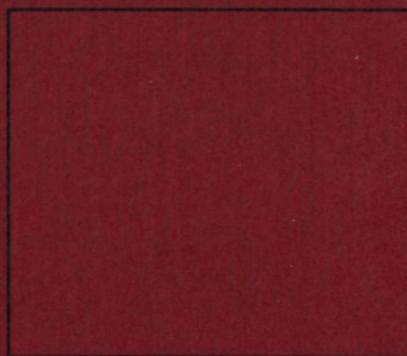
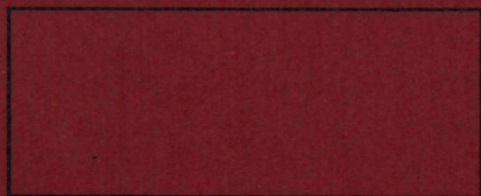
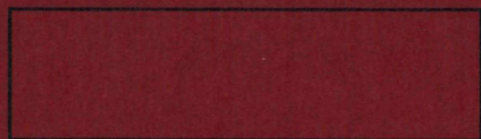
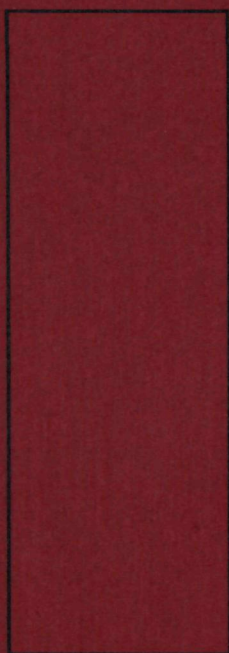
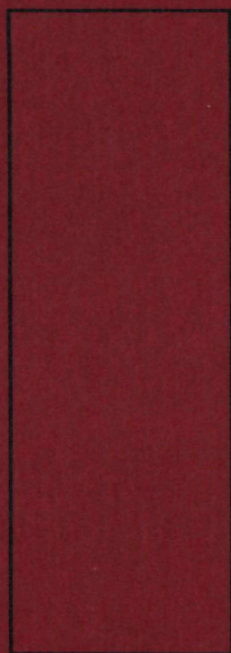
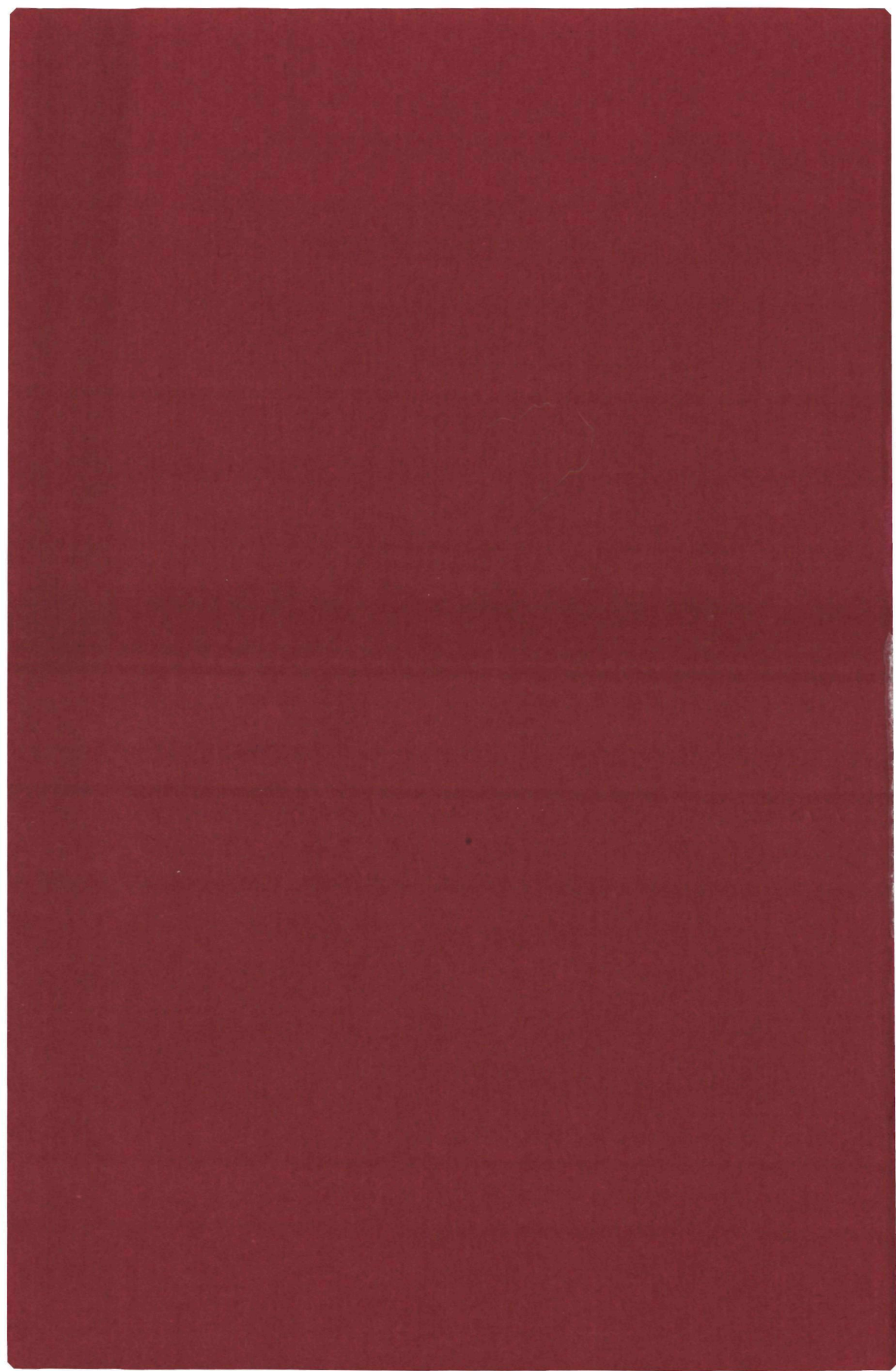


