + 2 \left( \frac{M_\odot}{M_d} \right)^5]$</td>
<td>0.87</td>
</tr>
<tr>
<td>He-HG</td>
<td>0.25</td>
<td>0.21</td>
</tr>
<tr>
<td>He-GB</td>
<td>1.28</td>
<td>0.87</td>
</tr>
<tr>
<td>WD</td>
<td>-</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Notes:
1) The symbol - indicates that no value is defined.
2) Based on Hurley et al. (2002).
3) Based on De Mink et al. (2007).
4) We distinguish between MS stars ($M > 0.7 M_\odot$) and low mass MS stars ($M < 0.7 M_\odot$), because the latter are almost completely convective, and react differently to mass loss and gain.
5) At solar metallicity $x \approx 0.3$ (Hurley et al., 2000, 2002).

To calculate the rate $\dot{M}_a$ at which material is accreted by the companion star, we use the Bondi-Hoyle prescription (Bondi & Hoyle, 1944; Hurley et al., 2002), more specifically,

$$\dot{M}_a = \min \left( 0.8 \dot{M}_d, -1 \left[ \frac{GM_a}{v_d^2} \right]^2 \frac{\alpha_{\text{BH}}}{2a^2 (1 + v^2_{\text{rel}})^{3/2}} \dot{M}_d \right),$$  \hspace{1cm} (4.6)

with $\dot{M}_d$ the mass loss rate from the donor star, $e$ the eccentricity, $a$ the semi-major axis, $v_d$ the wind velocity of the donor star which we assume equals 0.25 times the escape velocity as proposed by Hurley et al. (2002), and $v_{\text{rel}} = |v_{\text{orb}}/v_d|$, with $v_{\text{orb}}$ the orbital velocity. The Bondi-Hoyle accretion efficiency parameter, $\alpha_{\text{BH}}$, is set to 3/2 in our standard model (Table 4.1). When both stars lose mass through a wind they are treated independently, no interaction of the two winds is assumed.

Some observed RS CVn systems show that the least evolved star is the most massive of the binary before RLOF occurs, therefore it has been suggested that wind mass loss is tidally enhanced by a close companion (Tout & Eggleton, 1988). It is uncertain if this phenomenon occurs for other types of binaries. To approximate this effect the following formula is implemented (Tout & Eggleton, 1988),

$$\dot{M} = \dot{M}_R \left( 1 + B_{\text{wind}} \cdot \max \left[ \frac{1}{2}, \left( \frac{R}{R_L} \right)^{1/6} \right] \right),$$ \hspace{1cm} (4.7)

where $R_L$ is the Roche radius of the star (Eggleton, 1983). In our standard model $B_{\text{wind}} = 0$ (Table 4.1).

4.2.2.2 Stability of RLOF

Whether or not mass transfer is stable depends on: 1) the reaction of the donor, because a star with a convective envelope responds differently to mass loss than a star with a radiative envelope; 2) the evolution of the orbit, which itself depends on the accretion efficiency ($\beta = \dot{M}_a/\dot{M}_d$) and the mass ratio $Q$; and 3)
Chapter 4: Theoretical uncertainties of the type Ia supernova rate

the reaction of the accreting star. In our model the criterion to determine whether mass transfer is stable is given by a critical mass ratio \( Q_{\text{crit}} \), which depends on the types of donor and accreting stars (Table 4.2). During RLOF the mass ratio \( Q \) of the binary system is compared with \( Q_{\text{crit}} \): if \( Q < Q_{\text{crit}} \) mass transfer is dynamically unstable and a CE phase follows, otherwise RLOF is stable. In Table 4.2 we list the values of the critical mass ratios for the different phases of the evolution of a star during accretion onto either a non-degenerate or a degenerate star.

RLOF from a MS donor is generally stable, because a radiative MS star shrinks in reaction to mass loss. When material is transferred to a MS companion, RLOF is stable only for certain mass ratios because a radiative accreting star expands and can fill its own Roche lobe. If the mass ratio is less than 0.625 the system evolves into a contact system and we assume the two MS stars merge (De Mink et al., 2007). The stability criteria for the other types of donor stars transferring mass to a non-degenerate companion are calculated according to Hurley et al. (2002).

To calculate \( Q_{\text{crit}} \) of WDs, we assume that during RLOF barely anything is accreted onto the WD, \( \beta = 0.01 \), and the specific angular momentum of the ejected material is that of the orbit of the accreting star (Hachisu et al., 1996). Because of the low accretion efficiencies of WDs (Appendix 4.B), the critical mass ratio for stable mass transfer decreases. Our adopted critical mass ratios for mass transfer from non-degenerate donor stars onto a WD are calculated with the formulae from Soberman et al. (1997, Chiotellis priv. comm.).

4.2.2.3 Common envelope evolution

If the donor star is an evolved star and the mass ratio of the Roche-lobe overflowing system is less than its critical value (Table 4.2), the system evolves into a CE. We use the \( \alpha \)-prescription to describe this complex phase (Webbink, 1984), in which the binding energy \( (E_{\text{bind}}) \) of the CE is compared with the orbital energy \( (E_{\text{orb}}) \) of the system to calculate the amount the orbit should shrink in order to lose the envelope of the donor star. More specifically, \( \alpha_{\text{ce}} \) is defined by,

\[
E_{\text{bind},i} = \alpha_{\text{ce}}(E_{\text{orb},f} - E_{\text{orb},i}),
\]

where the subscripts \( i \) and \( f \) correspond to the states before and after the CE phase, respectively, and where

\[
E_{\text{bind},i} = -G \frac{M_d M_{\text{env},d}}{\lambda_{\text{ce}} R_d},
\]

and \( M_d, M_{\text{env},d} \) and \( R_d \) are the mass, the envelope mass, and the radius of the donor, respectively, and \( G \) is the gravitational constant.

The CE efficiency parameter, \( \alpha_{\text{ce}} \), describes how efficiently the energy is transferred from the orbit to the envelope. Its value is expected to be between 0 and 1, although it could be larger than 1 if another energy source is available, such as nuclear energy. In our standard model we take \( \alpha_{\text{ce}} = 1 \) (see Table 4.1). The parameter \( \lambda_{\text{ce}} \) depends on the relative mass distribution of the envelope, and is not straightforward to define (Ivanova, 2011). According to Ivanova (2011) its value is close to 1 for low-mass red giants and therefore in many studies taken as such. In our model \( \lambda_{\text{ce}} \) is variable and is dependent on the type of star, its mass and luminosity. We use a prescription based on Dewi & Tauris (2000, our Appendix 4.A), which gives a value of \( \lambda_{\text{ce}} \) between 0.25 and 0.75 for HG stars, between 1.0 and 2.0 for GB and AGB stars and \( \lambda_{\text{ce}} = 0.5 \) for helium stars. Because of the short timescale of the CE phase (e.g. Passy et al., 2012), we assume in our standard model that no mass is accreted by the companion star (Table 4.1).
Additional energy sources can boost the envelope loss, e.g. the ionization energy of the envelope. The extended envelope can become cool enough that recombination of hydrogen occurs in the outer layers. In our model the fraction of this energy which is used is expressed by $\lambda_{\text{ion}}$, which is between 0 and 1 (Appendix 4.A). In our standard model this effect is not considered and $\lambda_{\text{ion}} = 0$ (Table 4.1).

### 4.2.2.4 Stable RLOF

When mass transfer is stable ($Q > Q_{\text{crit}}$, Table 4.2), the mass transfer rate is calculated as a function of the ratio of the stellar radius of the donor $R_d$ and the Roche radius $R_L$ (based on Whyte & Eggleton, 1980),

$$M = f \times 3.0 \times 10^{-6} \left( \log \frac{R_d}{R_L} \right)^{3/2} \left( \min \left( \frac{M_d}{M_{\odot}}, 5.0 \right) \right)^2 M_{\odot} \text{yr}^{-1}.$$  

(4.10)

The last part of the equation is defined by Hurley et al. (2002) for stability reasons. In Hurley et al. (2002) the dimensionless factor $f$ is 1, however this underestimates the rate when mass transfer proceeds on the thermal timescale. In our model $f$ is not a constant, but depends on the stability of mass transfer. If $f$ is large, the radius stays close to the Roche radius and mass transfer is self-regulating. However, in our model numerical instabilities arise when $\log R_d/R_L < 10^{-3}$ and the envelope of the donor star is small. Therefore, a function is defined which forces the radius to follow the Roche radius more closely during thermal-timescale mass transfer ($f = 1000$) and more loosely during nuclear timescale mass transfer ($f = 1$). A smooth transition is implemented between the two extreme values of $f$. To test the stability of the function to calculate the mass transfer rate, a binary grid was run of $50^3$ binary systems with $\text{binary}_c$, initial primary masses between 2 and 10 $M_{\odot}$, secondary masses between 0.5 and 10 $M_{\odot}$ and initial separations between 3 and $10^4 R_{\odot}$. The mass transfer rates of systems evolving through the SD channel with a hydrogen-rich donor were used to optimize this function for computational stability. The resulting function is given by

$$f = \begin{cases} 1000 & Q < 1 \\ \max \left( 1, \frac{1000}{Q} \cdot \exp \left( -\frac{1}{2} \left( \frac{\ln Q}{0.15} \right)^2 \right) \right) & Q > 1. \end{cases}$$  

(4.11)

Because mass transfer is dynamically stable, the mass transfer rate is capped at the thermal timescale mass transfer rate $M_{\text{KH}}$, given by

$$M_{\text{KH},d} = \frac{M_d}{\tau_{\text{KH}}} M_{\odot} \text{yr}^{-1},$$  

(4.12)

with $\tau_{\text{KH}}$ the thermal timescale of the donor star,

$$\tau_{\text{KH}} = 1.0 \times 10^7 \frac{M M'}{M_{\odot}^2} \frac{R_{\odot} L_{\odot}}{R L} \text{yr},$$  

(4.13)

and $M'$ either the mass of the star, if the star is on the MS or He-MS, or the envelope mass of the star otherwise.

In order to test the mass transfer rate calculated with our BPS code, as a next step a set of binary systems and their mass transfer rates were calculated with the our BPS code and with a detailed binary stellar evolution code (STARS, Eggleton, 1971, 2006; Pols et al., 1995; Glebbeek & Pols, 2008) and compared. We simulated a grid with primary masses between 2 and 6 $M_{\odot}$, secondary masses varied between 0.7 and 3.5 $M_{\odot}$ and orbital periods varied between 2 and 4 days (to check binary systems with both MS and HG donors) and a more massive binary system of 12 and 7 $M_{\odot}$ at orbital periods of 3.5 and 4 days. We find that
the maximum resulting mass transfer rate computed with the BPS code is up to about a factor three in the MS and a factor five in the HG different to the maximum from the detailed stellar evolution code, with the duration of thermal timescale mass transfer accordingly shorter. Additionally, we find similar durations of the entire mass transfer phase calculated with both codes.

In addition, the accretor adjusts its structure to the accreted mass. If the mass transfer rate is higher than the thermal timescale rate of the accretor, the accreting star is brought out of thermal equilibrium, resulting in expansion and additional mass loss from the accretor. Consequently, during RLOF the fraction of transferred material that is accreted is not taken to be constant, but depends on the thermal timescale of the accretor. For moderately unevolved stars (stars on the MS, HG or helium stars) the accretion efficiency $\beta$ is calculated in our model as follows

$$\beta \equiv \frac{M_d}{M_d} = \min\left(\sigma \frac{M_{KH,a}}{M_d}, 1\right).$$

(4.14)

where $\sigma$ is a parameter for which we assume a value of 10 in our standard model (as in Hurley et al., 2002). Moreover, in the case of accretion onto a MS or He-MS star, rejuvenation is assumed (Hurley et al., 2002). The internal structure of the star is changed and new fuel is mixed into the burning region, which results in a star that appears younger. If the accretor is an evolved star on the GB or AGB, mass transfer is assumed to be conservative ($\beta = 1$) because a convective star shrinks as a reaction to mass gain. Accretion onto a WD is a special situation because of its degeneracy and this is be discussed separately below.

When material is lost from the system it removes angular momentum. Angular momentum loss is described with a parameter $\gamma$ that expresses the specific angular momentum of the lost material in terms of the average specific orbital angular momentum, as follows:

$$\frac{J_{\text{orb}}}{J_{\text{orb}}} = \gamma(1 - \beta) \frac{M_d}{M_d + M_a}.$$

(4.15)

In our standard model we assume that during stable RLOF material is lost by isotropic re-emission, removing the specific orbital angular momentum of the accretor ($\gamma = M_d/M_a$, Table 4.1).

### 4.2.2.5 Stable RLOF onto a WD

Because a WD is degenerate, it burns accreted material stably only over a small range of mass transfer rates which corresponds to approximately $10^{-7} M_\odot/\text{yr}$ when hydrogen-rich material is accreted and $10^{-6} M_\odot/\text{yr}$ when helium-rich material is accreted (Nomoto, 1982). If the mass transfer rate is too low, the material is not burnt immediately and a layer of material is deposited on the surface. This layer burns unstably, resulting in novae, and, if the mass transfer rate is too high, the WD cannot burn all the accreted material. According to Nomoto (1982) the accreted material forms an envelope around the WD and becomes a red giant-like stellar object with a degenerate core and, generally, a CE subsequently forms. Hachisu et al. (1996) propose that the burning material on top of the WD drives a wind which blows away the rest of the accreted material. The accreted material burns at the rate of stable burning, but contact is avoided and mass transfer remains stable. We take the latter possibility into account (for a description see Appendix 4.B). The material ejected through the wind from the WD removes specific angular momentum from the WD ($\gamma = M_d/M_a$). The above also holds for WDs accreting through a wind, but the material transferred to the WD is based on the Bondi-Hoyle accretion efficiency (Eq. 4.6).
4.2 Binary population synthesis

4.2.3 Binary population synthesis

We simulate $N_{M_{1,i}} \times N_{M_{2,i}} \times N_{a_i}$ binary systems in log $M_{1,i} - \log M_{2,i} - \log a_i$ space, with $M_{1,i}$ and $M_{2,i}$ the initial masses of the primary and secondary stars and $a_i$ the initial semi-major axis of the binary systems. The volume of each cell in the parameter space is $\delta M_{1,i} \delta M_{2,i} \delta a_i$. To compute the SN rate each system is assigned a weight $\Psi$ according to the initial distributions of binary parameters. We normalize the SN rate to the total mass of the stars in our grid,

$$M_{\text{total}} = \sum_{M_{1,i=1}}^{M_{1,i=\text{max}}} \sum_{M_{2,i=1}}^{M_{2,i=\text{max}}} \sum_{a_i=1}^{a_i=\text{max}} (M_{1,i} + M_{2,i}) \Psi \delta M_{1,i} \delta M_{2,i} \delta a_i,$$

(4.16)

where

- $M_{1,i=1} = 0.1 \, M_\odot$, $M_{1,i=\text{max}} = 80 \, M_\odot$,
- $M_{2,i=1} = 0.01 \, M_\odot$ (Kouwenhoven et al., 2007), $M_{2,i=\text{max}} = M_{1,i}$,
- $a_{i=1} = 5 \, R_\odot$, $a_{i=\text{max}} = 5 \cdot 10^6 \, R_\odot$ (Kouwenhoven et al., 2007) and

$\Psi$ is the initial distribution of $M_{1,i}$, $M_{2,i}$ and $a_i$.

We assume that $\Psi$ is separable, namely

$$\Psi(M_{1,i}, M_{2,i}, a_i) = \psi(M_{1,i}) \phi(M_{2,i}) \chi(a_i),$$

(4.17)

where

- $\psi(M_{1,i})$ is the initial distribution of primary masses from Kroupa et al. (1993, Table 4.1),
- $\phi(M_{2,i})$ is the initial secondary masses distribution, which we assume is flat in $M_{2,i}/M_{1,i}$ and
- $\chi(a_i)$ is the initial distribution of semi-major axes, which we assume is flat in log $a_i$ (Öpik, 1924b; Kouwenhoven et al., 2007).

We calculate the delay time distribution (DTD), which is the SN rate as a function of time per unit mass of stars formed in a starburst at $t = 0$, as follows:

$$\text{DTD}(t) = \frac{\sum_{M_{1,i=1}}^{M_{1,i=\text{max}}} \sum_{M_{2,i=1}}^{M_{2,i=\text{max}}} \sum_{a_i=1}^{a_i=\text{max}} \delta(\text{SNe Ia}) \Psi \delta M_{1,i} \delta M_{2,i} \delta a_i}{M_{\text{total}} \delta t},$$

(4.18)

where $\delta(\text{SNe Ia}) = 1$ if the binary system leads to a SN Ia event during a time interval $t$ to $t + \delta t$, otherwise $\delta(\text{SNe Ia}) = 0$. We assume all stars are formed in binaries, which is an overestimate of the binary fraction, because for low-mass stars the observed fraction of system in binaries is less than 50% (Lada, 2006). However, in intermediate mass stars, Kouwenhoven et al. (2007) find a best fit with 100% binaries and Duchêne & Kraus (2013) conclude that different surveys are consistent with a multiplicity higher than 50%. Additionally, Kobulnicky & Fryer (2007) and Sana et al. (2012) find that more than 70% of massive stars are in binary systems.

The results of Sect. 4.3 are calculated by simulating a grid with $M_{1,i}$ between 2.5 and $9 \, M_\odot$, $M_{2,i}$ between $1 \, M_\odot$ and $M_1$ and $a_i$ between 5 and $5 \cdot 10^3 \, R_\odot$, with $N_{M_{1,i}} = N_{M_{2,i}} = N_{a_i} = N = 150$.

67
Figure 4.2: Initial separation ($a_i$) versus initial mass of the primary star ($M_{i1}$, left) and versus initial mass ratio ($M_{2,i}/M_{1,i}$, right) of systems that form SNe Ia through the DD channel in our standard model (Sect. 4.2). Number density (greyscale) represents the number of systems normalized to the total number of systems forming a SN Ia through the DD channel. Lines show the minimum separation (assuming $q = 1$) for which the primary fills its Roche lobe in a certain evolution stage, as indicated. Symbols A-F indicate the number of CE phases necessary to evolve to a SN Ia and differences in the evolution stage when the primary fills its Roche lobe: A = HG, no CE; B = HG, 1 CE (the difference between B_1 and B_2 is based on which of the two stars first forms a WD); C = GB, 1 CE; D = E-AGB, 1 CE; E = TP-AGB, 1 CE; F = TP-AGB, 2 CEs.
4.3 Binarity progenitor evolution

In this section we discuss the general progenitor evolution of the different SN channels, SD and DD, and their contribution according to our standard model (Sect. 4.2). We describe binary evolution in terms of the number of stable or unstable phases of RLOF and the stellar types at the onset of mass transfer. The latter depicts the influence of different aspects of binary evolution and initial binary distributions (Sects. 4.5 and 4.6). In the following sections the mass ratio $q$ is $M_2/M_1$ were the suffixes denote the more and less massive star initially.

4.3.1 Double degenerate channel

The double degenerate channel (DD) needs two CO WDs, with a combined mass greater than $M_{Ch}$ in a short enough orbit ($a \leq 4R_\odot$), to merge within a Hubble time. We do not impose a restriction on the mass ratio of the two CO WDs. Consequently, the rate of this channel depends on the number of systems that, after phases of stable RLOF and/or CE evolution, are in such a short orbit.

Multiple formation channels can form close double WD systems (Mennekens et al., 2010; Toonen et al., 2012). Mennekens et al. (2010) distinguish between the common envelope channel, which needs two consecutive CE phases, and the Roche-lobe overflow channel, in which a phase of stable Roche-lobe overflow is followed by a CE phase. Additionally, Toonen et al. (2012) discuss the formation reversal channel where the secondary forms a WD first. Fig. 4.2 shows the distribution of systems that evolve towards SNe Ia according to the DD channel in terms of their initial separation $a_i$, initial primary mass $M_{1,i}$ and initial mass ratio $q_i$. We distinguish six main regions: two lower regions A and B, with $a_i/R_\odot \lesssim 300$, in which the primary fills its Roche lobe during the HG; two intermediate regions C and D, with $300 \lesssim a_i/R_\odot \lesssim 1000$, in which the primary fills its Roche lobe during the GB or E-AGB; and two upper regions E and F, with $a_i/R_\odot \gtrsim 1000$, in which one or more CE phases are needed and the primary fills its Roche lobe during the TP-AGB.

**Close systems: regions A and B.** If the initial separation is shorter than about $300R_\odot$ the primary first fills its Roche lobe on the HG when the star has a radiative envelope. The first phase of mass transfer is thus stable and a CE is avoided.

The separation and the mass of the secondary determine whether the second phase of mass transfer, from the secondary, is stable. The separation determines both the moment the secondary fills its Roche lobe and the stellar type of the secondary at RLOF. The initially closest systems with the least massive secondaries avoid a CE and two CO WDs form in a short orbit without any CE phase (region A), while the wider systems experience a CE phase when the secondary fills its Roche lobe (region B).

Region A consists of systems that avoid a CE phase during their entire evolution. The range of initial mass ratios that make SNe Ia is strongly restricted (Fig. 4.2), because systems with initial mass ratios larger than 0.46 form double WD systems in an orbit too wide to merge in a Hubble time. Because of this restriction, only 0.7% of the systems that become a SN Ia via the DD channel follow this evolutionary channel. However, some of these systems with a mass ratio smaller than 0.46 that contain an accreting MS-star are expected to form a contact system which eventually merges before the formation of two CO WDs when RLOF is followed with a detailed stellar evolution code (De Mink et al., 2007).
Chapter 4: Theoretical uncertainties of the type Ia supernova rate

Region B represents the most common evolutionary channel and corresponds to the Roche-lobe overflow channel (Mennekens et al., 2010; Toonen et al., 2012). In this region the primary starts mass transfer during the HG and a CE phase occurs when the secondary fills its Roche lobe. This channel accounts for 84% of SNe Ia formed through the DD channel in our model, a similar fraction to Mennekens et al. (2010). Binary systems with initial masses lower than 2.9 $M_\odot$ do not form a merger product with a combined mass greater than 1.4 $M_\odot$, while systems with primary masses higher than 8.2 $M_\odot$ form an ONe WD. Binary systems with mass ratios lower than 0.25 have an unstable first mass transfer phase and merge.

In region B we distinguish between regions B$_1$ and B$_2$, based on which of the two stars becomes a WD first. In general, the initially most massive star is expected to form the first WD of the binary system (region B$_1$). However, sometimes the evolution of the secondary catches up with the primary because of previous binary interaction and becomes a WD first (region B$_2$), which is the formation reversal channel (Toonen et al., 2012). Region B$_2$ contains 49% of the systems forming region B (41% of all DD systems).

The systems of region B$_2$ have primary masses between 2.9 and 5 $M_\odot$ and mass ratios between 0.67 and 1. When a hydrogen-rich star loses its envelope before the TP-AGB it becomes a helium star before evolving into a WD (Sect. 4.2.1). In this case the resulting helium stars have a mass between 0.5 and 0.85 $M_\odot$. The time it takes for these low-mass helium stars to become a WD is long (between about 40 and 160 Myr) which gives the secondary, which has increased in mass, time to evolve and fill its Roche lobe. It becomes the most massive helium star of the binary system after the subsequent CE phase. The short orbit after the CE phase allows the secondary to fill its Roche lobe again and evolve into the first WD of the binary system. Afterwards, the primary fills its Roche lobe for the second time and becomes the second WD of the binary system. The range of primary masses that make SNe Ia is restricted because the lifetime of the corresponding helium star has to be long enough for the secondary to evolve. This restriction originates from the necessity for the secondary to fill its Roche lobe before the primary becomes a WD.

Systems with initial separations in the gap between region B and regions C and D do not form a double WD system. The primary fills its Roche lobe at the end of the HG or on the GB, which leads to a CE phase in which the two stars merge.

Wide systems that undergo one CE phase: regions C, D and E. In systems with separations longer than about 300 $R_\odot$ the primary fills its Roche lobe after the HG while having a convective envelope which results in a CE phase. The primary becomes a WD immediately (region E) or after subsequent evolution as a helium star (regions C and D). Afterwards, the secondary fills its Roche lobe. In regions C, D and E mass transfer from the secondary is stable, permitting accretion onto the initial WD. However, the WD cannot reach $M_{Ch}$ before the secondary loses its entire envelope and the binary becomes a short-period double WD system. This imposes a restriction on the range of initial mass ratios that make SNe Ia, which determines the separation after the phase of stable RLOF, because the orbit should be short enough when the secondary becomes a WD. This channel is 12.8% of DD progenitors (1.9% region C, 6.3% region D and 4.6% region E). The difference between the three regions is the stellar type of the primaries at the onset of mass transfer: in region C the primary is on the GB, in region D on the E-AGB and in region E on the TP-AGB (Fig. 4.2).

The ranges of the initial masses of the three evolutionary channels are defined by the necessity for both stars to become a CO WD (upper limit of the masses) and form a massive enough merger (lower
4.3 Binary progenitor evolution

limit of the masses). In some systems with a mass ratio close to one ($q_i \gtrsim 0.93$) the secondary is already evolved at the moment the primary fills its Roche lobe and during the CE phase both envelopes are lost. The secondary still fills its Roche lobe after the primary becomes a WD, but as a helium star, viz. on the He-HG.

The systems with initial separation in the gap between regions D and E do not form double WD systems. These systems survive the first CE phase and form a helium star with a non-evolved companion. However, because of the longer initial separation, the helium star fills its Roche lobe again as an He-giant. This leads to a second CE phase, during which the two stars merge.

Wide systems that undergo two CE phases: region F. When the initial separation is longer than about 1000$R_\odot$, the first mass transfer phase starts when the primary is on the TP-AGB. This leads to unstable mass transfer and therefore a CE phase, during which the separation decreases. The primary becomes a CO WD immediately. Subsequently, the secondary fills its Roche lobe which also results in unstable mass transfer. This evolutionary channel is similar to the common envelope channel discussed in Mennekens et al. (2010) and Toonen et al. (2012). This channel is 2.4% of the DD systems. These systems have the first phase of mass transfer during the TP-AGB (Fig. 4.2), however this evolutionary channel also occurs when mass transfer starts during the E-AGB. Nevertheless, the systems which start RLOF during the E-AGB and have two consecutive CE phases generally merge before the formation of a double WD system and only account for 0.1% of all DD systems.

In systems with longer initial separations ($a \gtrsim 2500 R_\odot$), both stars do not fill their Roche lobe, and therefore do not form double WD systems in a short orbit (Fig. 4.2).

4.3.2 Single degenerate channel

The single degenerate channel needs a CO WD and a non-degenerate companion which provides enough mass to the WD at a high enough rate. We distinguish between hydrogen- and helium-rich companions.

4.3.2.1 SD with hydrogen-rich donor (SD$_{H_1}$)

A hydrogen-rich companion can be in any evolutionary stage between the MS and the AGB. The stability criterion for mass transfer (Table 4.2) and the rate at which mass is transferred determine which donor stars transfer enough material to the WD to make SNe Ia. Evolved stars (GB or AGB) have a convective envelope, which results in a smaller critical mass ratio for stable mass transfer compared to non-evolved stars (MS or HG) with a radiative envelope (Table 4.2). Consequently, donors on the MS can be more massive than evolved donors without the system evolving into a CE. In addition, the mass transfer rate determines the amount of material that is accreted by the WD. Ideally, the mass transfer rate is about $10^{-7} M_\odot\text{yr}^{-1}$ (Nomoto, 1982, Sect. 4.2). Mass transfer is generally faster from GB stars in comparison with a MS-star with the same mass. Consequently, initially less massive stars on the GB than stars on the MS can donate enough mass to a CO WD to grow to $M_{\text{Ch}}$ (Fig. 4.3).

The contours in Fig. 4.3 show the results of a simulation with our BPS code in log $M_{1,i}$ - log $M_{2,i}$ - log $a_i$ space as discussed in Sect. 4.2.3 with $N = 125$, in which one of two stars is a WD at $t = 0$ and the other is on the ZAMS. The initial masses range between 0.7 and 1.15 $M_\odot$ for the WD and between 0.7 and 6 $M_\odot$ for the companion MS star, the separation is varied between the minimum separation for a MS star to fill its Roche lobe and $10^3 R_\odot$ and we assume $e_i = 0$. The figure shows the possible ranges in donor
mass and separation for which WDs with a mass of 0.8, 1.0 or 1.15 \( M_\odot \) can grow to \( M_{\text{Ch}} \). The greyscale shows the regions which are populated in our standard model at the moment the WD is formed.

In Fig. 4.3 three main regions are distinguished with \( M_{\text{WD}} < 1.15 M_\odot \). The donor stars in the region with \( \log a/R_\odot \lesssim 1.2 \) and \( 1.5 \lesssim M_d/M_\odot \lesssim 5.2 \) start transferring mass to the WD during the MS, the donor stars in the region with \( 1.0 \lesssim \log a/R_\odot \lesssim 1.5 \) and \( 2.4 \lesssim M_d/M_\odot \lesssim 3.0 \) during the HG, and those in the region with \( 2.0 \lesssim \log a/R_\odot \lesssim 2.5 \) and \( 1.15 \lesssim M_d/M_\odot \lesssim 1.4 \) during the GB. For future reference we name these channels the WD+MS, WD+HG, WD+RG channel, respectively, as in Mennekens et al. (2010).

Fig. 4.3 in this article can be compared with Fig. 3 of Han & Podsiadlowski (2004), which was calculated with a detailed binary stellar evolution code and which discussed the WD+MS channel. Like Han & Podsiadlowski (2004) we find that SNe Ia can originate from accreting WDs with donor masses between 1.5 and 3.5 \( M_\odot \), but in addition we find systems with donor masses greater than 3.5 \( M_\odot \), which they find to become dynamically unstable and hence do not become SNe Ia. This arises because of a different treatment of the stability of RLOF between the two codes. For the WD+RG channel we compare our results with Fig. 2 of Wang & Han (2010) who find donor masses down to 0.6 \( M_\odot \). However, these systems do not become a SN Ia within a Hubble time and therefore are not shown in our Fig. 4.3.
4.3 Binary progenitor evolution

Figure 4.4: As Fig. 4.2 of systems that form SNe Ia through the SD$_H$ channel. Symbols A$_H$, B$_H$ and C$_H$ indicate groups which are distinguished based on the number of CE phases necessary to evolve to a SN Ia and the evolutionary stage when the primary fills its Roche lobe: $A_H = \text{HG, no CE}$; $B_H = \text{end of HG/GB, 1 CE}$; $C_H = \text{TP-AGB, 1 CE}$. 

100
The greyscale of Fig. 4.3 depicts the number density at WD formation of the systems evolving through the SD$_H$ channel with our standard model. To understand why our standard model does not form WD+MS systems over the entire mass and separation range shown in Fig 4.3, a closer look at the distribution of the initial characteristics of the systems which evolve into a SN Ia through the SD$_H$ channel is necessary (Fig. 4.4). In this figure three regions are distinguished: systems with short (A$_H$), intermediate (B$_H$) and long initial separations (C$_H$).

**Close systems: region A$_H$.** Systems with initial separation between between 20 and 70 $R_\odot$ have the first phase of RLOF during the HG, which is stable. Subsequently a WD forms in a short orbit because of the initially low mass ratio. Afterwards the systems evolve through the WD+MS channel (Fig. 4.3). Only 1% of the SD$_H$ systems evolve through this channel, because of the small initial mass ratios involved and the possibility of the formation of a contact system because of the reaction of the secondary during accretion (cf. region A of the DD channel, Sect. 4.3.1).

**Wide systems: regions B$_H$ and C$_H$.** The most common evolutionary channel (99%) of the SD$_H$ channel is through a CE after which the WD is in a short orbit with an unevolved companion. The systems of region B$_H$ follow the WD+MS channel, while the systems of region C$_H$ follow the WD+HG channel or the WD+RG channel.

In region B$_H$, systems have initial separations longer than 300 $R_\odot$ and the primary fills its Roche lobe at the end of the HG or the onset of the GB. After the CE phase a helium star forms, which subsequently fills its Roche lobe stably and evolves into a WD. The secondary fills its Roche lobe during the MS and follows the WD+MS channel. This evolutionary channel is followed by 99% of the SD$_H$ systems, with 21% starting RLOF at the end of the HG and 78% during the GB. A star with an initial mass between 5.7 and 8.2 $M_\odot$ evolves into a CO WD with a mass between 0.8 and 0.95 $M_\odot$ when stripped of its hydrogen envelope at the end of the HG or during the GB. This explains why the greyscale in Fig. 4.3 is limited in the mass range of the donors. Binary systems with initial separations longer than 400 $R_\odot$, after a CE phase, as well as systems with initial separations shorter than 200 $R_\odot$, after a phase of stable mass transfer, form a WD binary system that is too wide to go through the WD+MS channel (Fig. 4.3).

In region C$_H$, systems have initial separations longer than 1000 $R_\odot$ and the primary fills its Roche lobe during the TP-AGB and forms a CO WD immediately. After the CE phase the secondary fills its Roche lobe as an evolved star on the HG or GB. 0.3% of the SD$_H$ systems follow this evolutionary channel. The number of systems going through this evolutionary channel is limited because only systems with CO WDs with initial masses larger than about 1.1 $M_\odot$ can grow to $M_{Ch}$ with an evolved donor star. Additionally, the WD+RG channel is almost non-existent as binary systems with a massive WD ($\gtrsim 1.0 M_\odot$) and a low-mass MS star ($\lesssim 1.5 M_\odot$) rarely form with separations between 100 and 300 $R_\odot$ with the assumed CE efficiency and therefore cannot contribute to the WD+RG channel.

Systems with initial separations longer than about 3000 $R_\odot$ do not experience RLOF and therefore do not become a SN Ia. However, these systems still interact in the form of a stellar wind. Nevertheless, in our standard model wind mass transfer is insufficient for a CO WD to grow to $M_{Ch}$ (Sect. 4.2.2.1).
4.3 Binary progenitor evolution

4.3.2.2 SD with helium-rich donors (SD$_{He}$)

Helium rich donors must be massive enough ($M \gtrsim 1 M_\odot$) to transfer enough material to the WD to make a SN Ia. Consequently, their H-rich progenitor stars must be massive enough initially (at least $4 M_\odot$). The initial helium star can be non-evolved (He-MS) or evolved (He-HG or He-GB). Fig. 4.5 shows the results of a simulation similar to that of Sect. 4.3.2.1, with one of the stars at $t = 0$ a WD and the other star on the zero-age He-MS. The ranges of both masses, the separation and the resolution are equal to those discussed in Sect. 4.3.2.1.

Fig. 4.5 depicts the helium donor masses versus the separation of the systems with an initial WD mass $\leq 1.15 M_\odot$ which become SNe Ia. Two regions are distinguished. The left region with $\log a/R_\odot \lesssim 0.2$ and $0.9 \lesssim M_\odot/M_\odot \lesssim 4.5$ shows systems that start RLOF to the WD during the He-MS. The middle region with $0.0 \lesssim \log a/R_\odot \lesssim 0.85$ and $1.05 \lesssim M_\odot/M_\odot \lesssim 1.5$ shows donor stars that start RLOF to the WD during the He-HG. He-GB donors do not form SNe Ia when the initial WD mass is less than $1.15 M_\odot$, because the evolutionary timescale of these stars is too short to transfer much material to a WD.

He-MS stars more massive than $1.6 M_\odot$ explode as core collapse SNe (CC SNe) if they evolve as single stars (Pols et al., 1998). However, He-MS donors with these masses transfer much material to the WD and have masses lower than $1.6 M_\odot$ when the CO WD reaches $M_{Ch}$. In our model the structure of He-MS adapts during RLOF and the star evolves further as though it started its evolution with this new mass. These initially massive helium stars then become a WD instead of exploding as a CC SN. However, if the He-MS star does not adapt to its new mass during RLOF, a SN Ia with afterwards a CC SN occur in one binary system.

Figure 4.5: As Fig.4.3 for systems at formation of the WD and He-MS binary which produce SNe Ia through the SD$_{He}$ channel.
Figure 4.6: As Fig. 4.2 of systems that form SNe Ia through the SD_{He} channel. Symbols A_{He}, B_{He} and C_{He} indicate groups which are distinguished based on the number of CE phases necessary to evolve to a SN Ia, the evolutionary stage when the primary fills its Roche lobe and the type of donor star transferring mass to the CO WD: A_{He} = HG, no CE, with He-MS donors; B_{He} = HG, 1 CE, with He-HG donors; C_{He} = TP-AGB, 1 CE.
Our Fig. 4.5 should be compared with Fig. 8 in Wang et al. (2009b), which depicts the systems evolving through the WD+He-MS and WD+He-HG channel calculated with a detailed binary stellar evolution code. Both figures indicate that helium donors with masses down to about 0.9\,\,M_\odot transfer at least 0.25\,\,M_\odot to a 1.15\,\,M_\odot WD. However, we find that helium stars more massive than 3\,\,M_\odot can also transfer this amount stably to a WD, while Wang et al. (2009b) find that RLOF is dynamically unstable in these binary systems, because of differences in the stability criteria of RLOF between the two codes. In addition, they find slightly higher donor masses in the WD+He-HG channel, which in our model do not transfer enough material to the WD companion.

The greyscale of Fig. 4.5 represents the number density of the systems at formation of a WD and helium star binary for the SD\,He channel. Below we describe the different progenitor evolutionary channels, distinguishing between systems with initially short separations which have He-MS donors (A_{He}) or He-HG donors (B_{He}) and systems with initially long separations (C_{He}). These different groups can be distinguished in Fig. 4.6.

Close systems: regions A_{He} and B_{He}. Systems with initial separations between 25 and 300\,\,R_\odot have primaries that fill their Roche lobe during the HG when they have radiative envelopes which results in stable mass transfer. After mass transfer a helium star is formed which becomes a WD without interaction or after a phase of stable RLOF. Afterwards a CO WD and a MS star, which has increased in mass, remain. Because of the great difference in mass between the two stars a CE phase follows when the secondary fills its Roche lobe during the HG or beyond, but before the TP-AGB. Subsequently a CO WD and helium star in a short orbit remain. When the secondary fills its Roche lobe for the second time the WD increases in mass and reaches M_{Ch}.

The main difference between regions A_{He} and B_{He} is the moment the companion star fills its Roche lobe as a helium star, i.e. during the He-MS at the shortest separations (region A_{He}) and during the He-HG at the longest separations (region B_{He}).

The initial primary mass and mass ratio of both groups are determined by the need to form a CO WD (upper mass and mass ratio limit) and a massive enough merger product (lower mass and mass ratio limit). The lower primary mass boundary of region A_{He} is lower than of region B_{He}, because less massive WDs can grow to M_{Ch} with He-MS donors (Fig. 4.5).

Of the SD_{He} systems, 48\% follow the evolutionary channel of region A_{He}. The range of initial separations is limited by the small range of radii of He-MS stars (Fig. 4.5). Systems with shorter separations than 25\,\,R_\odot merge during the CE phase before the formation of a helium star, while systems with longer separations than about 55\,\,R_\odot the helium stars fill their Roche lobes after the He-MS. The evolutionary channel of region B_{He} is followed by 48\% of the SD_{He} systems. Systems with separations shorter than about 55\,\,R_\odot have the helium star filling its Roche lobe during the He-MS. While initially longer separations than 300\,\,R_\odot evolve into a CE phase when the primary fills its Roche lobe.

