Cyclotron resonance is observed for electrons confined in the InAs conduction band whose ground state is shifted to higher energy due to the periodic superlattice potential. These measurements confirm the calculated quantisation of the energy levels in a superlattice by ascertaining the effective mass directly.

A superlattice is a structure consisting of ultrathin, periodic layers of two different semiconductors (layer thickness in the order of 100 Å which is about 20 lattice constants per layer). This artificially introduced one-dimensional periodic potential leads to a strong modification of the original band structure of the host materials in the superlattice direction. The original Brillouin zone is divided into subzones with subzone boundaries at wave-vector values corresponding to the superlattice periodicity. The superlattice as a whole can be considered as a completely new, highly anisotropic material whose electronic and optical properties are determined not only by the band parameters of the basis material, but also by the superlattice periodicity, which can be varied at will. Most of the work up to now has been restricted to the relatively simple structures of GaAs-GaAlAs. Recently a new class of superlattices consisting of consecutive layers of InAs and GaSb has been considered. Shubnikov-de Haas measurements on this type of superlattice show clearly the extreme anisotropy of the superlattice system, and good agreement is found between the measured and the calculated Fermi energy. However, the agreement between the calculated effective mass and the effective mass determined indirectly from the temperature dependence of the Shubnikov-de Haas oscillations was not satisfactory for all samples.

In this paper we report the first observation of cyclotron resonance in a superlattice, and therefore the first direct and accurate determination of the effective mass. The superlattice under investigation was that of InAs and GaSb. This structure is especially interesting because due to the strong energy dependence of the effective mass, its determination provides direct information about the energy levels in the superlattice.

The periodic arrangement of two semiconductors in a superlattice gives rise to a one-dimensional periodic potential. Therefore the problem of calculating the band structure strongly resembles the well known Kronig-Penney problem for free electrons in a rectangular periodic potential. However, in the case of a superlattice, Bloch wave functions should be used instead of free electron wave functions. Taking into account this fact and matching the wave functions of different materials and their derivatives at the successive boundaries of the superlattice potential taken to be along the z-direction, one gets

\[ \cos(k_d) = \cos(k_{d_1}) \cos(k_{d_2}) - \frac{1}{2} \left( \frac{ik_{d_1} + u_{d_1}}{z_1} + \frac{ik_{d_2} + u_{d_2}}{z_2} \right) \sin(k_{d_1}) \sin(k_{d_2}) \]  

Here, the subscripts 1 and 2 refer to the two different materials, \( k_1, k_2 \) are the original \( k \) wave vectors of the host materials, \( k \) is the wave vector of the superlattice, \( d \) is the superlattice periodicity, \( d_1, d_2 \) are the layer thicknesses of the host materials, and \( u_1, u_2 \) are the cell-periodic parts of the Bloch function at an arbitrary boundary. Eq. (1) implies the well known fact that not all original \( k \) values of the host materials are allowed anymore by the superlattice periodicity. More specifically, the lowest allowed \( k \) value is non-zero. Knowing the dispersion relation of the basis materials, the energy bands can be calculated once the allowed \( k \) values are known. Since \( k = 0 \) is not allowed anymore, this leads to a shift in energy of the band edges in a superlattice. For an InAs-GaSb superlattice the lowest conduction subband and the highest valence subband calculated in this way results from matching at the boundaries of the light hole GaSb valence band wave function to the light electron InAs conduction band wave function. Since we are interested in electrons...
only we do not consider the heavy hole valence band which does not couple to the other two bands. The heavy hole bands in a superlattice can be considered separately. The Fermi energy can be obtained by integration of the density of states of each subband, knowing the total electron concentration $n$. This calculated Fermi energy can be verified by means of the Shubnikov-de Haas effect.

Table I summarizes the relevant characteristics of the investigated sample. The carrier concentration and the mobility were derived from Hall measurements. The subband energies and width of the two lowest conduction bands and the Fermi energy were calculated as described above. The material parameters used for this calculation are $0.142 \, \text{eV}$ and $0.81 \, \text{eV}$ for the energy gaps, band edge masses $m^*/m$ of electrons of $0.023$ and $0.048$, and for the light holes of $0.025$ and $0.052$ for InAs and GaSb respectively, while the bottom of the InAs conduction band is taken to be $0.1 \, \text{eV}$ below the top of the GaSb valence band. The Fermi energy is calculated as $E_F = 39 \, \text{meV}$ above the lowest subband edge, which implies that only this band is occupied. This band originates from the InAs conduction band which is shifted to higher energies by an amount $E_1$. This implies that in a cyclotron resonance experiment the observed shift in the InAs conduction band is related to the InAs conduction band mass. All other masses occur at energies relatively far away from the Fermi level. We now focus on this band to calculate the energy levels of the superlattice in a magnetic field. In a two-band model, using the $k \cdot p$ method, the approximate expression of the dispersion relation for the InAs conduction band valid over the energy range of the present experiments, is given by

$$\frac{\hbar^2 k_x^2}{2m_{\text{InAs}}} + \frac{\hbar^2 k_y^2}{2m_{\text{InAs}}} = E - E_1(1 + \frac{1}{E_1}),$$

(2)

where $k$ is the energy measured with respect to the InAs conduction band edge, and $E_1$ and $m_{\text{InAs}}$ are the band gap and the band edge effective mass. In a superlattice and in a magnetic field where both magnetic field and superlattice potential are in the $z$-direction, Eq. (2) is modified in the following way: The bottom of the band in a superlattice is given by Eq. (2) for $k_x = 0$, $k_y = 0$, and $k_z$ has the lowest value allowed by the superlattice periodicity. The energy corresponding to this level is $E_1$, which is just the shift in the InAs conduction band. In a magnetic field, the remaining quantum numbers $k_x$ and $k_y$ are replaced by $(N + \frac{1}{2}) \frac{\hbar}{eB}$; here $N$ is the Landau level number.