Modelling financial assets

in macroeconomic theory

Rudy van Stratum





Modelling financial assets in macroeconomic theory

druk: WIBRO dissertatiedrukkerij, Helmond

omslag: Rianne Wonnink (bedankt!)

MODELLING FINANCIAL ASSETS IN MACROECONOMIC THEORY

een wetenschappelijke proeve op het gebied van de
beleidswetenschappen

PROEFSCHRIFT

ter verkrijging van de graad van doctor
aan de Katholieke Universiteit te Nijmegen,
volgens besluit van het college van decanen
in het openbaar te verdedigen
op maandag 23 september 1991
des namiddags te 3.30 uur

door

Rudolf Marinus Gerardus van Stratum
geboren op 8 december 1960 te Den Helder

Promotores:

Prof.dr. P.J.L.M. Peters †

Prof.dr. Th.C.M.J. van de Klundert (K.U.B.)

Prof.dr. J.J. Sijben (K.U.B.)

ISBN: 90-9004266-0

VOORWOORD

Persoonlijk heb ik een hekel aan het lezen van voorwoorden. Ik wil het daarom kort houden.

Op de allereerste plaats wil ik mijn ouders, Jeanne en Jan, bedanken voor de stimulering en verzorging die ze me altijd met liefde gegeven hebben. Pas als je geen "echte" problemen hebt, kun je zoveel tijd steken in een proefschrift als dit.

Van de plotseling en veel te vroeg gestorven Piet Peters heb ik enorm veel geleerd. Ik beschouw hem dan ook, als ik dat ouderwetse woord mag gebruiken, als mijn leermeester.

De Tilburgse hoogleraren Van de Klundert en Sijben wil ik bedanken voor de snelle en correcte afwerking van de lopende gang van zaken.

Zonder Lex Meijdam was dit proefschrift, in deze vorm althans, geheel niet tot stand gekomen. Laten we voor het gemak maar stellen dat de goede ideeën van hem zijn en dat de slordige uitwerking ervan voor mijn rekening komt. Misschien vormt veelvuldig aanhalen van zijn werk een tegenprestatie mijnerzijds.