In our standard model some binary systems form WDs with masses greater than 1.15\,\,M_\odot because the core of the helium star forming the WD grows beyond 1.15\,\,M_\odot (see discussion Sect. 4.2.1.2). This results in higher mass donors which can transfer enough material to these massive WDs to reach M_{Ch} than indicated by the solid line of Fig. 4.5. This explains the high donor masses of the SD_{He} systems at 10\,\,R_\odot the moment the binary systems consist of a CO WD and a helium star as indicated by the greyscale in Fig. 4.5.
Wide systems: region $C_{\text{He}}$. SN Ia progenitor systems with initial separations longer than 1000 $R_\odot$ have an initial mass ratio close to one ($q \gtrsim 0.91$). The primary fills its Roche lobe during the TP-AGB which results in a CE and a CO WD is formed. Moreover, because the two stars stars have comparable masses, generally, the secondary is at the moment of RLOF an evolved star as well. Afterwards, a CO WD and a helium star in a short orbit are formed. However, some systems fill their Roche lobe during the GB shortly after the primary, which results in a second CE phase where after the secondary becomes a helium star. In both cases, the helium star (He-MS or He-HG) fills its Roche lobe afterwards, which increases the mass of the WD to $M_{\text{Ch}}$. Only a small range of mass ratios follow this channel because smaller companion masses evolve into too low mass helium donor stars. This channel accounts for only 4% of systems in the SD$_{\text{He}}$ channel.

4.4 COMPARISON WITH OBSERVATIONS

In this section we compare the rate of the previously discussed channels and the sum of the three channels, the overall SN Ia rate, with observations. The delay time distribution (DTD) represents the SN Ia rate per unit mass of stars formed as a function of time, assuming a starburst at $t = 0$. The DTD allows us to investigate the validity of the different progenitor models, by providing a direct comparison with observations.

In early studies the observed DTD has been described by a two-component model (Scannapieco &
4.4 Comparison with observations

Figure 4.8: Number of SNe Ia ($N_{SN}$) per unit mass vs. time, with our standard model (Sect. 4.2). Line styles have the same meaning as in Fig. 4.7. Data points show the integrated rate (over a Hubble time) determined by Maoz et al. (2011, diamond), Maoz et al. (2012, square) and Graur & Maoz (2013, triangle).

Bildsten, 2005; Mannucci et al., 2006). The first component accounts for the prompt SNe Ia before 300 Myr, while the second component accounts for the delayed SNe Ia which have delay times longer than 300 Myr. More recent observations show that the DTD is best described by a continuous power-law function with an index of $-1$ (Totani et al., 2008; Maoz et al., 2010, 2011; Maoz & Mannucci, 2012; Graur et al., 2011; Graur & Maoz, 2013), more specifically (Maoz & Mannucci, 2012)

$$\text{DTD}(t) = 0.4 \left(\frac{t}{\text{Gyr}}\right)^{-1} \text{SNuM},$$

(4.19)

where SNuM is the supernova rate per 100 yr per $10^{10} M_{\odot}$. According to Totani et al. (2008) this relation supports the DD channel and arises from a combination of the initial separation distribution of the systems ($a^{-1}_i$) and the timescale of gravitational radiation ($\tau_{GWR} \propto a^4$). Some groups find a different slope, e.g. Pritchet et al. (2008) find a $t^{-0.5 \pm 0.2}$-relation. To compare our models to observed DTDs, we use Eq. 4.18.

We also compare the integrated SN Ia rate, i.e. the number of SNe Ia ($N_{SN}$) per unit of stellar mass formed in stars over the history of the Universe. Maoz & Mannucci (2012) conclude that the number of SNe Ia between 35 Myr and the Hubble time, 13.7 Gyr is consistent with $2 \cdot 10^{-3} M_{\odot}^{-1}$. However, more recent determinations of the observed SN Ia rate show that the integrated rate may be lower than previously assumed. Different groups find an integrated rate between $0.33 \cdot 10^{-3}$ and $2.9 \cdot 10^{-3} M_{\odot}^{-1}$ (Maoz et al., 2011, 2012; Graur et al., 2011; Graur & Maoz, 2013; Perrett et al., 2012). Maoz et al. (2012) discuss that the divergence may be explained by enhancement of SNe Ia in galaxy cluster environments at long delay times compared to field environments. The lower limit on these observations, however, is found by Graur & Maoz (2013) by applying a $t^{-1}$ relation for the DTD and they did not consider the previously
discussed uncertainties of the slope. Because their integrated rate is based on SNe Ia with long delay times, a steeper power law results in a higher integrated rate. To compare with our models, we use an integrated rate of $2.3 \cdot 10^{-3} M_\odot^{-1}$ as found by Maoz et al. (2011). Below we describe the rate of the different channels and the overall SN Ia rate resulting from our standard model (Figs. 4.7 and 4.8).

4.4.1 Double degenerate rate

In our standard model, the DD channel begins after about 100 Myr and dominates the SN Ia rate from about 200 Myr up to a Hubble time (Fig. 4.7, dashed line). The delay time of the DD channel can be described by a continuous power-law function from about 400 Myr. However, because the DTD is the combination of different evolutionary channels, it does not exactly follow a $t^{-1}$-relationship, but rather a $t^{-1.3}$-relation. Region B (Fig. 4.2), the Roche-lobe overflow channel, contributes from about 100 Myr up to a Hubble time and is always the dominant evolutionary channel. Regions D and E, which form double WD systems after one CE phase followed by a phase of stable RLOF, also contribute with a few percent to the DTD after about 200 Myr up to a Hubble time. The rate following from our standard model for long delay times (averaged between 300 Myr and a Hubble time) is $0.031 \text{ SNuM}$ and peaks at about 400 Myr. The integrated rate is $4.3 \cdot 10^{-4} M_\odot^{-1}$.

Another observable prediction of the DD channel is the mass of the merger product, which can be up to $2.4 M_\odot$. A merger product of $2.4 M_\odot$ can lead to an overluminous SN Ia if all its mass is burned (e.g. Yoon & Langer, 2005), or –if only $1.4 M_\odot$ is burned– in about $1 M_\odot$ of carbon and oxygen which remains unburned, which can be observed in the spectrum. Fig. 4.9 shows that of all double WD systems with a combined mass higher than $M_{\text{Ch}}$, 50% have a combined mass larger than $1.65 M_\odot$. However, only about

---

**Figure 4.9:** Cumulative distribution of the combined mass of the two CO WDs merging within a Hubble time for WD mergers having a combined mass greater than $1.4 M_\odot$ in our standard model. Two lines indicate specific masses and cumulative fractions (Sect. 4.4.1).
3%, which have a combined mass greater than \(2 M_\odot\), will be classified as super-Chandrasekhar because of the current limitations in defining the total WD mass of the progenitor systems leading to observed SNe Ia (e.g. Howell et al., 2006).

### 4.4.2 Single degenerate rate

The delay time of the SD channel depends on the evolutionary timescale of the secondary, the donor star to the exploding CO WD. The range of initial masses of the different types of donor stars (Sect. 4.3.2) is apparent in their respective DTDs. The helium-rich donors are initially the most massive and therefore the resulting SNe Ia occur earlier than the SNe Ia formed through the SD\(H\) channel (Fig. 4.7, dash-dotted line and dotted line).

The SD channel with helium rich donors contributes between about 45 Myr and about 200 Myr. This short time frame arises because only initially massive secondary stars, which have a helium core greater than \(1 M_\odot\), can transfer enough material to the CO WD as a helium star. Assuming a starburst the average rate between 40 and 200 Myr is \(0.092 \, \text{SNuM}\) and the integrated rate is \(1.5 \times 10^{-5} M_\odot^{-1}\).

In the SD channel with hydrogen rich donors we distinguish between non-evolved and evolved donors. The shortest delay times occur for MS and HG donors, while the more delayed type Ia SNe originate from GB donors. SNe Ia formed through the WD+MS channel begins from about 170 Myr until 500 Myr, the WD+HG channel contributes at about 450 Myr. The WD+RG channel contributes from about 4000 Myr, but is not significant and cannot be distinguished in the DTD.

### 4.4.3 Overall SN Ia rate

The sum of the three channels results in a DTD best described by a broken power-law, slightly increasing before 100 Myr, a dip between 200 and 400 Myr and \(t^{-1.3}\) relation afterwards (Fig. 4.7). The dominant formation channel of prompt SNe Ia is the SD\(He\) channel, while the dominating channel of longer delay times is the DD channel. Assuming a starburst, the rate between 40 and 100 Myr is on average \(0.14 \, \text{SNuM}\) and is dominated by the SD\(He\) channel. The average rate between 100 and 400 Myr is \(0.22 \, \text{SNuM}\) with approximately equal contributions from the SD and DD channel. At longer delay times, the DD channel dominates (Table 4.5 and Fig. 4.7). The integrated rate is \(4.8 \times 10^{-4} M_\odot^{-1}\), with about 95% of the SNe Ia formed through the DD channel, and is approximately a factor five lower than the Maoz et al. (2011) rate but compatible with the lowest estimates for the SN Ia rate (Graur & Maoz, 2013, Fig. 4.8).

Additionally, we find that 2.4% of intermediate-mass stars with a primary mass between 3 and 8 \(M_\odot\) explode as a SN Ia. This is consistent with the lower limit, expressed as \(\eta\), given by Maoz (2008) based on several observational estimates, more specifically they find that \(\eta \approx 2 – 40\%\). However, Maoz (2008) discusses that \(\eta = 15\%\) is consistent with the different observational estimates discussed in the paper, which is about a factor six higher than our results.

The SN Ia rate versus the rate of core collapse SNe (CC SNe) is another observable prediction. We find that \(N_{\text{SNIa}}/N_{\text{SNCC}} = 0.07 – 0.14\), where the upper limit is determined by assuming that all primaries with a mass between 8 and 25 \(M_\odot\) explode as a CC SN and the lower limit is determined by assuming that all primaries and secondaries with a primary mass between 8 and 25 \(M_\odot\) explode as a CC SN. Mannucci et al. (2005) determines this ratio based on star forming galaxies and finds \(N_{\text{SNIa}}/N_{\text{SNCC}} = 0.35 \pm 0.08\). Cappellaro et al. (1999) estimate a lower ratio of \(0.28 \pm 0.07\), as well as Li et al. (2011b), who find that \(N_{\text{SNIa}}/N_{\text{SNCC}} = 0.220 \pm 0.067\) and \(0.248 \pm 0.071\), for the prompt and the delayed SN Ia component.
respectively. On the other hand De Plaa et al. (2007) finds a higher ratio of $0.79 \pm 0.15$ based on X-ray observation of the hot gas in clusters of galaxies. Our ratio is between two and four times smaller than the ratio found by Cappellaro et al. (1999), and more than a factor five lower than the ratio estimated by De Plaa et al. (2007).

### 4.5 Uncertainties in Binary Evolution

The theoretical rate and delay time distribution of type Ia SNe depends on many aspects of binary evolution, such as CE evolution, angular momentum loss and the stability criterion of Roche-lobe overflowing stars, some of which are not well constrained. In order to test the dependence of the progenitor evolution and the rate on the assumptions we performed 35 additional simulations with our BPS code, labelled #A to #R, with each letter indicating a variation in a parameter, and numerical subscripts indicating different values for important parameters. We only discuss the most important parameters, while in Table 4.4 and 4.5 we give an overview of our results. Variations of the initial distributions of binary parameters are discussed in Sect. 4.6. The results discussed in this section and the next are calculated with a resolution of $N = 100$ (Sect. 4.2.3), except for the simulation of conservative RLOF ($N = 150$) and for the simulation of $\gamma_{\text{wind}} = 2$ ($N = 125$), in which the ranges of both initial masses and separation are longer than in our standard model. For comparison we list in Tables 4.4 and 4.5 the results of our standard model with $N = 150$ to show that choosing a higher resolution only has a small effect.

#### 4.5.1 Common Envelope Evolution

In both the SD and DD channels the prescription of the CE phase is crucial because almost all progenitor systems go through at least one CE phase. This phase is modelled comparing the binding energy of the envelope and the orbital energy, the $\alpha_{\text{ce}}$-prescription, which is parametrized by the parameters $\alpha_{\text{ce}}$ and $\lambda_{\text{ce}}$ (Sect. 4.2.2.3). Below we describe the effect of varying both parameters separately.

##### 4.5.1.1 Common Envelope Efficiency

The CE efficiency $\alpha_{\text{ce}}$ plays a crucial role in the progenitor evolution. We vary $\alpha_{\text{ce}}$ between 0.2 and 10 (models #A$_{1}$ to #A$_{4}$). Different groups determine a CE efficiency smaller than 1 after fitting the $\alpha_{\text{ce}}$-prescription to a population of observed post-CE binaries (Zorotovic et al., 2010; De Marco et al., 2011; Davis et al., 2012). Zorotovic et al. (2010) found that only a CE efficiency between 0.2 and 0.3 reproduces their entire observed sample. A CE efficiency of 10 is an extreme assumption to demonstrate the effect of an efficient extra energy source during the CE phase.

**SD-channel** In our models with lower common envelope efficiencies than our standard model (models #A$_{1}$ and #A$_{2}$, with $\alpha_{\text{ce}} = 0.2$ and 0.5, respectively), systems with initially longer separations than in our standard model survive a CE phase and contribute to the SD$_{H}$ channel, more specifically to the WD+MS channel and the WD+HG channel. Stars which are stripped of their envelopes at a later stage of their evolution have more massive cores and form more massive WDs. Consequently, generally the mass range of accreting WDs increases with decreasing CE efficiency. More specifically, in the WD+MS channel for $\alpha_{\text{ce}} = 1$ the mass of initial CO WD ranges between 0.8 and 0.95 $M_{\odot}$, while for $\alpha_{\text{ce}} = 0.2$ (model #A$_{1}$) the mass of the initial CO WD ranges between 0.8 and 1.15 $M_{\odot}$, with an associated increase of the integrated
4.5 Uncertainties in binary evolution

![Figure 4.10](image)

**Figure 4.10**: DTD of the SD-channel for the assumptions discussed in Sect. 4.2 and with variable CE efficiency, $\alpha_{\text{ce}}$, which is 1 (solid line), 0.5 (dashed line) and 0.2 (dotted line). The thick line shows the SD$_{\text{He}}$ channel, while the thin line shows SD$_{\text{H}}$ channel. Data points have the same meaning as in Fig. 4.7.

rate (Fig. 4.3 and Table 4.4), from $9.9 \times 10^{-5} \, M_{\odot}^{-1}$ when $\alpha_{\text{ce}}$ equals 1, to $9.0 \times 10^{-5} \, M_{\odot}^{-1}$ when $\alpha_{\text{ce}}$ equals 0.2.

Additionally, the DTD changes significantly (Fig. 4.10 and Table 4.5). As the mass range of the donor stars of the WD+MS and WD+HG channels is enlarged, the rate of the SD channel increases and contributes for a longer time. The rate of the WD+RG channel does not increase in models #A$_1$ and #A$_2$, because binary systems formed after RLOF with massive CO WDs ($M_{\text{WD}} > 1.0 \, M_{\odot}$) and low mass MS stars ($M < 2.0 \, M_{\odot}$) have separations shorter than $30 \, R_{\odot}$ and therefore do not contribute to the WD+RG channel (Fig. 4.3).

In the case of higher CE efficiencies we find the opposite (models #A$_3$ and #A$_4$): the WD+MS and WD+HG channels decrease and the WD+RG channel increases (Table 4.4). In model #A$_4$ ($\alpha_{\text{ce}} = 10$) the DTD of the SD$_{\text{H}}$ channel mainly consists of a delayed component from SNe Ia formed through the WD+RG channel. Likewise the integrated rate of the SD$_{\text{He}}$ increases in models #A$_1$ and #A$_2$, however only by a factor two to three, because of the already large mass range of CO WDs formed when $\alpha_{\text{ce}}$ equals 1.

**DD-channel** Two effects play a role when changing the CE efficiency (Fig. 4.11 and Tables 4.4 and 4.5). The first effect is because of the selective evolution towards a formation reversal (Table 4.4, region B$_2$). In the formation reversal channel, the CE phase preceding the formation of the second helium star brings two helium stars together in a short orbit. If $\alpha_{\text{ce}} = 1$ the most massive helium star fills its Roche lobe when it is an evolved helium star. If the separation is too long after the CE phase and the helium star
Chapter 4: Theoretical uncertainties of the type Ia supernova rate

Figure 4.11: DTD of the DD-channel for our standard model (Sect. 4.2) and with variable CE efficiency, $\alpha_{ce}$, which is 3 (solid line), 1 (dashed line) and 0.5 (dotted line). Data points have the same meaning as in Fig. 4.7.

Figure 4.12: Cumulative distribution of the combined mass of the two CO WDs merging within a Hubble time, for WD mergers having a combined mass higher than $1.4 M_\odot$, assuming different CE efficiencies. Different lines have same meaning as Fig. 4.11. Vertical and horizontal lines indicate specific masses and cumulative fractions (Sect. 4.5.1.1).
4.5 Uncertainties in binary evolution

does not fill its Roche lobe, the subsequently formed WDs do not merge within a Hubble time. At small $\alpha_{ce}$ the most massive helium star fills its Roche lobe at an earlier evolutionary stage, during the He-MS, and the double WD system does not form a massive enough merger product. This evolutionary channel completely disappears at low CE efficiency (e.g. $\alpha_{ce} = 0.2$, model #A1) and almost completely for high CE efficiencies (e.g. $\alpha_{ce} = 10$, model #A4). It appears that $\alpha_{ce} = 1$ is an optimal value for this formation path (Table 4.4).

The second effect is that the lower and upper separation boundaries for the regions B to F would change to longer separations in the models with lower CE efficiencies compared to our standard model. However, systems with longer separations are less common and in some regions the movement of the upper boundary is limited, such as in region F. In region F, the initially widest binary systems in which both stars fill their Roche lobe already form SNe Ia when $\alpha_{ce} = 1$. These two implications decrease the DD rate assuming lower CE efficiencies (Table 4.4).

However, if the CE efficiency is very high (e.g. $\alpha_{ce} = 10$, Table 4.4), the orbital energy does not decrease much during a CE phase and fewer double WD systems evolve into a short enough orbit to merge within a Hubble time. Consequently, the rate of the DD channel decreases for very high CE envelope efficiencies. Regions C to E almost disappear when $\alpha_{ce} = 10$. The rate of region F increases when $\alpha_{ce} = 3$ and 10, however these SNe Ia originate from binary systems which have the first CE phase when the primary is on the E-AGB or GB, with initially shorter separations compared to our standard model.

The DTD of the DD channel is a combination of different evolutionary channels. Fig. 4.11 shows that the slope of the DTD changes with variation of the CE efficiency. When $\alpha_{ce} = 1$ the DTD can be approximated by a $r^{-1.3}$ power-law, however the slope flattens when $\alpha_{ce}$ increases ($r^{-0.5}$ when $\alpha_{ce} = 3$) or steepens when $\alpha_{ce}$ decreases ($r^{-1.5}$ when $\alpha_{ce} = 0.5$).

Moreover, when the CE efficiency decreases the double WD systems that merge originate from initially wider systems. Consequently, the resulting WDs are more massive and therefore also the merger product (Fig. 4.12). In our standard model, 50% of the merging DD systems have a mass lower than $1.65 \, M_\odot$, while this is $1.57 \, M_\odot$ in model #A3 with $\alpha_{ce} = 3$. Additionally, in our standard model only 3% of the merger products have a total mass higher than $2 \, M_\odot$, which is 9% when $\alpha_{ce} = 0.5$ (model #A2). The dominance of region B is less strong in the most extreme models #A1 and #A4 ($\alpha_{ce} = 0.2$ and 10), therefore these behave differently than the models discussed above, however the 50% and $2 \, M_\odot$ boundaries for both models are within the ranges discussed above.

Overall SN Ia rate: Decreasing the CE efficiency increases the rate of both SD channels and decreases the rate of the DD channel. The DD channel peaks when $\alpha_{ce} = 1$. Generally, the DD channel is the dominant formation channel, however the importance of the SD channel increases at lower CE efficiencies. The theoretical integrated SN Ia rate varies between $9 \cdot 10^{-5} \, M_\odot^{-1}$ ($\alpha_{ce} = 10$) and $45 \cdot 10^{-5} \, M_\odot^{-1}$ ($\alpha_{ce} = 1$), which is a factor 5 to 26 times below the Maoz et al. (2011) rate.

4.5.1.2 Mass distribution of the envelope

We investigate the influence of $\lambda_{ce}$, which describes the mass distribution of the envelope, on the SN Ia rate. We also consider a specific extra energy source which is expressed by $\lambda_{ion}$. In our standard model $\lambda_{ce}$ is a function of the type of star and its characteristics (Appendix 4.A). As this prescription is not available in all BPS codes, $\lambda_{ce}$ is often taken to be 1 in other BPS studies.
Compared to the results of our standard model, the results of model #B\textsubscript{1} with $\lambda_{ce} = 1$ show an increase of the integrated rate by only 3% and small changes in the different channels (Table 4.4). However, Table 4.5 shows that more SNe Ia occur at shorter delay times compared to the DTD from our standard model. If we assume $\lambda_{ce}$ equals 1 for all stars, the binding energy of HG and helium stars is lower than in our standard model, while the binding energy of stars on the GB and AGB is higher. This assumption leaves the systems interacting during the GB or AGB in a shorter orbit after the CE phase compared to our standard model. In double WD systems formed through the dominant evolutionary channel, corresponding to region B, the secondary generally fills its Roche lobe for the first time during the GB or beyond. Consequently, the double WD systems formed through this evolutionary channel merge at relatively shorter delay times compared to our standard model.

During the CE phase, extra energy sources may facilitate the loss of the envelope. One example is the ionization energy which is modelled with $\lambda_{ion}$. In model #B\textsubscript{2} we assume that 50% of the ionization energy is used to eject the envelope during the CE phase. This has the largest effect on the most massive progenitor systems and the systems which interact during the AGB. In general, the rate of the progenitor channels in which one star interacts during the AGB increases because AGB stars lose their envelope more easily during the CE phase. Consequently, SNe Ia formed through these progenitor channels originate from systems with shorter separations than in our standard model (e.g. region F of the DD channel and regions C\textsubscript{H} and C\textsubscript{He} of both SD channels, Table 4.4). Even though a variation of $\lambda_{ion}$ has a large effect on individual systems, the SN Ia rate increases by only 5% compared to our standard model.

### 4.5.2 Stability criterion of Roche-lobe overflowing stars

#### 4.5.2.1 Helium stars

The evolution of helium stars is ill-constrained and therefore there remain uncertainties in this evolutionary phase. We updated the stability criterion of Roche-lobe overflowing helium stars in the HG. In our standard model the stability criterion of He-HG stars is the same as the criterion of hydrogen-rich stars in the HG, which is expected as both types of stars have a similar structure of their outer layers and therefore react similarly to mass loss. In older results published with our code the stability criterion of He-HG stars is equal to the criterion of He-GB stars (model #C, Table 4.2). With this assumption stable RLOF from helium stars in the HG is less likely. As many results are published with the latter assumption, we compare the effect of the variation of the stability criterion of Roche-lobe overflowing helium stars.

One might expect that a different prescription for Roche-lobe overflowing helium stars mainly affects the SD\textsubscript{He} channel, more specifically the progenitors with He-HG donors, i.e. region B\textsubscript{He}. Table 4.4 shows model #C alters region B\textsubscript{He} and the integrated rate of the SD\textsubscript{He} channel decreases from $1.50 \cdot 10^{-5} M_{\odot}^{-1}$ in our standard model to $1.40 \cdot 10^{-5} M_{\odot}^{-1}$ in model #C. In our standard model only a small number of WD and helium star systems with a mass ratio smaller than 0.87 form a SN Ia through the He-HG + WD channel because of the high mass transfer rates involved, which explains the small difference in the rate of the SD\textsubscript{He} channel (Fig. 4.5).

The helium star stability criterion mainly affects the DD channel, more specifically region B\textsubscript{2} (the formation reversal channel, Table 4.4). This is related to the crucial role of the stability criterion of helium stars in the formation reversal channel because these systems go through a phase where both stars are helium stars simultaneously. In this evolutionary channel, the difference between the stability criteria
4.5 Uncertainties in binary evolution

Determines if a double WD system is formed in a short orbit or the binary system merges during a CE phase. No SNe Ia are formed through the formation reversal channel in model #C and the integrated rate of region B decreases to $1.4 \cdot 10^{-4} M_{\odot}^{-1}$, compared to $3.6 \cdot 10^{-4} M_{\odot}^{-1}$ with our standard model.

Another difference between the results of our standard model and model #C is the DTD (Figs. 4.7 and 4.13 and Table 4.5). The DTD of systems at short delay times changes substantially. The average rate at delay times between 100 and 300 Myr is 0.052 SNUm with our standard model, while it is 0.31 SNUm with model #C (Table 4.5). Our results suggest a $t^{-1.3}$-relationship from about 400 Myr with the former model, and a $t^{-1.1}$-relationship from about 115 Myr with the latter. These differences originate from double WD systems that are formed after an evolved helium star has interacted with the first formed WD. In our standard model, when the helium star fills its Roche lobe mass transfer is stable. In model #C, RLOF is unstable for some of these systems. In both cases a double WD system forms but systems that result from a CE phase are in a shorter orbit afterwards than those that result from a phase of stable RLOF and therefore merge within a shorter time.

In conclusion, a different stability criterion of Roche-lobe overflowing helium stars affects: 1) the DTD, more specifically the shorter delay times and 2) the integrated rate with a factor two. Model #C has an integrated SN Ia rate of $2.3 \cdot 10^{-4} M_{\odot}^{-1}$ (Table 4.5), which is a factor ten lower than the observed Maoz et al. (2011) rate.

4.5.2.2 Stars in the Hertzsprung gap

A non-degenerate accreting star can be out of thermal equilibrium, which leads to its thermal expansion. The binary possibly evolves into a contact system and afterwards into a CE and/or merger. This process is more likely to occur in unequal mass binary systems (De Mink et al., 2007). Because in our model the

Figure 4.13: Delay time distribution of the different channels with model #C which assumes $Q_{\text{crit,He–HG}} = Q_{\text{crit,He–GB}}$. Line styles and data points have the same meaning as in Fig. 4.7.
formulae used to determine the evolution of a star are based on stars in thermal equilibrium, we do not take thermal expansion into account. It is expected that this increases the critical mass ratio \( Q_{\text{crit}} \) for donor stars in the HG (Table 4.2). However, this depends on the accretion efficiency and how energy is transported in the accreting star.

To estimate the effect of this uncertainty on the rate and the DTD, we consider model #D with a different stability criterion of donor stars in the HG and He-HG with a non-degenerate accretor, \( Q_{\text{crit, HG}} = 0.5 \), compared to our standard model \( (Q_{\text{crit, HG}} = 0.25) \). This implies that stable RLOF of HG donors is less likely to occur, and therefore influences regions A, B, A\(_{\text{H}}\), B\(_{\text{H}}\), A\(_{\text{He}}\) and B\(_{\text{He}}\) (Table 4.4). Because the systems that start RLOF during the HG have short initial separations and more binary systems evolve into a CE in model #D compared to our standard model, fewer systems survive the first RLOF phase and form a SN Ia. In model #D the integrated rate decreases by 23% compared to our standard model (Table 4.5).

### 4.5.3 Accretion efficiency

Apart from the stability criterion for RLOF, one of the main uncertainties in binary evolution is the RLOF accretion efficiency \( \beta \). The RLOF accretion efficiency depends on the reaction of the accretor to mass accretion and on the angular momentum of the accreted material, which determines how fast the accretor spins up in reaction to accretion. It is uncertain how efficiently the star can lose the gained angular momentum. In our standard model the accretion efficiency is a function of the thermal timescale of the accretor (Eq. 4.14) in which the uncertainty is expressed by a parameters \( \sigma \) which equals 10 in our standard model.

In models #E\(_1\) and #E\(_2\) we assume \( \sigma \) is 1 and 1000, respectively. A variation of the accretion efficiency of non-degenerate accretors has an impact on regions A to B of the DD-channel and A\(_{\text{H}}\), B\(_{\text{H}}\), A\(_{\text{He}}\) and B\(_{\text{He}}\) of the SD channels. In our standard model, mass transfer during RLOF is approximately conservative for high mass ratios because there is only a small difference between the thermal timescales of the two stars. In model #E\(_2\) RLOF is approximately conservative for all mass ratios, while in model #E\(_1\) RLOF is non-conservative over the entire mass ratio range. Consequently, a variation of \( \sigma \) mainly affects the binary systems with unequal masses. Increasing the upper limit of accretion efficiency increases the mass of initially lowest mass companions after RLOF, which results in a higher SN Ia rate, and vice-versa. However, region A of the DD channel disappears in model #E\(_2\) because the two WDs are in too long an orbit after conservative mass transfer to merge within a Hubble time. Compared to our standard model, the total integrated rate decreases by 12% assuming that the accretor can only accept accreted material with mass transfer rates lower than its thermal timescale (model #E\(_1\), Table 4.4) and it increases by 14% assuming RLOF is approximately conservative (model #E\(_2\), Table 4.4).

#### 4.5.3.1 Unlimited accretion onto WDs

The models that determine the WD retention efficiency assume a WD that has cooled for \( 10^8 \) yrs (Appendix 4.B and Nomoto, 1982). These do not consider WD models with different temperatures, e.g. because of previous nova outbursts, which can alter the accretion efficiency. However, the retention efficiency of WDs is uncertain and several models exist which describe this efficiency (Bours et al., 2013, and reference therein). We do not investigate the effect of different retention efficiencies such as Bours et al. (2013), but we consider unlimited accretion onto a WD, hence all mass transferred to the WD remains on the WD and is burnt into carbon and oxygen (model #F\(_1\)). We do not consider this as a realistic model but it represents an upper limit of the SD channel. During CE evolution we assume nothing is accreted. During
phases of wind accretion, the amount accreted is calculated according to the Bondi-Hoyle prescription (Eq. 4.6) for both degenerate and non-degenerate stars. The critical mass ratio \( Q_{\text{crit}} \) for stable RLOF onto WDs is the same as onto non-degenerate stars (Table 4.2).

**DD channel** Unlimited accretion onto a WD does not significantly affect the rate of this channel, because this channel is not defined by the amount of material which is accreted but by how close together two WDs are formed. However, conservative RLOF results in changes in the initial parameter space compared to our non-conservative model (Table 4.4). The systems which undergo two CE phases (region F) are unaffected. The systems forming regions C to E in our standard model –which have stable mass transfer when the secondary fills its Roche lobe– disappear in model \( F_1 \). In the latter model, after the primary fills its Roche lobe during the GB or E-AGB, either RLOF is unstable when the secondary fills its Roche lobe and the binary evolves into a CE and merges, or RLOF is stable and everything is accreted by the WD which explodes as a SN Ia through the SD\( _H \) channel. The DTD of the DD channel does not alter significantly, however the overall rate decreases compared to our standard model by about 10% (Table 4.4 and Fig. 4.7 and 4.14).

**SD channel** For both SD channels large changes are expected because of the prominent role of the accretion efficiency in these channels. Figs. 4.3 and 4.5 change significantly, because lower masses can provide enough mass towards the WD through conservative mass transfer. This is partly counteracted by the fact that stable RLOF is only possible for somewhat lower donor masses.
Chapter 4: Theoretical uncertainties of the type Ia supernova rate

SD_H Assuming conservative mass transfer towards the WD decreases the lower limit of donor masses to 0.45 \( M_\odot \) (Fig. 4.3) and the minimum mass of WDs at formation to 0.3 \( M_\odot \). Consequently, the rate increases significantly in both models with conservative mass transfer. The SD_H channel is dominant for delay times longer than 200 Myr (Fig. 4.14 and Table 4.5). The integrated rate is about three times larger than the rate of the DD channel with our standard model (Table 4.4).

SD_He The minimum He-rich donor mass decreases to 0.7 \( M_\odot \), significantly lower than without conservative mass transfer (Fig. 4.5). The SD_He channel still starts contributing to the DTD from about 45 Myr and contributes until about 300 Myr in model #F_1 (Fig. 4.14 and Table 4.5). It remains the dominant channel for short delay times and increases by a factor 10 compared to our standard model (Table 4.4).

Overall SN Ia rate: The DTD is completely dominated by the SD channel. Prompt SNe Ia are mainly produced through the SD_He channel, while delayed SNe Ia are mainly produced through the SD_H channel. The delayed component approximately follows a \( r^{-1} \)-relation from 300 Myr, however the rate drops slightly from about 5 Gyr. In the two models with conservative mass transfer the observed DTD is well reproduced. The theoretical DTD has a rate averaged between 0 and 13.7 Gyr of 0.14 SNuM in model #F_1 (Figs. 4.14). The resulting integrated rate from the theoretical models which assume conservative mass transfer is compatible with the Maoz et al. (2011) rate (Table 4.4).

4.5.4 Angular momentum loss

When material is lost from a binary system it also removes angular momentum. Several prescriptions for angular momentum loss exist and influence the evolution of a binary system in a different way, depending on whether mass is lost through a stellar wind or RLOF. Below we discuss the effect of the uncertainties in angular momentum loss on both types of mass loss from a binary system (models #G, #H_1 and #H_2).

4.5.4.1 Stellar wind

When mass is lost in a stellar wind, it is generally assumed that this material does not interact with the system when it is lost and that it is lost through a spherically symmetric way. However this assumption only holds when winds are fast compared to the orbital velocity of the system. AGB stars have slow winds (speed \( \approx 10-15 \) kms\(^{-1} \), see e.g. Vassiliadis & Wood, 1993) and when they are in a short orbit the wind can interact with the orbit and remove specific orbital angular momentum from the binary system (Jahanara et al., 2005; Izzard et al., 2010).

In model #G we assume that the material lost through a stellar wind carries twice the specific orbital angular momentum of the binary system. Some systems which do not interact in our standard model do interact in model #G, which increases the rate of regions F, C_H and C_He (Table 4.4). Additionally, even though the DD channel remains dominant, the SD_H channel, more specifically the WD+RG path, is more important at longer delay times (Table 4.5). Because angular momentum loss through a stellar wind does not affect the most common regions, the integrated rate of the two models differs by only 15%.

4.5.4.2 Roche-lobe overflow

Variation of the prescription of angular momentum loss when material is removed during a phase of stable RLOF affects the channels which have stable RLOF that is far from conservative. Our model of accreting
4.5 Uncertainties in binary evolution

WDs that drive an optically thick wind requires that the material lost removes the specific orbital angular momentum of the accreting star (Sect. 4.2.2.5) therefore we fix this assumption of accreting WDs when varying the prescription of angular momentum loss during RLOF.

A different assumption of angular momentum loss during RLOF affects regions A and B of the DD channel and A_H, B_H, A_He and B_He of the SD channels. In general, when more angular momentum is lost compared to our standard model initially wider systems become a SN Ia after a phase of stable RLOF, and vice versa. In model #H_1 mass lost takes the specific orbital angular momentum of the donor star, while in model #H_2 it is twice the specific orbital angular momentum of the binary. Although angular momentum loss during RLOF influences the most common evolutionary channels, it does not alter the rate drastically (Table 4.4).

4.5.5 Wind prescription

In our standard model, wind mass loss from stars up to the E-AGB is described by Reimers (1975, Eq. 4.1, with $\eta = 0.5$) and by Vassiliadis & Wood (1993) for the TP-AGB. Wind from helium stars is described by Reimers (1975) or Hamann & Koesterke (1998) depending on which of the two is stronger (Eq.4.5). Although the general trend of the evolution of the wind is known, the rate is not well constrained (Wachter et al., 2002). Below we discuss the effect of this uncertainty on the SN Ia rate.

4.5.5.1 Stars on the E-AGB and helium stars

In model #I the strength of the wind of stars up to the E-AGB and helium stars is increased ($\eta = 5$, Eq. 4.1) compared to our standard model ($\eta = 0.5$). This affects all the evolutionary channels, even those which show interaction on the TP-AGB because prior evolution is altered. The DD channel is mainly affected in regions D to F because these binary systems have the strongest winds. Region F increases significantly, because more systems survive two CE phases as more material is lost before RLOF sets in. The SD channel is mainly affected in region C_H and the SD channel in regions B_Tree and C_He. The rate of both channels decreases because less material is accreted than through stable RLOF. In conclusion, the rate of the SD channel decreases in model #I compared to our standard model. The opposite is true for the DD channel. The integrated SN Ia rate changes by only 6%.

4.5.5.2 TP-AGB

Several prescriptions for wind mass loss during the TP-AGB phase are used to describe this evolution phase. Changing the wind prescription of stars on the TP-AGB only affects regions E and F of the DD channel and regions C_H and C_He of the SD channels (Table 4.5, models #J_1 to #J_4). The high mass loss rate of Bloecher (1995, model #J_3) results in the shortest TP-AGB phase and forms the lowest mass WDs, which accordingly results in the lowest SN Ia rate of all the prescriptions for the TP-AGB under considerations. The prescription of Reimers (1975, with $\eta = 1$ during the TP-AGB, model #J_2) describes the longest wind phase, which results in the highest SN Ia rate. Because a change in the wind prescription of stars on the TP-AGB does not affect the largest regions, a variation of it only changes the integrated rate up to 5%.
Chapter 4: Theoretical uncertainties of the type Ia supernova rate

4.5.6 Combining two binary parameters

In the above sections we tested the influence on the SN Ia rate of different binary evolution aspects separately. However, we do not necessarily expect that a combination of changing two parameters has the same effect as the sum of the effect of changing the two parameters separately. Therefore we change some parameters under study at the same time and investigate their combined effect. We combine parameters which have the largest influence on the SN Ia rate, to determine the range of variation because of uncertainties in binary evolution.

4.5.6.1 Common envelope efficiency and stability criterion of RLOF

Model #P combines a low CE efficiency ($\alpha_{\text{ce}} = 0.2$) and a different stability criterion of stars on the He-HG ($Q_{\text{crit,He-HG}} = Q_{\text{crit,He-GB}}$). Both assumptions separately decrease the rate (models #A_1 and #C). When combined, the rate also decreases, but not by as much as the sum of the two effects separately. This is because model #A_1 and #C both affect region $B_2$ of the DD channel, moreover in both models this region disappears. In model #P, as in model #A_1, the SD channel dominates. The DTD is similar to that of model #A_1, except for an additional decrease of the DD channel because of the extra change of the stability criterion of Roche-lobe overflowing helium stars. The integrated rate decreases by about 65% compared to our standard model.

4.5.6.2 Accretion efficiency and angular momentum loss during wind mass transfer

Model #Q combines a high Bondi-Hoyle accretion efficiency ($\alpha_{\text{BH}} = 5$, Eq. 4.6) and a high angular momentum loss during wind mass loss ($\gamma_{\text{wind}} = 2$, Eq. 4.15). It increases the SN Ia rate by 13%.