Combining with Eq. (2) one obtains the energy levels of a superlattice in a magnetic field

$$\left(\frac{N + \frac{1}{2}}{2} \right) \frac{\hbar}{eB} = \frac{E}{E_0} \left(1 + \frac{1}{E_1} - \frac{1}{E_2} \right),$$

(3)

and an effective mass

$$m^*(E_{1,B}) = m_{\text{InAs}} \left(1 + \frac{E_{1,B}}{E_1} \right),$$

(4)

where $m^*$ is again the effective mass at the bottom of the conduction band of the bulk InAs; $E_{1,B}$ can be calculated from Eq. (3). Therefore, a determination of the effective mass provides a direct means to verify the calculation.

Table I

<table>
<thead>
<tr>
<th>Superlattice sample characteristics</th>
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<tbody>
<tr>
<td>InAs layer thickness</td>
<td>$d_1$ 65 Å</td>
</tr>
<tr>
<td>GaSb layer thickness</td>
<td>$d_2$ 80 Å</td>
</tr>
<tr>
<td>number of periods</td>
<td>$n$ 125</td>
</tr>
<tr>
<td>carrier concentration (Hall effect)</td>
<td>$n_e$ 4.8 $\times 10^{17}$ cm$^{-3}$</td>
</tr>
<tr>
<td>carrier mobility (Hall effect)</td>
<td>$\mu$ 7300 cm$^2$/Vs</td>
</tr>
<tr>
<td>first subband energy (subband width)</td>
<td>$E_1$ $(\Delta E_1)$ 186 (10) meV</td>
</tr>
<tr>
<td>second subband energy (subband width)</td>
<td>$E_2$ $(\Delta E_2)$ 465 (6) meV</td>
</tr>
<tr>
<td>calculated Fermi energy above first subband</td>
<td>$E_F$ 39 meV</td>
</tr>
<tr>
<td>measured Fermi energy above first subband</td>
<td>$E_F$ calc 40 meV</td>
</tr>
</tbody>
</table>

where $E_F$ is the energy measured with respect to the InAs conduction band edge, and $E_1$ and $m_{\text{InAs}}$ are the band gap and the band edge effective mass. In a superlattice and in a magnetic field where both magnetic field and superlattice potential are in the $z$-direction, Eq. (2) is modified in the following way: The bottom of the band in a superlattice is given by Eq. (2) for $k_x = 0$, $k_y = 0$, and $k_z$ has the lowest value allowed by the superlattice periodicity. The energy corresponding to this level is $E_1$, which is just the shift in the InAs conduction band. In a magnetic field, the remaining quantum numbers $k_x$ and $k_y$ are replaced by $(N + \frac{1}{2}) \frac{\hbar}{eB}$; here $N$ is the Landau level number.

Combining with Eq. (2) one obtains the energy levels of a superlattice in a magnetic field

$$\left(\frac{N + \frac{1}{2}}{2} \right) \frac{\hbar}{eB} = \frac{E}{E_0} \left(1 + \frac{1}{E_1} - \frac{1}{E_2} \right),$$

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![Image of Table I](image)
Transmission directly demonstrates the lifting of the bulk conduction band due to the superlattice periodicity.

In order to identify the relevant cyclotron resonance transitions, we show in Fig. 2 the calculated splitting of the subbands as a function of magnetic field using Eq. (3). Transitions occur between conduction band states below and above the Fermi level, and the observed transitions are indicated in the figure. The effective masses at the three resonant fields can be calculated by using Eq. (4), and can be compared with the observed values (Table II). Considering that the calculation contains no adjustable parameters (only commonly accepted literature values for the bulk material are used) and that spin splitting and energy dependence of the $g$-factor are completely neglected, the agreement between measured and calculated masses is quite satisfactory.

![Diagram](image)

**Fig. 1**: Transmission signal for an InAs-GaSb superlattice as a function of magnetic field for different frequencies. The field is parallel to the direction of the superlattice potential. The inset shows the transmission minima as a function of the magnetic field and photon energy. The drawn line represents the fit of an effective mass of $0.043 \, m_0$ to the data.

Effective masses at the three resonant fields can be compared with the observed values (Table II). Considering that the calculation contains no adjustable parameters (only commonly accepted literature values for the bulk material are used) and that spin splitting and energy dependence of the $g$-factor are completely neglected, the agreement between measured and calculated masses is quite satisfactory.

**Table II**

<table>
<thead>
<tr>
<th>Resonance Field $B$ (Tesla)</th>
<th>$m^e$ calculated</th>
<th>$m^e$ measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.95</td>
<td>0.0476</td>
<td>0.044 ± 0.001</td>
</tr>
<tr>
<td>6.39</td>
<td>0.0468</td>
<td>0.042 ± 0.001</td>
</tr>
<tr>
<td>8.16</td>
<td>0.0480</td>
<td>0.044 ± 0.001</td>
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</table>
It should be noted that both the calculated and observed masses are somewhat lower for 6.39 T than for the other two fields. The reason for this may lie in the relative positions of the Fermi energy with respect to the Landau levels: The transition at this field occurs at a somewhat lower energy than those at the other two fields, as can be seen from Fig. 2.

In conclusion it may be stated that for the first time cyclotron resonance in a superlattice has been observed. The measured effective mass can only be explained by taking into account the shift in the energy band brought about by the quantisation caused by the superlattice periodicity. This observation therefore is a direct demonstration of the quantisation of the energy bands in a superlattice.

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