Een inwijding in het gebruik van de programmatuur voor het oplossen van de tweepunts-randvoorwaarde problemen die ten grondslag liggen aan de numerieke simulaties in dit proefschrift, werd geleverd door Gerard Staarink en wederom Lex Meijdam. De hierbij gebruikte techniek van het "veelvuldig schieten" staat nader beschreven in Ascher et.al.(1988). Deze verwijzing schoont mij van de plicht in de lopende tekst nader op de gebruikte techniek in te gaan.

Jan van der Hilst tenslotte, heeft me, waarschijnlijk zonder het te willen, in zijn prikkelende colleges op het spoor van de financierings-macro-synthese gezet.

Den Haag, juni 1991.

CONTENTS

Chapter 0	1
Introduction	1
1 Scope and subject of the study	1
2 Overview	5
3 A simple numerical example regarding chapter 4	11
Chapter 1	25
Introducing financial assets in Crusoe and barter economies	25
1 Introduction	25
2 The separation theorem of Fisher	26
3 The Crusoe model	30
4 The barter economy with shares as a financial asset	35
5 A monetary economy	42
6 Conclusions	54
Appendix	56
Chapter 2	60
The banking firm: the case of perfect competition	60
1 Introduction	60
2 The banking firm	63
3 The goods-producing firm	69
4 The representative consumer	71
5 The clearing of markets	75
6 Simulation of a technological shock	76

7 An increase in the required reserve ratio	83
8 Financial innovations in the banking model	84
9 Conclusions	89
Appendix	90
Chapter 3	94
The banking firm: the case of monopolistic competition	94
1 Introduction	94
2 The banking firm	96
3 The representative consumer	103
4 The markets	105
5 Some remarks about the stationary state	105
6 Simulation of a technological shock	111
Simulation assuming no adjustment costs	112
Simulation in the case of adjustment costs	114
7 Conclusions	116
Appendix	118
Chapter 4	121
The choice between debt and equity	121
1 Introduction	121
2 Firm behaviour	124
3 Consumer behaviour	134
4 Aggregate behaviour	140
5 Introducing a tax on profits	142
Firm behaviour	142

Consumer behaviour	148
Aggregate behaviour	149
6 Some remarks about the numerical simulation	150
7 Some remarks about the stationary state	151
8 Numerical simulation experiments	153
Consumers show greater risk-aversion	154
Firms are faced with a smaller probability of managerial breakdown	156
Firms are faced with a more efficient production process . .	159
A tax on profits is levied on firms	161
9 Conclusions	164
Appendix	166
Chapter 5	169
An open economy: a financial perspective on debt accumulation	169
1 The closed model	169
2 Turning the closed economy into a (small) open one	174
Appendix	182
Chapter 6	184
Summary	184
References	187

INTRODUCTION

1 Scope and subject of the study

This study is focused on applying well developed parts of financial theory (or for that matter: the theory of finance)¹ to the area of macroeconomics. Financial decisions are modelled according to the view that individuals and firms allocate (risky) cash-flows through time to achieve a desired goal. When actors decide on behalf of cash, time and risk, financial assets and capital markets come into play. A financial asset is a claim against some other economic unit, such as an individual or a firm.

It is the synthesis of financial theory and macroeconomics that ties together the different parts of this study. The asset-approach to macroeconomics looks at the behaviour of asset prices as determined by the different market forces. In contrast to the larger part of financial theory, the scope of macroeconomics is of a general character. Macroeconomics is concerned with the interaction between markets, whereas finance is (mainly) con-

¹ There are several textbooks that deal with the matters I refer to. One good example of an overview of "the theory of finance" (as I use the phrase) is Allen(1983).

cerned with price-formation at financial markets.

It is generally believed that financial markets clear faster than non-financial ones. A simplifying assumption of our study is that as a general rule all markets clear all the time. This strong assumption excludes all kinds of price-stickiness which together have formed the subject of another branch in the field of macroeconomics.² This assumption has certainly not been made for analytical convenience. Numerical simulation of theoretical models of the kind studied here is far more complex in the case of clearing markets.