However, the rate of model #Q is lower than that of model #G. When more material is accreted during wind mass transfer and less material is lost, less angular momentum is lost from the system. Therefore fewer double WD systems form in a short enough orbit to merge within a Hubble time in model #Q compared to model #G. The same holds for regions $C_{\text{H}}$ and $C_{\text{He}}$ of both SD channels, where at WD formation the binary system is in a longer orbit in model #Q than in model #G. One might expect that the same reasoning explains model #L, in which less material is lost from the binary system compared to our standard model. However, in model #L and our standard model when material is removed from the binary systems specific angular momentum of the donor star is lost which is smaller for our progenitor systems than in models #G and #Q. Therefore, the rate of model #L increases compared to our standard model because more material is accreted by the companion star.

4.6 Influence of initial binary distributions

There are limitations to the different techniques used to determine initial distribution functions, such as difficulties in resolving binary companions or the incorrect determination of masses because of rotation (Scalo, 1998; Kroupa, 2001; Duchêne & Kraus, 2013). In addition, it is not clear if the distribution functions are universal. Therefore we compare the theoretical SN Ia rates calculated assuming different initial binary distribution functions.
4.6 Influence of initial binary distributions

Table 4.3: The fraction $\eta$ of the binary systems with a primary mass between 3 and 8 $M_\odot$ that result in a SN Ia and the expected SN Ia rate versus CC SN rate of different models with varying IMFs compared with the observations.

<table>
<thead>
<tr>
<th>Model</th>
<th>IMF</th>
<th>$\eta$ (%)</th>
<th>$N_{\text{SNeIa}}/N_{\text{CCSN}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>Kroupa et al. (1993)</td>
<td>2.4</td>
<td>0.07 - 0.14</td>
</tr>
<tr>
<td>#O₁</td>
<td>Scalo (1998)</td>
<td>2.4</td>
<td>0.05 - 0.09</td>
</tr>
<tr>
<td>#O₂</td>
<td>Kroupa (2001)</td>
<td>2.3</td>
<td>0.04 - 0.09</td>
</tr>
<tr>
<td>#O₃</td>
<td>Chabrier (2003)</td>
<td>2.5</td>
<td>0.05 - 0.09</td>
</tr>
<tr>
<td>#O₄</td>
<td>Bell et al. (2003)</td>
<td>2.3</td>
<td>0.04 - 0.09</td>
</tr>
<tr>
<td>Observed</td>
<td></td>
<td>15 (2-40)</td>
<td>0.28 ±0.07(3)</td>
</tr>
</tbody>
</table>

Notes:
1. The upper and lower limit are determined by assuming that only the primary or both stars explode as a CC SNe in a binary system with a primary mass between 8 and 25 $M_\odot$.
3. Cappellaro et al. (1999)

4.6.1 Initial mass ratio distribution

The initial mass ratio distribution of intermediate-mass stars is highly uncertain because of difficulties in the observations of the companion star (Duchêne & Kraus, 2013). A flat distribution function of the initial mass ratio is widely used for BPS studies. However, some observations suggest a different distribution function of intermediate-mass stars, e.g. $\phi(q) \propto q^{-0.4}$ (Kouwenhoven et al., 2007). We investigate in models #N₁ to #N₅ the initial mass ratio distribution with a slope between -1 and 1 (Table 4.1).

Figs. 4.2, 4.4 and 4.6 show that most evolutionary channels favour binary systems with a high initial mass ratio. Therefore the rate strongly depends on the distribution of the initial mass ratio and peaks when equal masses are favoured. Table 4.4 shows that the rate of every channel increases if the models assume that more initial binary systems have equal masses, excepts in the most extreme case of model #N₅. In model #N₅ the rate corresponding to the regions which do not favour equal masses decreases compared to the rate of model #N₁, such as regions B_H and B_He of the SD channels. Although the overall rate of model #N₅ is higher compared to models #N₁ to #N₄. The integrated rate differs by a factor 4.5 between the two extremes and is a factor 4 to 17 times lower than the Maoz et al. (2011) rate. The most extreme model which strongly disfavours equal masses (model #N₁) is probably not realistic, as in this model about 50% of the systems with an initial primary mass between 2.5 and 10 $M_\odot$ have a secondary mass lower than 0.2 $M_\odot$.

4.6.2 Initial mass function

The initial mass function (IMF) of Kroupa et al. (1993) is widely used, especially in SN Ia studies. However, other prescriptions for the IMF exist. The IMF chosen defines the normalization of the population under study, therefore it is important to know which IMF is assumed when comparing results from different BPS codes and to realize how it affects the SN Ia rate (model #O₁ to #O₄). In general the IMF is described as a broken power-law function $M^{-\Gamma}$, with $\Gamma$ the slope of the power law. Kroupa et al. (1993) determine the IMF based on the low-mass stellar population in the Galactic disc. A break in the slope is
observed around $0.5 \, M_\odot$, leading to $\Gamma = 1.3$ for systems with masses lower than $0.5 \, M_\odot$ and $\Gamma = 2.2$ for higher masses. An additional break arises around $1.0 \, M_\odot$, resulting in a slope $\Gamma = 2.7$ above this mass.

Scalo (1998, model #O1) defines an IMF mainly based on different galaxies and stellar associations, and his IMF is commonly used in older BPS studies. He finds that a stellar population contains a similar number of intermediate-mass stars with a mass between $0.1$ and $10 \, M_\odot$ compared to the IMF of Kroupa et al. (1993), however it contains fewer low-mass stars. As the number of intermediate mass stars remains constant, while it decreases for the low-mass stars, the overall rate of model #O1 increases compared to our standard model (Table 4.4).

Kroupa (2001, model #O2) and Bell et al. (2003, model #O4) find similar IMFs, based on the Galactic field and a large galaxy sample from the local universe, respectively. Both groups find a Salpeter-like IMF with $\Gamma \approx 2.35$ for intermediate and massive stars, and a flatter slope below $0.5$ or $0.6 \, M_\odot$, respectively. A name commonly used for the IMF determined by Bell et al. (2003) is the ‘diet Salpeter’ IMF and this is used for example by Maoz et al. (2012) and Graur & Maoz (2013) to extract the SN Ia rate. Models #O2 and #O4 show an increase of the SN Ia rate of approximately 40% compared to our standard model. Because the slope in the mass range between $2.5$ and $10 \, M_\odot$ is different from that in our standard model, the rate corresponding to the different regions does not increase by the same amount, e.g. regions C and D of the DD channel change differently because the former favours less massive primaries than the latter.

Chabrier (2003, model #O3) compares different present-day IMFs and shows that low-mass primaries ($< 1 \, M_\odot$) are distributed according to a log-normal function and the best fit to the more massive stars is a power-law function in between the Salpeter-function and the function determined by Scalo (1998) for the mass range between $1$ and $10 \, M_\odot$. This results in a stellar population which contains less low-mass primaries than our standard model. In model #O3 our integrated rate increases by about 45% compared to our standard model and is about a factor 3.5 lower than the Maoz et al. (2011) rate.

Even though other prescriptions of the IMF result in an increase of the SN Ia rate, the rate of CC SNe increases as well. In models #O1 to #O4 the ratio of the SN Ia rate and the CC SN rate decreases compared to our standard model and is about a factor four to nine lower than the observationally estimated ratio by Cappellaro et al. (1999, Table 4.3). The fraction $\eta$ of intermediate mass systems leading to a SN Ia varies between 2.3 and 2.5 between the different models (Table 4.3) and is about about a factor six smaller than the observational estimate (Maoz, 2008, Table 4.3). Both comparisons in Table 4.3 indicate that although a variation of the IMF increases the integrated SN Ia rate, it does not reproduces other observational predictions regarding SN Ia.

### 4.6.3 Combining two distribution functions

In a population of stars, both the IMF and the the initial mass ratio distribution can differ simultaneously from our standard model. To test an extreme situation we change both initial binary distribution functions simultaneously. We assume in model #R an IMF according to Chabrier (2003) and an initial mass ratio distribution function with $\phi(q) \propto q$ (see Table 4.4 and 4.5).

The rate corresponding to all the regions increases in model #R, similar to the amount expected from the combination of the two distributions separately. The overall rate is about a factor three lower than the Maoz et al. (2011) rate and is compatible with the Maoz et al. (2012) rate (Fig. 4.15).
4.7 Discussion & conclusion

4.7.1 Progenitor evolution

We find that the dominant progenitor evolutionary path for the DD channel is the Roche-lobe overflow path, in which the primary WD forms after a phase of stable RLOF and the secondary WD forms after a CE phase. In our standard model, this path accounts for 84% of the systems evolving through the DD channel, which is comparable to the fraction determined by Mennekens et al. (2010). In addition, 41% of the DD channel evolves through the formation reversal path, in which the initially least massive star forms the first WD (Toonen et al., 2012). This path is less common according to the results of Toonen et al. (2012), mainly because stable RLOF of helium stars in the HG is less likely to occur in the model of Toonen et al. (2012). The common envelope channel, which forms a double WD system after two CE phases, is uncommon and only accounts for about 2.4% of the DD channel in our standard model. This is lower than the fraction of about 17% found by Mennekens et al. (2010), which is partly explained by the fact that they only consider two formation scenarios for the DD channel. We also find another formation channel, in which the first WD forms after a CE phase and the secondary WD forms after a phase of stable RLOF, which accounts for about 13% of the DD systems in our standard model.

The dominant progenitor evolutionary path for the SD channel with hydrogen-rich donors (SD\textsubscript{H}) involves the formation of a WD after a CE phase, and subsequent evolution through the WD+MS path. The least common in our standard model is the WD+RG path, which is opposite to the results of Ruiter et al. (2009b), who find that this path is more common than the WD+MS path. A possible origin of the differences between the results is that stable RLOF of stars in the HG is less likely and of stars on the early
GB is more likely to occur in the models of Ruiter et al. (2009b) compared to our model (Toonen et al., 2013). Mennekens et al. (2010) find a DTD from the SD$_H$ channel also dominated by the WD+MS path.

The dominant progenitor evolutionary path for the SD channel with helium-rich donors (SD$_{He}$) is through a phase of stable RLOF, which forms the WD, followed by a CE phase which forms the helium star donor. In our standard model and in most other models, the WD+He-MS path and WD+He-HG path contribute equally to the rate.

### 4.7.2 General characteristics of theoretical rate

The DD channel contributes from about 100 Myr to a Hubble time. The different models agree that the respective DTD follows a \( t^{-x} \) power law, with \( x = 1.3 \) in our standard model. The DD channel does not contribute at the shortest delay times. The DTD of SNe Ia at short delay times, \( \leq 100 \) Myr, is formed by the SD$_{He}$ channel. Our models show that SD$_H$ channel may contribute through the WD+MS path from about 70 to 3500 Myr, depending on the mass range of the initially formed CO WDs, and until about 8000 Myr through the WD+RG path. The SD channel does not contribute to the longest delay times, \( \gtrsim 8000 \) Myr. In our standard model, however, the SD$_H$ channel mainly contributes between 100 and 500 Myr. Generally, the DD channel is the dominant formation channel, compassing 95% in our standard model, however cannot reproduce the prompt channel. Only the SD$_{He}$ channel can account for SNe Ia at short delay times.

We produce SNe Ia both at short and long delay times, although we do not reproduce the observed number of SNe Ia. We find an integrated rate which is a factor 3 to 26 times lower than Maoz et al. (2011) rate. However, the highest integrated rate we find, model #R, is compatible with the Maoz et al. (2012) rate.

### 4.7.3 Uncertainties in binary evolution

The influence of uncertain aspects of single and binary star evolution on the SN Ia rate and DTD are studied in this paper. We find that most uncertainties only have a small effect on the SN Ia rate (\(< 15\%\)). This is because in our standard model the DD channel dominates, which is less sensitive to the masses of the WDs. However, the DD channel mainly depends on the spiral-in time after the formation of the two WDs and most uncertainties only marginally affect the spiral-in time. The uncertainties with the largest effect are the CE efficiency and helium star evolution, more specifically the stability of Roche-lobe-overflowing helium stars. Both result in a variation of at least a factor two in the integrated rate (Table 4.5). Moreover, the ratio of the SD channel and the DD channel changes significantly when the CE efficiency is varied. The DD channel peaks when \( \alpha_{ce} = 1 \) and the SD channel increases with decreasing CE efficiency. In the model with a low CE efficiency (\( \alpha_{ce} = 0.2 \)) the SD channel even dominates.

Additionally, our models show that the \( t^{-1} \)-relation is not a standard characteristic of the DD channel. Because the DD channel is a combination of different progenitor channels, the DTD depends on the contribution of each of them (Sect. 4.5.1.1).

Other uncertainties are the initial binary distributions, where the two most extreme models give an integrated rate which is a factor five different. The integrated rate of the model which combines the most optimistic distribution function of the initial primary mass and initial mass ratio is only a factor three lower than the Maoz et al. (2011) rate. In addition, the number of SN Ia versus the number of CC SN remains at least a factor 2.5 lower than estimated by Cappellaro et al. (1999).
We also consider the extreme situation of unlimited accretion onto a WD (model \#F_1), which we do not believe to be realistic, but is adopted to determine an upper limit of the SD channel. We find that rate corresponding to model \#F_1 reproduces the Maoz et al. (2011) rate. This does not indicate that the SD channel can be excluded, only that with normal assumptions it is hard to reproduce the observed rate with only the SD channel.

We mainly vary one parameter at every time to show their effects on the progenitor study of SN Ia separately. We demonstrate that the theoretical rate of changing two parameters at the same time is not always equal to the rate of adding the effect of the two separately (Sect. 4.5.6).

In addition, as mentioned in Sect. 4.4, the observed rate is also uncertain. Most recent observations show a rate which is about a factor two and maybe a factor four lower than the previously determined rate (Maoz et al., 2012; Graur & Maoz, 2013; Perrett et al., 2012). Maoz et al. (2012) conclude that their rate at long delay times based on a sample dominated by field galaxies is more than 2σ lower than the rate at long delay times of Maoz et al. (2011) based on galaxies in cluster environments. In addition, the metallicities of clusters of galaxies (De Plaa et al., 2007) indicate a higher fraction of SNe Ia than in our own Galaxy. Maoz et al. (2012) suggests a possible enhancement of SNe Ia in cluster environments, while Sarazin (1986) suggests a difference in the IMF between the two types of environments. Even though it is not clear exactly where the differences come from, we point out that the theoretical rate found by our standard model is compatible with the lower limit of the observed rate.

4.7.4 Comparison with other work

The results of other groups are similar. Ruiter et al. (2009b) find that the DD channel reproduces the observed \( t^{-1} \) relation, but is not able to reproduce the observed height of the DTD. Wang et al. (2009a) also discuss the SD$_{He}$ channel and find that the SD$_{He}$ channel is the progenitor channel responsible for the prompt SNe Ia. Greggio (2010) determines the DTD with an analytical approach. Her models which assume contributions of both the SD and the DD channel indicate that SNe Ia with delay times shorter than 0.1 Gyr originate from a combination of both channels, while at longer delay times the DD channel dominates. Greggio (2010) finds that the SD channel cannot reproduce the observed SN Ia rate at delay times longer than 10 Gyr.

Hachisu et al. (2008) describe the possibility of a radiation driven wind from the WD that strips extra material from the donor star and stabilizes RLOF more than the model adopted in this research. This results in more massive donor stars which can steadily transfer material to the WD. Mennekens et al. (2010) consider this model to derive the DTD with a BPS code, but they cannot reproduce the observed rate. Their rate is at least a factor three too low. In addition, the delayed component, produced by the SD WD+RG channel, drops at about 10 Gyr.

4.7.5 Other uncertainties in the results

We show that variation of binary physics has a great influence on the rate. However, our research is not a complete list of the uncertainties that dominate the SN Ia rate. Nelemans et al. (2013) show that different BPS codes show conflicting results, mainly for the SD channel. Recently four different BPS codes have been compared (Toonen et al., 2013), including ours. Toonen et al. (2013) show that the results found with the four codes are similar, with the same approximate assumptions. The differences that they find are because of differences in the inherent assumptions of the code, such as the initial-final mass relation,
helium star evolution, the stability criterion of Roche-lobe overflowing stars and the mass transfer rate. In addition, Bours et al. (2013) show that the lack of understanding of the retention efficiency of WDs gives an integrated rate which varies between $< 10^{-7}$ and $1.5 \cdot 10^{-4} M_\odot^{-1}$.

Varying the binary fraction affects the rate as well. Our models assume a binary fraction of 100%, which is a reasonable assumption for O and B stars (e.g. Kouwenhoven et al., 2007; Sana et al., 2012), however an overestimation for lower mass stars (e.g. Raghavan et al., 2010). As we do not expect that SNe Ia originate from single stars, the rate lowers adopting more realistic binary fractions. In addition, differences in the metallicity of the progenitor systems can alter the observed SN Ia rate. Toonen et al. (2012) discuss that a lower metallicity than Solar does not affect the DTD from the DD channel, however the integrated SN Ia rate increases with 30 to 60%. Meng et al. (2011) indicate that the DTD of the SD$_{\mathrm{H}}$ channel is more delayed in models with lower metallicities than Solar.

There are also newly proposed channels possibly leading to SNe Ia which we did not consider for the work presented here, such as the core degenerate model (Kashi & Soker, 2011; Ilkov & Soker, 2012, 2013), in which a WD merges with the core of an AGB star during the CE phase; the double-detonation sub-Chandrasekhar explosion (Fink et al., 2010; Kromer et al., 2010), in which a WD detonates after the detonation of a thin helium layer accreted onto the WD with a low mass transfer rate; the violent merger model (Pakmor et al., 2010, 2011), in which a massive enough WD explodes because of the accretion of material of another WD with an almost equal mass as the exploding WD; and the spin-up/spin-down model (Di Stefano et al., 2011; Hachisu et al., 2012), in which a WD gains angular momentum with the accreted material which spins-up the WD, which can possible lead to super-Chandrasekhar WD and a delay between the explosion of the WD and the accretion of the material. Additionally, we do not consider triple star evolution, which produces double WD mergers in eccentric orbits (Hamers et al., 2013) and, according to Rosswog et al. (2009), an alternative scenario to produce SNe Ia. Even though we do not investigate the rate of these channels, it is expected that similar uncertainties as discussed in this work influence the rate.

4.7.6 Outlook

Our models indicate that the progenitor evolution of SNe Ia does not consist of one evolutionary channel, but has many different branches with the relevance of each depending on different aspects of binary evolution. With the upcoming surveys we get more detail of the differences between SNe Ia separately. Therefore it is possible to gain insight in the sub-populations of SNe Ia. As a next step the characteristics of the newly proposed channels should be investigated in more detail, such as the merger products or the remaining companion stars. This combined with the results of this paper can be linked separately with the different sub-populations of SNe Ia. However, when studying the rate of the other channels the uncertainties discussed in this work should always be kept in mind.
Table 4.4: Number of SNe Ia within a Hubble time per $10^5 M_\odot$ of stars formed, corresponding to the different regions discussed in Sect. 4.3 and the different channels, calculated with our standard model (Sect. 4.2) and the models from our parameter study (Table 4.1).

<table>
<thead>
<tr>
<th>N°</th>
<th>Model</th>
<th>DD</th>
<th>SDH</th>
<th>SDHe</th>
<th>SN</th>
<th>Ia Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>A</td>
<td>B1</td>
<td>B2</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.29</td>
<td>18.4</td>
<td>17.5</td>
<td>0.82</td>
<td>2.70</td>
</tr>
<tr>
<td>1</td>
<td>Standard (N = 150)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.29</td>
<td>18.4</td>
<td>17.5</td>
<td>0.82</td>
<td>2.53</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>0.01</td>
<td>0.86</td>
<td>0.002</td>
<td>0.87</td>
<td></td>
</tr>
<tr>
<td></td>
<td>#A1</td>
<td>$\sigma_c = 0.2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.29</td>
<td>2.6</td>
<td>0.0</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>#A2</td>
<td>$\sigma_c = 0.5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.29</td>
<td>13.5</td>
<td>3.8</td>
<td>0.12</td>
<td>1.96</td>
</tr>
<tr>
<td></td>
<td>#A3</td>
<td>$\sigma_c = 3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.29</td>
<td>6.4</td>
<td>7.6</td>
<td>0.91</td>
<td>2.33</td>
</tr>
<tr>
<td></td>
<td>#A4</td>
<td>$\sigma_c = 10$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.29</td>
<td>4.0</td>
<td>0.1</td>
<td>0.22</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>#B1</td>
<td>$\lambda_{CE} = 1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.29</td>
<td>21.7</td>
<td>15.8</td>
<td>0.65</td>
<td>2.80</td>
</tr>
<tr>
<td></td>
<td>#B2</td>
<td>$\lambda_{SN} = 0.5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.29</td>
<td>17.1</td>
<td>17.6</td>
<td>0.82</td>
<td>2.34</td>
</tr>
<tr>
<td></td>
<td>#C</td>
<td>$Q_{crit, H - HG} = Q_{crit, H - GB}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.29</td>
<td>13.9</td>
<td>0.0</td>
<td>0.82</td>
<td>1.89</td>
</tr>
<tr>
<td></td>
<td>#D</td>
<td>$Q_{crit, H} = 0.5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.00</td>
<td>13.8</td>
<td>12.6</td>
<td>0.82</td>
<td>2.53</td>
</tr>
<tr>
<td></td>
<td>#E1</td>
<td>$\sigma = 1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.17</td>
<td>19.7</td>
<td>11.1</td>
<td>0.75</td>
<td>2.42</td>
</tr>
<tr>
<td></td>
<td>#E2</td>
<td>$\sigma = 1000$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.20</td>
<td>22.6</td>
<td>19.7</td>
<td>0.82</td>
<td>2.48</td>
</tr>
<tr>
<td></td>
<td>#F1</td>
<td>$\eta_1 = 1$, $\eta_{BH} = 1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.16</td>
<td>0.34</td>
<td>37.5</td>
<td>0.32</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>#F2</td>
<td>$\eta_1 = 1$, $\eta_{BH} = 1$, $\sigma \rightarrow \infty$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.16</td>
<td>18.5</td>
<td>18.1</td>
<td>0.00</td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td>#G</td>
<td>$\eta$ (Reimers, 1975) = 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.28</td>
<td>18.8</td>
<td>17.0</td>
<td>0.84</td>
<td>2.77</td>
</tr>
<tr>
<td></td>
<td>#H1</td>
<td>$\eta$ (Reimers, RLOF) = $M_*/M_H$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.11</td>
<td>0.87</td>
<td>0.001</td>
<td>0.98</td>
<td></td>
</tr>
<tr>
<td></td>
<td>#H2</td>
<td>$\eta$ (Reimers, RLOF) = 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.84</td>
<td>15.1</td>
<td>0.00</td>
<td>2.48</td>
<td>2.08</td>
</tr>
<tr>
<td></td>
<td>#I</td>
<td>$\eta$ (Scalo, 1998) = 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.29</td>
<td>16.3</td>
<td>17.6</td>
<td>0.82</td>
<td>2.54</td>
</tr>
<tr>
<td></td>
<td>#K</td>
<td>CE accretion = 0.05 $M_\odot$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.29</td>
<td>18.7</td>
<td>20.0</td>
<td>0.85</td>
<td>2.60</td>
</tr>
<tr>
<td></td>
<td>#L</td>
<td>$\alpha_{BH} = 5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.29</td>
<td>18.5</td>
<td>17.6</td>
<td>0.82</td>
<td>2.48</td>
</tr>
<tr>
<td></td>
<td>#M</td>
<td>CRAP = 1c3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.29</td>
<td>16.2</td>
<td>16.7</td>
<td>0.81</td>
<td>2.82</td>
</tr>
<tr>
<td></td>
<td>#N1</td>
<td>$\psi(q_i)$ $\propto q_i^{-1}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.14</td>
<td>6.4</td>
<td>4.2</td>
<td>0.31</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>#N2</td>
<td>$\psi(q_i)$ $\propto q_i^{-0.5}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.26</td>
<td>14.0</td>
<td>10.9</td>
<td>0.66</td>
<td>1.62</td>
</tr>
<tr>
<td></td>
<td>#N3</td>
<td>$\psi(q_i)$ $\propto q_i^{1.4}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.28</td>
<td>15.2</td>
<td>12.3</td>
<td>0.71</td>
<td>1.82</td>
</tr>
<tr>
<td></td>
<td>#N4</td>
<td>$\psi(q_i)$ $\propto q_i^{0.5}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.26</td>
<td>19.9</td>
<td>23.1</td>
<td>0.82</td>
<td>3.22</td>
</tr>
<tr>
<td></td>
<td>#N5</td>
<td>$\psi(q_i)$ $\propto q_i^{1.2}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.24</td>
<td>19.7</td>
<td>27.6</td>
<td>0.75</td>
<td>3.74</td>
</tr>
<tr>
<td></td>
<td>#O1</td>
<td>$\psi(M_i)$ (Scalo, 1998)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.37</td>
<td>23.8</td>
<td>22.7</td>
<td>1.06</td>
<td>3.27</td>
</tr>
</tbody>
</table>

4.7 Discussion & Conclusion
Table 4.4: continued.

<table>
<thead>
<tr>
<th>N°</th>
<th>Model</th>
<th>DD</th>
<th>SD$_{H}$</th>
<th>SD$_{He}$</th>
<th>SNe Ia</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>A</td>
<td>B$_1$</td>
<td>B$_2$</td>
<td>C</td>
</tr>
<tr>
<td>#O$_2$</td>
<td>$\psi(M_{1_{1}})$ (Kroupa, 2001)</td>
<td>0.46</td>
<td>26.5</td>
<td>22.2</td>
<td>1.32</td>
</tr>
<tr>
<td>#O$_3$</td>
<td>$\psi(M_{1_{1}})$ (Chabrier, 2003)</td>
<td>0.48</td>
<td>27.9</td>
<td>23.4</td>
<td>1.39</td>
</tr>
<tr>
<td>#O$_4$</td>
<td>$\psi(M_{1_{1}})$ (Bell et al., 2003)</td>
<td>0.45</td>
<td>26.7</td>
<td>22.8</td>
<td>1.31</td>
</tr>
<tr>
<td>#P</td>
<td>$Q_{\text{core}-\text{HG}} = Q_{\text{core}-\text{GB}}, \alpha_{\text{c}} = 0.2$</td>
<td>0.29</td>
<td>1.8</td>
<td>0.0</td>
<td>0.00</td>
</tr>
<tr>
<td>#Q</td>
<td>$\gamma (J_{\text{orb}}, \text{wind}) = 2, \alpha_{\text{BH}} = 5$</td>
<td>0.28</td>
<td>18.8</td>
<td>17.2</td>
<td>0.84</td>
</tr>
<tr>
<td>#R</td>
<td>$\psi(M_{1})$ (Chabrier, 2003), $\phi(q) \propto q^{\frac{1}{2}}$</td>
<td>0.34</td>
<td>29.4</td>
<td>36.9</td>
<td>1.27</td>
</tr>
</tbody>
</table>
Table 4.5: The averaged SNe Ia rate for the different channels separately and combined (the overall SNe Ia rate) for different time intervals, assuming a starburst, calculated with our standard model (Sect. 4.2) or the models from our parameter study (Table 4.1).

<table>
<thead>
<tr>
<th>N°</th>
<th>Time-interval (Gyr)→</th>
<th>DD</th>
<th>SDH</th>
<th>SD toilet</th>
<th>SNe Ia</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.0-0.1</td>
<td>0.1-0.3</td>
<td>0.3-1</td>
<td>1-10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SNuM</td>
<td>SNuM</td>
<td>SNuM</td>
<td>SNuM</td>
</tr>
<tr>
<td>-----</td>
<td>----------------------</td>
<td>----------</td>
<td>----------</td>
<td>-----------</td>
<td>-------------</td>
</tr>
<tr>
<td></td>
<td>Standard (N = 150)</td>
<td>2.2e-5</td>
<td>0.052</td>
<td>0.27</td>
<td>0.023</td>
</tr>
<tr>
<td></td>
<td>Standard (N = 100)</td>
<td>0.0</td>
<td>0.056</td>
<td>0.27</td>
<td>0.023</td>
</tr>
<tr>
<td>#A1</td>
<td>α_{ec}=0.2</td>
<td>0.0</td>
<td>0.026</td>
<td>0.034</td>
<td>0.0015</td>
</tr>
<tr>
<td>#A2</td>
<td>α_{ec}=0.5</td>
<td>0.0</td>
<td>0.12</td>
<td>0.17</td>
<td>0.0078</td>
</tr>
<tr>
<td>#A3</td>
<td>α_{ec}=3</td>
<td>0.0020</td>
<td>0.047</td>
<td>0.059</td>
<td>0.017</td>
</tr>
<tr>
<td>#A4</td>
<td>α_{ec}=10</td>
<td>6.9e-5</td>
<td>0.0026</td>
<td>0.021</td>
<td>0.0057</td>
</tr>
<tr>
<td>#B1</td>
<td>λ_{ec} = 1</td>
<td>0.0</td>
<td>0.16</td>
<td>0.34</td>
<td>0.018</td>
</tr>
<tr>
<td>#B2</td>
<td>λ_{ec} = 0.5</td>
<td>0.0</td>
<td>0.067</td>
<td>0.27</td>
<td>0.025</td>
</tr>
<tr>
<td>#C</td>
<td>ξ_{crit,He-He} = ξ_{crit,He-GB}</td>
<td>0.059</td>
<td>0.31</td>
<td>0.061</td>
<td>0.0087</td>
</tr>
<tr>
<td>#D</td>
<td>ξ_{crit,He-He} = 0.5</td>
<td>0.0</td>
<td>0.037</td>
<td>0.19</td>
<td>0.019</td>
</tr>
<tr>
<td>#E1</td>
<td>σ = 1</td>
<td>0.0</td>
<td>0.053</td>
<td>0.20</td>
<td>0.022</td>
</tr>
<tr>
<td>#E2</td>
<td>σ = 1000</td>
<td>0.000075</td>
<td>0.070</td>
<td>0.34</td>
<td>0.024</td>
</tr>
<tr>
<td>#F1</td>
<td>η_{H} = 1, η_{He} = 1</td>
<td>0.0</td>
<td>0.055</td>
<td>0.26</td>
<td>0.017</td>
</tr>
<tr>
<td>#F2</td>
<td>η_{H} = 1, η_{He} = 1, σ → ∞</td>
<td>0.0</td>
<td>0.060</td>
<td>0.36</td>
<td>0.015</td>
</tr>
<tr>
<td>#G</td>
<td>γ (L_{esc, wind} = 2)</td>
<td>0.0</td>
<td>0.074</td>
<td>0.31</td>
<td>0.026</td>
</tr>
<tr>
<td>#H1</td>
<td>γ (L_{esc, RLOF}) = M_{J}/M_{d}</td>
<td>0.0</td>
<td>0.066</td>
<td>0.30</td>
<td>0.026</td>
</tr>
<tr>
<td>#H2</td>
<td>γ (L_{esc, RLOF}) = 2</td>
<td>7.6e-5</td>
<td>0.075</td>
<td>0.28</td>
<td>0.019</td>
</tr>
<tr>
<td>#I</td>
<td>η (Reimers, 1975) = 5</td>
<td>1.4e-4</td>
<td>0.081</td>
<td>0.36</td>
<td>0.20</td>
</tr>
<tr>
<td>#J1</td>
<td>TP-AGB (Karakas et al., 2002)</td>
<td>0.0</td>
<td>0.058</td>
<td>0.27</td>
<td>0.023</td>
</tr>
<tr>
<td>#J2</td>
<td>TP-AGB (Reimers, 1975, η = 1)</td>
<td>0.0</td>
<td>0.067</td>
<td>0.29</td>
<td>0.024</td>
</tr>
<tr>
<td>#J3</td>
<td>TP-AGB (Bloeker, 1995)</td>
<td>0.0</td>
<td>0.057</td>
<td>0.26</td>
<td>0.022</td>
</tr>
<tr>
<td>#J4</td>
<td>TP-AGB (Van Loon et al., 2005)</td>
<td>0.0</td>
<td>0.063</td>
<td>0.27</td>
<td>0.022</td>
</tr>
<tr>
<td>#K</td>
<td>CE accretion = 0.05 M_{o}</td>
<td>0.0</td>
<td>0.058</td>
<td>0.27</td>
<td>0.026</td>
</tr>
<tr>
<td>#L</td>
<td>α_{BH} = 5</td>
<td>0.0</td>
<td>0.061</td>
<td>0.29</td>
<td>0.023</td>
</tr>
<tr>
<td>#M</td>
<td>B (CRAP) = 1e3</td>
<td>2.8e-4</td>
<td>0.061</td>
<td>0.31</td>
<td>0.017</td>
</tr>
<tr>
<td>#N1</td>
<td>φ(q_{i}) = q_{i}^{-1}</td>
<td>0.0</td>
<td>0.015</td>
<td>0.075</td>
<td>0.0070</td>
</tr>
<tr>
<td>#N2</td>
<td>φ(q_{i}) = q_{i}^{-0.5}</td>
<td>0.0</td>
<td>0.038</td>
<td>0.18</td>
<td>0.016</td>
</tr>
<tr>
<td>#N3</td>
<td>φ(q_{i}) = q_{i}^{-0.4}</td>
<td>0.0</td>
<td>0.042</td>
<td>0.20</td>
<td>0.018</td>
</tr>
<tr>
<td>#N4</td>
<td>φ(q_{i}) = q_{i}^{-0.4}</td>
<td>0.0</td>
<td>0.068</td>
<td>0.34</td>
<td>0.027</td>
</tr>
<tr>
<td>#N5</td>
<td>φ(q_{i}) = q_{i}^{-1}</td>
<td>0.0</td>
<td>0.077</td>
<td>0.39</td>
<td>0.029</td>
</tr>
</tbody>
</table>
### Table 4.5: continued.

<table>
<thead>
<tr>
<th>#O₁</th>
<th>SNuM</th>
<th>#O₂</th>
<th>SNuM</th>
<th>#O₃</th>
<th>SNuM</th>
<th>#O₄</th>
<th>SNuM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model</td>
<td></td>
<td>Model</td>
<td></td>
<td>Model</td>
<td></td>
<td>Model</td>
</tr>
<tr>
<td></td>
<td>$\psi(M_{1,3})$ (Scalo, 1998)</td>
<td></td>
<td>$\psi(M_{1,3})$ (Kroupa, 2001)</td>
<td></td>
<td>$\psi(M_{1,3})$ (Chabrier, 2003)</td>
<td></td>
<td>$\psi(M_{1,3})$ (Bell et al. 2003)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0</td>
<td>0.072 0.35 0.030</td>
<td>0.0</td>
<td>0.079 0.36 0.032</td>
<td>0.0</td>
<td>0.083 0.38 0.034</td>
<td>0.0</td>
<td>0.079 0.37 0.033</td>
</tr>
<tr>
<td></td>
<td>0.0 0.036 0.0059</td>
<td>0.0</td>
<td>0.044 0.0074</td>
<td>0.0</td>
<td>0.047 0.0078</td>
<td>0.0</td>
<td>0.044 0.0074</td>
</tr>
<tr>
<td></td>
<td>0.011 0.041 0.0</td>
<td>0.0</td>
<td>0.045 0.0</td>
<td>0.0</td>
<td>0.054 0.0</td>
<td>0.0</td>
<td>0.051 0.0</td>
</tr>
<tr>
<td></td>
<td>0.11 0.15 0.36 0.030</td>
<td>0.14 0.17 0.37 0.032</td>
<td>0.15 0.18 0.39 0.034</td>
<td>0.14 0.17 0.37 0.033</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#P</td>
<td>$Q_{\text{crit}, \text{He}} = Q_{\text{crit}, \text{GB}}, \alpha_{ce} = 0.2$</td>
<td>#Q</td>
<td>$\gamma (J_{\text{orb, wind}}) = 2, \alpha_{BH} = 5$</td>
<td>#R</td>
<td>$\psi(M_f)$ (Chabrier, 2003), $\phi(q) = q^4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0</td>
<td>0.012 0.028 0.0015</td>
<td>0.0</td>
<td>0.076 0.31 0.025</td>
<td>0.0</td>
<td>0.11 0.53 0.043</td>
<td>0.0</td>
<td>0.011 0.53 0.043</td>
</tr>
<tr>
<td></td>
<td>0.018 0.30 0.038 1.4e-4</td>
<td>0.0</td>
<td>0.028 0.0046 1.1e-4</td>
<td>0.0</td>
<td>0.040 0.0051 0.0</td>
<td>0.0</td>
<td>0.051 0.0</td>
</tr>
<tr>
<td></td>
<td>0.17 0.069 0.0</td>
<td>0.10 0.036 0.0</td>
<td>0.17 0.051 0.0</td>
<td>0.17 0.20 0.53 0.043</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.19 0.38 0.066 0.0016</td>
<td>0.10 0.14 0.32 0.025</td>
<td>0.17 0.20 0.53 0.043</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:** The rates are given in SNuM = rate per 100 yr per $10^{10} M_\odot$.

(1): The rate of the SD\text{He} channel between 1 and 10 Gyr is not given because always 0
4.8 Acknowledgements

JSWC would like to thank Alex Chiotellis for the discussions about effects of wind mass loss on the progenitor evolution of SNe Ia and for the help to determine the critical mass ratios of accreting WDs. The work of JSWC was supported by the Netherlands Research School for Astronomy (NOVA). RGI thanks Alexander von Humboldt Foundation for funding his current position.

4.A Mass distribution of the envelope

A method similar to Dewi & Tauris (2000) is used to calculate $\lambda_{ce}$, by fitting to detailed models from the STARS code Eggleton (1971). These formulae are also given and discussed in Izzard (2004).

\[
\lambda_{ce} = \begin{cases} 
\lambda_3 \cdot (\log_{10} L - 2) & M_{env} = 0 M_\odot, \\
\lambda_3 + M_{env}^{0.5}(\lambda_1 - \lambda_3) & 0 \leq M_{env} / M_\odot \leq 1, \\
\lambda_1 & M_{env} \geq 1 M_\odot.
\end{cases}
\]  

(4.20)

with

\[
\lambda_1 = \begin{cases} 
2 \lambda_1 & MS, GB, \\
2 \min(\lambda_1, \lambda_2) & CHeB, E-AGB, \\
2 \min(1.0, \max[\lambda_1, \lambda_2]) & TP-AGB.
\end{cases}
\]  

(4.21)

In general

\[
\lambda_1 = \min \left( \frac{3}{2.4 + m_1^{3/2}} - 0.15 \log_{10} L, 0.80 \right).
\]  

(4.22)

where

\[
m_1 = \begin{cases} 
M & MS, HG, GB, \\
100 & CHeB, E-AGB.
\end{cases}
\]  

(4.23)

And

\[
\lambda_1 = -3.5 - 0.75 \log_{10} M + \log_{10} L,
\]  

(4.24)

for stars on the TP-AGB.