The kind of model best suited to the needs of modelling financial assets in macro perspective is in my opinion the class of intertemporal optimizing models. Two such models are presented in Blanchard/Sachs(1982) and Van de Klundert/Peters(1986). These two models were chosen as a point of reference regarding method and scope. Some ingredients of these models will be discussed now.

Though the roots of the intertemporal optimizing models can be found way back in the sixties,³ it was only in the eighties that the different pieces were put together in one model. The drawback of the approach is the enormous complexity of the models which led to numerical simulation experi-

² See the dissertation of Meijdam(1991b), which deals with matters of price-stickiness, using the same family of models as used in this study.

³ Ramsey(1928) is one of the forerunners using calculus of variations. It was only after the 1962-appearance of Pontryagin's translated work that optimal control techniques could be used. Some examples of the sort are Shell(1967), Fair(1974) and Mussa(1976).

ments. This complexity probably caused the literature to dismiss the approach to a great extent. With the benefit of hindsight it is probably legitimate to say that a more partial approach to the problem has reaped more fruits in the literature. In my opinion the beauty of the Blanchard/Sachs-model lies in the strict adherence to microfounding the actions of different agents in a market context. The price to pay is loss in analytical rigour, but one can easily argue that a lot of rigour can be "observed" from numerical simulation experiments. Parameter sets are not chosen as arbitrarily as may seem at first sight. Sensitivity analyses of the results derived in the Van de Klundert/Peters paper were carried out by Meijdam(1986) and showed some of the "empirical" robustness to be got from simulation. Also, looking at the different tables with simulation results in this study, the same patterns of simulation can be seen again and again regardless of the exact nature of the model.

The most apparent feature of the models is that they are simple and abstract in the sense that as actors in the economic playground they discern producers and consumers only. On top of that, it is assumed that producers and consumers are of the representative type, which does away with much of what should be the matter of macroeconomics. The representative producer and consumer are assumed to be infinitely lived and to have perfect foresight with respect to all variables concerning their optimizing plans. The assumption of perfect foresight is closely related to the rational expectations postulate. Both producers and consumers use all relevant information to reach their decisions. In the absence of uncertainty rational expectations lead to the same thing as perfect foresight. The only thing the representative actors forget is the fact that every now and then the economy is hit by some "unexpected" event. The intertemporally opti-

mizing agents have the goals of utility maximization and profit maximization. These goals are as standard as can be and are copied from any micro-economic textbook available.

The papers of Blanchard/Sachs and Van de Klundert/Peters to a great extent deal with the matters of price-stickiness. As said before, the assumption of price-stickiness is traded for the assumption of clearing markets throughout. Since solving these models analytically is complex and probably impossible within life-time limits, the authors of both papers use numerical simulation as a way out (both papers do not regret this to the same degree, it seems).

To make a study of intertemporal price formation of financial assets a non-trivial one, the concept of adjustment costs is introduced at several places in our study. To assume a combination of clearing markets and no adjustment costs whatsoever would exclude all interesting dynamics. Adjustment costs are introduced only at the level of product markets. It is therefore possible to have markets where real assets are traded showing adjustment costs, while financial markets are perfectly efficient. The process of real investment can be described in this way: the introduction of adjustment costs (it takes time and trouble to install real capital goods) causes a slowdown in the process of reaping the (windfall) profits from new opportunities. At the same time, the price of the relevant security reflects the existence of all future cashflows immediately and correctly. When the prices of securities reflect the values of these securities as implied by the relevant information set, we have efficient capital markets. All relations of the models are stated in continuous time. All numerical simulation experiments carried out in this thesis, use the same parameter set. All models were simulated using the same technological shock to

obtain the possibility of comparing the model-outcomes directly with one another. It should be noted at this point that (traditional) monetary policy has no (real) effects whatsoever in any of the models used.

2 Overview

In chapter 1 we formulate a model where no financial assets exist. This is the famous Robinsonade, where borrowing and lending is not possible. A number of introductory finance books start with the Crusoe case as a means of illustrating the individual consumption/investment decision (see for example Bishop, Crapp, Twite(1984)). The formulation of this model serves as a blue-print for all other models to be discussed. This material is covered at the start of chapter 1.

Allowing Crusoe to borrow and lend to other people in the same circumstances as he is (the postulate of the representative consumer/producer), is modelled by the introduction of a first financial asset called a share of the firm "coconut incorporated". The "cash-flows" in this model are expressed in terms of goods, because there is not yet such a thing as money. The scope of this Crusoe-model (and its extension to borrowing and lending) can also be found in Abel/Blanchard (1983).

The introduction of money allows individuals to store financial wealth in either of two forms: shares or money. A direct consequence of the introduction of money is that actors are confronted with nominal magnitudes rather than real ones. Chapter 1, which deals with the topics discussed sofar, excludes all forms of uncertainty. It is therefore a matter of conven-

ience that the first financial asset is called a share, while it could have been labelled a bond as well. As the concept of dividend plays a central role in clarifying matters, equity seems more appropriate a label. As we distinguish between equity and bonds in chapter 4, it becomes feasible to see the consequences of introducing bonds at the firms level.

In the absence of uncertainty or risk (treated here as different words for the same thing) the appropriate goal of the firm is to maximize the value of equity. Introducing bonds as a separate means of finance to the firm at this level is of no use. Pointing towards the results of Modigliani and Miller (1958) is superfluous, since these results apply to the case where shares are a risky asset from the start. The irrelevance of finance structure in the case of certainty is of pre-Modigliani-Miller status.

In chapter 1 the demand for money is modelled quite crudely, but in broad accordance with two main lines of reasoning in the recent literature. One approach of formally deriving a demand for money is taking it as an argument in the utility-function as if real cash-balances are goods as any other goods. The interpretation of money in the utility-function is that having real cash at hand provides liquidity services. Another approach claims that money is needed to buy goods. The latter approach is catered for by implementing a continuous-time cash-in-advance condition for consumers and firms. Some writers argue that both approaches come to the same thing (of course using the "right" assumptions, see for instance Feenstra(1986)), while it is argued in the book that represents the state-of-the-art at the time of writing that the distinction can be a most sensible one (see Blanchard/Fisher(1989)). In any case, numerical simulations did not show any differences between the two modes of modelling worth mentioning.

The monetary model of chapter 1 incorporates all relevant variables to discuss the matters in the chapters that follow. The simulations of tables 3 and 4 serve as a reference for simulations in later chapters. For simplicity, in later chapters the demand for money will be derived either from the utility-function approach or from the cash-in-advance approach (and not a mixture of both). The supply of money in chapter 1 is assumed to be exogenous to the model.

The subject of chapters 2 and 3 is to make the supply of money an endogenous variable. A banking sector is introduced and the goal of the representative bank is to maximize the market-value of bank-equity. In this manner, another financial asset is introduced apart from shares of goods-producing firms and money. In the spirit of corporate finance, the bank is modelled as any other firm. In order to "produce" money, a bank has access to a production technology with labour and a banking licence as inputs and credit as output. By introducing a banking licence, we cope with the problem of price-level indeterminacy in the economy. The introduction of a banking licence forms another financial asset again. To start a banking business one needs a banking licence and as long as profits can be reaped from banking, the licence has some positive market-value. The problem of price-level indeterminacy is circumvented due to decreasing returns to scale in labour. The latter can be obtained by postulating a constant-returns-to-scale technology in both inputs but assuming the number of banking licences to be of a fixed magnitude.

Chapter 2 assumes perfect competition at the credit-market. The market for credit is cleared by the rate of interest charged on credit. Chapter 3 assumes monopolistic competition at the credit market. Credit is the output of the banking sector, so the credit market is best seen as a product

market. This imperfection at the credit market introduces some exceptions to the general rules described above. In the first place, banking behaviour is to some extent myopic since every representative bank acts as if it is unique in offering a specific product. Secondly, the credit-market is no longer a clearing market by assumption. The phenomenon of non-clearing is not obtained by implementing some arbitrary price-adjustment mechanism however. It appears that the regime of credit rationing is not possible in the absence of adjustment costs to changing the rate of interest. Introducing adjustment costs shows that credit rationing is theoretically possible, but simulation "experience" shows that credit rationing hardly ever shows up.