For stars on the horizontal branch (CHeB), E-AGB and TP-AGB we define

\[
\lambda_2 = \min(-0.9, 0.58 + 075 \log_{10} M) - 0.08 \log_{10} L,
\]  

(4.25)

and for the other stellar types

\[
\lambda_2 = 0.
\]  

(4.26)

The loss of the envelope can be enhanced by ionization energy. In the code this is expressed by $\lambda_{ion}$.

\[
\lambda_1 = \lambda_1 + \lambda_{ion}(\lambda_3 - \lambda_1),
\]  

(4.27)
where
\[
\lambda_3 = a + \arctan(b[c - \log_{10} L]),
\]
(4.28)

where
\[
a = \begin{cases} 
\min(1.2 [\log_{10} M - 0.25]^2 - 0.7, -0.5) & \text{MS, GB}, \\
\max(-0.2 - \log_{10} M, 0.5) & \text{CHeB, E-AGB, TP-AGB},
\end{cases}
\]
(4.29)

\[
b = \max(3 - 5 \log_{10} M, 1.5),
\]
(4.30)

and
\[
c = \max(3.7 + 1.6 \log_{10} M, 3.3 + 2.1 \log_{10} M).
\]
(4.31)

A fudge for the pre-helium-burning stars is
\[
\lambda_3 = \begin{cases} 
\lambda_3 + d \cdot (\log_{10} L - 2) & \text{MS, HG, GB}, \\
\lambda_3 & \text{CHeB, E-AGB, TP-AGB},
\end{cases}
\]
(4.32)

where
\[
d = \max(0, \min[0.15, 0.15 - 0.25 \log_{10} M]).
\]
(4.33)

Then
\[
\lambda_3 = \max\left(\frac{1}{\max[\lambda_3, 0.01]}, \lambda_1\right).
\]
(4.34)

If the envelope mass is \( \leq 1 M_\odot \) then
\[
\lambda_3 = 0.42 \left(\frac{R_{\text{zams}}}{R}\right)^{0.4}.
\]
(4.35)

This results in typically \( \lambda_{ce} \approx 1.0 - 2.0 \) for stars on the GB or AGB, and \( \lambda_{ce} \approx 0.25 - 0.75 \) for HG stars. For helium stars no fit is available and therefore \( \lambda_{ce} = 0.5 \).

### 4.B Accretion Efficiency of WDs

Material that is transferred to a WD, \( \dot{M}_t \), can only be burnt by the WD at a specific rate (Nomoto, 1982). If the mass transfer rate is lower than this specific rate the material is added onto the surface and later on ejected in a nova explosion. When the material is transferred to the WD at a higher rate, we assume that the material which is not burnt is blown away through an optically thick wind from the accreting WD (Hachisu et al., 1996).

When He-rich material is accreted, it burns into carbon and oxygen. The net accretion efficiency \( (\eta_{He}) \) is written
\[
\dot{M}_{WD} = \eta_{He} \dot{M}_t,
\]
(4.36)

where \( \dot{M}_t \) is the mass transfer rate and \( \dot{M}_{WD} \) the net mass growth of the WD.

When hydrogen-rich material is transferred to the WD the net accretion efficiency \( (\eta_{He}\eta_H) \) is written
\[
\dot{M}_{WD} = \eta_{He}\eta_H \dot{M}_t,
\]
(4.37)
4.B Accretion efficiency of WDs

because first hydrogen is burned into helium, and subsequently helium is burnt into carbon and oxygen.

The accretion efficiencies for hydrogen and helium burning, $\eta_H$ and $\eta_{He}$, are calculated following Hachisu et al. (1999a)

$$\eta_H = \begin{cases} 
\frac{M_{\text{cr},H}}{M_{\text{tr}}} & M_{\text{tr}} > M_{\text{cr},H}, \\
1 & M_{\text{cr},H} > M_{\text{tr}} > M_{\text{cr},H}/8, \\
0 & M_{\text{tr}} < M_{\text{cr},H}/8, 
\end{cases}$$

where

$$M_{\text{cr},H} = 5.3 \cdot 10^{-7} \left(\frac{1.7 - X}{X}\right) \left(\frac{M_{\text{WD}}}{M_{\odot}} - 0.4\right) M_{\odot} \text{ yr}^{-1},$$

where $X$ is the hydrogen abundance and

$$\eta_{He} = \begin{cases} 
\frac{\dot{M}_{\text{up}}/\dot{M}_{\text{tr}}}{\eta_{KH04}} & \dot{M}_{\text{tr}} > \dot{M}_{\text{up}} , \\
1 & \dot{M}_{\text{up}} > \dot{M}_{\text{tr}} > M_{\text{cr},He} , \\
\eta_{KH04} & M_{\text{cr},He} > M_{\text{tr}} > M_{\text{low}} , \\
0 & \dot{M}_{\text{tr}} < M_{\text{low}} , 
\end{cases}$$

where

$$\dot{M}_{\text{up}} = 7.2 \cdot 10^{-6} \left(\frac{M_{\text{WD}}}{M_{\odot}} - 0.6\right) M_{\odot} \text{ yr}^{-1},$$

$$M_{\text{cr},He} = 10^{-5.8} M_{\odot} \text{ yr}^{-1},$$

$$\dot{M}_{\text{low}} = 10^{-7.4} M_{\odot} \text{ yr}^{-1}.$$
Chapter 5

PopCORN: Hunting down the differences between binary population synthesis codes

S. Toonen, J. S. W. Claeys, N. Mennekens, and A. J. Ruiter

Submitted to A&A

Abstract

Binary population synthesis (BPS) modelling is a very effective tool to study the evolution and properties of various types of close binary systems. The uncertainty in the parameters of the model and their effect on a population can be tested in a statistical way, which then leads to a deeper understanding of the underlying (sometimes poorly understood) physical processes involved. Several BPS codes exist that have been developed with different philosophies and aims. Although BPS has been very successful for studies of many populations of binary stars, in the particular case of the study of the progenitors of supernovae type Ia, the predicted rates and ZAMS progenitors vary substantially between different BPS codes. To understand the predictive power of BPS codes, we study the similarities and differences in the predictions of four different BPS codes for low- and intermediate-mass binaries. We investigate the differences in the characteristics of the predicted populations, and whether they are caused by different assumptions made in the BPS codes or by numerical effects, e.g. a lack of accuracy in BPS codes. We compare a large number of evolutionary sequences for binary stars, starting with the same initial conditions following the evolution until the first (and when applicable, the second) white dwarf (WD) is formed. To simplify the complex problem of comparing BPS codes that are based on many (often different) assumptions, we equalise the assumptions as much as possible to examine the inherent differences of the four BPS codes. We find that the simulated populations are similar between the codes. Regarding the population of binaries with one WD, there is very good agreement between the physical characteristics, the evolutionary channels that lead to the birth of these systems, and their birthrates. Regarding the double WD population, there is a good agreement on which evolutionary channels exist.
to create double WDs and a rough agreement on the characteristics of the double WD population. Regarding which progenitor systems lead to a single and double WD system and which systems do not, the four codes agree well. Most importantly, we find that for these two populations, the differences in the predictions from the four codes are not due to numerical differences, but because of different inherent assumptions. We identify critical assumptions for BPS studies that need to be studied in more detail.

\section{Introduction}

Binary population synthesis codes (hereafter BPS codes) enable the rapid calculation of the evolution of a large number of binary stars over the course of the binary lifetime. With such models, we can study the contribution of binary stars to an environment, e.g. the chemical enrichment of a region, or the frequency of an astrophysical event. We can learn about and study the formation and evolution of stellar systems that are important for a wide range of astronomical topics: novae, X-ray binaries, symbiotics, subdwarf B stars, gamma ray bursts, R Coronae Borealis stars, AM CVn stars, type Ia and type Ib/c supernovae, runaway stars, binary pulsars, blue stragglers, etc.

To carefully study binary populations, in principle it is necessary to follow the evolution of every binary system in detail. However, it is not feasible to evolve a population of binary stars from the zero-age main-sequence (ZAMS) to remnant formation with a detailed stellar evolution code. Such a task is computationally expensive as there are many physical processes which must be taken into account over large physical and temporal scales: tidal evolution, Roche lobe Overflow (RLOF), mass transfer. Moreover not all processes can be modelled with detailed codes, e.g. common envelope evolution, contact phases. Therefore, simplifying assumptions are made about the binary evolution process and many of its facets are modelled by the use of parameters. This process is generally known as binary population synthesis. Examples of such parametrisation are straightforward descriptions for the stability of mass transfer, the accretion efficiency during mass transfer, the angular momentum loss during non-conservative mass transfer, etc. For the evolution of an individual system, the above can of course be an oversimplification. However, for the treatment of the general characteristics of a large population of binaries this process works very well (e.g. Eggleton et al., 1989).

The use of BPS codes dates back several decades (e.g. Iben & Tutukov, 1984; Melnick et al., 1985; Eggleton et al., 1989; Meurs & Van den Heuvel, 1989; Yungelson et al., 1993, for some early examples) and they were used to calculate the most diverse properties of binary populations (for a very thorough review, see Han et al., 2001). A number of BPS codes are being used in the field, and (sometimes large) differences exist in the way the codes are designed to treat various stages of binary evolution. The physics of binary star evolution is not clear-cut, since many mechanisms that govern important processes are quite uncertain (e.g. mass transfer and transport of angular momentum in and from the binary). In all BPS codes, certain physical processes are modelled in some detail, while others are modelled using simple approximations, to e.g. save computational time. To some degree, the effects that are most important for the problem being studied will be more elaborately included in the corresponding BPS codes.

Recently, several BPS codes have been used to study the progenitors of type Ia supernovae (e.g. Yungelson et al., 1994; Han et al., 1995; Jorgensen et al., 1997; Yungelson & Livio, 2000; Nelemans et al., 2001b; Han & Podsiadlowski, 2004; De Donder & Vanbeveren, 2004; Yungelson, 2005; Lipunov et al.,
5.2 Binary evolution

In this section we will give a rough outline of binary evolution and the most important processes that take place in low and intermediate mass binaries. The actual implementation in the four BPS codes under consideration in this study is described in Appendix 5.A and 5.5.

5.2.1 Roche lobe overflow

Low-and intermediate-mass systems with initial periods less than approximately 10 years and primary masses above approximately 0.8 $M_\odot$, will come into Roche lobe contact within a Hubble time. The stars in
a binary system evolve effectively as single stars, slowly increasing in radius and luminosity, until one or both of the stars fills its Roche lobe. At this point mass from the outer layers of the star can flow through the first Lagrangian point leaving the donor star.

Depending on the reaction of the star upon mass loss and the reaction of the Roche lobe upon the rearrangement of mass and angular momentum in the system, mass transfer can be stable or unstable. When mass transfer becomes unstable, the loss of mass from the donor star will cause it to overfill its Roche lobe further. In turn this increases the mass loss rate leading to a runaway process. In comparison, when mass transfer is stable, the donor star will stay approximately within the Roche lobe. Mass transfer is maintained by the expansion of the donor star, or the contraction of the Roche lobe from the rearrangement of mass and angular momentum in the binary system.

RLOF influences the evolution of the donor star by the decrease in mass. The evolution of the companion star is affected too if some or all of the mass lost by the donor is accreted. This is particularly true if some of the accreted (hydrogen-rich) matter makes its way to the core through internal mixing, where it will thus lead to replenishment of hydrogen, a process known as rejuvenation (e.g. Vanbeveren & De Loore, 1994).

Orbits of close binaries are affected by angular momentum loss (AML) from gravitational wave emission (e.g. Peters, 1964), possibly magnetic braking (Verbunt & Zwaan, 1981; Knigge et al., 2011, for an overview) and tidal interaction. Magnetic braking extracts angular momentum from a rotating star by a stellar wind that is magnetically coupled to the star. If the star is in corotation with the orbit, angular momentum is essentially also removed from the binary orbit. Tidal interaction plays a crucial role in circularising binaries and will strive to synchronise the rotational period of each star with the orbital period. While it is known that tidal effects will eventually achieve tidal locking of both components, the strength of tidal effects is still subject to debate (e.g. Zahn, 1977; Hut, 1981).

### 5.2.1.1 Stable mass transfer

In the case of conservative RLOF the variation in the orbital separation $a$ during the mass transfer phase is dictated solely by the masses. If the gainer star accretes mass non-conservatively, there is a loss of matter and angular momentum from the system. We define the accretion efficiency:

$$\beta = |M_d/M_a|,$$

where $M_d$ is the mass of the donor star and $M_a$ is the mass of the accreting companion. If $\beta < 1$, it is also necessary to make an assumption about how much angular momentum is carried away with it. We define this with a parameter $\eta$ such that:

$$\frac{j}{J} = \eta \frac{M}{M_d + M_a}(1 - \beta).$$

The amount of angular momentum that is lost from the system due to mass loss has a large influence on the evolution of the binary. Several prescriptions for AML exist. They can be divided in four modes of AML or combinations of these modes (e.g. Soberman et al., 1997, for an overview of the effect of the different prescriptions on the stability of the system).

- Orbital angular momentum loss mode;
  In this mode the mass is assumed to leave the binary system, with (a multiple of) the specific orbital angular momentum of the binary, i.e $\eta = constant$. 

110
5.2 Binary evolution

Figure 5.1: Angular momentum loss (in terms of $\frac{M(M - \rho)}{M + M_d}$) as a function of mass ratio for four modes: specific angular momentum loss mode (solid, for $\eta = 1$), Jeans mode (dotted), isotropic re-emission mode (dashed) and in the case of a circumbinary ring (dashed-dotted, for $a_{\text{ring}}/a = 2.3$). See text for definition and explication of modes.

- **Jeans mode;**
  Mass is assumed to leave the system from the vicinity of the donor star in a fast spherically symmetric wind. In this mode, the wind matter does not interact with the system. It takes with it the specific orbital angular momentum of the donor in its relative orbit around the centre of mass. Making the assumption that the donor star can be approximated by a point mass, the specific angular momentum loss is as in eq. 5.2 with:

  $$\eta = \frac{M_d}{M_a}. \tag{5.3}$$

- **Isotropic re-emission;**
  In this case mass is assumed to leave the system from the position of the gainer in a spherically symmetric way (or at least symmetric with respect to the equatorial plane of the star). Possible scenarios are an enhanced stellar wind or bipolar jets. Further assumptions are as in the previous case, resulting in:

  $$\eta = \frac{M_d}{M_a}. \tag{5.4}$$

- **Circumbinary ring;**
  Finally, it is possible to assume that the matter will leave the binary through the formation of a non-corotating circumbinary ring, after passing through the second Lagrangian point $L_2$. The amount of angular momentum lost then depends on the radius of this ring $a_{\text{ring}}$ compared to the orbital separation $a$:

  $$\eta = \sqrt{\frac{a_{\text{ring}}}{a} \left(\frac{M_d}{M_a} + \frac{M_d}{M_a}\right)^2 M_d M_a}. \tag{5.5}$$

While an absolute minimum for $a_{\text{ring}}$ is the distance from the center of mass to $L_2$ (which can be shown to vary only very slightly during a mass transfer episode), it was shown by Soberman et al. (1997) that a more realistic value is 2.3 times the orbital separation.
Fig. 5.1 shows, for the four different AML modes, the angular momentum loss $\dot{J}/J$ as a function of mass ratio $q = M_d/M_a$. It is clear that the assumption of AML from a circumbinary ring always leads to the largest AML. The Jeans mode causes the least AML for systems with low mass ratios $q < 1$, because the donor is then close to the center of mass of the system. As the mass ratio increases during mass transfer, the AML increases as well since the donor recedes from the center of mass. Conversely, the isotropic re-emission mode causes a large AML for low mass ratio systems, as the gainer is far from the center of mass. As the mass ratio rises, the gainer closes in on the center of mass and AML decreases. The orbital AML assumption results in an intermediate case between the two.

The choice of AML mode is critical for both the stability and the orbital evolution of mass transfer. To illustrate, in the case of the circumbinary ring mode (extracting the most angular momentum), a given amount of mass loss will lead to much more AML than in the case of Jeans mode AML. The former mode will thus result in a far greater number of systems that merge than the latter.

Matter and angular momentum can also be lost through stellar winds. As these are usually assumed to be spherically symmetric, they will extract the specific orbital angular momentum of the donor star i.e. Jeans mode, and result in an increase in the orbital period. If, however, the wind is allowed to interact with the orbit of the binary, the result is entirely dependent on this interaction.

### 5.2.1.2 UNSTABLE MASS TRANSFER

During unstable mass transfer, the envelope of the donor star engulfs the companion star. Therefore this phase is often called the common envelope (CE) phase (Paczynski, 1976). A merger of the companion and the core of the donor star can be avoided, if the gaseous envelope surrounding them is expelled e.g. by viscous friction that heats the envelope. Because of the loss of significant amounts of mass and angular momentum the CE-phase can have a very strong effect on the binary orbit. In particular it plays an essential role in the formation of short period systems containing at least one compact object. Despite this, the phenomenon is not yet well understood, see Ivanova et al. (2013) for an overview.

There are several formalisms available to treat the orbital evolution during CE-evolution. The most popular ones are the $\alpha$-formalism (Tutukov & Yungelson, 1979) and the $\gamma$-formalism (Nelemans et al., 2000). The first considers the energy budget of the initial and final configuration, while the latter is based on the angular momentum balance. Both prescriptions include a parameter after which they are named, which determines the efficiency to remove the envelope. Because such an unstable mass transfer phase occurs on a short timescale, it is often assumed that the gainer does not have the time to gain an appreciable amount of mass during a CE-phase.

The $\alpha$-parameter describes the efficiency of which orbital energy is consumed to unbind the common envelope according to:

$$E_{gr} = \alpha_{ce}(E_{orb,i} - E_{orb,f})$$

where $E_{orb}$ is the orbital energy, $E_{gr}$ is the binding energy of the envelope and $\alpha_{ce}$ is the efficiency of the energy conversion. The subscript i and f represent the parameter before and after the CE-phase respectively. Several prescriptions for the quantities $E_{orb,i}$ and $E_{gr}$ have been proposed (Webbink 1984, Iben & Livio 1993, Hurley et al. 2002, see Zorotovic et al. 2010 for an overview) resulting in de facto different $\alpha$-formalisms. We assume $E_{orb,i}$ and $E_{gr}$ as given in the $\alpha$-formalism of Webbink (1984), such that

$$E_{orb,i} = \frac{GM_d M_a}{2a_i}$$

112
5.3 Binary population synthesis codes

\[ E_{\text{gr}} = \frac{G M_d M_{d,\text{env}}}{\lambda_{\text{ce}} R}, \]  

(5.8)

where \( R \) is the radius of the donor star, \( M_{d,\text{env}} \) is the envelope mass of the donor and \( \lambda_{\text{ce}} \) depends on the structure of the donor (De Kool et al., 1987; Dewi & Tauris, 2000; Xu & Li, 2010; Loveridge et al., 2011).

In the case of mass transfer between two giants with loosely bound envelopes, some codes employ a formalism different from the canonical CE-descriptions. The envelopes are expelled according to

\[ E_{\text{gr,d1}} + E_{\text{gr,d2}} = \alpha(E_{\text{orb,i}} - E_{\text{orb,f}}), \]  

(5.9)

analogous to eq. 5.6, where \( E_{\text{gr,d1}} \) and \( E_{\text{gr,d2}} \) represents the binding energy of the envelope of the two donor stars. This mechanism is termed a double common envelope phase (Brown, 1995).

5.3 Binary population synthesis codes

In this paper we compare the results of the simulations of four different BPS codes. These codes have been developed throughout the years with different scientific aims and philosophies, which has resulted in different numerical treatments and assumptions to describe binary evolution. An overview of the methods that are inherent to and the typical assumptions in the four BPS codes can be found in Appendix 5.A and 5.5. Below a short description is given of each code in alphabetical order:

5.3.1 Binary_c/nucsyn

Binary_c/nucsyn (binary_c for future reference) is a rapid binary population synthesis code with binary evolution based on Hurley et al. (2000, 2002). Updates and relevant additions are continuously made (Izzard et al., 2004, 2006, 2009, Claeys et al., subm.) to improve the code and to compare the effects of different prescriptions for ill-constrained physical processes. The most recent updates (Claeys et al., subm.) that are relevant for this paper are a new formulation to determine the mass transfer rate, the accretion efficiency of WDs and the stability criteria for helium star donors and accreting WDs. The code uses analytical formulae based on detailed single star tracks at different metallicities (based on Pols et al., 1998; Karakas et al., 2002), with integration of different binary features (based on BSE, Hurley et al., 2002). In addition, the code includes nucleosynthesis to follow the chemical evolution of binary systems and their output to the environment (Izzard et al., 2004, 2006, 2009).

The code is used for different purposes, from the evolution of low-mass stars to high-mass stars. This includes the study of carbon or nitrogen-enhanced metal-poor stars (CEMP/NEMP-stars Izzard et al., 2009; Pols et al., 2012; Abate et al., 2013), the evolution of Barium stars (Bonačić Marinović et al., 2006; Izzard et al., 2010), progenitor studies of SNe Ia (Claeys et al., subm.), the study of rotation of massive stars (De Mink et al., 2013) and recently the evolution of triple systems (Hamers et al., 2013). Although the code has different purposes, the main strength of the code is the combination of a binary evolution code with nucleosynthesis which enables the study of not only the binary effects on populations, but also the chemical evolution of populations and its output to the environment.
5.3.2 **THE BRUSSELS CODE**

The Brussels binary evolution population number synthesis code has been under development for the better part of two decades, primarily to study the influence of binary star evolution on the chemical evolution of galaxies. A thorough review of the Brussels PNS code is given by De Donder & Vanbeveren (2004).

The population code uses actual binary evolution calculations (not analytical formulae) performed with the Paczyński-based Brussels binary evolution code, developed over more than three decades at the Astrophysical Institute of the Vrije Universiteit Brussel. An important feature is that the effects of accretion on the further evolution of the secondary star are taken into account. The population code interpolates between the results of several thousands of actual binary evolution models, calculated under the assumption of the “snowfall model” by Neo et al. (1977) in the case of direct impact, and assuming accretion induced full mixing (Vanbeveren & De Loore, 1994) if accretion occurs through a disk. The actual evolution models have been published by Vanbeveren et al. (1998). The research done with the Brussels code mainly focuses on the chemical enrichment of galaxies caused by intermediate mass and massive binaries. Therefore the interpolations contained in the population code do not allow for the detailed evolution of stars with initial masses below $3 M_\odot$.

In recent years, the code was mainly used to study the progenitors of type Ia supernovae (Mennekens et al., 2010, 2012), the contribution of binaries to the chemical evolution of globular clusters (Vanbeveren et al., 2012) and the influence of merging massive close binaries on type II supernova progenitors (Vanbeveren et al., 2013).

5.3.3 **SeBa**

SeBa is a fast binary population synthesis code that is originally developed by Portegies Zwart & Verbunt (1996) with substantial updates from Nelemans et al. (2001b) and Toonen et al. (2012). Recent updates include the metallicity dependent single stellar evolution tracks of Hurley et al. (2000) for non-degenerate stars, updated wind mass loss prescriptions and improved prescriptions for hydrogen and helium accretion, and the stability of mass transfer.

The philosophy of SeBa is to not a priori define evolution of the binary, but rather to determine this at runtime depending on the parameters of the stellar system. When more sophisticated models become available of processes that influence stellar evolution, these can be included, and the effect can be studied without altering the formalism of binary interactions. An example of this is the stability criterion of mass transfer and the mass accretion efficiency.

SeBa has been used to study a large range of stellar populations: high mass binaries (Portegies Zwart & Verbunt, 1996), double neutron stars (Portegies Zwart & Yungelson, 1998), gravitational wave sources (Portegies Zwart & Spreeuw, 1996; Nelemans et al., 2001a), double white dwarfs (Nelemans et al., 2001b), AM CVn systems (Nelemans & Van den Heuvel, 2001), sdB stars (Nelemans, 2010), SNIa progenitors (Toonen et al., 2012; Bours et al., 2013) and ultracompact X-ray binaries Van Haften et al. (2013).

As part of the software package Starlab, it has been used to simulate the evolution of dense stellar systems (Portegies Zwart et al., 2001, 2004). Recently, SeBa is incorporated in the Astrophysics Multi-purpose Software Environment, or AMUSE. This is a component library with a homogeneous interface structure, and can be downloaded for free at amusecode.org (Portegies Zwart et al., 2009).
5.3.4 StarTrack

StarTrack is a Monte Carlo-based single and binary star rapid evolution code. Stars are evolved at a given metallicity (range: $Z = 0.0001 - 0.03$) by adopting analytical fitting formulae from evolutionary tracks of detailed single stellar models (Hurley et al., 2000), and modified over the years in order to incorporate the most important physics for binary evolution. The orbital parameters (separation, eccentricity and stellar spins) $a$, $e$, $\omega_1$ and $\omega_2$ are solved numerically as the system evolves, and re-distribution of angular momentum determines how the orbit behaves. As physical insights regarding various aspects of stellar and binary evolution become available in the literature, new input physics can be implemented into the code, and thus the code is continuously being updated and improved.

The StarTrack code was originally used to predict physical properties of compact objects such as single and double black holes and neutron stars, as well as gamma ray bursts and compact object mergers in context of gravitational wave detection with LIGO (Belczynski et al., 2002b,a; Abbott et al., 2004). In more recent years, studies with the code have grown to include compact binaries in globular clusters (Ivanova et al., 2005), X-ray binary populations (Belczynski et al., 2004; Ruiter et al., 2006), sources of gravitational wave radiation for ground-based and space-based gravitational wave detectors (Ruiter et al., 2009a, 2010; Belczynski et al., 2010a,c), gamma ray bursts (Belczynski et al., 2007; O'Shaughnessy et al., 2008; Belczynski et al., 2008b), type Ia supernovae progenitors (Belczynski et al., 2005; Ruiter et al., 2009b, 2011, 2013) and core-collapse supernova explosion mechanisms (Belczynski et al., 2012). The most comprehensive description of the code to date can be found in Belczynski et al. (2008a), with some updates described in Ruiter et al. (2009b) (SNe Ia), Belczynski et al. (2010b) (stellar winds), and Dominik et al. (2012) (wind mass-loss rates, CE).

5.4 Method

To examine the inherent differences of the four BPS codes, we compare the results of a simulation made by these codes in which the assumptions are equalised as far as possible (Sect. 5.4.1). We consider two populations of binaries:

- Single WDs with a non-degenerate companion (hydrogen-rich or helium-rich star) (SWDs)
- Double WD systems (DWDs)

Of both populations we investigate the initial distributions and the distributions at the moment that the SWD or DWD system forms. At these specific times we distinguish the different evolutionary paths, including (possibly several) phases of mass transfer. We establish the similarities between the results of the different BPS codes. If we notice differences between the results, we analyse these in greater detail by comparing e.g. individual systems, their evolutionary path, but also the mass transfer rate and/or wind mass loss rate; or the stability criteria of a group of systems.

In the simulation, we assume an initial primary mass $M_{1,zams}$ between $M_{1,zams,min} = 0.8 M_\odot$ and $M_{1,zams,max} = 10 M_\odot$, an initial mass ratio $q_{zams} = M_{2,zams}/M_{1,zams}$ between $q_{zams,min} = 0.1 M_\odot/M_{1,zams}$ and $q_{zams,max} = 1$ and an initial semi-major axis $a_{zams}$ between $a_{zams,min} = 5 R_\odot$ and $a_{zams,max} = 10^4 R_\odot$. Furthermore we assume an initial eccentricity $e_{zams}$ of zero. We consider SWDs and DWDs that are formed within a Hubble time, more specifically 13.7 Gyr. The initial distribution of the primary masses follows
Kroupa et al. (1993), the initial mass ratio distribution is flat\(^1\), and the initial distribution of the semi-major axis is flat in a logarithmic scale.

Not every BPS research group focuses on the full range of stellar masses. Consequently in their codes there are no (valid) prescriptions available for all stellar masses. The research group that uses the Brussels code, mainly focuses on the chemical enrichment of galaxies and therefore is not interested in the evolution of stars with a mass lower than \(3 \, M_\odot\) (Sect. 5.3.2). Consequently, in order to make the comparison with the results of the Brussels code we only compare with a subset of the SWD and DWD populations. We define this subset as the ‘intermediate mass range’, while the entire populations is considered as the ‘full mass range’. The ‘intermediate mass range’ is defined in the two populations as follows:

- for the \textit{SWD population} we only consider WDs originating from initial primary masses higher than \(3 \, M_\odot\).
- for the \textit{DWD population} we only consider WDs originating from initial primary and secondary masses both higher than \(3 \, M_\odot\).

In addition, we refer to the ’low mass range’ or ’low mass primaries’ which encompasses the systems with an initial primary mass lower than \(3 \, M_\odot\).

BPS codes are ideal to investigate the effect of different assumptions on populations, since a different assumption can cause a shift in e.g. the mass or separation of the population under investigation. We do not have to agree on the exact evolution of individual systems. As long as the shift is small the characteristics of the population do not change. Keeping this in mind when comparing the results of the different BPS codes we define them to agree when similar evolutionary paths are recovered at the same regions in the mass and separation space.

5.4.1 Assumptions for this project

In order to compare the codes we make the most simple assumptions. These are not necessarily believed to be realistic, but are taken to make the comparison feasible. The assumptions for this project are discussed below and shown in Table 5.1. The typical assumptions taken by the authors in the corresponding BPS codes in their previous research projects are summarised in Table 5.5 in Appendix 5.5. For simplicity and brevity, we do not study the effect of different assumptions on the characteristics of SWD and DWD populations in this project.

- Mass transfer is assumed to be conservative (\(\beta = 1\)) during stable RLOF towards all types of objects. We emphasise that this is not a realistic assumption, especially in the case of a WD accretor. During the CE-phase no material is assumed to be accreted by the companion star (\(\beta = 0\)).

In the Brussels code a constant accretion efficiency of a WD-accretor cannot be implemented and therefore for this study mass transfer to all compact objects is assumed to be unstable and evolve into a common envelope in this code.

\(^1\)Note that the initially imposed constraint on the mass ratio (i.e. \(q_{\text{zams, min}} = 0.1 \, M_\odot/(M_1\text{,zams})\)) affects the overall shape of the resulting \(q_0\)-distribution. Even though the probability of drawing a mass ratio anywhere is equal, this is strictly only true between \(q_{\text{zams}} \approx 0.1 \sim 1\). Mass ratios lower than approximately 0.1 are drawn less often, since the primary masses cluster around \(1 \, M_\odot\) due to the IMF, and the lower mass limit of the secondary is assumed to be \(0.1 \, M_\odot\).
5.4 Method

Table 5.1: Equalised initial distribution and range of binary parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{1,zams}$ ($M_\odot$)</td>
<td>KTG93</td>
</tr>
<tr>
<td>$a_{zams}$ ($R_\odot$)</td>
<td>Flat</td>
</tr>
<tr>
<td>$q_{zams}$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{1,zams,min}$</td>
<td>0.8 (0.1)(^{(1)})</td>
</tr>
<tr>
<td>$M_{1,zams,max}$</td>
<td>10 (100)(^{(1)})</td>
</tr>
<tr>
<td>$a_{zams,min}$</td>
<td>5</td>
</tr>
<tr>
<td>$a_{zams,max}$</td>
<td>1e4 (1e6)(^{(1)})</td>
</tr>
<tr>
<td>$q_{zams,min}$</td>
<td>0.1/$M_{1,zams}$</td>
</tr>
<tr>
<td>$q_{zams,max}$</td>
<td>1</td>
</tr>
<tr>
<td>$e_{zams}$</td>
<td>0</td>
</tr>
<tr>
<td>Max time (Gyr)</td>
<td>13.7</td>
</tr>
<tr>
<td>Binary fraction (%)</td>
<td>100</td>
</tr>
<tr>
<td>$\beta$ (RLOF)</td>
<td>1</td>
</tr>
<tr>
<td>$\alpha_{ce,\lambda_{ce}}$</td>
<td>1</td>
</tr>
<tr>
<td>Physics</td>
<td>Assumption</td>
</tr>
<tr>
<td>AML (RLOF)</td>
<td>Orbit ($\eta = 1$)</td>
</tr>
<tr>
<td>CE</td>
<td>$\alpha^{(2)}$</td>
</tr>
<tr>
<td>Wind accretion</td>
<td>No</td>
</tr>
<tr>
<td>Tides</td>
<td>No</td>
</tr>
<tr>
<td>Magn. braking</td>
<td>No</td>
</tr>
</tbody>
</table>

Notes:
(1) The values outside and inside the brackets represent the values for the simulated and entire stellar population, respectively.
(2) prescription based on Webbink (1984).


- We assume that during mass transfer the angular momentum lost is specific orbital angular momentum of the binary (with $\eta = 1$, eq. 5.2).

  It is not possible to equalise the assumptions for AML during wind mass loss between the codes (for an overview of the assumptions see Sect. 5.A.5).

- We use the $\alpha$-prescription of Webbink (1984) to describe the CE-phase (eq. 5.6, 5.7 and 5.8). We assume that the parameters $\alpha_{ce}$ and $\lambda_{ce}$ are equal to one, mainly for simplicity, but also because the prevalence of this choice in the literature allows for comparison between this and other studies.

- We assume that matter lost through winds cannot be accreted by the companion star.

- Due to the diversity of the prescriptions for magnetic braking and tides, we do not consider these effects and they are turned off for this paper. However, in StarTrack, spin-orbit coupling is still taken into account, as it is firmly integrated with the binary evolution equations.
5.4.2 Normalisation

When calculating birthrates of evolutionary channels, the simulation has to be normalised to an entire stellar population (Table 5.1). For this work the initial distribution and ranges of $M_{1,\text{zams}}$, $q_{\text{zams}}$ and $a_{\text{zams}}$ are as discussed in Sect. 5.4 with the exception of the initial primary masses of a stellar population to vary between 0.1 and 100 $M_\odot$, and the semi-major axis between 5 and $10^6$ $R_\odot$. We assume a binary fraction of 100%.

If the star formation rate $S$ in $M_\odot$ yr$^{-1}$ is independent of time, the birthrate of an evolutionary channel $X$ is given by:

$$\text{Birthrate}(X) = S \frac{\phi(X)}{M_{\text{tot}}},$$

(5.10)

with $\phi(X)$ the total number of systems evolving through evolutionary channel $X$ in the simulation, and $M_{\text{tot}}$ the total mass of all stellar systems in the entire stellar population. More specifically,

$$\phi(X) = \int_{0.1}^{100} \int_{0.1/M_{1,\text{zams}}}^{1} \int_{5}^{1e6} x \Psi dM_{1,\text{zams}} dq da,$$

(5.11)

with $x = x(M_{1,\text{zams}}, q, a)$ equals 1 for the binary systems evolving through evolutionary channel $X$, and zero otherwise and $\Psi$ is the initial distribution function of $M_{1,\text{zams}}$, $q_{\text{zams}}$ and $a_{\text{zams}}$. Note that in this project we assume that the initial distribution for $M_{1,\text{zams}}$, $q_{\text{zams}}$ and $a_{\text{zams}}$ are independent (Table 5.1), such that $\Psi$ is separable:

$$\Psi(M_{1,\text{zams}}, q_{\text{zams}}, a_{\text{zams}}) = \psi(M_{1,\text{zams}}) \varphi(q_{\text{zams}}) \chi(a_{\text{zams}}).$$

(5.12)

The total mass of all stellar systems assuming a 100% binary fraction is:

$$M_{\text{tot}} = \int_{0.1}^{100} \int_{0.1/M_{1,\text{zams}}}^{1} \int_{5}^{1e6} M_{1,\text{zams}} \Psi dM_{1,\text{zams}} dq da,$$

(5.13)

where $M_{1,\text{zams}} = M_{1,\text{zams}} + M_{2,\text{zams}}$.

For this project a constant star formation rate of 1 $M_\odot$ yr$^{-1}$ is assumed. This simple star formation rate is chosen to make the comparison with other codes easier.

5.5 Results

5.5.1 Single white dwarf systems

Systems containing a WD and a non-degenerate companion have typically undergone a one-directional mass transfer event i.e. one star has lost mass and possibly the other gained mass. The mass transfer event may consist of one or two episodes, either of which may have been stable or unstable. The characteristics of the population of SWD systems show the imprint of the mass transfer episodes. Fig. 5.2 and 5.3 show the orbital separation $a_{\text{swd}}$ as a function of primary mass $M_{1,\text{swd}}$ at the moment of WD formation for the full and intermediate mass range respectively. Likewise Fig. 5.4 and 5.5 show the secondary mass $M_{2,\text{swd}}$ as a function of primary mass at WD formation for the full and intermediate mass range. These figures show that in general the codes find very similar SWD systems.
Table 5.2: Birthrates in $\text{yr}^{-1}$ for different evolutionary channels (described in Sect. 5.5) of single and double white dwarf systems for the three BPS codes for the full mass range and the intermediate mass range.

<table>
<thead>
<tr>
<th>Evolutionary channels</th>
<th>Full mass range</th>
<th>Intermediate mass range</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>binary_c</td>
<td>SeBa</td>
</tr>
<tr>
<td>SWD systems</td>
<td>0.048</td>
<td>0.052</td>
</tr>
<tr>
<td>Channel I</td>
<td>0.026</td>
<td>0.026</td>
</tr>
<tr>
<td>Channel 2a</td>
<td>6.9e-3</td>
<td>6.5e-3</td>
</tr>
<tr>
<td>Channel 2b</td>
<td>5.7e-4</td>
<td>5.8e-4</td>
</tr>
<tr>
<td>Channel 3a</td>
<td>1.4e-3</td>
<td>4.2e-3</td>
</tr>
<tr>
<td>Channel 3b</td>
<td>5.7e-4</td>
<td>4.6e-4</td>
</tr>
<tr>
<td>Channel 4a</td>
<td>0.012</td>
<td>0.012</td>
</tr>
<tr>
<td>Channel 4b</td>
<td>1.8e-4</td>
<td>8.9e-5</td>
</tr>
<tr>
<td>Channel 5</td>
<td>2.4e-4</td>
<td>5.6e-4</td>
</tr>
<tr>
<td>DWD systems</td>
<td>0.012</td>
<td>0.014</td>
</tr>
<tr>
<td>Channel I</td>
<td>8.4e-3</td>
<td>8.8e-3</td>
</tr>
<tr>
<td>Channel II</td>
<td>2.0e-3</td>
<td>1.3e-3</td>
</tr>
<tr>
<td>Channel III</td>
<td>1.3e-3</td>
<td>3.0e-3</td>
</tr>
<tr>
<td>Channel IV</td>
<td>1.6e-4</td>
<td>5.5e-5</td>
</tr>
</tbody>
</table>
Figure 5.2: Orbital separation versus WD mass for all SWDs in the full mass range at the time of SWD formation. The contours represent the SWD population from a specific channel: channel 1 (solid line), channel 4a (thin dashed line), channel 4b (thick dashed line) and channel 5 (dash-dotted line).
Figure 5.3: Orbital separation versus WD mass for all SWDs in the intermediate mass range at the time of SWD formation. The contours represent the SWD population from a specific channel: channel 1 (solid line), channel 4a (thin dashed line), channel 4b (thick dashed line) and channel 5 (dash-dotted line).
Figure 5.4: Secondary mass versus WD mass for all SWDs in the full mass range at the time of SWD formation. The contours represent the SWD population from a specific channel: channel 1 (solid line), channel 4a (thin dashed line), channel 4b (thick dashed line) and channel 5 (dash-dotted line).
Figure 5.5: Secondary mass versus WD mass for all SWDs in the intermediate mass range at the time of SWD formation. The contours represent the SWD population from a specific channel: channel 1 (solid line), channel 4a (thin dashed line), channel 4b (thick dashed line) and channel 5 (dash-dotted line).
Figure 5.6: Initial orbital separation versus initial primary mass for all SWDs in the full mass range. The contours represent the SWD population from a specific channel: channel 1 (solid line), channel 4a (thin dashed line), channel 4b (thick dashed line) and channel 5 (dash-dotted line).
Figure 5.7: Initial orbital separation versus initial primary mass for all SWDs in the intermediate mass range. The contours represent the SWD population from a specific channel: channel 1 (solid line), channel 4a (thin dashed line), channel 4b (thick dashed line) and channel 5 (dash-dotted line).
In more detail, at large separations ($a_{\text{wd}} \gtrsim 500 \, R_\odot$ for the full mass range, and $a_{\text{wd}} \gtrsim 2000 \, R_\odot$ for the intermediate mass range) all codes find systems in which the stars do not interact. The population of SWDs with WD masses in the low mass range is very comparable in orbital separation, primary and secondary mass between the codes binary, SeBa and StarTrack. Intermediate mass systems can be divided in two groups, either in separation and/or in secondary mass. According to all codes, intermediate mass systems that undergo a CE-phase (for the first mass transfer episode) are compact with $a_{\text{wd}} \lesssim 200 \, R_\odot$ and have secondary masses up to $10 \, M_\odot$. Furthermore, the codes agree that in the intermediate mass range, systems for which the first phase of mass transfer is stable are in general more compact than non-interacting systems and less compact than the systems undergoing a CE-phase. The secondary mass is between 3 and $18 \, M_\odot$ as it accretes conservatively during stable mass transfer. The ZAMS configurations for progenitors of SWDs are shown in Fig. 5.6 and 5.7 with the separation $a_{\text{zams}}$ versus primary mass $M_{1,\text{zams}}$. There is a general agreement between the codes about which progenitor systems lead to a SWD system and which systems do not. According to all codes, compact progenitor systems ($a_{\text{zams}} \lesssim 400 \, R_\odot$ for the intermediate mass range, while $a_{\text{zams}} \lesssim 30 \, R_\odot$ for the low mass range) undergo stable mass transfer for the initial mass transfer episode. Furthermore the codes agree that for most progenitor systems with orbital separations in the range $a_{\text{zams}} \approx (0.1 - 3) \cdot 10^3 \, R_\odot$ the first phase of mass transfer is unstable. Systems with orbital separations that lie between the ranges described above lead to a merging event, thus no SWD system is formed. Progenitor systems with $a_{\text{zams}} \gtrsim 700 \, R_\odot$ for the intermediate mass range ($a_{\text{zams}} \gtrsim 250 \, R_\odot$ for the low mass range) are too wide for the primary star to fill its Roche lobe.

![Figure 5.8: Initial-final mass relation of single stars that become WDs for the different groups, dotted line shows the results of binary, solid line the results of the Brussels code, the dashed line the results of SeBa, and the dash-dotted line the results of StarTrack.](image)

The initial-final mass (MiMf)-relation of single stars (Fig. 5.8) is very similar between binary, SeBa and StarTrack, but different than the one from the Brussels code due to different single star prescriptions that are used in the latter code (Sect. 5.5.1.1 for a discussion). The effect on the population of SWD progenitors can be seen in Fig. 5.7 in the maximum mass of the primary stars which is extended from
about $8 \, M_\odot$ in binary_c, SeBa and StarTrack to about $10 \, M_\odot$ in the Brussels code. For binary stars the relation between WD mass and the initial mass is hereafter called the initial-WD mass (MiMwd)-relation (Sect. 5.5.1.2 for a discussion). Differences in the MiMwd-relation lead to an increase of systems at small WD masses $\lesssim 0.64 \, M_\odot$ in Fig. 5.3 in the Brussels code compared to the other codes. The gap in WD masses between 0.7-0.9 $M_\odot$ in the Brussels data in Fig. 5.3 is a result of a discontinuity in the MiMwd-relation between the WD masses of primaries that fill their Roche a second time, and those that do not. In the other codes, the primary WD masses of binaries that evolve through these two evolutionary channels are overlapping. Differences in the stability criteria of mass transfer can be seen in Fig. 5.3 and 5.5, most pronouncedly via the greyscales where the StarTrack code shows a decrease of systems that underwent stable mass transfer (Sect. 5.5.1.3). Mass transfer is modelled differently in the codes (Sect. 5.A) leading to an extension to small separations in the Brussels data compared to the other codes (Fig. 5.7), and an increase in systems that underwent stable mass transfer at $a_{\text{zams}} \approx 10 \, R_\odot$ for $M_{\text{1,zams}} \gtrsim 4 \, M_\odot$ in Fig. 5.5 (Sect. 5.5.1.5).

In the next sections, we make a more detailed comparison between the simulated populations of SWDs of the four codes. We distinguish between the most commonly followed evolutionary paths with birthrates larger than $1.0 \cdot 10^{-3} \, \text{yr}^{-1}$. We describe each evolutionary path, the similarities and differences, and investigate the origin of these differences. Specific examples are given and discussed for the most common paths. Abbreviations of stellar types are shown in Table 5.3. Paragraphs explaining the evolutionary path, an example evolution and the comparison of the simulated populations for each evolutionary channel are indicated with *Evolutionary path*, *Example* and *Population*, respectively. For some channels, causes for differences between the populations are discussed separately in paragraphs that are indicated by *Effects*. Masses and orbital separations according to each code are given in vector form $[c_1, c_2, c_3, c_4]$ where $c_1$ represents the value according to the binary_c code, $c_2$ according to the Brussels code, $c_3$ according to SeBa, and $c_4$ according to StarTrack. The examples are given to illustrate the evolutionary path and relevant physical processes. However, note that when comparing different BPS codes, achieving similar results for specific binary populations is more desirable and important than achieving a perfect match between specific, individual binary systems.

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Type of star</th>
</tr>
</thead>
<tbody>
<tr>
<td>MS</td>
<td>Main-sequence star</td>
</tr>
<tr>
<td>HG</td>
<td>Hertzsprung-gap star</td>
</tr>
<tr>
<td>GB</td>
<td>Star on the first giant branch (red giants)</td>
</tr>
<tr>
<td>AGB</td>
<td>Star on the asymptotic giant branch</td>
</tr>
<tr>
<td>He-MS</td>
<td>Star on the equivalent of the main-sequence for hydrogen-poor helium-burning stars</td>
</tr>
<tr>
<td>Ev. He-star</td>
<td>Evolved hydrogen-poor helium-burning star</td>
</tr>
<tr>
<td>WD</td>
<td>White dwarf</td>
</tr>
</tbody>
</table>

### 5.5.1.1 Channel 1: Detached Evolution

*Evolutionary path* Most SWD binaries are non-interacting binaries where the stars essentially evolve as single stars. Most binary processes that are discussed in Sect. 5.2 do not play a role in channel 1.
Example As an example of a system in channel 1, we discuss the evolution of a system that initially contains a 5 $M_\odot$ and 4 $M_\odot$ star in an orbit of $10^4 R_\odot$ (and $e_{\text{zams}} = 0$ by assumption). When the primary star becomes a WD its mass is $[1.0, 0.94, 1.0, 1.0] M_\odot$ in an orbit of $[1.8, 1.8, 1.8, 1.8] \cdot 10^4 R_\odot$. The differences in the resulting SWD system from different BPS codes are small and mainly due to different initial-final mass (MiMf)-relations (Fig. 5.8). The maximum progenitor mass to form a WD from a single star is $7.6, 10, 7.9, 7.8$ $M_\odot$ and corresponding maximum WD mass of $[1.38, 1.34, 1.38, 1.4] M_\odot$ according to the four codes. The MiMf-relations of the binary_c code, SeBa and StarTrack are very similar. The similarities are not surprising as these codes are based on the same single stellar tracks and wind prescriptions of Hurley et al. (2000). However, small differences arise in the MiMf-relation as the prescriptions for the stellar wind are not exactly equal. The Brussels code is based on different models of single stars e.g. different stellar winds and a different overshooting prescription (Appendix 5.A). The result is that the core mass of a specific single star is larger according to the Hurley tracks. In other words, the progenitor of a specific single WD is more massive in the Brussels code.

Population Despite differences for individual systems, the population of non-interacting binaries at WD formation is very similar. The previously mentioned differences in the MiMf-relations are noticeable in the maximum initial primary mass in Fig. 5.6 and 5.7. The distribution of separations at WD formation (Fig. 5.2 and 5.3) are very similar between the codes. For the intermediate mass range, the separations at SWD formation are $> 4.5 \cdot 10^3 R_\odot$ for the Brussels code and extend to slightly lower values of $> 2.0 \cdot 10^3 R_\odot$ for binary_c, SeBa, and StarTrack. For the full mass range, the latter three codes agree that the separations can be as low as $5.0 \cdot 10^2 R_\odot$. The progenitor systems of channel 1 have similar separations of $> 3.0 \cdot 10^2 R_\odot$ for low mass primaries. For intermediate mass stars binary_c, SeBa and StarTrack find that the initial separation is $> 0.7 \cdot 10^3 R_\odot$ where the Brussels code finds a slightly higher value of $> 1.6 \cdot 10^3 R_\odot$ (Fig. 5.6 and 5.7). The minimum separation (at ZAMS and WD formation) for a given primary mass depends on whether or not the primary fills its Roche lobe, which in turn depends on the maximum radius for that star according to the particular single star prescriptions that are used. Even though the progenitor populations are not 100% equal, the characteristics of the SWD population and the birthrates (Table 5.2) in this channel are in excellent agreement.

5.5.1.2 Channel 2: Unstable Case C

Evolutionary path One of the most common evolutionary paths of interacting binaries is channel 2, of which an example is shown in Fig. 5.9. In this channel, the primary star fills its Roche Lobe when helium is exhausted in its core, so-called case C mass transfer (Lauterborn, 1970). As the envelope of the donor star is deeply convective at this stage, generally mass transfer leads to an unstable situation and a CE-phase develops. While the orbital separation shrinks severely, the primary loses its hydrogen envelope. By assumption in this project, the secondary is not affected during the CE-phase. The primary can either directly become a WD or continue burning helium as an evolved helium star as shown in the example of Fig. 5.9. If the primary becomes a WD directly, or indirectly but without further interaction, the evolutionary path is called channel 2a. Evolution according to channel 2b occurs if the primary fills its Roche lobe for a second time when it is a helium star. The second phase of mass transfer can be either stable or unstable.

Example As an example of channel 2a, we discuss the evolution of the binary system in Fig. 5.9 with initial parameters $M_{1,\text{zams}} = 3.5 M_\odot$, $M_{2,\text{zams}} = 3 M_\odot$ and $a_{\text{zams}} = 350 R_\odot$ in more detail. The primary star fills the Roche lobe early on the AGB before thermal pulses and superwinds occur. Wind mass loss
5.5 Results

prior to the CE-phase is small, $[4.4, 0, 4.3, 4.9] \cdot 10^{-2} M_\odot$. After the CE-phase the orbital separation is reduced to $[14, 9.1, 14, 14] R_\odot$. In this example the primary continuous burning helium as an evolved helium star of mass $[0.78, 0.55, 0.78, 0.78] M_\odot$. When the primary exhausts its fuel, it becomes a WD of $[0.76, 0.51, 0.77, 0.76] M_\odot$ in an orbit of $[14, 9.1, 14, 14] R_\odot$ with a $3 M_\odot$ MS companion. The most important differences, to be seen between the Brussels code and the other codes, arises from the different single star prescriptions that are used. This affects the resulting mass of a WD from a specific primary, and the resulting orbital separation. Note that while the MiMf-relation for single stars depends on the single star prescriptions (i.e. core mass growth and winds), the MiMwd-relation is also affected by the companion mass and separation (which determine when and which kind of mass transfer event takes place), and the single star prescriptions for helium stars. In other words, the MiMwd-relation represents how fast the core grows on one hand, and the envelope is depleted by mass transfer and stellar winds on the other hand.

**Population** Despite the differences between individual systems, the different BPS codes agree in which regions of phase space ($M_{1,swd}$, $M_{2,swd}$, $a_{swd}$) in Fig. 5.10, 5.11, 5.12 and 5.13 the systems from channel 2 lie. The systems of channel 2 evolve towards small separations, with the majority in the range $0.2 - 150 R_\odot$ at WD formation. In addition, the codes agree on the masses of both stars at formation of the single WD system. In the low mass range binary_c, SeBa and StarTrack find and agree that $M_{1,swd} \approx 0.5 - 0.7 M_\odot$ and $M_{2,swd} \approx 0.1 - 2.7 M_\odot$. In the intermediate mass range the different codes find that $M_{1,swd} \geq 0.64 M_\odot$, however the Brussels code finds primary WD masses down to $0.5 M_\odot$ due to differences in MiMwd-relation. For secondary masses the codes find $M_{2,swd} \approx 0.1 - 7.0 M_\odot$. The binary_c, SeBa, and StarTrack codes agree on the initial separation for low mass binaries, which is between $(0.6 - 12) \cdot 10^2 R_\odot$ (Fig. 5.14), $M_{1,zams} \approx 1.0 - 3.0 M_\odot$ and $M_{2,zams} \approx 0.1 - 3.0 M_\odot$. For intermediate mass binaries in channel 2, there is an agreement between all codes that the initial primary masses lie between $M_{1,zams} \approx 3 - 8.5 M_\odot$ and $M_{2,zams} \approx 0.1 - 7.7 M_\odot$. Due to the MiMwd-relation, the maximum initial primary mass extends to slightly higher values for the Brussels code in comparison with the other codes (Fig. 5.15). However, for massive primary progenitors e.g. $M_{1,zams} > 9 M_\odot$ in the Brussels code, the envelope mass of the donor is large and

![Figure 5.9: Example of the evolution of a single white dwarf system in channel 2a. Abbreviations are as in Table 5.3.](image-url)
therefore a merger is more likely to happen in the simulations of the Brussels code compared to those of the other three codes. The initial orbital separation lies between \((0.1 - 2.4) \cdot 10^3 \, R_\odot\) (Fig. 5.15) according to \textit{binary\_c, SeBa and StarTrack}, however the range is extended to \(3.2 \cdot 10^3 \, R_\odot\) in the Brussels code due to the single star prescriptions of stellar radii.

**Effects** Comparing channel 2a and 2b separately, the birthrates of SWDs (Table 5.2) in the full mass range are close between the codes \textit{binary\_c, SeBa, and StarTrack}. In the intermediate mass range for channel 2a, the birthrates of \textit{binary\_c, SeBa, and StarTrack} are essentially identical, and within a factor of 2.5 lower compared to that of the Brussels code. The larger difference with the Brussels code are caused because this code assumes a priori that a white dwarf is formed without a second interaction, thus there is no entry for the Brussels code in Table 5.2 for channel 2b. The birthrates for channel 2b are very similar within a factor of about 1.2 between \textit{binary\_c, SeBa and StarTrack}. Comparing the total birthrate in channel 2 between all codes, the rate of \textit{binary\_c, SeBa and StarTrack} is only lower by about a factor 1.5 compared to the Brussels code, as some systems merge in the second interaction in the simulations of the former codes. Other differences in the simulated populations from this channel are due to the MiMwd-relation as seen in the example, but also due to differences in the criteria for the stability of mass transfer and the prescriptions for the wind mass loss (see below).

**Effects** The effect of the stellar wind in the example above is negligible, but the effect of wind mass loss becomes more important for systems with more evolved donors. Mass loss from the primary either in the CE-phase or in foregoing wind mass loss episodes affects the maximum orbital separation of the SWD systems directly and through angular momentum loss. In the simulations of the Brussels code, the maximum orbital separations at WD formation are lower \((a_{\text{swd}} \lesssim 80 \, R_\odot\) compared to \(\lesssim 150 \, R_\odot\) for the main group of systems in \textit{binary\_c, SeBa and StarTrack}), as winds are not taken into account and more mass is removed during the CE-phase in this code. More mass loss during a CE-phase leads to a greater shrinkage of the orbit, where as more wind mass loss with the assumption of specific angular momentum loss from the donor (Jeans-mode, eq. 5.3), leads to an orbital increase.

Another effect arises from the stellar wind in combination with the stability criterion of mass transfer. For systems with high wind mass losses in which the mass ratio has reversed, the first phase of mass transfer can become stable according to \textit{binary\_c, SeBa, and StarTrack}. Systems in which this happen are not included in channel 2, however the birthrates are low \([1.3, - , 6.5, 4.7] \cdot 10^{-4} \, \text{yr}^{-1}\) in the full mass range and \([5.4, - , 10, 9.1] \cdot 10^{-5} \, \text{yr}^{-1}\) in the intermediate mass range). In general, when a AGB star initiates mass transfer, stable mass transfer is more readily realised in \textit{SeBa and StarTrack} than in \textit{binary\_c}. Therefore the maximum separation of SWDs in channel 2 is highest in the \textit{binary\_c} data (up to \(650 \, R_\odot\)). However, only about 1% of systems in channel 2 in the \textit{binary\_c} code lie in the region with a separation larger than \(70 \, R_\odot\) and a WD mass higher than \(0.6 \, M_\odot\).

The stability of mass transfer is another important effect for the population of systems in channel 2b during the second phase of mass transfer. We only compare the \textit{binary\_c} code, \textit{SeBa}, and \textit{StarTrack}, as the Brussels code does not consider this evolutionary path. Whether or not the second phase of mass transfer is stable affects the resulting distribution of orbital separations. This effect is shown in Fig. 5.10 as an extension to lower separations \(a_{\text{swd}} \lesssim 10 \, R_\odot\) for \(M_{1,\text{swd}} \gtrsim 0.8 \, M_\odot\) in the \textit{binary\_c} data due to unstable mass transfer.
Figure 5.10: Orbital separation versus WD mass for all SWDs in the full mass range at the time of SWD formation. The contours represent the SWD population from a specific channel: channel 2a (thin line) and channel 2b (thick line).
Figure 5.11: Orbital separation versus WD mass for all SWDs in the intermediate mass range at the time of SWD formation. The contours represent the SWD population from a specific channel: channel 2a (thin line) and channel 2b (thick line).
Figure 5.12: Secondary mass versus WD mass for all SWDs in the full mass range at the time of SWD formation. The contours represent the SWD population from a specific channel: channel 2a (thin line) and channel 2b (thick line).
Figure 5.13: Secondary mass versus WD mass for all SWDs in the intermediate mass range at the time of SWD formation. The contours represent the SWD population from a specific channel: channel 2a (thin line) and channel 2b (thick line).
Figure 5.14: Initial orbital separation versus initial primary mass for all SWDs in the full mass range. The contours represent the SWD population from a specific channel: channel 2a (thin line) and channel 2b (thick line).
Figure 5.15: Initial orbital separation versus initial primary mass for all SWDs in the intermediate mass range. The contours represent the SWD population from a specific channel: channel 2a (thin line) and channel 2b (thick line).
5.5 Results

There is a difference between StarTrack on one hand, and binary_c and SeBa on the other hand regarding the survival of systems in channel 2b during the first phase of mass transfer. Due to a lack of understanding of the CE-phase, generally BPS codes assume for simplicity that when the stars fit in their consecutive Roche lobes after the CE is removed, the system survives the CE phase. However this depends crucially on the evolutionary state of the stars after the CE. For channel 2b in which the primary continues helium burning in a shell as a non-degenerate helium star, the response of the primary to a sudden mass loss in the CE is not well known. The StarTrack code assumes the stripped star immediately becomes an evolved helium star and corresponding radius, while binary_c and SeBa assume the stripped star is in transition from an exposed core to an evolved helium star with a radius that can be a factor of about 1-15 smaller. The uncertainty in the radii of the stripped star mostly affect systems with $M_{1,zams} \gtrsim 5 M_\odot$ at separations $\gtrsim 450 R_\odot$ that merge according to StarTrack, and survive according to binary_c and SeBa.

Included in channel 2 are systems that evolve through a double CE-phase in which both stars lose their envelope described in Sect. 5.2 and in eq. 5.9. The double CE-mechanism is taken into account by the binary_c, SeBa and StarTrack code. However there is a difference between StarTrack on one hand, and binary_c and SeBa on the other hand regarding the binding energy of the envelope of the secondary star. In StarTrack the binding energy is calculated according to eq. 5.8) with $R_2 = R_{RL,2}$, whereas in binary_c and SeBa the instantaneous radius at the start of the double CE-phase is taken for the secondary radius. This can have a significant effect on the orbit of the post-double CE-system, leading to an increase of systems at low separations (approximately 1 $R_\odot$) in the binary_c and SeBa data compared to the StarTrack data.

5.5.1.3 Channel 3: stable case B

Evolutionary path For channel 3, mass transfer starts when a hydrogen shell burning star fills its Roche lobe in a stable way before core helium-burning starts (Kippenhahn & Weigert, 1967, case Br). This can occur when the envelope is radiative or when the convective zone in the upper layers of the envelope is shallow. In this project we assume that stable mass transfer proceeds conservatively and so the secondary significantly grows in mass. Because mass transfer is conservative, the orbit first shrinks and when the mass ratio has reversed the orbit widens. Mass transfer continues until the primary has lost (most of) its hydrogen envelope. At this stage the primary can become a helium WD or, if it is massive enough, ignite helium in its core. In the latter scenario the primary is a He-MS star. Like for channel 2, there are two sub-channels depending on whether the primary star fills the Roche lobe for a second time as a helium star. If the primary does not go through a helium-star phase or does not fill its Roche lobe as a helium star, the system evolves according to channel 3a. In channel 3b there is a second phase of mass transfer.

Example of channel 3a Fig. 5.16a shows an example of the evolution of a binary system of channel 3a with initial parameters $M_{1,zams} = 4.8 M_\odot$, $M_{1,zams} = 3 M_\odot$ and $a_{zams} = 70 R_\odot$. The masses of He-MS and secondary star are very similar in the BPS codes [0.82, 0.83, 0.82, 0.82] $M_\odot$ and [6.9, 7.0, 7.0, 7.0] $M_\odot$ respectively. The separations at the moment the helium star forms are [4.2, 4.3, 4.3, 4.7] $\cdot 10^2 R_\odot$ and are similar as well. In the subsequent evolution, the primary star effectively evolves as a single helium star before becoming a carbon-oxygen WD and loses [0.038, 0.14, 0.043, 0.038] $M_\odot$ during that time and the orbit does not change significantly.

---

2Note that systems in which the double CE-phase results directly in a DWD system are not taken into account for the comparison of SWD systems.
Population from channel 3a

Regarding channel 3a, not all codes agree on the ranges of separation and masses (Fig. 5.21 and 5.22). However, there is an agreement between binary_c, the Brussels code and SeBa that majority of intermediate mass systems originate from systems with $M_{1,\text{zams}}$ between 3 and 5 $M_\odot$ and $a_{\text{zams}}$ between 10 and 100 $R_\odot$. The SWD population at WD formation is centred around systems with $M_{1,\text{swd}} \approx 0.6 M_\odot$ for the binary_c, Brussels and SeBa codes, and with the majority of separations between about 20 – 1000 $R_\odot$. The SWD systems and their progenitors that are just described are not SWD progenitors according to StarTrack. According to this code, mass transfer is unstable and the system merges. The birthrates of binary_c, the Brussels code and SeBa differ within a factor of about 4 (Table 5.2). In addition binary_c, SeBa and StarTrack show a good agreement on the different sub-populations for the full mass range. At WD formation these codes show a subpopulation between 15 to about 200 $R_\odot$ with WD masses of between 0.17 and 0.35 $M_\odot$. There is a second subpopulation at about 1 $R_\odot$ with most systems having a WD between 0.4 and 0.8 $M_\odot$. A third population shows mainly WD masses of more than 0.8 $M_\odot$ at separations of more than 300 $R_\odot$, where the population is extended to higher separations and WD masses in the results of SeBa and StarTrack. The third population is also visible in the progenitor population in Fig. 5.21 with primary masses of more than 5 $M_\odot$ and separations of more than about 70 $R_\odot$. Again this population is more extended to high masses and separations according to SeBa and StarTrack. The low mass range of the progenitor population shows predominantly systems in orbits of 5-15 $R_\odot$. SeBa and
StarTrack agree that there is an extra group at high orbital separations \( a_{\text{zams}} \approx (1.3 - 4.6) \cdot 10^2 R_\odot \).

**Example of channel 3b** An example of the evolution in channel 3b is shown in Fig. 5.16b. Initially the system has \( M_1 = 7 M_\odot \), \( M_2 = 5 M_\odot \) and \( a = 65 R_\odot \). After the first phase of mass transfer the primary masses \( M_1 = [1.4, 1.5, 1.4, 1.4] M_\odot \), the secondary masses \( M_2 = [11, 11, 11, 11] M_\odot \) and separations \( a = [3.8, 3.3, 3.8, 4.1] \cdot 10^2 R_\odot \). When the primary fills its Roche lobe again, it has lost \( [5.8, -6.8, 7.3] \cdot 10^{-2} M_\odot \) in the wind. The mass transfer phase is stable and the secondary increases in mass to \([11, 11, 11, 11] M_\odot \). The primary becomes a WD of \([1.1, 1.0, 0.99, 1.0] M_\odot \) in an orbit of \([4.5, 6.5, 5.9, 6.2] \cdot 10^2 R_\odot \).

**Population from channel 3b** The binary_c, Brussels and SeBa codes agree well on the initial systems leading to SWDs through channel 3b. This holds for both the initial mass, namely between about 5 and 10 \( M_\odot \) and the initial separation between \( 0.1 - 3.0 \cdot 10^2 R_\odot \). The population of progenitors of channel 3b according to the StarTrack code lies inside the previously mentioned ranges, however the parameter space is smaller. In addition the four codes agree that at WD formation the majority of companions that are formed through channel 3b are massive, about 6 to 18 \( M_\odot \) (for StarTrack 8-18 \( M_\odot \)). The orbits of these systems are wide around \( 10^3 R_\odot \), however the ranges in separation and WD mass differs between the codes and will be discussed in the next paragraphs. Binary_c, SeBa and StarTrack also show a group of lower mass companions. For binary_c and SeBa these lie in the range \( 0.8-4.5 M_\odot \) with separations of \( 0.5-30 R_\odot \) and \( M_{1, \text{swd}} \) mainly between 0.6 and 1.0 \( M_\odot \). The population of StarTrack agrees with these ranges, however the parameter space for this population is smaller.

**Effects** The population of SWDs from channel 3a and 3b are influenced by the MiMwd-relation. An important contribution to the MiMwd-relation comes from the assumed mass losses for helium stars and its mechanism, i.e. in a fast spherically symmetric wind or in planetary nebula (Appendix 5.A). There is not much known about the mass loss from helium stars either observationally or theoretically. The differences in the MiMwd-relation affect for example the distribution of separations in Fig. 5.18. For channel 3b the separation is \( \lesssim 1400 R_\odot \) for binary_c, SeBa, and StarTrack, but is extended to \( 6600 R_\odot \) in the Brussels code. Binaries become wider in the Brussels code, as the WD masses in channel 3 are in general smaller compared to the other three codes.

There is also a difference in the MiMwd-relation between StarTrack on one hand, and binary_c and SeBa on the other hand regarding primaries that after losing their hydrogen envelopes become helium stars. For massive helium stars, binary_c and SeBa find that these stars will collapse to neutron stars, where as in StarTrack these stars form WDs. For channel 3a the difference occurs for the range of helium star masses of 1.6-2.25 \( M_\odot \). As a result, systems containing massive helium stars are not considered to become SWD systems in binary_c and SeBa. These systems are present in the SWD data of StarTrack at \( M_{1, \text{swd}} \gtrsim 1.38 M_\odot \) in Fig. 5.4 for channel 3a and 3b. The progenitors lie at \( M_{1, \text{zams}} \gtrsim 8 M_\odot \) with mostly \( a_{\text{zams}} \approx 65 - 220 R_\odot \) for channel 3a and 3b.
Figure 5.17: Orbital separation versus WD mass for all SWDs in the full mass range at the time of SWD formation. The contours represent the SWD population from a specific channel: channel 3a (thin line) and channel 3b (thick line).
Figure 5.18: Orbital separation versus WD mass for all SWDs in the intermediate mass range at the time of SWD formation. The contours represent the SWD population from a specific channel: channel 3a (thin line) and channel 3b (thick line).
Figure 5.19: Secondary mass versus WD mass for all SWDs in the full mass range at the time of SWD formation. The contours represent the SWD population from a specific channel: channel 3a (thin line) and channel 3b (thick line).
Figure 5.20: Secondary mass versus WD mass for all SWDs in the intermediate mass range at the time of SWD formation. The contours represent the SWD population from a specific channel: channel 3a (thin line) and channel 3b (thick line).
Figure 5.21: Initial orbital separation versus initial primary mass for all SWDs in the full mass range. The contours represent the SWD population from a specific channel: channel 3a (thin line) and channel 3b (thick line).
Figure 5.22: Initial orbital separation versus initial primary mass for all SWDs in the intermediate mass range. The contours represent the SWD population from a specific channel: channel 3a (thin line) and channel 3b (thick line).
Another effect on the SWD population is the modelling of the mass transfer phases which is inherent to the BPS codes. The value of the mass transfer rate or the length of the mass transfer phase, however, do not have a large effect on the population or the evolution of individual systems from channel 3b in the set-up of the current study. This is because a priori conservative mass transfer is assumed, and therefore the accretion efficiency is not affected by the mass transfer rate. The mass transfer timescale only affects the binary evolution when other evolutionary timescales (such as the wind mass loss timescale or nuclear evolution timescale) are comparable. For example, while for $M_1 \ll M_2$ the orbit increases strongly during RLOF, the orbit increases only moderately during wind mass loss assuming Jeans mode angular momentum loss. The range of separations in Fig. 5.18 is therefore, besides the MiMwd-relation, also affected by the amount of wind mass and wind angular momentum leaving the system during RLOF. The binary_c, SeBa, and StarTrack codes assume wind mass takes with it the specific angular momentum of the donor star (Jeans mode), where as the Brussels code does not take wind mass loss into account during stable mass transfer.

Generally, no significant evolution of the donor star takes place during the mass transfer phase. Therefore with the current set-up, the post-mass transfer masses are determined by their initial mass and for binary_c, SeBa and StarTrack also the evolutionary moment the donor star fills its Roche lobe. However, an exception to this occurs for channel 3b during the second phase of mass transfer. Here the length of the mass transfer phase is important, as the evolutionary time scale of an evolved helium star is very short (of the order of few Myr) and the core grows significantly during this period. As a result the duration of the mass transfer phase becomes important for the resulting WD mass and separation in binary_c, SeBa and StarTrack (see e.g. the example of channel 3b).

A crucially important assumption for the evolutionary outcome of channel 3 are the adopted stability criteria. Despite the importance of the stability criteria, the various implementations have not been compared until this study. We find a clear disagreement between the codes; stable mass transfer is possible in systems with mass ratios $q_{\text{zams}} \gtrsim 0.6$ according to StarTrack, in SeBa $q_{\text{zams}} > 0.35$, in binary_c $q_{\text{zams}} > 0.25$ and $q_{\text{zams}} > 0.2$ in the Brussels code. The effect of the relative large critical mass ratio for StarTrack results in a low birthrate in particular in the intermediate mass range (Table 5.2 and Fig. 5.22), which results in a lack of SWD systems lower than $300 \, R_\odot$ (Fig. 5.17) and fewer SWD systems with $M_{2,\text{swd}} \gtrsim 1.0 \, M_\odot$ (Fig. 5.19). The effect of the relative low critical mass ratio for the Brussels code can be seen in Fig. 5.20 as an extension in the Brussels code to lower separations $a_{\text{swd}} \lesssim 50$. Systems with lower mass companions initially, go through mass ratio reversal and subsequent expansion of the orbit later in the mass transfer phase.

In the low mass range we find that the stability criteria vary most strongly for donor stars that are early on the first giant branch when they have shallow convective zones in the upper layers of the envelope. In general, stable mass transfer from this type of donor for the same conditions is more readily realised in StarTrack than in binary_c, and it is even more readily realised in SeBa. Systems with this kind of donor show in Fig. 5.21 at $M_{1,\text{zams}} < 3 \, M_\odot$ a larger range in initial separations for SeBa (5-25 $R_\odot$) than for binary_c and StarTrack (5-18 $R_\odot$). There is also an extra population of SWD systems in the SeBa and StarTrack data with high initial separation $q_{\text{zams}} \approx (1.3 - 4.6) \cdot 10^2 \, R_\odot$ and high initial mass ratio $q_{\text{zams}} \approx M_{2,\text{zams}} / M_{1,\text{zams}} > 0.8$. In these systems the primary fills its Roche lobe stably on the giant branch after the mass ratio has reversed due to wind mass losses. When donors with shallow convective zones are excluded, the birthrate in the full mass range in channel 3a decreases to $1.4 \cdot 10^{-3} \, \text{yr}^{-1}$ for SeBa and
7.3 \cdot 10^{-4} \text{ yr}^{-1} for StarTrack, which is comparable to the birthrate predicted by binary_c (Table 5.2).

The long-term behaviour of the orbit can be affected by tides. If energy is dissipated, angular momentum can be exchanged between the orbit and the spin of the stars. For this project the binary_c, Brussels and SeBa code assume that the spin angular momentum of the stars can be neglected compared to the orbital angular momentum\(^3\). As such in their simulations orbital angular momentum is conserved. In the StarTrack code, the orbital and spin angular momentum combined are conserved, under the assumption that the stars are and remain in a synchronized orbit. As a consequence after the first phase of mass transfer in channel 3, the orbits are slightly larger in StarTrack compared to those of the other codes (see the example system of channel 3a and 3b).

Whether or not a primary fills its Roche lobe for a second time is modelled different in the Brussels code than in the other three codes. In the Brussels code stars with an initial mass less than 5 \(M_\odot\) are assumed to evolve through channel 3a, and stars with a higher mass evolve through channel 3b. The binary_c and SeBa simulations roughly agree with this, see Fig. 5.21. However, the boundary between channel 3a and 3b is determined at run time in binary_c, SeBa, and StarTrack. It is dependent on the evolution of the radii and wind mass loss of helium stars, the stability criterion and the separation after the first phase of mass transfer. Therefore differences exist between these codes in the upper limits for ZAMS masses and separations in channel 3a in Fig. 5.21 and at WD formation in Fig. 5.19. Binary_c, SeBa, and StarTrack also find that systems that evolve through channel 3a or 3b overlap in WD mass at WD formation (Fig. 5.18 and 5.20). In the data from the Brussels code, the boundary at WD formation is discontinuous in primary mass causing a gap between 0.7 and 0.9 \(M_\odot\) (Fig. 5.18 and 5.20). The gap in WD mass in the data from the Brussels code originates as a considerable amount of mass is lost during the planetary nebula phase of a star that does not initiate a second mass transfer phase. In the other three codes, the mass loss in winds from helium stars is less strong compared to the mass loss in the planetary nebula phase of helium stars in the Brussels code.

The evolution of helium stars (their radii, core masses, wind mass losses, and if they fill their Roche lobes also the stability and mass transfer rates) are important in channel 3. A difference arises between the Brussels code and the others, because of the way helium stars are simulated. In binary_c, SeBa, and StarTrack it is possible that after the first phase of mass transfer, the secondary fills its Roche lobe before the primary moves off the He-MS and becomes a white dwarf. Subsequently the primary becomes a WD before the secondary evolves significantly\(^4\). This reversal can occur because the evolutionary timescale of a low-mass helium -star is very long (about 10\(^8\)yr), while that of the secondary that gained much mass is reduced. As a result, when the first WD is formed, the mass of the secondary and the orbital separation has decreased substantially. These systems lie according to binary_c, SeBa and StarTrack at separations \(\lesssim 20 \, R_\odot\), primary WD masses of \(\lesssim 1.0 \, M_\odot\) and secondary masses of \(\lesssim 4.5 \, M_\odot\) in Fig. 5.17 and 5.19. The birthrates of the systems in binary_c, SeBa and StarTrack, are low (\([1.1, - , 8.6, 0.4] \cdot 10^{-4} \text{ yr}^{-1}\) in the full mass range). In the Brussels code, the evolution of the stars is not followed in time, and this evolutionary track is not considered. As a result in the range of 0.45-0.7 \(M_\odot\) for the WD mass, the range in secondary masses is broader in the Brussels code.

\(^3\)In binary_c it is possible to take into account spin angular momentum into the total angular momentum of the system.

\(^4\)Note that it is also possible that the secondary becomes a WD before the primary does (Toonen et al., 2012, Claeys et al., subm.). Because of the evolutionary reversal, these systems are not shown in Fig. 5.17 to 5.22 nor included in channel 3. The birthrates, however, are low (\([1.4, - , 5.6, 0.7] \cdot 10^{-4} \text{ yr}^{-1}\) in the full mass range).
5.5.1.4 Channel 4: unstable case B

Evolutionary path In this path, a hydrogen shell burning star fills its Roche lobe (Kippenhahn & Weigert, 1967, case Bc), but the mass transfer is unstable. After the CE-phase the primary becomes a helium WD or a He-MS. Again, we differentiate two evolutionary paths within a channel. In channel 4a, the primary becomes a WD directly or the primary becomes a helium star that will evolve into a WD without any further interaction with the secondary. If the primary star fills its Roche lobe for a second time, the system evolves through subchannel 4b. An example of the evolutionary path of channel 4b in shown in Fig. 5.23.

Example Fig. 5.23 shows the evolution of a system of channel 4b, that starts its evolution with \( M_{1,zams} = 6 M_\odot \), \( M_{1,zams} = 3 M_\odot \) and \( a_{zams} = 320 R_\odot \). The primary fills its Roche lobe as it ascends the first giant branch. After mass transfer ceases the primary has become a He-MS of mass \([1.1, 1.1, 1.1, 1.1] M_\odot\) in an orbit with a separation of \([7.0, 7.1, 7.0, 7.0] R_\odot\). As the helium star evolves and increases in radius, it initiates the second phase of mass transfer. Soon after mass transfer ceases, the primary becomes a WD with \( M_{1,swd} = [0.81, 0.91, 0.77, 0.79] M_\odot\). The secondary is still on the MS with \( M_{2,swd} = [3.2, 3.2, 3.3, 3.3] M_\odot\) and the orbital separation is \( a_{swd} = [10, 9.4, 11, 11] R_\odot\). The differences in this example are caused by effects discussed before; the MiMwd-relation including the mass transfer rates from helium rich donors.