Chapter 4 introduces debt and equity at the firm's level in a non-trivial way. The Yaari-Blanchard life-time model (see Yaari(1965) and Blanchard(1985)) is used to discuss problems of optimal leverage for the firm. It is only in this chapter that some form of explicit risk is considered. The assumption of perfect foresight is not applicable for this chapter. It is assumed that though any risk is absent at the aggregate level, consumers (think they) are small enough not to be able to buy a riskless portfolio with equity.

The Modigliani-Miller result that the financial structure is irrelevant to the value of the firm in a world without taxes is shown to hold in the model. Because debt is assumed to be of absolute risk-free quality, there is no possibility for management to make debtholders worse off. Therefore the goal of the firm can be expressed as either maximizing the value of equity or maximizing the total value of the firm, since the behavioural relations of the firm will be the same under both regimes.

Introducing a corporate tax (allowing for deductability of interest payments) shows that a tax-shield can be obtained by using leverage. The higher the amount of debt the higher the value of the levered firm will be. The amount of debt is bounded from above by introducing a so-called capital-in-advance condition throughout chapter 4. This additional constraint imposed on management springs from a lack of belief that debt will be repaid totally in the case of financial breakdown of the firm. The capital-in-advance condition guarantees that debt will be repaid without remorse.

Debt is modelled as paying a variable rate of interest during its life-time as would be the case with a floating rate note. As a consequence the market-value of debt always equals the contractual amount of debt (that is: debt can not be sold at a discount).⁴ The framework adopted does not easily allow for studying the impact of changing yield curves on optimal repayment schedules of outstanding debt. The fact that debt can be of different maturity (measured by the concept of duration), though one of the most relevant parts of financial theory, is left out of the analysis altogether. It is superfluous to say that here is ample scope for future research.⁵ When dealing with equity, it is assumed that the firm never issues new shares. Since we do not impose beforehand that dividend payments must be of

⁴ Alternatively, one could say that debt is recontracted infinitely fast. The term to maturity equals zero in such a case.

⁵ One could think of the introduction of flotation costs associated with the issuance of debt. Discriminating between different vintages of debt could make sense in that case.

positive value, in our model paying a negative dividend comes formally to the same thing as issuing new shares.

Other financial assets dominating the finance literature and left out of the analysis altogether, are options (on shares) and futures. Implementing these financial assets at the level of macroeconomic models is a real challenge for future research. I am convinced however that the complexity of the models has reached awful degrees right now. The complexity not only refers to analytical matters but to numerical matters as well. It proved a reasonably hard task to interpret the simulation results of chapter 4.

Reference model:	chapter 1: table 3,4	chapter 2: table 5	chapter 5: table 14
cash-in-advance			
-consumers	yes	yes	no
-producers	yes	no	no
money-in-utility	yes	no	yes
compare with:	chapter 1: table 1,2	chapter 2: table 6,7 chapter 3: table 8,9	chapter 4: table 10,11 12,13 chapter 5: table 15

Finally chapter 5 takes as its starting point that the value of a (small) country can be measured by the net exports of that country. It is shown that

looking at a small open economy as being a big firm financed with shares of equity possessed by the outer world, can deliver a neat balance-of-payments condition. The no-Ponzi-game condition of a small open economy implies for the model of chapter 5 that the shares must have value zero at the beginning of time. Chapter 5 is based on a paper by Meijdam and myself (Meijdam/Van Stratum(1990)).

3 A simple numerical example regarding chapter 4

In order to illustrate some of the points to be made in chapter 4, we follow a numerical example adopted from Moyer, McGuigan and Kretlow(1987). Of course this example is arbitrary to a great extent, but many of these examples can be found in other textbooks on managerial finance, all being the same in spirit.