Population The codes agree well on the location of the SWDs at WD formation from channel 4a and 4b in Fig. 5.2, 5.3, 5.4 and 5.5, their progenitor systems in Fig. 5.6 and 5.7 and the birthrates of the channels (Table 5.2). The low mass SWDs at WD formation have WDs of \( 0\cdot48 M_\odot\) in an orbit of \( 0\cdot11 R_\odot\), and progenitor systems with \( a_{zams} \approx (0.3 - 4.0) \cdot 10^2 R_\odot\) for \( M_{1,zams} \approx 1 - 2 M_\odot\).

The population of systems that evolve through channel 4b are primarily intermediate mass binaries of mass \( M_{1,zams} \approx 4.5 - 10 M_\odot\) that become WDs of \( M_{1,swd} \approx 0.7 - 1.3 M_\odot\). The majority of systems have initial separations of \( a_{zams} \approx (0.2 - 1.0) \cdot 10^3 R_\odot\). At WD formation the range of separations according to binary_c, SeBa and StarTrack is \( 4 - 1.0 \cdot 10^2 R_\odot\), however for the Brussels code it is extended to \( 0.9 - 1.4 \cdot 10^2 R_\odot\).

Effects There is a difference between the Brussels code on one hand and the other three codes on the other hand, regarding the survival of systems with low initial secondary masses \( M_{2,zams} < 3 M_\odot\) in channel 4b. This is predominantly due to the difference in the single star prescriptions for the radii of stars. The radius of low-mass secondary-stars are in general larger in binary_c, SeBa and StarTrack than in the Brussels code. Therefore in the former three codes, the stars are more likely to fill their Roche lobe at the end of the CE-phase resulting in a merger. In the Brussels data, these systems survive at small separations (\( \leq 10 R_\odot\) at \( 1 M_\odot\), Fig. 5.3). Note that the Brussels code was written for intermediate mass stars (Sect. 5.3), and in principal the code does not allow for the detailed evolution of stars with initial masses below \( 3 M_\odot\).

In addition, the stability of the second phase of mass transfer affects the SWD population of channel 4. If this phase is unstable, the system will evolve to a merger. In the Brussels code, it is assumed that the second phase of mass transfer is always stable, however this is not the case in the three other codes. Differences in the stability criteria affect the orbital separation of SWD formation for all codes.

5.5.1.5 Channel 5: case A

Evolutionary path In channel 5 mass transfer starts during the core hydrogen burning phase of the donor (Case A, Kippenhahn & Weigert, 1967).
Figure 5.23: Example of the evolution of a single white dwarf system in channel 4b. The primary fills its Roche lobe a second time. The top and bottom parts of the figure have different scales due to a common envelope phase, denoted as CE in the figure. Abbreviations are as in Table 5.3.
Chapter 5: PopCORN

Population

The birthrates in the full mass range differ within a factor 2.5 between binary_c, SeBa and StarTrack (Table 5.2). According to these codes, the progenitors of the primaries in channel 5 are stars of low mass (1-4 $M_\odot$) in small orbits (5-13 $R_\odot$), see Fig. 5.6. There is a good agreement that the majority of SWDs from channel 5 at WD formation consists of a primary of mass 0.2-0.35 $M_\odot$, a secondary of mass 1.8-5.5 $M_\odot$ in an orbit with a separation of 30-240 $R_\odot$ (Fig. 5.2 and 5.4). Binary_c, SeBa and StarTrack further agree on a subchannel ($a_{\text{swd}} \approx 0.4 R_\odot$ and $M_{1,\text{swd}} \approx 0.3 M_\odot$ in Fig. 5.2) in which the secondary is a hydrogen-poor helium-star at WD formation (see also channel 3). The birthrates of this subchannel are low ([4.5, −, 4.8, 18] $\cdot 10^{-6}$ yr$^{-1}$).

The birthrate of channel 5 in the Brussels code is higher by over a factor 20 compared to binary_c, SeBa and StarTrack (Table 5.2). For the Brussels code the intermediate mass primaries have an initial mass $M_{1,\text{zams}} \approx 3 – 10 M_\odot$ and WD mass $M_{1,\text{swd}} \approx 0.45 – 1.3 M_\odot$, while the other codes show smaller ranges: for the main group of progenitors $M_{1,\text{zams}} \approx 3 – 4 M_\odot$ and WD mass $M_{1,\text{swd}} \leq 0.35 M_\odot$ (Fig. 5.2 and 5.6). The initial separation in the Brussels code $a_{\text{zams}} \approx 5 – 22 R_\odot$, while in binary_c, SeBa and StarTrack $a_{\text{zams}} \approx 8 – 13 R_\odot$. The separation at SWD formation $a_{\text{swd}}$ in the Brussels code is between 20-350 $R_\odot$, while in the other codes the separation is mainly between 100-250 $R_\odot$. The range of secondary masses is $M_{2,\text{swd}} \approx 3 – 18 M_\odot$ in the Brussels code, but only $M_{2,\text{swd}} \approx 4 – 6 M_\odot$ in the other codes. Note that the region indicated by the dash-dotted contours in Fig. 5.2, contains systems from channel 5 as well as from channel 3, however this does not change our conclusion regarding the extended range and birthrates in the Brussels code compared to the other codes.

Effects

The differences between the Brussels code and the other codes is caused by the fact that the Brussels population code does not follow the mass transfer event and its mass transfer rate in detail. It considers only the initial and final moment of the mass transfer phase, therefore any intermediate steps in which the system can be closer are disregarded. For example, during conservative mass transfer to an initially less massive companion, the orbital separation first decreases and then increases again after mass ratio reversal. As the orbital separation decreases, the secondary can fill its Roche lobe leading to a contact system, especially as it grows in mass and radius due to the accretion. In the binary_c, SeBa and StarTrack code, it is assumed that the contact phase will lead to a merger or CE-phase for evolved secondaries. The Brussels code assumes that for shallow contact, the merger can be avoided. In other words, the codes have different assumptions for the stability of mass transfer.
5.5.2 Double white dwarfs

In this section we compare and discuss the population of DWDs as predicted by binary_c, the Brussels code, SeBa and StarTrack. Prior to the formation of a second degenerate component, DWDs undergo the evolution as described in the previous section (channel 1-5). Subsequently, they undergo a second intrusive (series of) event(s) at the time the secondary fills its Roche lobe. As a consequence the processes that influence the evolution of SWDs influence the DWD population as well. Here we will point out the evolutionary processes that are specifically important for DWDs.

The population of DWDs at DWD formation is shown in Fig. 5.24, 5.25, 5.26, and 5.27 where orbital separation and secondary mass respectively is shown as a function of primary mass for the full and intermediate mass range. The ZAMS progenitors of these systems are shown in Fig. 5.28 and 5.29 for the full and intermediate mass range respectively.

In the full mass range the population of DWDs is comparable between binary_c, SeBa and StarTrack with white dwarf masses of $M_{1,dwd} \approx 0.2 - 1.4 M_\odot$ and $M_{2,dwd} \approx 0.1 - 1.4 M_\odot$. At large separations ($0.1 - 5 \times 10^4 R_\odot$) the codes find systems which are formed without any interaction, see Fig. 5.24. This figure also shows a population of interacting systems at lower separations, where the majority has separations of $a \approx 0.1 - 10 R_\odot$. Furthermore there is a good agreement on which progenitors lead to a DWD system and which do not. Fig. 5.28 shows several subpopulations of DWD progenitors with comparable binary parameters for binary_c, SeBa and StarTrack; a group of non-interacting systems at $a_{dwd} \gtrsim 5 \times 10^2 R_\odot$, a group of systems for which the first phase of mass transfer is stable at $a_{dwd} \lesssim 25 R_\odot$ for low mass primaries and $a_{dwd} \lesssim 2.5 \times 10^2 R_\odot$ for the full mass range, and a group of systems at intermediate separations that predominantly undergoes a CE-phase for the first phase of mass transfer.

An effect that plays a role for DWDs, has already been noted for SWDs, namely that stable mass transfer is more readily realized in binary_c and SeBa compared to StarTrack. It plays a role for example in Fig. 5.28 in the lack of systems at $M_{1,zams} > 3 M_\odot$ and $a_{zams} < 2.5 \times 10^2 R_\odot$ in the StarTrack data compared to binary_c and SeBa, and in Fig. 5.26 in the lack of systems with $M_{2,dwd} > M_{1,dwd}$. Differences in the interpretations of the double CE-phase, in which both stars lose their envelopes (eq. 5.8), result in larger separations at DWD formation and less mergers in StarTrack compared to binary_c and SeBa (Fig. 5.25 and Sect. 5.5.2.2). At the same time, the initial separations of systems evolving through a double CE-phase can be smaller in the StarTrack data compared to binary_c and SeBa ($a_{dwd} \approx 25 - 100 R_\odot$, Fig. 5.28). The DWD populations will be discussed in more detail in the following sections.

In the intermediate mass range at DWD formation, two groups of systems can be distinguished in all simulations (Fig. 5.25). Similar to the full mass range, there is one group of non-interacting systems at separations higher than $6 \times 10^3 R_\odot$ and a group of interacting systems with separations $\lesssim 20 R_\odot$. However, the distribution of systems in the latter range, corresponding to different evolutionary paths, varies between the codes. Most DWD systems have primary and secondary WD masses above $0.6 M_\odot$ in all the codes. The progenitors in the intermediate mass range show the same division in separation in three groups as the progenitors in the full mass range. DWD progenitors with separations $a_{zams} \lesssim 3 \times 10^2 R_\odot$ undergo a stable first phase of mass transfer. The components of DWD progenitors with $a_{zams} \gtrsim 1.5 \times 10^3 R_\odot$ do not interact. At intermediate separations the first phase of mass transfer is predominantly a CE-phase.
Figure 5.24: Orbital separation versus primary WD mass for all DWDs in the full mass range at the time of DWD formation. The contours represent the DWD population from a specific channel: channel I (dash-dotted line), channel II (solid line), channel III (dashed line) and channel IV (dotted line).
Figure 5.25: Orbital separation versus primary WD mass for all DWDs in the intermediate mass range at the time of DWD formation. The contours represent the DWD population from a specific channel: channel I (dash-dot line), channel II (solid line) and channel III (dashed line). The contours of the DWD population from channel III according to StarTrack and channel IV according to all codes are not shown, as the birthrate from this channel is too low.
Figure 5.26: Secondary WD mass versus primary WD mass for all DWDs in the full mass range at the time of DWD formation. The contours represent the DWD population from a specific channel: channel I (dash-dotted solid line), channel II (solid line), channel III (dashed line) and channel IV (dotted line).
Figure 5.27: Secondary WD mass versus primary WD mass for all DWDs in the intermediate mass range at the time of DWD formation. The contours represent the DWD population from a specific channel: channel I (dash-dotted solid line), channel II (solid line) and channel III (dashed line). The contours of the DWD population from channel III according to StarTrack and channel IV according to all codes are not shown, as the birthrate from this channel is too low.
Figure 5.28: Initial orbital separation versus initial primary mass for all DWDs in the full mass range. The contours represent the DWD population from a specific channel: channel I (dash-dotted solid line), channel II (solid line), channel III (dashed line) and channel IV (dotted line).
Figure 5.29: Initial orbital separation versus initial primary mass for all DWDs in the intermediate mass range. The contours represent the DWD population from a specific channel: channel I (dash-dotted solid line), channel II (solid line) and channel III (dashed line). The contours of the DWD population from channel III according to StarTrack and channel IV according to all codes are not shown, as the birthrate from this channel is too low.
Comparing the Brussels code with binary_c and SeBa (differences with StarTrack have the same origin as discussed in the previous paragraph), the most important causes for differences in the DWD population in the intermediate mass range, are the MiMwd-relation, the MiMf-relation, the modelling of the stable mass transfer phase and the survival of mass transfer. The effect of the first three causes on the DWD population, is similar to the effect on the SWD population. Firstly, the differences in the MiMf-relation can be seen in the progenitor population of non-interacting binaries in Fig. 5.29 as an extension to higher primary masses in the Brussels data (8-10 $M_\odot$, Sect. 5.5.2.1). Secondly, differences in the MiMwd-relation can be seen in Fig. 5.25 as an extension to lower primary WD masses $M_{1,dwd} \leq 0.64 M_\odot$ and the discontinuity in primary WD masses around 0.7-0.9 $M_\odot$ (Sect. 5.5.2.2 and 5.5.2.3). The MiMwd-relation also effects the orbital separation distribution at DWD formation and results in a higher maximum separation in the Brussels code compared to binary_c and SeBa. Finally, due to the method of modelling of mass transfer there is a disagreement between the codes regarding which systems survive mass transfer, see Fig. 5.29 at $a_{dwd} \leq 20 R_\odot$ (Sect. 5.5.2.3). The survival of mass transfer is more important for the DWD population than for the SWD population, as the average orbital separation of DWDs is lower (Sect. 5.5.2.2 and 5.5.2.3). As the formation of DWDs involves more phases of mass transfer than the SWDs, the differences in the SWD population carry through and are larger in the DWD population.

In the next sections, we differentiate four different evolutionary paths of DWDs. This is based on whether or not mass transfer occurs and if so, if the mass transfer initiated by the primary and secondary is stable or unstable. For clarity we do not distinguish the evolutionary path further e.g. by separating channel 3 and 5, nor channel 3a and 3b. Channel I, II and III represent the most commonly followed evolutionary paths with birthrates larger than 1.0 $\cdot 10^{-3}$ yr$^{-1}$. Channel IV is included because it stands out in Fig. 5.24 and 5.26, even though the birthrates in this channel are low (Table 5.2). In each section we describe a specific evolutionary path (marked as Evolutionary path), we compare the simulated populations from each code (marked as Population) and investigate where differences between the populations come from (marked as Population and Effects).

5.5.2.1 Channel I: detached evolution

Evolutionary path Channel I involves non-interacting binaries. An example of a system was given for channel 1 in Sect. 5.5.1.1: a 5 $M_\odot$ and 4 $M_\odot$ star in a circular orbit of $10^4 R_\odot$. When the first WD is born, the orbit has increased to [1.8, 1.8, 1.8, 1.8] $\cdot 10^4 R_\odot$. When the second WD is born, the orbit has increased even more to [4.9, 5.0, 4.9, 4.9] $\cdot 10^4 R_\odot$ with primary and secondary masses of [1.0, 0.94, 1.0, 1.0] $M_\odot$ and [0.87, 0.86, 0.87, 0.87] $M_\odot$ respectively.

Population There is a good agreement between the codes on the separations and masses of non-interacting DWDs, initially and at DWD formation. In the full mass range, initial separations are $a_{zams} \approx (0.5 – 10) \cdot 10^3 R_\odot$. The codes binary_c, SeBa, and StarTrack find non-interacting DWDs with WD masses between 0.5-1.4 $M_\odot$ in wide orbits of $a_{dwd} \approx (0.1 – 5.4) \cdot 10^4 R_\odot$. In the intermediate mass range, the initial separations are $a_{zams} \geq 1.5 \cdot 10^3 R_\odot$ for binary_c, SeBa and StarTrack. Both WD masses are $\geq 0.75 M_\odot$ and orbits are wide with separations $a_{dwd} \approx (0.6 – 5.4) \cdot 10^4 R_\odot$ for binary_c, SeBa and StarTrack. For the Brussels code, the separations are slightly higher at $a_{zams} \geq 2.8 \cdot 10^3 R_\odot$ and $a_{dwd} \approx (1.3 – 7.2) \cdot 10^4 R_\odot$, and both WD masses extend to slightly lower values of $\geq 0.65 M_\odot$. Small differences between the populations are due to different MiMf-relations and different descriptions for single stars (e.g. stellar radii), as for SWDs from channel 1. The birthrates in channel I are very similar in the full mass range as well as in the
intermediate mass range (Table 5.2).

5.5.2.2 Channel II: CE + CE

Evolutionary path The classical formation channel for close DWDs involves two CE-phases. First the primary star evolves into a WD via a phase of unstable mass transfer, i.e. via the evolutionary path described in Sect. 5.5.1.2 and 5.5.1.4 as channel 2 or 4 respectively. Subsequently the secondary initiates a CE-phase. It should be noted that for binary_c, SeBa and StarTrack this channel includes, systems that evolve through one CE-phase in which both stars lose their (hydrogen) envelope, the so-called double common envelope phase described in Sect. 5.2 and in eq. 5.9. Note that in the Brussels code, the double CE phase is not considered.

Population In the full mass range there is a good agreement between the progenitors according to the binary_c and SeBa code and a fair agreement with the StarTrack code. These three codes find that primaries of $M_{\text{zams}} \approx 1 - 8 M_\odot$ contribute to this channel. For the majority the primaries have initial separations of $a_{\text{zams}} \approx (0.1 - 2.5) \cdot 10^3 R_\odot$. The DWD populations as predicted by binary_c and SeBa are similar, and comparable with the population of StarTrack. WD masses range from 0.35-1.4 $M_\odot$ for primaries and 0.19-0.9 $M_\odot$ for secondaries for binary_c and SeBa, and slightly larger ranges for StarTrack of 0.2-1.4 $M_\odot$ for primaries and 0.1-0.8 $M_\odot$ for secondaries. The orbital separation of DWDs from channel II is between a few tenths of solar radii to a few solar radii, however the specific ranges of the three codes differ. The birthrates in channel II are similar between the three codes (Table 5.2). In the intermediate mass range the codes agree that primaries and secondaries with initial mass between about 3 to 8 $M_\odot$ can contribute to channel II. In the Brussels code the mass range is slightly extended to higher masses of 10 $M_\odot$ for primaries due to the MiMwd-relation. There is an agreement on the initial separation of the majority of system, although the range of separations differs between the codes. For binary_c and SeBa $a_{\text{zams}} \approx (0.7 - 2.5) \cdot 10^3 R_\odot$, however the range for StarTrack is extended to lower values of $a_{\text{zams}} \approx (0.4 - 20) \cdot 10^2 R_\odot$ as noted above. Comparing with the Brussels code, the range is extended to lower as well as higher values ($[0.3 - 3.2] \cdot 10^3 R_\odot$). The higher maximum initial separation depends on the maximum radius in the single star prescriptions as discussed in channel 1. The difference in the lower minimum initial separation for the Brussels code has been noted for the SWDs in channel 2 as well. The Brussels code assumes that the primaries in these systems become WDs without a second interaction, where as in binary_c, SeBa and StarTrack these systems merge in the second interaction of the primary star. The separations of DWDs are centred around 0.5 $R_\odot$, however the distribution of separations is different between the codes: 0.17-10 $R_\odot$ for binary_c, 0.06-1.18 $R_\odot$ for the Brussels code, 0.14-3.6 $R_\odot$ for SeBa and 0.05-11 $R_\odot$ for StarTrack. Primary WD masses are comparable between the codes, $[0.8 - 1.4, 0.5 - 1.3, 0.7 - 1.4, 0.7 - 1.4] M_\odot$ where the ranges are the largest for the Brussels code. The maximum WD mass in the Brussels code is lower compared to the other codes due to the MiMwd-relation, see channel 1. The secondary WD masses at a given primary WD mass are lower in binary_c, SeBa and StarTrack ($\lesssim 0.9 M_\odot$) compared to the Brussels code ($\lesssim 1.3 M_\odot$).

Effects Several effects influence the distribution of separations in Fig. 5.25. Even though the codes agree that the majority of DWDs from channel II have separation around 0.5 $R_\odot$, the spread around this value varies between the codes. In the full mass range the maximum separation is 8 $R_\odot$ in the SeBa data, 22 $R_\odot$ in the StarTrack data and 31 $R_\odot$ for binary_c. In the intermediate mass range it is 1 $R_\odot$ for the Brussels results, 4 $R_\odot$ in the SeBa data, 10 $R_\odot$ for binary_c and 11 $R_\odot$ in the StarTrack data. The maximum
Chapter 5: PopCORN

separation is affected by the MiMwd-relation and winds. As seen in channel 2, the maximum orbital separation in the Brussels code is lower as winds are not taken into account and more mass is removed during the CE. The distribution of orbital separation in the Brussels data is also affected in a different way than in the others codes as this code assumes that AGB donors become WDs directly without a second phase of interaction (see also channel 2). In binary_c, SeBa and StarTrack AGB donors can become helium stars, that fill their Roche lobes for a second time, resulting in lower average masses. This effect can be seen in Fig. 5.27 where the secondary mass in binary_c, SeBa and StarTrack is \( \lesssim 0.9 M_\odot \) where as it is extended to \( \lesssim 1.3 M_\odot \) in the Brussels data. Mass loss in combination with the stability criteria, as also discussed for channel 2 causes high separations in the binary_c data. However, the relatively high maximum separations found by the StarTrack code is not affected much by the difference in the MiMwd-relation and winds, but are affected by differences in the double CE-formalism (see below).

All codes find that initially many DWD systems have high mass ratios, that in binary_c, SeBa and StarTrack lead to a double CE-phase. As discussed for channel 2, there is a difference in the formalism of the double-CE phase between StarTrack on one hand, and binary_c and SeBa on the other hand. As a result the separation after the double CE-phase is smaller according to the latter two codes, and a merger is more likely to happen. The birthrates of systems in the full (intermediate) mass range that evolve through a double CE is \( 7.2 \times 10^{-4} \text{ yr}^{-1} \) (7.9 \( \times 10^{-5} \text{ yr}^{-1} \) according to StarTrack, while it is \( 4.6 \times 10^{-5} \text{ yr}^{-1} \) (2.5 \( \times 10^{-5} \text{ yr}^{-1} \) and \( 1.1 \times 10^{-4} \text{ yr}^{-1} \) (3.2 \( \times 10^{-5} \text{ yr}^{-1} \) for binary_c and SeBa respectively. An example of systems that merge according to binary_c and SeBa, but form a DWD according to StarTrack are the systems at \( a_{\text{zams}} \lesssim 120 R_\odot \) in Fig. 5.28 which lie at \( a_{\text{dwd}} \approx 0.07 - 1.2 R_\odot \) for \( M_{1,\text{dwd}} \lesssim 0.35 M_\odot \) in Fig. 5.24. An example of systems that survive according to all codes, however at smaller separations for binary_c and SeBa compared to StarTrack, are systems with \( M_{1,\text{dwd}} \gtrsim 0.7 M_\odot \) and \( a_{\text{dwd}} \approx 4 - 10 R_\odot \) for StarTrack and \( a_{\text{dwd}} \lesssim 2 R_\odot \) for binary_c and SeBa.

An effect that plays a role in channel II concerns the survival of a system during the mass-transfer event. As explained for channel 2, BPS codes compare the radius of the stripped star (i.e. exposed cores) to the corresponding Roche lobe to determine whether or not a merger takes place during the CE event. For donor stars that become WDs directly after mass transfer ceases, i.e. without a hydrogen-poor helium burning phase, the Brussels and StarTrack code assume a zero-temperature WD where as binary_c and SeBa assume the exposed core is expanded due to previous nuclear shell burning. The effect of this is that the radius of the stripped star is a factor of about 5 smaller in the Brussels and StarTrack code than in binary_c and SeBa. Therefore a merger is less likely to take place. Therefore the minimum separation in the intermediate mass range is \( 0.06 \) and \( 0.05 M_\odot \) in the Brussels code and the StarTrack code, respectively. While the minimum separation is about \( 0.15 R_\odot \) in binary_c and SeBa.

5.5.2.3 Channel III: stable + CE

Evolutionary path In channel III, the primary initiates stable mass transfer (alike channel 3 or 5 which are described in Sect. 5.5.1.3 and 5.5.1.5). When the secondary fills its Roche lobe mass transfer is unstable\(^5\)

\(^5\)Note that there are two variations of this evolutionary path that are not included in this channel and Fig. 5.24 to 5.29. First, systems in which the secondary becomes a WD before the primary are excluded in this channel. The birthrates of this evolutionary path are low (\( 8.6, -27, 5.1 \times 10^{-5} \text{ yr}^{-1} \) in the full mass range. See also the discussion and footnote for channel 3 in Sect. 5.5.1.3 on this evolutionary path. Secondly, for systems with AGB donors that have suffered severe wind mass loss such that the mass ratio has reversed, the first phase of mass transfer can become stable as well. However, consequently the orbit widens to separations
5.5 Results

Population There is an agreement between the codes about the main parameter space occupied by the DWDs from channel III and their progenitors, however the codes do not agree completely. The causes for differences in channel III have been discussed previously in the context of SWDs (see the discussion on channel 3 and 5), but they lead to more pronounced differences in the DWD population than in the SWD population.

In the intermediate mass range, the binary_c, Brussels and SeBa code agree on the orbital characteristics of the main progenitors, $M_{1,zams} \approx 4 - 9 \, M_\odot$ and $a_{zams} \approx (0.2 - 2) \cdot 10^2 \, R_\odot$. There is also a rough agreement between these codes on the range of masses of both WDs. For primaries binary_c and SeBa find $M_{1,dwd} \geq 0.65 \, M_\odot$ and the Brussels code $M_{1,dwd} \geq 0.45 \, M_\odot$ due to differences in the MiMwd-relation. For secondaries these three codes find $M_{2,dwd} \geq 0.7 \, M_\odot$. The maximum mass of the primary and secondary white dwarfs varies between 1.2 and 1.4 $M_\odot$. The birthrates are high (a few times $10^{-4} \, \text{yr}^{-1}$) in this channel according to binary_c, the Brussels code and SeBa, however the birthrate is a factor 1000 lower according to StarTrack. In the StarTrack simulation there are only two systems in channel III in the intermediate mass range, and therefore we refrain from showing contours for this channel for the StarTrack data in Fig. 5.25, 5.27 and 5.29. Fig. 5.29 shows an increase of progenitor systems at separations $a_{zams} \lesssim 20 \, R_\odot$ and primary masses $M_{1,zams} \approx 3 - 5.5 \, M_\odot$ in the Brussels simulation compared to those from the other codes. The effect carries through into the DWD population as the increase of systems in the data from the Brussels code with WD primary masses between 0.45-0.7 $M_\odot$. The orbital separation of DWDs in channel III is very similar between binary_c and SeBa, $a_{dwd} \approx 0.1 - 1.1 \, R_\odot$, however for the Brussels code $a_{dwd} \approx 0.3 - 20 \, R_\odot$. The existence of wide systems in the Brussels code is not surprising, as this code also finds the widest SWDs from channel 3 in comparison with binary_c and SeBa. As discussed previously in Sect. 5.5.1.3, this is related to differences in the MiMwd-relation and angular momentum loss from winds. The gap at $M_{1,dwd} \approx 0.7 - 0.9 \, M_\odot$ in Fig. 5.27 in the data from the Brussels code, is caused by the boundary between channel 3a and 3b, as in Fig. 5.20 (see also Sect. 5.5.1.3).

Regarding the populations of progenitors for low mass primaries, binary_c, SeBa and StarTrack agree reasonably well. They both show that most DWDs in channel III have initial separations of 5-20 $R_\odot$. However the range of initial separations is extended to 25 $R_\odot$ in the population simulated by SeBa. SeBa and StarTrack also show an extra population compared to binary_c ($a_{zams} \approx 140 - 270 \, R_\odot$ and $M_{1,zams} \lesssim 1.2 \, M_\odot$). These two differences are due to differences in the stability of mass transfer for donors with shallow convective envelopes, as discussed for channel 3. Comparing the population of DWDs itself for low mass primaries, binary_c, SeBa and StarTrack agree well. The codes show a population of DWDs with primary mass $M_{1,dwd} \approx 0.2 - 0.44 \, M_\odot$ at a separation of $a_{dwd} \approx 0.1 - 1.5 \, R_\odot$, with secondary masses $M_{2,dwd} \approx 0.6 \, M_\odot$. The extra population in the SeBa and StarTrack data lies at $a_{dwd} \approx 10 - 50 \, R_\odot$ and $M_{1,dwd} \approx 0.4 - 0.47 \, M_\odot$. The three codes show systems at $M_{2,dwd}$ about 0.3 $M_\odot$, where in the binary_c data this group is extended to higher primary WD masses of $M_{1,dwd} \approx 0.2 - 0.7 \, M_\odot$ in Fig. 5.26. These systems in binary_c mainly evolve through a specific evolutionary path in which there is a phase of stable mass transfer between a He-MS and a WD, a so-called AM CVn-system. The birthrate of these systems is $5 \cdot 10^{-4} \, \text{yr}^{-1}$ according to binary_c and negligible according to the other codes.

Effects The extremely low birthrate of StarTrack in the intermediate range is caused by a combination of effects discussed previously. Firstly, stable mass transfer is more readily realised in the other codes comparable to the separations of Channel I such that the secondary will not fill its Roche lobe. The birthrates of this evolutionary path are low as well ([9.4, −, 6.6, 5.3] · $10^{-4} \, \text{yr}^{-1}$ in the full mass range.
Chapter 5: PopCORN

compared to StarTrack (see channel 3). Only systems with \( q_{\text{zams}} \geq 0.6 \) undergo stable mass transfer and become SWD systems according to StarTrack. For about 60% of these systems, the secondary becomes massive enough to collapse to a neutron star after nuclear burning ceases (in accordance with the other codes). Secondly, the remaining systems merge when the secondary star fills its Roche lobe. For AGB donors this is more likely to happen in the StarTrack data, because of the difference in the radii of stripped stars compared to binary\_c and SeBa (see channel 2b).

The different methods of calculating mass transfer between the Brussels code and the other codes, cause an increase in systems in the data from the Brussels code, similar to channel 5. In particular for DWDs, it is important how the secondary responds to mass gain. The systems that survive in the Brussels code have \( q_{\text{zams}} > 0.85 \), such that the orbit widens severely due to the mass transfer. However, according to binary\_c and SeBa, when the secondary accretes a significant amount of mass and is rejuvenated, its evolutionary timescale is reduced. As the secondary evolves, the system comes in contact and merges. The Brussels code assumes that the merger can be avoided for phases of shallow contact.

The evolution of and mass transfer rates from evolved helium stars donors (see channel 3) are important for channel III. It affects the DWD systems with high masses of the primary progenitor and primary WD, see Fig. 5.27 and 5.29. The range of primary WD masses is extended to 1.2 \( M_\odot \) according to SeBa, and 1.3 \( M_\odot \) according to the Brussels code and 1.4 \( M_\odot \) according to binary\_c. Contrary to stable mass transfer from hydrogen rich donors, the core of evolved helium stars can grow significantly during stable mass transfer phases as the timescale for mass transfer can become comparable to the timescale of wind mass loss or nuclear evolution. If the mass transfer phase is relatively short, the core of the donor star does not have time to grow significantly and little mass is lost in the wind. With the assumption of conservative mass transfer, most of the envelope is then transferred to the secondary star which then is more likely to become a neutron star instead of a WD.

Differences in the radii of stripped stars causes a relative lack of close systems for the Brussels code compared to the other codes. For channel II this was discussed in the context of donor stars that become a WD directly. However, in channel III in the intermediate mass range many donor stars continue burning helium after the mass transfer event ceases. The radius of the stripped donor star depends on its mass, and for binary\_c, SeBa and StarTrack also on the evolutionary state of the donor stars (see also channel 2). When the donor star is stripped of its envelope before the AGB-phase, the core radius is a factor of about 4-5 larger in the Brussels code compared to binary\_c, SeBa and StarTrack. Therefore a merger is more likely to take place in the Brussels code.

5.5.2.4 Channel IV: CE + stable

Evolutionary path In the final evolutionary channel for DWDs, when the primary fills its Roche lobe, mass transfer proceeds in an unstable manner (according to channel 2 or 4 which are described in Sect. 5.5.1.2 and 5.5.1.4). However, when the secondary fills its Roche lobe mass transfer to the primary is stable. As a result the primary accretes mass.

Population The systems of channel IV lie in a small and specific region of DWD parameter space (see Fig. 5.24 and 5.26). The birthrates are low, 1.6\( \cdot 10^{-4} \) yr\(^{-1} \) and 5.5\( \cdot 10^{-5} \) yr\(^{-1} \) for binary\_c and SeBa respectively in the full mass range. We do not compare the population of this channel with the Brussels code as the progenitors according to binary\_c and SeBa are low mass binaries and the birthrate in the Brussels code is zero per definition (see also Sect. 5.4.1). We cannot compare the characteristics of the
5.6 Summary of critical assumptions in BPS studies

In the previous section we compared simulations from four different BPS codes and investigated the causes for the differences. The causes are not due to numerical effects, but are inherent to the codes and the underlying physical principles of the differences are listed below and discussed. The implementations of these principles in each code are described in Appendix 5.A.

- Initial-WD mass-relation;
  For single stars or non-interacting stars, the initial-final mass relation for WDs (see Fig. 5.8) is determined by the trade-off between the growth of the core and how much mass is lost in stellar winds and the planetary nebula phase. The amount of mass a low or intermediate mass star loses in a stellar wind is small on the MS, but significant in later stages of its evolution. The WD mass of primary stars is further affected by the mass transfer event, the moment and the timescale of the removal of the envelope mass. If the primary star becomes a hydrogen-poor helium burning star before turning into WD, the MiMwd-relation is influenced by helium star evolution. Of particular importance are the core mass growth versus the mass loss from helium stars and a possible second phase of mass transfer. A related issue, of particular importance for supernova type Ia rates, concerns the composition of WDs; what is the range of initial masses for carbon-oxygen WDs or other types of WDs?
  The amount of mass that is lost in the wind and in the planetary nebula influences the orbit directly, and indirectly through angular momentum loss. Questions remain about how much mass is lost in the wind of stars, and how much angular momentum is lost with the wind.

- The stability of mass transfer;
  For which systems does mass transfer occur in a stable manner and for which systems is it unstable? As binary population synthesis codes do not solve for the stellar structure equations, and cannot model stars that are not in hydrostatic or thermal equilibrium, BPS codes rely on parametrisations or interpolations to determine the stability of mass transfer. Theoretical stability criteria for polytropes exist (Hjellming & Webbink, 1987), but are lacking for most real stars (but see De Mink et al., 2007; Ge et al., 2010, 2013, for MS stars).
The critical mass ratio for stable mass transfer with hydrogen shell-burning donors differs between the codes from $q \geq 0.2$ in the Brussels code to $q \geq 0.6$ in StarTrack. A difference for low mass stars between binary_c, SeBa and StarTrack arises from the uncertainty of the mass transfer stability of donors with shallow convective envelopes. In a recent paper, Woods et al. (2012) show that mass transfer between a hydrogen shell-burning donor ($M_{1,zams} = 1 - 1.3 \, M_\odot$) and a main-sequence star ($q = 0.83 - 0.92$) can be stable when non-conservative. The effect on the orbit is a modest widening.

- **Survival of mass transfer;**
  For which systems does mass transfer lead to a merger and which system survive the mass transfer phase, in particular when mass transfer is unstable? As for the stability of mass transfer, the inability of BPS codes to model stars that are not in hydrostatic and thermal equilibrium, makes BPS codes rely on additional prescriptions, that are not (fully) available in literature. Regarding the survival of mass transfer, different assumptions for the properties (e.g. radii) of stripped stars lead to differences in the results of the four BPS codes, see e.g. channel II and III. For donor stars in which the removal of the envelope due to mass transfer leads to an end in nuclear (shell) burning and a WD is formed directly, it is unclear how much the core is bloated just after mass transfer ceases compared to a zero-temperature WD (Hurley et al., 2000). For donor stars that are stripped of their hydrogen envelopes due to mass transfer, but helium burning continues, it is unclear how fast the transition takes place from an exposed core to an (evolved) helium star (see channel 2b).

- **Stable mass transfer;**
  Modelling of the stable mass transfer phase in great detail is not possible in BPS codes, as for the stability of mass transfer. Therefore BPS codes rely on simplified methods to simulate stable mass transfer events. In this project, the different approaches to model the event do not lead to large differences in the synthetic stellar populations, however differences do exist most strongly for channel 3b and 5.

  The evolution of the mass transfer rate during the mass transfer phase can have a strong effect on the resulting binary. However in the current set-up of this project that assumes conservative mass transfer, the importance is greatly reduced. The mass transfer rates are only important when timescale of other effects e.g. wind mass loss or nuclear evolution, become comparable to the mass transfer timescale, as in channel 3b.

  An effect of the approach to model the stable mass transfer phases, is that mergers are less likely to happen in the Brussels code compared to the other codes. The approach of binary_c, SeBa and StarTrack is to follow the mass transfer phase in time, with approximations of the mass transfer rate. In the Brussels code, the mass transfer phases are not followed in detail. Instead it only considers the initial and final situation from interpolations of a grid of detailed calculations. Regarding channel 5 mass transfer, it is important to better understand which contact systems lead to a merger and which not. From observations, many Algol systems are found which have undergone and survived a phase of shallow contact.

- **The evolution of helium stars;**
  A large fraction of interacting systems go through a phase in which one of the stars is a helium star, for SWDs roughly 15% in the full mass range and roughly 50% in the intermediate mass range. Not
5.6 Summary of critical assumptions in BPS studies

Much is known about these objects about e.g. the stellar evolution, winds or mass transfer stability. Also the mass transfer rate is important, in particular for evolved helium stars whose evolutionary and wind loss timescales can become comparable to the mass transfer timescales. Therefore small differences in the mass transfer rate can lead to large differences in the resulting WD. This is especially important for massive WDs, e.g. supernova type Ia rates.

The assumptions that were equalised for this project will lead to a larger diversity in the simulated populations as different codes make different assumptions (see Appendix 5.5) and these should be taken into account when interpreting BPS results. The influence of the parameters that were equalised has not been studied here, not qualitatively nor quantitatively. The assumptions are:

- The CE-prescription and efficiency;
- Accretion efficiency during stable mass transfer;
- Angular momentum loss;
- Tidal effects;
- Magnetic braking;
- Wind accretion;
- The initial distribution of primary mass, mass ratio and orbital separation.

Despite the significance of these phenomena to binary evolution and the enormous effort of the community, all efforts so far have not been successful in understanding them. Several prescriptions exist for the first six phenomena and the effect on the evolution of the binary can be severely different. For example regarding the CE-phase, it is unclear how efficient orbital energy can be used to expel the envelope and if other sources of energy can be used, such as recombination, rotational, tidal or magnetic energy (Iben & Livio, 1993; Han et al., 1995; Politano & Weiler, 2007; Webbink, 2008; Zorotovic et al., 2010; De Marco et al., 2011; Zorotovic et al., 2011; Davis et al., 2012; Ivanova et al., 2013). Also, predictions for the efficiency of mass accretion onto WDs vary strongly and the supernova type Ia rate is severely affected by this uncertainty (Bours et al., 2013). Furthermore, the adopted mode of angular momentum loss has a strong effect on the evolution of the orbit (see Fig. 5.1 and Sect. 5.2.1.1). It is also not clear how the different prescriptions for tidal evolution affect the populations. However, in Sect. 5.5.1.3 we found that spin-orbit coupling (assuming orbits are continuously synchronised), only has a small effect on the final separation of the SWD systems. The effect of different initial distributions (see e.g. Duquennoy & Mayor, 1991; Kouwenhoven et al., 2007) of binary parameters can be severe with respect to the importance of a certain evolutionary channel, e.g. birthrates, number density or events per solar mass of created stars (see e.g. Eggleton et al., 1989; De Kool & Ritter, 1993; Davis et al., 2010). Furthermore, the importance of a certain channel is affected by the boundaries of the distribution through the normalisation of the simulation. In Appendix 5.B.7 we show that for the typical assumptions of the four codes this effect can be a factor 0.5-2 on the birthrates. The typical assumptions for each code for all equalised parameters are described in Appendix 5.5.
Chapter 5: PopCORN

5.7 Conclusion

In this paper we studied and compared four binary population synthesis codes. The codes involved are the binary_c code (Izzard et al., 2004, 2006, 2009, Claeys et al., subm.), the Brussels code (De Donder & Vanbeveren, 2004; Mennekens et al., 2010, 2012), SeBa (Portegies Zwart & Verbunt, 1996; Nelemans et al., 2001b; Toonen et al., 2012) and StarTrack (Belczynski et al., 2002b, 2008a; Ruiter et al., 2009b; Belczynski et al., 2010a). We focused on low and intermediate mass binaries that evolve into single white dwarf systems (containing a WD and a non-degenerate companion) and double white dwarf systems. These populations are interesting for e.g. post-common envelope binaries, cataclysmic variables, single degenerate as well as double degenerate supernova type Ia progenitors. For this project input assumptions in the BPS codes were equalised as far as the codes permit. This was done to simplify the complex problem of comparing BPS codes that are based on many (often different) assumptions. In this manner inherent differences between and numerical effects within the codes were investigated.

Regarding the single white dwarf population, we identified five evolutionary paths. There is a general agreement on what initial parameters of \( M_{1,\text{zams}}, M_{2,\text{zams}} \) and \( a_{\text{zams}} \) lead to SWD binaries through each formation channel. When the SWD system is formed, there is an agreement on the orbital separation range for those systems having undergone stable or unstable mass transfer. Furthermore there is a general agreement on the stellar masses after a phase of stable or unstable mass transfer.

Regarding the double white dwarf population, similar evolutionary paths can be identified in the various codes. There is an agreement on which primordial binaries lead to DWD systems through stable and unstable mass transfer respectively, and a rough agreement on the characteristics \( (M_{1,\text{dwd}}, M_{2,\text{dwd}} \text{ and } a_{\text{dwd}}) \) of the DWD population itself. Double white dwarfs go through more phases of evolution than single degenerate systems. The uncertainty in their evolution builds up through each mass transfer phase. The white dwarfs are formed with comparable masses, but at different separations.

We found that differences between the simulated populations arise not due to numerical differences, but due to different inherent assumptions. The most important ones that lead to differences are the MiMf-relations (of single stars), the MiMwd-relation (of binary stars), the stability of mass transfer, the mass transfer rate and in particular helium star evolution. Different assumptions between the codes are made for these topics as theory is poorly understood and sometimes poorly studied. Therefore we suggest further research into these topics e.g. with a detailed (binary) stellar evolution code to eliminate the differences between BPS codes.

In addition some assumptions were equalised for the comparison that can affect the results of the comparison. These are the initial binary distributions, the common envelope prescription and efficiency, the accretion efficiency, angular momentum loss during RLOF, tidal effects, magnetic braking and wind accretion. We leave the study of their effects on stellar populations for another paper.

In Sect. 5.3 a short description is given of each code. In Appendix 5.A and 5.5, a more detailed overview is given of the typical assumptions of each code outside the current project. These should be taken into account when interpreting results from the BPS codes. Furthermore, we recommend using these sections as a guideline when deciding which code or results to use for which project. Finally we would like to encourage other groups involved in BPS simulations, to do the same test as described in this paper and compare the results with the figures given in this paper. More detailed figures are available on request.

Concluding, we found that when the input assumptions are equalised as far as possible within the
5.A Backbones of the BPS codes

codes, we find very similar populations and birthrates. Differences are caused by different assumptions for the physics of binary evolution, not by numerical effects. So although the four BPS codes use very different ways to simulate the evolution of these systems, the codes give similar results and are adequate.

Acknowledgements
The authors would like to thank G. Nelemans, M. van der Sluys, O. Pols, D. Vanbeveren, W. van Rensbergen, & R. Voss for very helpful discussions on this project. ST thanks S. Portegies Zwart, JSWC thanks R.G. Izzard thanks I. Seitenzahl and K. Belczynski for thoughtful comments and general discussion. The authors are very grateful to those involved in the Lorentz Center workshops “Stellar mergers” (2009) and “Observational signatures of type Ia supernova progenitors” (2010), who provided the inspiration to carry out this long-awaited work. Furthermore the authors are grateful for the contributions to an early version of this project in 2002-2003 from K. Belczynski, C. Fryer, J. Hurley, V. Kalogera, G. Nelemans, S. Portegies Zwart and C. Tout, and in 2009-2010 from A. Bogomazov, P. Kiel, B. Wang and L. Yungelson. The work of ST and JSWC was supported by the Netherlands Research School for Astronomy (NOVA).

5.A Backbones of the BPS codes
The structure of BPS codes can vary strongly, which complicates the process of comparing BPS codes. Some aspects of the code are relatively simple to adapt in order to let assumptions of different groups converge, where as other aspects are inherent to the code and are not straightforward to change. For example, where some codes use results from detailed single star evolution codes, written down in analytical formulae (e.g. Eggleton et al., 1989; Hurley et al., 2000) to compute stellar parameters, others use the results of detailed binary evolution codes – a grid over which one can interpolate – and those results are integrated into the population code (e.g. De Donder & Vanbeveren, 2004). The inherent differences will create differences between the results of the different groups. The main differences are summarised in Table 5.4 and a more complete overview is given below. For most of the points the influence on a population it not immediately clear, therefore their effects are discussed in Sect. 5.6.
Table 5.4: Numerical treatments in the different codes which are inherent to them. (Further explanation can be found in Sect. 5.A)

<table>
<thead>
<tr>
<th></th>
<th>binary_c</th>
<th>Brussels code</th>
<th>SeBa</th>
<th>StarTrack</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single star prescriptions</td>
<td>HPT00</td>
<td>VB98</td>
<td>HPT00</td>
<td>HPT00</td>
</tr>
<tr>
<td>Stability of RLOF</td>
<td>( q_{\text{crit}} )</td>
<td>( R_{\text{conv}} ), ( q_{\text{crit}} )</td>
<td>( \zeta )</td>
<td>( \zeta, \frac{q_{\text{dd}}}{\tau} )</td>
</tr>
<tr>
<td>Mass transfer rate</td>
<td>( R_{d}/R_{\text{RL}}^{(2)} )</td>
<td>( R_{d}/R_{\text{RL}}^{(2)} )</td>
<td>( \zeta \rightarrow \frac{M^{(3)}}{\tau} )</td>
<td>( \zeta \rightarrow \frac{M^{(3)}}{\tau} )</td>
</tr>
<tr>
<td>Wind (AGB)</td>
<td>R75, VW93, HPT00</td>
<td>HG97</td>
<td>R75, VW93, HPT00</td>
<td>R75, VW93, HPT00</td>
</tr>
<tr>
<td>AML (wind)</td>
<td>Donor (HTP02)</td>
<td>No</td>
<td>Donor</td>
<td>Donor</td>
</tr>
<tr>
<td>Helium star evolution</td>
<td>Yes</td>
<td>Not explicit</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Population synthesis</td>
<td>Grid based</td>
<td>Grid based</td>
<td>Monte Carlo</td>
<td>Monte Carlo</td>
</tr>
</tbody>
</table>

Notes:
2. \( R_{\text{RL}} \) is the Roche radius of the donor star.
3. \( \tau \) = Characteristic timescale of mass transfer. Can be nuclear, Kelvin-Helmholtz, timescale of magnetic braking or of gravitational wave radiation.

5. A Backbones of the BPS codes

5.A.1 Single star prescriptions

The single star prescriptions, either given by analytical formulae or included in a grid of binary systems over which can be interpolated, determine which mass the WD will have when the star loses its envelope. Furthermore they determine the radii during the evolution of the star and therefore the moment at which the star fills its Roche lobe.

- **binary_c, SeBa, StarTrack**: the codes use analytical fitting formulae (Hurley et al., 2000) from detailed single star evolution tracks, with an overshooting constant $\delta_{ov} = 0.12$ (based on Pols et al., 1998). In binary_c different AGB models can be used, based on detailed models of Karakas et al. (2002) for thermally pulsating AGB stars (TP-AGB). However, these are not used for this work. Prior to the work of Toonen et al. (2012), the single star prescriptions in SeBa were based on Eggleton et al. (1989).

- **Brussels code**: intermediate mass single star prescriptions are taken from Schaller et al. (1992). These tracks include convective overshooting by means of the following prescription: the overshooting distance $d_{over}$ is directly proportional to the pressure scale height $H_p$ according to $d_{over} = 0.2H_p$. This corresponds to a slightly lower degree of overshooting than in the codes that use the overshooting constant $\delta_{ov} = 0.12$ in the stability criteria, the latter corresponding to a $d_{over}/H_p$ between 0.22 and 0.4 depending on mass (see Hurley et al., 2000). Stellar parameters which do not depend on whether the star is part of an interacting binary system are taken directly from this reference. Other stellar parameters, such as the remnant mass after RLOF, are taken from the detailed binary evolution code.

5.A.2 Stability of mass transfer

At the moment that one of the stars fills its Roche lobe mass transfer can proceed in a stable manner or the system can evolve into a CE-phase (see Sect. 5.2.1). In the simulation of the evolution of a binary system the entire stellar structure is not explicitly followed in detail, and consequently, ‘stability checks’ must be built-in to BPS codes to determine if RLOF will lead to a CE-phase.

- **binary_c**: for every type of donor star and type of accretor star a critical mass ratio ($q_{crit}$) is given. The mass ratio of the system during mass transfer is compared with the critical mass ratio for stable mass transfer and determines if mass transfer will proceed in a stable manner or not. An overview can be found in Claeyss et al. (subm.). Note that in that paper two possibilities are described for the stability of Roche lobe overflowing helium stars to non-degenerate accretors. For this project the criterion as described in Hurley et al. (2002) is used.

- **Brussels code**: the boundary between stable and unstable RLOF is determined by whether the outer layers of the donor star are radiative or deeply convective respectively. For each stellar mass, the minimum stellar radius $R_{conv}$ is given for which the envelope is convective. If the orbital period of the system under investigation is smaller than the theoretical orbital period at the time when $R_{RL} = R_{conv}$, mass transfer will proceed in a stable way.

If the mass ratio between the two stars is extreme ($q = M_d/M_d < 0.2 = q_{crit}$) at the onset of mass transfer, this will result in an instability (Darwin, 1879). The donor star will be unable to extract...
sufficient angular momentum from the orbit to remain in synchronized rotation, resulting in the mass transfer episode quickly becoming dynamically unstable. Tidal interaction will cause the secondary to spiral into the donor’s outer layers, a process that is treated identically to the common envelope evolution (hence with $\beta = 0$).

- **SeBa**: the stability and rate of mass transfer are dependent on the reaction to mass change of the stellar radii and the corresponding Roche lobes. The change in the Roche radius $R_{RL}$ due to loss and transfer of mass $M$ is given by

$$
\zeta_{RL} \equiv \frac{d \ln R_{RL}}{d \ln M},
$$

(5.14)

the adiabatic (i.e. immediate) response of the donor star’s radius $R$ is given by

$$
\zeta_{ad} \equiv \frac{d \ln R}{d \ln M}.
$$

(5.15)

For every Roche lobe filling system, $\zeta_{RL}$ and $\zeta_{ad}$ are compared at every timestep. If $\zeta_{RL} < \zeta_{ad}$ we assume mass transfer proceeds in a stable manner (e.g. Webbink, 1985; Pols & Marinus, 1994). When $\zeta_{RL} > \zeta_{ad}$, mass transfer is dynamically unstable leading to a CE-phase.

The value of $\zeta_{RL}$ is calculated numerically by transferring a test mass of $10^{-5} M_\odot$. The advantage of this is that, because $\zeta_{RL} = \zeta_{RL}(M_d, M_a, a)$ and so $\zeta_{RL}$ is dependent on the mass accretion efficiency of the secondary, the (de)stabilising effect (see Soberman et al., 1997) of non-conservative stable mass transfer is taken into account automatically. Appropriate recipes of $\zeta_{ad}$ are implemented in the code for every type of donor star. An overview can be found in Toonen et al. (2012), appendix A3 therein.

Furthermore, the orbital angular momentum is compared with the stellar spin angular momenta, to check whether a Darwin instability is encountered (Darwin, 1879).

- **StarTrack**: When a non-degenerate star fills its Roche lobe, $\zeta_{ad}$ and $\zeta_{RL}$ are calculated, similar to the case of SeBa. The value of $\zeta_{ad}$ is determined by removing mass from the star over a 1-year timestep (Belczynski et al., 2008a). The value of $\zeta_{RL}$ is determined by transferring a small amount (1%) of the star’s mass toward the companion. In cases where the mass loss is so rapid such that the star loses thermal equilibrium, a ‘diagnostic diagram’ is used to predict the stability of mass transfer (see description in Belczynski et al., 2008a, sect. 5.2). The diagnostic diagram is a numerical tool that was first calibrated using detailed stellar evolution calculations of massive stars, and is currently being updated to include a range of stellar models for low- and intermediate-mass stars.

In addition, there is also a check for a possible delayed dynamical instability. This occurs for stars with $M_d/M_a > q_{ddi}$, with $q_{ddi}$ based on Hjellming & Webbink (1987), or when a Darwin instability is encountered, or when the trapping radius of the accretion stream (King & Begelman, 1999) exceeds the Roche radius of the accreting star (see Ivanova et al., 2003; Belczynski et al., 2008a, sect. 5.4). This latter point however is not considered for this work.

### 5.A.3 Stable mass transfer

To take into account various driving mechanism of stable RLOF, such as thermal readjustment or nuclear evolution of the donor, approximate prescriptions are used to determine the mass transfer rate. Note that
mass transfer rate refers to the mass lost by the donor, which will always be equal to or greater than the mass accretion rate, which refers to the mass gained by the companion.

- **binary.c**: the mass transfer rate is calculated as a function of the ratio of the stellar radius and the Roche radius (based on Whyte & Eggleton, 1980). A function is generated which follows the radius more closely during mass transfer on a thermal timescale and more loosely when the star is in thermally equilibrium. A smooth transition is build-in between the two. The formulation can be found in Claeys et al. (subm.). That paper also shows that the resulting mass transfer phases are comparable to that of the detailed binary stellar evolution code STARS (based on Eggleton, 1971) in the duration of the mass transfer phases and the mass transfer rates for a set of models. This method indirectly considers mass transfer on the nuclear and thermal timescale, but also on the timescale of gravitational wave radiation or magnetic braking are considered.

- **Brussels code**: the mass transfer rates are not explicitly calculated in the population code. It considers merely the initial and final masses. These are interpolated from the results of the detailed binary evolution code. The latter calculates the mass transfer rate during stable RLOF iteratively, by investigating how much mass needs to be lost during the current timestep for the donor star to remain confined by its Roche lobe (within the order of a few percent).

- **SeBa**: $\zeta_{RL}$ is compared with appropriate values of $\zeta_{eq}$ to determine if mass transfer is driven by the thermal readjustment or the nuclear evolution of the donor star. $\zeta_{eq}$ represents the response of the donor star’s radius $R$ as it adjusts to the new thermal equilibrium:

$$\zeta_{eq} = \left( \frac{d \ln R}{d \ln M} \right)_\text{th}.$$  

Appropriate recipes of $\zeta_{eq}$ are implemented for every type of donor star. If $\zeta_{RL} < \min(\zeta_{ad}, \zeta_{eq})$, mass transfer is driven by the nuclear evolution of the donor star and we assume mass transfer proceeds on the nuclear timescale of the donor star (e.g. Webbink, 1985; Pols & Marinus, 1994). If $\zeta_{eq} < \zeta_{RL} < \zeta_{ad}$ RLOF is dynamically stable and driven by thermal readjustment of the donor, so that mass transfer proceeds on the thermal timescale of the donor star.

In addition, stable mass transfer can be driven by angular momentum loss from magnetic braking or gravitational wave emission. When the timescale of angular momentum loss is shorter than the mass loss timescale determined above, we assume mass transfer is driven by angular momentum loss. For more detail see Appendix A.3 of Toonen et al. (2012).

- **StarTrack**: For non-degenerate donors $\zeta_{RL}$ and $\zeta_{ad}$ are calculated, along with the thermal timescale $\tau_{KH}$ (based on Kalogera & Webbink, 1996). Additionally, the equilibrium mass transfer timescale $\tau_{eq}$ is calculated as a combination of RLOF both driven by angular momentum loss and the nuclear evolution of the star and/or the changes due to magnetic braking and gravitational wave radiation (see Belczynski et al., 2008a). If $\tau_{eq} > \tau_{KH}$ the mass losing star is in thermal equilibrium and mass transfer proceeds on $M_{eq} = M/\tau_{eq}$. If $\tau_{eq} \leq \tau_{KH}$ mass transfer proceeds on a thermal timescale, given by $M_{KH} = M/\tau_{KH}$. If $\dot{M}_{eq}$ becomes positive the star falls out of equilibrium and the stability of mass transfer is determined by the diagnostic diagram (see Belczynski et al., 2008a). In the case of WD donors, the mass transfer rate is always driven by gravitational radiation.
5.A.4 Wind mass loss

The driving mechanisms of the wind and the explicit rate at which this material is lost are not yet completely understood. This results in different prescriptions to describe the rate of wind mass loss and the amount that can be lost (e.g. Wachter et al., 2002). We only discuss the wind-prescriptions that are relevant for low and intermediate mass stars.

- **binary_c, SeBa, StarTrack:** for stars up to the early AGB the prescription of Reimers (1975) is adopted (with $\eta = 0.5$). To describe the wind mass loss of stars on the TP-AGB a prescription based on Vassiliadis & Wood (1993) is implemented. Both prescriptions are defined in Hurley et al. (2000). In binary_c and StarTrack different prescriptions for the wind mass loss are available that used by different users of the respective codes.

- **Brussels code:** For intermediate mass interacting binaries, the initial-final mass relation of WDs is determined by assuming the wind prescription of Van den Hoek & Groenewegen (1997). However, it should be noted that in the BPS code a star in an interacting binary does not have wind mass loss. For the most massive stars, wind mass loss is as is described in Vanbeveren et al. (1998) - how massive?

5.A.5 Angular momentum loss from winds

Sect. 5.2.1.1 describes the importance of angular momentum loss (AML) and the effect on the orbit. Not only mass lost during RLOF, but also wind carries angular momentum, which is lost when it leaves the system. The same prescriptions as described in Sect. 5.2.1.1 can be applied to AML when material is lost through a wind and different prescriptions are used in the BPS codes.

- **binary_c:** different prescriptions of angular momentum loss through a stellar wind are available in binary_c. In this study as in Claeys et al. (subm.), wind angular momentum loss is as described in Hurley et al. (2002). When no material is accreted by the companion star, the wind takes specific angular momentum of the donor.

- **Brussels code:** Mass lost by a stellar wind in non-interacting systems is lost through the Jeans mode. Interacting systems do not have wind mass loss prior to interaction.

- **SeBa, StarTrack:** the material lost by a wind that is not accreted by the companion is lost from the system with specific angular momentum from the donor.

5.A.6 Evolution of helium stars

A helium star is formed after a hydrogen-rich star with a helium core loses its hydrogen-rich envelope. When the core is not degenerate at that time, the evolution of the star continuous as a helium-burning star. Uncertainties in the evolution of helium stars encompasses the growth of this star, the wind mass loss and mass transfer phase, such as the stability and rate.

- **binary_c, SeBa, StarTrack:** the evolutionary tracks and wind prescription are based on Hurley et al. (2000). The stability of mass transfer and the rate are described in previous sections.
5.B Typical variable assumptions in BPS codes

- **Brussels code**: helium star evolution is not explicitly included in the code. It is assumed that the donor star always loses its entire H-rich envelope in one episode and becomes a white dwarf afterwards, except in the case where a donor fills its Roche lobe for a second time as a helium star. In this case mass transfer is followed as described in Sect. 5.A.3, however time-dependent evolutionary aspects of the helium star are not followed. This simplification is made because the intermediate step is not believed to have a large influence on the eventual masses and separation. However, this implicitly means that the most massive star will always become a white dwarf first, which is not necessarily the case when helium star evolution is explicitly followed. For stars that lose mass during the planetary nebula phase, no resulting angular momentum loss is taken into account.

5.A.7 Generating the initial stellar population

The initial population can be chosen by a Monte Carlo method, or the choice can be made grid-based. Nevertheless, if the method is well performed both methods should give the same results for a high enough resolution.

- **binary_c**: \(N_{M_{1,\text{zams}}} \cdot N_{M_{2,\text{zams}}} \cdot N_{q_{\text{zams}}}\) binaries are simulated, with \(M_{1,\text{zams}}, M_{2,\text{zams}}, a_{\text{zams}}\) chosen in logarithmic space. A probability is calculated for every system determined by the defined initial distributions.

- **Brussels code**: the code works with a three-dimensional grid of initial parameters: primary mass \(M_{1,\text{zams}}\), mass ratio \(q_{\text{zams}}\) and orbital period \(P_{\text{zams}}\). According to the initial mass function, initial mass-ratio distribution and initial orbital period (or separation) distribution, each grid point is assigned a certain weight. Every system corresponding to such a grid point is then taken through its evolution.

- **SeBa, StarTrack**: initial parameters \(M_{1,\text{zams}}, M_{2,\text{zams}}, a_{\text{zams}}\) and the initial eccentricity \(e_{\text{zams}}\) are chosen randomly on a Monte-Carlo based-approach where the probability functions are given by the initial distributions. With this method, the resolution is highest in those regions of parameter space where most systems lie.

5.B Typical variable assumptions in BPS codes

Some aspects of the codes that are not straightforward to change have been discussed in the previous section. However, other aspects of the codes are relatively simple to adapt. These aspects are often contained in relatively isolated and parametrised functions. For this project we equalised these aspects in the codes as far as possible. However, we do not believe that all the assumptions made for this project are realistic. Previous publications of results from these BPS codes are based on different assumptions. Although we do not compare the effect of the different assumptions on stellar populations in this work, it is good to realise which assumptions are generally used. Therefore the usual assumptions made by the authors in their corresponding BPS code are summarised in Table 5.5 and are discussed in more detail below. Typical assumptions may vary between different users of the BPS codes.
Table 5.5: Equalised assumptions for this research and the usual assumptions of the authors in the corresponding BPS codes.

<table>
<thead>
<tr>
<th></th>
<th>binary_c</th>
<th>Brussels code</th>
<th>SeBa</th>
<th>StarTrack</th>
<th>This research</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$ (RLOF)</td>
<td>Variable</td>
<td>Conditional$^{(1)}$</td>
<td>Variable</td>
<td>Conditional$^{(1)}$</td>
<td>1</td>
</tr>
<tr>
<td>AML (RLOF)</td>
<td>Isotropic re-emission</td>
<td>$\alpha$ (v2)</td>
<td>Orbit$^{(2)}$ ($\eta = 2.5$)</td>
<td>Orbit$^{(2)}$ ($\eta = 1$)</td>
<td>Orbit ($\eta = 1$)</td>
</tr>
<tr>
<td>CE$^{(3)}$</td>
<td>$\alpha$ (v1)</td>
<td>$\gamma\alpha$ (T12)</td>
<td>$\alpha$ (v1)</td>
<td>$\alpha$ (v1)</td>
<td></td>
</tr>
<tr>
<td>$\alpha_{ce}l_{ce}/\gamma$</td>
<td>Variable$^{(4)}$</td>
<td>1</td>
<td>2/1.75</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Wind accretion</td>
<td>B-H$^{(5)}$</td>
<td>No</td>
<td>B-H$^{(5)}$</td>
<td>No$^{(6)}$</td>
<td>No</td>
</tr>
<tr>
<td>Tides</td>
<td>Z77, H81, HTP02</td>
<td>Z77</td>
<td>PZV96</td>
<td>Z77, H81, HTP02, C07</td>
<td>No</td>
</tr>
<tr>
<td>Magn. braking</td>
<td>RVJ83</td>
<td>No</td>
<td>RVJ83</td>
<td>IT03</td>
<td>No</td>
</tr>
</tbody>
</table>

Notes:
(1) Constant for non-degenerate accretors, variable for accretion onto a WD.
(2) Except during accretion onto a compact object, AML = isotropic re-emission.
(3) $v1$ = prescription Webbink (1984), $v2$ = prescription Hurley et al. (2002).
(4) Based on detailed stellar structure models (for the description see Izzard, 2004, Claeys et al., subm.).
(5) B-H = Prescription based on Bondi & Hoyle (1944).
(6) Wind accretion is taken into account for neutron star and black hole accretors assuming B-H-accretion$^{(5)}$.

Table 5.6: Equalised initial distribution and range of binary parameters and the usual distributions and ranges of the authors for the corresponding BPS codes.

<table>
<thead>
<tr>
<th>What?</th>
<th>binary_c</th>
<th>Brussels code</th>
<th>SeBa</th>
<th>StarTrack</th>
<th>This research</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(M_{1\text{,zams}})$</td>
<td>KTG93</td>
<td>KTG93</td>
<td>KTG93</td>
<td>KTG93</td>
<td>KTG93</td>
</tr>
<tr>
<td>$M_{1\text{,zams}}$ (M$_\odot$)</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.08</td>
<td>0.1</td>
</tr>
<tr>
<td>$M_{\text{max}}$ (M$_\odot$)</td>
<td>80</td>
<td>120</td>
<td>100</td>
<td>150</td>
<td>100</td>
</tr>
<tr>
<td>$f(a_{\text{zams}})$</td>
<td>$\propto a^{-1}$</td>
<td>$\propto a^{-1}$</td>
<td>$\propto a^{-1}$</td>
<td>$\propto a^{-1}$</td>
<td>$\propto a^{-1}$ (A83)</td>
</tr>
<tr>
<td>$a_{\text{zams}}$ (R$_\odot$)</td>
<td>max(5, [R_a + R_b]/[1 - e_0])</td>
<td>2 - 12^{(1)}(P = 1d)</td>
<td>(R_a + R_b)/(1 - e_0)</td>
<td>2(R_a + R_b)/(1 - e_0)</td>
<td>5</td>
</tr>
<tr>
<td>$a_{\text{max}}$ (R$_\odot$)</td>
<td>5e6</td>
<td>5.8e2 - 2.2e3^{(1)}(P = 3650d)</td>
<td>1e6</td>
<td>1e6</td>
<td>1e6</td>
</tr>
<tr>
<td>$f(q_{\text{zams}})$</td>
<td>Flat</td>
<td>Flat</td>
<td>Flat</td>
<td>Flat</td>
<td>Flat</td>
</tr>
<tr>
<td>$q_{\text{zams}}$</td>
<td>0.01 M$<em>\odot$/M$</em>{1\text{,zams}}$</td>
<td>0.1 M$<em>\odot$/M$</em>{1\text{,zams}}$</td>
<td>0</td>
<td>0.08 M$<em>\odot$/M$</em>{1\text{,zams}}$</td>
<td>0.1 M$<em>\odot$/M$</em>{1\text{,zams}}$</td>
</tr>
<tr>
<td>$q_{\text{max}}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$f(e_{\text{zams}})$</td>
<td>-</td>
<td>-</td>
<td>H75</td>
<td>H75</td>
<td>-</td>
</tr>
<tr>
<td>$e_{\text{zams}}$</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>$e_{\text{max}}$</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>Max time (Gyr)</td>
<td>13.7</td>
<td>15</td>
<td>13.5</td>
<td>15</td>
<td>13.7</td>
</tr>
<tr>
<td>Binary fraction (%)</td>
<td>100</td>
<td>100</td>
<td>50-100</td>
<td>50</td>
<td>100</td>
</tr>
</tbody>
</table>

Notes: $f(\xi)$ is the distribution of parameter $\xi$. '-' Indicates that no distribution of initial eccentricities is considered, instead $e_{\text{zams}} = 0$ a priori. Otherwise the distribution of initial eccentricities is $f(e_{\text{zams}})$ with $e_{\text{zams}}$ between $e_{\text{zams}}$,min and $e_{\text{zams}}$,max.

(1) Separations given for the binary masses under investigation.

References: KTG93 = Kroupa et al. (1993), A83 = Abt (1983), H75 = Heggie (1975)
Chapter 5: PopCORN

5.B.1 ACCRETION EFFICIENCY

In this project mass transfer is assumed to be conservative to all types of stars. However in general, the accretion efficiency depends on the type of accreting star and the mass transfer rate.

- binary_c, SeBa: in the case of non-degenerate accretors with radiative envelopes, the accretion efficiency mainly depends on the mass transfer rate and the thermal timescale of the accreting star. In the case of non-degenerate objects with convective envelopes, mass is transferred conservatively. In the case of a degenerate accretors, the accretion efficiency depends on the mass of the degenerate object and the mass transfer rate.

- Brussels code: the accretion efficiency onto a non-degenerate object is taken to be constant. If the mass ratio is below 0.2, mass transfer is unstable and the accretion efficiency is assumed to be zero (see Sect. 5.A.2). To ensure continuity, between mass ratios 0.2 and 0.4 a linear interpolation is used for the accretion efficiency, between 0 and $\beta$ (usually 1). Note that for popcorn this transition was not implemented and the accretion efficiency is one between 0.2 and 0.4. In case of a degenerate accreting object, the regions in the (companion mass, orbital period)-parameter space from Hachisu et al. (2008) are used to determine in which cases the white dwarf can stably accrete up to $1.4 M_\odot$. In all other cases, mass transfer towards white dwarfs is assumed to become unstable, and is treated as a common envelope phase.

- StarTrack: the accretion efficiency onto a non-degenerate object is taken to be constant. In the case of a degenerate accreting object, the accretion efficiency depends on the mass of the accreting object and the mass transfer rate (see Belczynski et al., 2008a, sect. 5 therein).

5.B.2 ANGULAR MOMENTUM LOSS DURING RLOF

In BPS codes a wide range of prescriptions are used to describe angular momentum loss when material is lost in a phase of stable RLOF. See Sect. 5.2.1.1 for the different prescriptions of angular momentum loss and a discussion of the importance of the effect on the orbit.

- binary_c: in this work and the standard model in Claeys et al. (subm.) the material not accreted during the stable RLOF phase is lost as isotropic re-emission.

- Brussels code: the material is lost through the second Lagrangian point such that angular momentum is lost from a circumbinary ring with $a_{\text{ring}} = 2.3$.

- SeBa: when the accretor is a non-degenerate star, the material lost carries 2.5 times the specific orbital angular momentum of the binary (Portegies Zwart, 1995; Nelemans et al., 2001b). In the case of a degenerate accretor, the material lost carries specific orbital angular momentum of the accreting star.

- StarTrack: when the accretor is a non-degenerate star, the material lost carries one time the specific orbital angular momentum. In the case of a degenerate accretor, the material lost carries specific orbital angular momentum of the accreting star.
5.B.3 Common envelope evolution

The evolution of a CE-phase is highly uncertain. For this reason, various BPS codes employ different CE-prescriptions (see Sect. 5.2.1.2) and CE-efficiencies (such as $\alpha_{\text{ce}}$) and both aspects are often varied within a BPS study for comparison. Here, we briefly describe the CE-parametrisations that are implemented most often by the authors in the four different codes.

- **binary_c**: to describe CE-evolution the prescription based on Hurley et al. (2002) is used. In the standard model of Claeyss et al. (subm.) $\alpha_{\text{ce}}$ is one, while $\lambda_{\text{ce}}$ depends on the type of star, its mass and luminosity (see Izzard, 2004, Claeyss et al., subm.). However, in the BPS code also the $\gamma$-prescription can be used.

- **Brussels code, StarTrack**: For standard calculations, the prescription based on Webbink (1984) is used, where $\alpha_{\text{ce}}$ and $\lambda_{\text{ce}}$ are both one. In both codes different values for $\alpha_{\text{ce}}$ and $\lambda_{\text{ce}}$ can be implemented, as well as the $\gamma$-prescription (for further information about the version of the $\gamma$-prescription implemented in StarTrack see Belczynski et al., 2008a; Ruiter et al., 2011).

- **SeBa**: the standard model for simulating CE-evolution in SeBa is the $\gamma$-prescription, unless the binary contains a compact object or the CE is triggered by a Darwin instability (Darwin, 1879) for which the $\alpha$-formalism based on Webbink (1984) is used. The $\gamma$-formalism is introduced by Nelemans et al. (2000) in order to better reproduce the mass ratio distribution of observed double white dwarfs. The mass loss reduces the angular momentum of the system according to:

$$\frac{J_i - J_f}{J_i} = \gamma \frac{M_{d,\text{env}}}{M_d + M_a} \quad (5.17)$$

where $J_i$ and $J_f$ are the angular momenta of the pre- and post-mass transfer binary respectively. The motivation for this formalism is the large amount of angular momentum available in binaries with similar mass objects that possibly can be used to expel the envelope. In SeBa $\gamma$ is taken to be equal to 1.5, and $\alpha_{\text{ce}} \times \lambda_{\text{ce}}$ is equal to two.

5.B.4 Wind accretion

Material that is lost in the form of a stellar wind can be partly accreted by the companion star. The amount depends on properties of the wind (e.g. the velocity), the accreting star and the binary system (e.g. the separation). However, the exact amount accreted is ill-constrained.

- **binary_c, SeBa**: the accretion efficiency of wind material is determined by the Bondi-Hoyle prescription (Bondi & Hoyle, 1944). In binary_c the accretion efficiency based on the wind Roche-lobe overflow model can be used (Mohamed & Podsialdowski, 2007, 2012; Abate et al., 2013), however is not used for this work.

- **Brussels code**: no material lost in the form of a stellar wind is accreted by the companion star.

- **StarTrack**: material lost through a wind is in general not accreted by the companion star, except when the companion star is a neutron star or a black hole.
5. B. 5 Tides

The general picture of tidal effects is clear, however uncertainties remain due to missing knowledge about for example some dissipative processes.

- **binary_c**: tidal evolution is implemented as described by Hurley et al. (2002), which is based on Hut (1981); Zahn (1977).
- **Brussels code**: tidal evolution is a described by Zahn (1977).
- **SeBa**: tidal evolution is implemented as described by Portegies Zwart & Verbunt (1996).
- **StarTrack**: tidal evolution is implemented as described by Claret (2007), as well as Hurley et al. (2002), which is based on Hut (1981); Zahn (1977).

5. B. 6 Magnetic braking

Magnetic braking is important for low mass stars with convective envelopes. Nevertheless, this process is not fully understood and different prescriptions co-exist.

- **binary_c, SeBa**: both codes use the prescription of Rappaport et al. (1983).
- **Brussels code**: the code is not used for the evolution of stellar objects with a mass lower than $3 \, M_\odot$, therefore magnetic braking is not considered.
- **StarTrack**: the prescription of Ivanova & Taam (2003) is used in standard calculations.

5. B. 7 Initial population

**Table 5.7**: Effect on the normalisation of different boundaries for the initial distribution of binary parameters. The initial distributions are as in Table 5.1. The normalisation factor is defined as the ratio of the number of systems for the given boundary conditions and the total number of systems using the typical boundary conditions (as given in the top line). The normalisation factor is used to convert the number of systems in the simulation to physical quantities such as birthrates, number density, events per solar mass of created stars.

<table>
<thead>
<tr>
<th>Normalisation factor</th>
<th>min $M$/$M_\odot$</th>
<th>max $M$/$M_\odot$</th>
<th>min $a$/$R_\odot$</th>
<th>max $a$/$R_\odot$</th>
<th>min $q$</th>
<th>max $q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.1</td>
<td>100</td>
<td>5</td>
<td>1e6</td>
<td>0.1 $M_\odot$/M1,zams</td>
<td>1</td>
</tr>
<tr>
<td>1.0</td>
<td>0.1</td>
<td>150</td>
<td>5</td>
<td>1e6</td>
<td>0.1 $M_\odot$/M1,zams</td>
<td>1</td>
</tr>
<tr>
<td>0.6</td>
<td>0.1</td>
<td>100</td>
<td>5</td>
<td>1e4</td>
<td>0.1 $M_\odot$/M1,zams</td>
<td>1</td>
</tr>
<tr>
<td>1.4</td>
<td>0.08</td>
<td>100</td>
<td>5</td>
<td>1e6</td>
<td>0.08/M1,zams</td>
<td>1</td>
</tr>
</tbody>
</table>

The choice for an initial distribution and the respective boundaries can severely affect the importance of a certain evolutionary channel through the normalisation of the simulation. Table 5.7 shows the effect of
5. B Typical variable assumptions in BPS codes

different boundary conditions considered in the standard simulations of the respective BPS codes compared to the boundary conditions assumed in this study. It shows that different boundary conditions affect the birthrates by a factor of about 0.5 to 2. The assumptions made by the authors with their respective codes are summarised in Table 5.6. Different aspects which need extra clarification are discussed below. Note that other users of the BPS codes under study here, other than the authors, may use different distribution functions and/or ranges.

- **binary_c**: the initial eccentricity is zero, based on the work of Hurley et al. (2002). The minimum initial separation is varied between $5 \, R_\odot$ or the minimum separation at which a binary system with a certain mass is initially detached. The minimum and maximum masses and separations are based on the work of Kouwenhoven et al. (2007).

- **Brussels code**: the initial eccentricity is zero. No minimum and maximum separation is assumed for binaries, but a minimum and maximum initial orbital period, more specifically one day and 3650 days. In order to compare with the other codes, a conversion of orbital period to separations is given in Table 5.6.

- **SeBa, StarTrack**: a distribution for the initial eccentricities is assumed (see Table 5.6). The initial semi-major axis is chosen between $10^6 \, R_\odot$, and the minimum initial separation is the minimum separation at which a binary system with a certain mass is initially detached.
SUMMARY AND PROSPECTS

In this thesis we show the results of theoretical studies of the progenitor evolution of different types of supernovae (Chapter 2 to 4), and of a study of the validity of the method used for part of the research (Chapter 5). The entire work gives an overview of both the binary progenitor evolution of these supernova types and the unresolved issues in binary evolution studies.

6.1 TYPE IIb supernovae

In Chapter 2 we describe the evolution towards the transitional type IIb supernova (SN). We find that only considering wind mass loss cannot explain the observed rate and the individual observed SNe of this type. We show that late case B mass transfer, in which a red giant transfers material to a companion star, explains the full range of observed characteristics, such as the (non-) detections of the companion star. We determine the evolutionary path for the SN giving rise to the SN remnant Cassiopeia A, in which no companion star has been observed. We also derive the progenitor evolution of SN 1993J and its probable observed companion star, a blue supergiant, which is by previous authors the first binary path put forward for this SN type. We identify three evolutionary paths of the companion star: towards a main sequence star, a blue supergiant or a red supergiant. However, we find that the evolutionary path towards a blue supergiant companion is the least likely path, which raises the question if this blue supergiant is really the surviving companion star of SN 1993J.

The observed rate remains difficult to reproduce even with extreme initial binary distributions, such an initial mass ratio distribution which favours equals masses, although the observe rate has a large uncertainty as it is based on only a small number of observations. In this chapter we indicate to relevance of studying SNe type IIb, as they are the border case between type II SNe and type Ib/c SNe. We expect that upcoming surveys will quantify the rate of this SN type and of the different types of remaining companions, which has the potential to constrain some single stars and binary physics, such as the wind mass loss rate of massive stars and the boundary conditions of convection.