Looking at the table ("corporate structure without a corporate income tax"), the question is whether the unlevered firm (U) can be better off by going into some degree of leverage. To answer this question, another firm is constructed. This firm L is different from firm U only in capital structure. Both the levered and unlevered firm have equal levels of operating risk and have the same earning power as measured by the net operating income (*NOI*). Firm L is levered with a perpetual debt, B , of 2,000 (say dollars). The setting of the numerical example is a Modigliani-Miller (MM) world without taxes. It is assumed that the financial data presented in the table stay this way for eternity (assuming zero growth).

Capital structure without a corporate income tax

	firm U	firm L
Equity amount	10,000	8,000
Cost of equity	10%	11.25%
Debt amount	--	2,000
Cost of debt	--	5%
NOI	1,000	1,000
Interest payments	--	100
Dividend	1,000	900

We start to calculate the present value of firm U. The cost of equity is assumed to be 10% as a starting point for the example. This cost of equity represents the required rate of return associated with the risk of the unlevered firm. The following perpetuity valuation formula can be applied:

$$(0.3.1) \quad V_U = E = \frac{D}{R_e} = \frac{NOI}{R_e} = \frac{1,000}{0.10} = 10,000 .$$

V_U is the market-value of the unlevered firm and equal to the market-value of equity E . The symbol D represents the dividend of the firm and the symbol R_e represents the return on equity, which equals the cost of equity for the time being.

Turning to the levered case, we have to know what the required return on equity is since the higher debt/equity ratio involves more risk to holders of

equity. MM(1958) and MM(1963) argue with help of the principle of "home-leverage" that the required rate of return on equity depends linearly on the debt/equity ratio according to:

$$(0.3.2) \quad R_e = R_u + (R_u - R_f) \cdot \frac{B}{E} .$$

Here I introduce the symbol R_u to represent the cost of capital in the unlevered case. This symbol serves as a benchmark for the MM-analysis. Assuming no taxes, this cost of capital R_u must be the same as the return on the portfolio of assets in the levered case. Formula (0.3.2) simply is a rewritten version of the definition of the return on the portfolio (consisting of debt and equity). The symbol R_f represents the risk-free rate of return, assumed to be equal to the cost of debt.

We can not compute the cost of equity in the levered case since we do not know the market value of equity. We know that in case of the levered firm:

$$(0.3.3) \quad E = \frac{D}{R_e} = \frac{NOI - R_f \cdot B}{R_e} = \frac{900}{R_e} .$$

Combining the latter two equations, we obtain for the required rate on equity and the value of equity:

$$(0.3.4) \quad R_e = 0.1125 , \quad E = \frac{900}{0.1125} = 8,000 .$$

The value of the levered firm (V_L) equals the market-value of equity plus the market-value of debt, so:

$$(0.3.5) \quad V_L = E + B = 10,000 .$$

The conclusion is that the value of the firm is unaffected by the financial structure. The value of the firm (in either case) is determined by the *NOI* divided by the weighted average cost of capital (*WACC*). Both the *NOI* and the *WACC* can not be changed by leverage. Checking this for the numerical example in case of the levered firm:

$$(0.3.7) \quad WACC = \frac{E}{V} \cdot R_e + \frac{B}{V} \cdot R_f = \frac{8}{10} \cdot 0.1125 + \frac{2}{10} \cdot 0.05 = 0.10 ,$$

$$(0.3.8) \quad V_L = \frac{NOI}{WACC} = \frac{1,000}{0.10} = 10,000 .$$

We see that the return on the portfolio equals the cost of capital of the unlevered firm in the case without taxes.

The next step is the introduction of a corporate tax, as indicated in the following table ("capital structure with a corporate income tax").

As should be clear from the table, interest payments to debtholders is tax-deductible while the corporate tax rate (τ) is chosen to be 40%. In all other respects the firms U and L are the same as the ones from the table without taxes.

The value of the unlevered firm in the case of a corporate income tax is:

Capital structure with a corporate income tax

	firm U	firm L
Equity amount	6,000	4,000
Cost of equity	10%	11.25%
Debt amount	--	2,000
Cost of debt	--	5%
NOI	1,000	1,000
Interest payments	--	100
Corporate tax (40%)	400	360
Dividend	600	540

$$(0.3.9) \quad V_U = \frac{D}{R_e} = \frac{D}{R_u} = \frac{(1 - \tau) \cdot NOI}{R_u} = \frac{600}{0.10} = 6,000 .$$

No comments are necessary here.