The next step in theoretical studies of the progenitor evolution of core collapse SNe is the combination of binary physics and rotation (Langer, 2012). Hirschi et al. (2004) show that the structure of rotating single stars at the moment of explosion can differ severely from non-rotating single stars. Additionally, De Mink et al. (2013) demonstrate that binary interaction produces about 20% of rapidly rotating massive main-
Chapter 6: Summary and prospects

sequence stars, which confirms the importance of studying both binary evolution and rotational effects together.

6.2 Type Ia supernovae

Howell (2011) discussed the unresolved issues of type Ia SNe at that time. As the SN Ia community is a very active research field, many of these issues have been addressed in recent years and a few of them in this thesis (Chapters 3 and 4).

The main argument against the single degenerate channel as the main progenitor channel is the non-detection of the surviving companion stars. Large SN surveys detected possible indirect proof of a companion star in a large number of SNe Ia (Patat et al., 2007a; Sternberg et al., 2011; Dilday et al., 2012). However, supporters of the double degenerate channel demonstrate that these detections can also be explained by the double degenerate channel (Kashi & Soker, 2011; Ilkov & Soker, 2013; Pakmor et al., 2013; Shen et al., 2013). Chapter 3 discusses the accretion process of the single degenerate scenario, in which it is expected that a white dwarf not only accretes material but also angular momentum which spins up the white dwarf. As a result white dwarfs reaching the Chandrasekhar mass may have to spin down before they can explode, creating a delay between the accretion process and the explosion. The spin-down process itself is still unclear as is the delay time between the end of the accretion process and the explosion, but it can diminish the signal of the single degenerate model and making the donor star harder to detect, so the SN appears as originating from the double degenerate channel because of the non-detection of the companion. Additionally, this scenario possibly provides a natural explanation for the set of anomalously bright SNe Ia, which have been interpreted as super-Chandrasekhar explosions. Even though this research only discusses the main ideas of the effect of spinning white dwarfs, and more theoretical studies are necessary to get a full picture of the resulting characteristics, it demonstrates the importance of considering angular momentum. Finally, we put forward some observational signatures which can indicate the occurrence of this spin-up/spin-down process.

The single degenerate nor the double degenerate channel can reproduce the observed SN Ia rate. Additionally different studies show conflicting results, both regarding the rate and the importance of the different channels. Chapter 4 discusses the progenitor evolution of SNe Ia, through both the single and the double degenerate channel. We remark that both channels are necessary to recover the rate. The single degenerate with helium rich donor stars is the only channel that can account for the prompt population of SNe Ia, while the double degenerate channel is mainly responsible for the delayed population. Our study indicates that many of the ill-constrained aspects of binary evolution have only a small effect on our theoretical predictions and do not influence the general characteristics of our delay time distribution. However, we demonstrate the large impact of a variation of the common envelope efficiency on the rate, and on the ratio of the number of SNe Ia formed through the single degenerate and double degenerate channels. Additionally, different assumptions regarding helium star evolution change both the rate and the number of SNe occurring shortly after the formation of the binary system. The relevance of helium star evolution has barely been studied previously, and our results indicate the need for more detailed studies of this stellar phase. Also the initial binary distribution functions have a large impact on the overall rate. The determination of the distribution functions of intermediate mass stars in binaries has various difficulties which will not be resolved in the near future, therefore the large dependence of the SN Ia rate on these
functions should be considered when comparing theoretical models with both other theoretical studies and observations.

The large number of detections of SNe Ia, which will even increase significantly in the coming years, will not only place stronger constraints on the rate, but will also create the possibility to further distinguish and quantify different subtypes of SN Ia, which can be linked with different progenitor paths, helping to finally solve the mystery of the progenitor evolution of SNe Ia.

6.3 Uncertainties in stellar and binary physics

As mentioned above, different binary population synthesis studies over the last few years generated inconsistent results. Both Chapters 4 and 5 show that the differences between the results arise because of different assumptions.

In Chapter 5 we test the inherent assumptions of the code used for the research shown in Chapter 4. In this work we compare four different binary population synthesis codes, which are used to study the progenitor evolution of SNe Ia. We study two stellar populations, the population of single white dwarf systems with a non-degenerate companion and the population of double white dwarf systems. The results of the different codes are similar for the single white dwarf population, however small differences are noted. In the double white dwarf population, which is formed after more phases of Roche-lobe overflow, the differences are larger, although we find great similarities between the evolutionary paths leading to this population distinguished by the four groups. We conclude that the diverging results are not because of differences in the numerical treatments but because the codes use different inherent assumptions, such as the criterion to determine whether mass transfer is stable or unstable and different aspects of helium star evolution.

This study helps researchers to understand the conflicting results and to use the differences to constrain binary physics, by hunting for observations which can disprove one of the models. Because the last two chapters provide an overview of which ill-constrained stellar physical aspects have the largest influence on the results of binary population synthesis studies, it should motivate theoreticians to investigate these aspects in more detail. As computer power increases it becomes possible for example to follow the evolution of a common envelope phase in more detail, which this thesis showed is an important facet of population studies. Also observational studies are now trying to better constrain this stellar phase (Davis et al., 2010; De Marco et al., 2011; Zorotovic et al., 2011).

This work should encourage other groups performing binary population synthesis studies to check their inherent assumptions. Binary population synthesis codes contain many ingredients from stellar and binary physics, which influence on stellar populations should be understood to constrain the progenitor evolution of this population. This work gives insight in the stellar and binary aspects which are important also for other stellar populations than SN progenitors.
Anand, S. P. S. 1965, Proceedings of the National Academy of Science, 54, 23
Benetti, S., Cappellaro, E., Turatto, M., & Pastorello, A. 2000, IAU Circ., 7375, 2
Bibliography

Bogomazov, A. I. & Tutukov, A. V. 2009, Astronomy Reports, 53, 214
Bogomazov, A. I. & Tutukov, A. V. 2011, Astronomy Reports, 55, 497
Böhm-Vitense, E. 1958, Zeitschrift fur Astrophysik, 46, 108
Caughlan, G. R. & Fowler, W. A. 1988, Atomic Data and Nuclear Data Tables, 40, 283
Caughlan, G. R., Fowler, W. A., Harris, M. J., & Zimmerman, B. A. 1985, Atomic Data and Nuclear Data Tables, 32, 197
Darwin, G. 1879, Phil. Trans. Roy. Soc., 170, 447
Howell, D. A. 2011, Nature Communications, 2
Bibliography

Laueterborn, D. 1970, Astronomy and Astrophysics, 7, 150
Lindblom, L. 1999, Phys. Rev. D, 60, 064007
Lorimer, D. R. 2008, Living Reviews in Relativity, 11, 8
Maoz, D. & Mannucci, F. 2012, PASA, 29, 447
Maund, J. R. & Smartt, S. J. 2009, Science, 324, 486
Mohamed, S. & Podsidiadowski, P. 2012, Baltic Astronomy, 21, 88
Nariai, K., Nomoto, K., & Sugimoto, D. 1980, PASJ, 32, 473
Neo, S., Miyaji, S., Nomoto, K., & Sugimoto, D. 1977, PASJ, 29, 249
Öpik, E. 1924a, Publications of the Tartu Astrofizica Observatory, 25, 1
Öpik, E. 1924b, Publications of the Tartu Astrofizica Observatory, 25, 1
Bibliography

Reimers, D. 1975, Memoires of the Societe Royale des Sciences de Liege, 8, 369
Roxburgh, I. W. 1965, ZAp, 62, 134
Sarazin, C. L. 1986, Reviews of Modern Physics, 58, 1
Bibliography

Toonen, S., Claeyss, J. S. W., Mennekens, N., & Ruiter, A. J. 2013, ArXiv e-prints
Wang, B. & Han, Z.-W. 2010, Research in Astronomy and Astrophysics, 10, 235
Warner, B. 2003, Cataclysmic Variable Stars
Williams, J. P., Blitz, L., & McKee, C. F. 2000, Protostars and Planets IV, 97
Bibliography

(Vlaamse) Samenvatting

Op 1 mei 1006 verscheen een helder licht aan de hemel, een licht dat zowel overdag als 's nachts zichtbaar was en bleef voor enkele maanden. Dit licht was zichtbaar over de hele wereld en was afkomstig van een explosie van een ster, een supernova. En hoewel deze explosie plaats vond op een afstand 500 miljoen keer verder dan onze zon, kon het licht voor lange tijd als extra nachtlampje gebruikt worden. Het was de helderste supernova ooit waargenomen. De evolutie van sterren naar deze energetische explosies, zoals deze supernova met de naam SN 1006, wordt bestudeerd in deze thesis.

De massa van sterren kan sterk variëren, gaande van een tiende tot meer dan honderd keer de massa van onze zon ($M_\odot$). Hoewel een ster met het blote oog als een onveranderlijke entiteit wordt waargenomen, is deze gedurende haar evolutie onderhevig aan tal van fysische processen zoals bijvoorbeeld opwarmen, afkoelen, uitzetten en exploderen. Daarnaast hebben de meeste sterren een andere ster als begeleider. Hoe massiever de ster hoe groter de kans dat deze een metgezel heeft (Raghavan et al., 2010; Duchêne & Kraus, 2013). Sterren in een dubbelstersysteem kunnen elkaar heel sterk beïnvloeden. Ze kunnen bijvoorbeeld elkaars rotatiesnelheid aanpassen, materiaal afnemen of laten verdwijnen van het dubbelstersysteem en ze kunnen zelfs samensmelten tot één stellair object.

Maar wat is het verwachte einde van een ster? Explosief of vervagen de sterren eerder door ouderdom? En wat is de invloed van een metgezel? Kan deze het leven van zijn kameraad rekken of verkorten of zelf zijn lot helemaal veranderen? Dit is het onderwerp van deze thesis: het explosieve einde van een ster en de invloed van zijn metgezel.

De levensloop van een enkele ster

Sterren worden geboren uit het ineenstorten van een gigantische gaswolk, waaruit honderden tot (soms) miljoenen sterren gevormd worden. Deze gaswolken bestaan voornamelijk uit waterstof, iets minder helium en nog minder zwaardere elementen. Een ster is gevormd wanneer de dichtheid en temperatuur in de kern van de ster hoog genoeg zijn voor de fusie van waterstof naar helium. Dit noemen we in de sterrenkunde waterstofverbranding. Tijdens dit proces komt veel energie vrij. In de ster is er een evenwicht tussen de energieproductie en de hoeveelheid energie die verloren raakt aan het oppervlak van de ster, wat wij zien als het licht van de ster. Er is ook een evenwicht tussen de binnenwaartse kracht, veroorzaakt door
(VLAAMSE) SAMENVATTING

de zwaartekracht, en de buitenwaartse kracht, voornamelijk veroorzaakt door de druk van het gas waaruit
de ster bestaat. De evolutie van een enkelvoudige ster en dus ook het einde van zijn levensloop worden
voornamelijk bepaald door zijn oorspronkelijke massa.

Alle sterren beginnen in hun binnenste lagen (de kern) met de fusie van waterstof naar helium. In
sterren met een massa groter dan 0.8 $M_\odot$ wordt dit vervolgd door de fusie van helium naar koolstof en
zuurstof. Sterren met een beginmassa groter dan ongeveer acht keer de massa van onze zon, massieve
sterren, fuseren het gevormde koolstof dan verder. De fusie van almaar zwaardere elementen gaat door tot
uiteindelijk ijzer gevormd is in de kern, aangezien bij de fusie van ijzer geen energie vrijkomt maar energie
nodig is. Aan het einde van hun leven hebben zware sterren een ajuin-achtige structuur, met de zwaarst
gevormde elementen meer aan de binnenkant van de ster (Fig. 1).

**Figuur 1:** Schematische voorstelling van de ajuin-achtige structuur van een massieve ster aan het einde van
zijn leven, waarbij zwaardere elementen zich in diepere lagen van de ster bevinden. (credits: James
B. Kaler, Scientific American Library, 1992.)

Sterren met een intermediaire massa, met een beginmassa kleiner dan acht keer de massa van de zon,
beginnen ook met de fusie van waterstof in hun kern, maar bereiken de condities niet voor de aanmaak van
ijzer. De meeste intermediaire sterren ondergaan fusie tot ze een kern bestaande uit koolstof (C) en zuurstof
(O) gevormd hebben. Op het moment dat sterke winden al de buitenste lagen hebben weggeblazen blijft
enkel deze compacte CO-kern over. Deze compacte stellaire objecten hebben een hoge dichtheid en zijn
gedegenerereerd ofwel ontaard, wat betekent dat de elektronen een kwantummechanische druk uitoefenen
wat tevens de tegendruk levert voor de zwaartekracht. Hierdoor zijn deze compacte stellaire objecten ook
in evenwicht. De massa van deze objecten is vergelijkbaar met de massa van de zon, terwijl hun grootte
gelijkbaar is met de grootte van de aarde. Vandaar de toepasselijke benaming witte dwergen. Deze
ontaarde kernen koelen geleidelijk af waardoor ze steeds minder zichtbaar worden.

DE INVLOED VAN EEN BEGELEIDER

Dubbelsterstystemen komen voor in verscheidene combinaties, van heel wijde systemen waar de twee
sterren jaren doen over één baanomloop, tot heel dichte systemen waar dit slechts enkele minuten duurt.
Ook de massa’s van de twee sterren ten opzichte van elkaar variëren heel sterk, van twee sterren met een gelijkaardige massa tot een dubbelstersysteem waarbij de lichtste slechts een tiende (of zelfs minder) van de massa heeft van de zwaarste. Waarnemingen tonen aan dat meer dan 70% van de massieve en intermediaire sterren deel uitmaken van een dubbelstersysteem (Kouwenhoven et al., 2007; Sana et al., 2012).

Het al dan niet hebben van een begeleider kan een grote invloed hebben op de levensloop van een ster en dus ook op het einde van het leven van een ster. Zoals de maan invloed heeft op de aarde door getijdewerking, kan ook een ster in een compact dubbelstersysteem effect hebben op zijn metgezel. Het grootste effect wordt verwacht wanneer de sterren materiaal naar elkaar beginnen overdragen. Tijdens deze massa overdracht verandert niet alleen de massa, maar kan ook de rotatiesnelheid van beide sterren veranderen.

**DE EXPLOSIES VAN STERREN**

Er zijn twee soorten supernovae, core collapse supernovae en supernovae van het type Ia. De eerste zijn de explosies van massieve sterren aan het einde van hun leven en de tweede zijn explosies van witte dwergheren.

**CORE COLLAPSE SUPERNOVAE**

‘Core collapse’ betekent het ineenstorten van de kern en dat is dus ook wat er gebeurt aan het einde van het leven van massieve sterren. Wanneer een ijzerkern gevormd wordt, na verscheidene fases van fusie-processen, is verdere verbranding naar zwaardere elementen niet meer energetisch voordelig. Wanneer de temperatuur in de kern hoog genoeg is, desintegreren de ijzeratomen tot uiteindelijk een neutron-rijke kern gevormd wordt. Aangezien bij dit proces energie nodig is, implodeert de kern. Deze implosie kan tegengehouden worden indien de druk van de neutron-rijke kern groter is dan de druk van de buitenste lagen. In dat geval worden de buitenste lagen weggeblazen wat waargenomen wordt als een supernova en er blijft een ‘neutronenster’ achter. Indien de druk van de kern niet groot genoeg is, implodeert de kern verder en wordt er een zwart gat gevormd (Fryer, 2003).

In de loop der jaren werd duidelijk dat er verschillende types bestaan van explosies van massieve sterren. Het eerste onderscheid is gebaseerd op het al dan niet waarnemen van waterstof in de explosie. Dit leidde tot de classificatie van type II en type I supernovae. De volgende onderverdeling kwam door de aan- of afwezigheid van helium in de explosie. Dit leidde tot de classificatie van type Ib en Ic supernovae. Recentelijk kwam ook de ontdekking van supernovae die kort na de explosie gelijkenissen vertoonden met type II supernovae en later meer en meer vergelijkbaar werden met een type Ib supernovae (bijv. Filippenko et al., 1993). Deze werden geclasseerd als type IIb supernovae.

De aangenomen verklaring voor deze verschillen is hoeveel een ster voor zijn explosie van zijn mantel verloren heeft. Aangezien een massieve ster op het einde van zijn leven een ajuin-achtige structuur heeft, worden almaal zwaardere elementen zichtbaar naargelang er meer materie is afgehaald. Dus als bijvoorbeeld de gehele waterstofmantel er is afgepel, verschijnt het heliumlaag en ga zo maar door. Hierdoor kwam men tot het volgende lijstje waarbij de rangschikking zegt hoeveel van de mantel van de ster verwijderd is op het moment dat deze explodeert, met SN II (supernova van het type II) het type explosie waarbij het minste verwijderd is,

\[
\text{SNII} \rightarrow \text{SNIIb} \rightarrow \text{SNIb} \rightarrow \text{SNIc}.
\]
Echter kan de evolutie van enkelvoudige sterren alleen de waargenomen verhouding van de verschillende supernova types niet verklaren (Smith et al., 2011; Langer, 2012). Een andere reden voor de grote diversiteit is dubbelsterevolutie, waar beide sterren materie van elkaar kunnen stelen en laten verdwijnen. In hoofdstuk 2 bestuderen we de evolutie van dubbelstersystemen naar supernovae van het type IIb en vergelijken we dit met modellen voor enkele sterren.

**Supernovae type Ia**

Sterren met een intermediaire massa vormen een witte dwerg aan het einde van hun leven, een object met een hoge dichtheid waarbij de materie ontwaait. In een ontwaaid stellair object is de druk onafhankelijk van de temperatuur, waardoor als er verbranding plaatsvindt de druk zich niet aanpast aan de vrijgekomen energie. Hierdoor blijft de verbranding heel onstabil doorgaan zolang het object ontwaait is.

Er wordt aangenomen dat supernovae van het type Ia het resultaat zijn van de explosies van witte dwergen bestaande uit koolstof en zuurstof. Tijdens de explosie wordt de gehele witte dwerg uiteengeblazen waardoor niets ervan achterblijft. Ook wordt er in de explosie veel ijzer geproduceerd wat met een hoge snelheid verspreid wordt in het heelal. In 1993 kwam men tot de ontdekking dat er een correlatie bestaat met de tijdschaal waarmee de helderheid afneemt van een supernova van het type Ia (Phillips, 1993). Deze ontdekking droeg bij tot de Nobelprijs in 2011, aangezien met deze kennis kon bepaald worden dat het universum niet alleen uitdijt maar ook versneld uitdijt.

Deze evenementen zijn dus heel belangrijk in de astrofysica. Desondanks zijn er nog altijd onde- lijkheden over de ontstaansgeschiedenis van deze types supernova. Een manier om witte dwergen te laten exploderen is wanneer zij de kritische dichtheid bereiken waarbij koolstofzuurstof plaatsvindt. Deze kritische dichtheid komt bij niet-roterende witte dwergen overeen met ongeveer 1.4 $M_\odot$, de Chandrasekhar massa. Waarnemingen en gedetailleerde stellaire modellen tonen echter aan dat pas gevormde witte dwergen bestaande uit koolstof en zuurstof een massa hebben tussen 0.6 en 1.2 keer de massa van de zon (Weidemann, 2000). Vandaar is een begeleider nodig om dit tekort aan massa te compenseren.

![Figure 2: Schematische voorstelling van de twee vooropgestelde evolutiepaden die leiden tot de explosie van een witte dwerg, namelijk a) het single degenerate scenario en b) het double degenerate scenario. (credits: NASA/CXC/M.Weiss)](image)

Verscheidene scenario’s zijn voorgesteld waarmee een witte dwerg 1.4 $M_\odot$ zou kunnen bereiken (Fig. 2). Het eerste houdt in dat de witte dwerg massa ontvangt van een andere ster die nog geen witte dwerg is en zo aangroeit tot de kritische massa. Dit model heet het ‘single degenerate’ scenario (SD, Whelan & Iben, 1973; Nomoto, 1982). Het andere scenario zegt dat er twee witte dwergen, met een massa samen groter dan 1.4 $M_\odot$ samensmelten en dan exploderen. Hier wordt naar gerefereerd als het ‘double degenerate’ sce-
nario (DD, Webbink, 1984; Iben & Tutukov, 1984). Jammer genoeg is het nog niet duidelijk of slechts één van deze of allebei deze scenario’s supernovae type Ia vormen en als beide bijdragen welke van de twee het dominante scenario is (bijv. Ruiter et al., 2009b; Mennekens et al., 2010). Ook is het niet duidelijk of beide scenario’s überhaupt genoeg explosies opleveren en of de WD misschien op een andere manier kan exploderen. Deze prangende vragen wordt behandeld in de hoofdstukken 4 en 5.

**HOOFDSTUKKEN VAN DEZE THESIS**

De hoofdstukken van dit proefschrift geven inzicht in de evolutiepaden die leiden naar verschillende types supernovae, zowel van massieve als intermediaire sterren. Daarbij geven ze ook een overzicht van de onzekerheden in onze kennis van dubbelsterevolutie en de invloed op voorafgaande voorspellingen.

**Hoofdstuk 2: De evolutie via een dubbelstersysteem naar een supernova van het type IIb**

Een supernova van het type IIb wordt de eerste dagen na de explosie waargenomen als een type II supernova en na enkele dagen als een type Ib supernova, wat betekent dat er de eerste dagen waterstof zichtbaar is die later vervaagt en uiteindelijk niet meer zichtbaar is. Dit wijst op een lage hoeveelheid waterstof in de ster op het moment van explosie.

![Diagram](image.png)

**Figuur 3:** Twee mogelijkheden waarop een ster zijn mantel kan verliezen, (a) door een stellaire wind of (b) door massa overdracht naar zijn metgezel (credits: Selma de Mink).

Een ster kan op twee manieren het waterstof in zijn mantel verliezen, namelijk door een stellaire wind of door massa overdracht naar zijn metgezel (fig. 3). Gedurende hun leven hebben massieve sterren heel sterke winden die delen van hun buitenste lagen, de mantel, kunnen wegbreken, en zo diepere lagen tentoon spreiden waar zwaardere elementen liggen. Niettemin worden deze hoeveelheid waterstof over te houden op het moment van explosie is fine-tuning nodig. Moest dit de enige verklaring zijn, zou dit type supernovae zelden voorkomen. In dit hoofdstuk focussen we daarom ook op de dubbelsterevolutie om dit overgangstype te verklaren.

Voor dit onderzoek hebben we een groot aantal dubbelstersysteem gesimuleerd met een gedetailleerde stellaire evolutie code, een code die de gehele evolutie en structuur van een enkele of dubbelster berekent van de geboorte van de ster tot bijna aan het einde van de ster. We vinden dat dubbelsterevolutie de karakteristieken en het aantal waargenomen supernovae van dit type beter kan verklaren dan enkelvoudige
Sterren. Echter voorspellen we via dubbelsterevolutie nog steeds een aantal lager dan waargenomen, zelfs met extreme aannames.

Verder kijken we ook naar de karakteristieken van de al dan niet resterende metgezel en bepalen we de specifieke evolutiepaden naar waargenomen type IIb supernovae. Een voorbeeld hiervan is SN 1993J, de tiende (J) supernova waargenomen in 1993. Deze is geclassificeerd als een supernova van het type IIb (bijv. Woosley et al., 1994) en in 2004 werd de mogelijke verandeleven metgezel ontdekt (Maund et al., 2004), die blauwer was en dus heter dan men verwacht van enkelvoudige sterren. Onze modellen tonen aan dat dit kan verklard worden doordat deze ster heel efficiënt materiaal heeft gestolen (zonder veel te verliezen tijdens de diefstal) van zijn exploderende metgezel.

Door het groot aantal supernovae die men de volgende jaren verwacht te detecteren met de nieuwste telescopen en technieken, kan ons model beter getest worden en daarbij ook onzekerheden in dubbelsterevolutie behandeld worden. Een voorbeeld hiervan is de efficiëntie waarmee een ster het ontvangen/vergelijk materiaal vermengt.

Hoofdstuk 3: Modellen voor het sneller en/of trager roteren van de voorlopers van supernovae van het type Ia
In het SD scenario (zie Fig. 2), wanneer een witte dwerg explodeert na het stelen van voldoende materiaal van zijn metgezel, blijft deze laatste alleen achter. Kort na de explosie verwacht men deze achterblijvers niet te zien, aangezien de helderheid van de explosie vele malen hoger is dan van een ster. Maar de kans deze waar te nemen neemt toe naargelang de helderheid van de explosie afneemt. Verscheidene groepen hebben gezocht naar deze achterblijvers, maar tot nog toe zonder enig succes (Ruiz-Lapuente et al., 2004; Kerzendorf et al., 2009; Schaefer & Pagnotta, 2012). In dit hoofdstuk bespreken we een mogelijke oorzaak waarom deze achterblijvers nog niet zijn waargenomen.

Een witte dwerg die massa ontvangt van zijn metgezel ontvangt ook impulsmoment, wat de snelheid waarmee een witte dwerg roteert, verhoogt. Aangezien de dichtheid van een roterende witte dwerg lager is dan van een niet-roterende witte dwerg, wordt verwacht dat de snel roterende witte dwerg niet direct explodeert wanneer deze 1.4 $M_{\odot}$ bereikt. De witte dwerg explodeert pas wanneer zijn rotatiesnelheid voldoende is afgenomen. Hierdoor ontstaat er een tijdsverschil tussen wanneer de witte dwerg 1.4 $M_{\odot}$ bereikt en explodeert.

Dit heeft niet alleen theoretische gevolgen maar ook observationele, aangezien dit tijdsverschil ook belangrijk is voor de evolutie van de metgezel. Deze kan hierdoor verder evolueren en zal misschien een koele witte dwerg zijn op het moment van explosie. Aangezien de helderheid van een afgekoelde witte dwerg zeker tienduizend keer lager is dan deze van een evoluerende ster, kan dit een verklaring zijn waarom deze achterblijvers niet waargenomen worden.

Hoofdstuk 4: De theoretische onzekerheden in de voorspellingen van het aantal supernovae van het type Ia
Zowel het SD als het DD kanaal (Fig. 2) kunnen het aantal waargenomen type Ia supernova moeilijk verklaren. Daarbij komt nog dat verscheidene theoretische studies conflicterende voorspellingen doen inzake het aantal dat ze voorspellen, maar ook inzake het belang van beide kanalen ten opzichte van elkaar.

Met behulp van een code die dubbelsterpopulaties simuleert (zie uitleg bij hoofdstuk 5), berekenen we
hoeveel supernovae van het type Ia we kunnen verwachten door het SD en het DD scenario. Wij vinden dat het SD kanaal domineert voor supernova type Ia die plaatsvinden kort na de vorming van beide sterren in het dubbelstelsysteem (minder dan 300 Myr), terwijl het DD kanaal domineert op langere tijdschalen (meer dan 300 Myr).

Echter zijn er verschillende onzekerheden in de evolutie van zowel enkelvoudige sterren als dubbelsterren, alsook in de verdeling van beginmassa’s en baanperiodes van de dubbelstelsystemen. In dit hoofdstuk onderzoeken we de effecten van deze onzekerheden op de voorspellingen van supernova Ia. We vinden dat vooral de onzekerheid in de fase van onstabiele massa overdracht grote impact heeft op de resultaten en zelfs op de ratio van het SD versus DD kanaal.

Doordat het aantal waargenomen type Ia supernova almaar stijgt en zo ook het detail dat we krijgen in de verschillen tussen deze supernovae onderling, verwachten we dat we met behulp van dit onderzoek definitieve conclusies kunnen maken over de bijdrage van de verschillende scenario’s. Daarbovenop geeft dit hoofdstuk een overzicht van de effecten van al de onderzochte onzekerheden, wat bijdraagt tot een beter begrip van deze.

**Hoofdstuk 5: PopCORN: de zoektocht naar de verschillen tussen populatie synthese codes**

De resultaten besproken in hoofdstuk 4 werden berekend met behulp van een ‘binary population synthesis’ (BPS) code. Dit type code berekent niet de evolutie en structuur van een ster in detail, maar gebruikt de resultaten van gedetailleerde stellaire codes voor het berekenen van de gehele evolutie van een ster. Dit maakt dat evolutie van een dubbelstelsysteem veel sneller kan berekend worden dan indien deze zou berekend worden met een gedetailleerde code, zoals deze die gebruikt werd voor de berekeningen in hoofdstuk 2 (een milliseconde versus één uur). Dit maakt dat een BPS code veel minder tijdrovend is en ook veel flexibeler. Daar tegenover staat dat een BPS code een toegifte doet op vlak van accuraatheid ten opzichte van gedetailleerde stellaire evolutie codes. Dit maakt dit type code ideaal om naar populaties van sterren te kijken, zoals de populatie van de sterren die evolueren naar een supernova type Ia, maar ook een goed beeld kunnen geven van de onzekerheden in de stellaire fysica en hun effect op de onderzochte populaties.

De laatste drie decennia werden verscheidene BPS codes ontwikkeld, elk met hun eigen filosofieën. Echter geven deze codes onderling vaak conflicterende resultaten. In dit hoofdstuk proberen we te begrijpen wat de oorsprong is van deze conflicten. Worden deze veroorzaakt door verschillen in de aannames van onzekerheden in ster- en dubbelsterevolutie, of door verschillende numerieke methodes waarop de levensloop van deze sterren berekend wordt?

We vergelijken vier verschillende BPS codes, waaronder de code gebruikt voor hoofdstuk 4. Dit project heet het PopCORN-project, wat staat voor ‘Population synthesis of Compact Objects Research Network’, het onderzoeksnetwerk voor populatie synthese van compacte stellaire objecten. Als eerste stap in dit project zijn de aannames in de vier codes zo ver mogelijk gelijkgesteld zodat de inherente verschillen tussen de codes kunnen vergeleken worden. Er worden twee dubbelsterpopulaties bestudeerd, namelijk de populaties waarbij één van de sterren een witte dwerg is en waarbij beide een witte dwerg zijn.

We vinden dat de gesimuleerde populaties tussen de vier codes goed overeenkomen. De kleine verschillen die we zien, kunnen we herleiden tot verschillen in de aannames voor bepaalde (astro-)fysische processen, zoals de stabilititeit van massa overdracht, en deze zijn dus niet te wijten aan verschillen in de numerieke methodes.
(Vlaamse) Samenvatting

Dit onderzoek is een belangrijke stap voor het begrijpen van de conflicterende resultaten. Ook moedigt dit werk andere groepen aan die onderzoek doen met BPS codes om dezelfde controles uit te voeren om zo een overzicht te krijgen van ook hun inherente aanname. Verder in dit hoofdstuk wordt een concreet overzicht gegeven van fysische processen die een grote impact kunnen hebben op de voorspellingen van niet alleen supernovae van het type Ia, maar ook andere populaties en dus beter bestudeerd moeten worden in de toekomst, met behulp van gedetailleerde codes en/of waarnemingen.
MY PUBLICATIONS

REFEREED JOURNALS


• R. Di Stefano, R. Voss, and J. S. W. Claeys. Spin-up/Spin-down Models for Type Ia Supernovae. APJ, 738:L1, September 2011. (chapter 3)


CONFERENCE PROCEEDINGS


MY PUBLICATIONS


I was born the 7th of April 1986 in Ghent, Belgium. I attended the primary school ‘De Bron’ in Loven- degem. This is the time my interest in astronomy started as almost all presentations I gave these years discussed an astronomy-related topic. I continued my school-career in Ghent at the Sint-Bavo Humaniora, which I finished with the specialization in math and science. In addition I followed for eleven years drama school, which included poetry reading (great distinction), elocution (greatest distinction) and repertoire study (greatest distinction).

In 2004 I started the studies Physics and Astronomy at the university of Ghent. I received my Bachelor degree in 2007 with distinction. My bachelor research topic was ‘spectroscopy of Saturn’ under the supervision of Sven De Rijcke. As Ghent University is not specialized in the subject of stellar evolution, where my main interest lies, I did an Erasmus project to follow several courses in the master Astrophysics at the University of Utrecht, the Netherlands. At this university I also started my master research project, with the title ‘Binary progenitor models of type IIb supernovae’ under the shared supervision of Onno Pols (Utrecht University) and Maarten Baes (University of Ghent). I received my master degree with greatest distinction in June 2009. Afterwards I continued working on my thesis research, at first a month at Ghent University, which led to my first publication (Chapter 2). During my university education I was a member of the student association for physics. I was for 1 year vice-president of this association and for two years president of the astronomy group, for which I organized public lectures, observing nights and a trip to several research institutes.

I started my PhD in September 2009 at the University of Utrecht under the supervision of Onno Pols, with Frank Verbunt as my promotor and Jacco Vink as my co-supervisor. I would continue working on the progenitor evolution of supernovae, with the main focus on type Ia supernovae. In 2011 the board of Utrecht University made the terrible decision to close the department of Astrophysics for no good reason. Therefore I continued my PhD from February 2012 at the Radboud University of Nijmegen.

During my PhD I guided two bachelor students en helped two master students with their research projects, where one already has led to a publication (not included in this thesis). Two years on a row I was the main organizer of a prize for high school students which made a research project about astronomy. I was during my PhD involved in several collaborations. For example a collaboration with Rosanne Di Stefano and Rasmus Voss, which started at a workshop in Leiden in 2010. At this workshop we started discussing the possibilities of spinning white dwarfs, which led to a publication (Chapter 3) and a press
Curriculum vitæ

release in the US and the Netherlands which led to an article in NRC Handelsblad, an important newspaper in the Netherlands. At the same workshop me and three other researchers re-started another project (which was never successful so far), the PopCORN-project, where we compared the results of four BPS research groups. I presented my research results in talks or by posters at conferences and workshops in Brussels and Lommel (Belgium), Mykonos (Greece), Paris (France), Leiden and Noordwijkerhout (The Netherlands), Viña del Mar (Chile), Sydney (Australia), Padova (Italy), Garching (Germany). In addition, I gave colloquia at several institutes in England, Germany and other institutes in the Netherlands.
ACKNOWLEDGMENTS

This thesis carries my name, but it is the product of the support of so many people! I would like to thank everyone of you, for all the great discussions, the encouragements and much more.

Vooraleerst wil ik mijn begeleider bedanken voor de vijf jaar lange samenwerking, Onno Pols. Gedurende mijn master- en promotieonderzoek heb je me het vertrouwen gegeven en gemotiveerd om zelf mijn weg te bepalen. Maar je deur stond ook altijd open voor discussie en begeleiding wanneer ik het nodig had. Je hebt mij geleerd hoe een goede en vooral een kritische onderzoeker te zijn.

Natuurlijk wil ik ook mijn promotor bedanken, Frank Verbunt, voor de leuke discussies, zowel over sterrenkunde, geschiedenis en politiek. deze gaven altijd stof tot nadenken. Ook wil ik je bedanken voor het grote vertrouwen die ik van jou gekregen heb.

Ook mijn mede-begeleider in Utrecht, Jacco Vink, die mij ook veel geholpen heeft tijdens mijn promotie. Jij kwam vaak met een andere kijk op sterevolutie waar ik veel van geleerd heb.

I also want to thank the people I collaborated with, to perform the research and write the articles shown in this thesis. Selma de Mink, one of the most important people, who helped me a lot the first years of my PhD. You are like my second supervisor, who guided, trained and most importantly believed in me. You made me a better astrophysicist. Silvia Toonen, like you said we probably annoyed many office mates with all our short meetings, but I really enjoyed working with you and you have become much more than a colleague. Of course many thanks to Rob Izzard who made me a better programmer and (I hope) a better scientific writer. I enjoyed also working with you Rasmus Voss and Rosanne Di Stefano, because I learned so much from our spin-up/spin-down project.

Also special thanks to Carlo Abate, Alex Chiotellis, Evert Glebbeek, John Eldridge, Marc van der Sluys, Jelle de Plaa and Dani Maoz. You gave or helped me work out several ideas which greatly aided my work.

I want to thank all my office mates who I could annoy when I was annoyed (or mainly for the great office-discussions): Bob van Veelen, Carlo Abate, Ines Brott, Jorien van den Bor, Jos de Boer, Emilio Enriquez, Sander ter Veen, Stijn Buitink, Harry Balster and Sander Walg.

All my ex-colleagues from Utrecht who were there at the end and who were a delight to work with: Eveline Helder, Ana Chies-Santos, Esteban Silva Villa, Héctor Cánovas, TIM van Werkhoven, Maria De Juan Ovelar, Alex Chiotellis, Sjors Broersen, Gerard van Harten. I miss all the discussions, the pie-eating, the football and of course the hugs ;-) The astrophysical department of Utrecht was a great one!
Acknowledgments

Of course it was great to start in the astrophysical department of Nijmegen. I was so happy to start in such a nice, joyful group. Everyone was so enthusiastic, there was and is so much discussion and interaction between the different research-fields. Nijmegen provided a very pleasant environment to work. I want to mention everyone, however as we are so many I will definitely forget someone, so I just keep it to ‘thanks to all of you’! However, a special thanks to Sally Macfarlane whose great shoe-support was very important for all the work I have done ;-) . Also thank you Martin van den Akker en Pim schellart for all the help with my (sometimes stupid I have to admit) computer problems. Thank you for the help to remove the annoying Red Hat distribution and replace it by Ubuntu. It made my life so much easier! In addition, Martin van de Akker I also want to thanks for this great thesis-template and for all the times I could bother you when I was finishing the thesis. The secretaries and it-supporting staff both from Utrecht and Nijmegen I also want to thank: Marion Wijburg, Cisca Custers, Marja van Duijvenvoorde, Esther Gebhardt, Sake Hogeveen and Wim Janssen.

Ik wil ook al mij vrienden, familie en schoonfamilie bedanken voor jullie steun de laatste jaren. Sofie Cammaert, zonder jou zou mijn thesis er niet zo leuk uitzien. Super bedankt voor het maken van deze leuke cover! Mijn grootouders: opa van Sleine die mij mijn eerste kleine telescoop kocht die niet echt goed werkte om naar de sterren te kijken maar mij toch motiveerde om meer te lezen over sterrenkunde; opi van Gent, die heel enthousiast was om mij iets bij te brengen over techniek. Natuurlijk waren mijn ouders heel erg belangrijk voor al de dingen die ik de laatste jaren tot stand heb gebracht. Jullie lieten mij de vrijheid om mijn eigen keuzes te maken, soms met vallen en opstaan, en motiveerden met dan ook bij alles wat ik koos. Jullie beiden waren een groot voorbeeld voor mij, waar ik zo trots op ben en waar ik zoveel van geleerd heb. Mijn paranimfen, mijn zussen, dank je voor jullie voortdurende steun!

Als laatste wil ik natuurlijk mijn schattie bedanken, mijn ventje. Ik vind het super dat jij vaak aanmoe-digde om thuis te discussiëren over mijn onderzoek. Jij hebt mijn ups en downs ter lijve ondervonden en overleefd en tegelijk mij zo vaak gemotiveerd, waardoor ik veel beter en sterker ben geworden. Dank je schattie!