Again we have a problem determining the required rate of return on equity in the levered case. Referring to MM(1958 and 1963) once again, the following is stated for the case with a corporate profit tax:

$$(0.3.10) \quad R_e = R_u + (R_u - R_f) \cdot (1 - \tau) \cdot \frac{B}{E} .$$

Though the derivation of this relation is by no means clear from any article that I know of, it is a most important one. A number of remarks con-

cerning this relationship will be made in due time.

The value of equity in the case of the levered firm is:

$$(0.3.11) \quad E = \frac{D}{R_e} - \frac{(1 - \tau) \cdot (NOI - R_f \cdot B)}{R_e} .$$

Using the last two equations solves the required rate of return and the value of equity for the numerical example:

$$(0.3.12) \quad R_e = 0.1125 , \quad E = \frac{540}{0.1125} = 4,800 .$$

The amount of equity for the levered firm in the table (4,000) is not a market-value, since the latter is higher due to the existence of a so-called tax-shield. The fact that the required rate of return on equity is the same for the levered firm with and without taxes is not a matter of coincidence here. Given the exogeneity of the amount of debt, the net operating income, the risk-free rate of interest and the cost of capital for the unlevered firm, the corporate tax rate does not figure in the expression that ultimately determines the return on equity. MM show that the value of the levered firm equals the value of the unlevered firm plus the tax-shield as follows:

$$(0.3.13) \quad V_L = V_U + \tau \cdot B .$$

The fact that interest on debt is tax-deductible leads to a gain of 800 for holders of equity of the levered firm. In fact, the difference in value

between the levered and the unlevered firm is equal to the present value of the (per period) tax-shield from the perpetual debt (see Moyer p.432). This present value is expressed as:

$$(0.3.14) \quad \frac{R_f \cdot B \cdot \tau}{R_f} = \tau \cdot B .$$

The weighted average cost of capital is in the levered case with a corporate tax:

$$(0.3.15) \quad WACC = \frac{E}{V} \cdot R_e + (1 - \tau) \cdot \frac{B}{V} \cdot R_f .$$

This can be calculated for the example as:

$$(0.3.16) \quad WACC = \frac{48}{68} \cdot 0.1125 + \frac{20}{68} \cdot 0.6 \cdot 0.05 = \frac{6}{68} .$$

The value of the levered firm in the case of a corporate tax can also be computed as:

$$(0.3.17) \quad V_L = \frac{(1 - \tau) \cdot NOI}{WACC} = \frac{600}{1} \cdot \frac{68}{6} = 6,800 .$$

Now there can be a problem in the case with taxes. It is not clear from the start what rate of return can be expected from holding the levered firm, being the portfolio of debt and equity. A starting point in making matters clear is the fact that (for the first time) there are three parties that receive

money springing from the earning capacity of the firm. These three parties are holders of debt and equity and (the third party) the government that levies the taxes. At this stage we introduce some imaginary new financial asset that gives right to the reception of the money levied from taxes. The market value of this asset could be computed theoretically from the integral of all transfers over time discounted by the relevant rate of return. Denoting the market value of the transfers by S , and the required rate of return by the symbol R_{ov} , the steady state formula for the third asset is:

$$(0.3.18) \quad S = \frac{T}{R_{ov}} = \frac{\tau \cdot (NOI - R_f \cdot B)}{R_{ov}} .$$

The symbol T represents the per-period amount of tax assumed to be redistributed by government to consumers. For the numerical example discussed so far S has a value of 3,200 (the total of assets has to be of value 10,000). The per-period value of taxes is 360, so the computed required rate of return for the transfer asset is 11.25%.

The next that can be stated without problem is that the total return of holding all three assets must be equal to the cost of equity of the unlevered firm. Adding the values of three relevant assets in the case of debt and taxes should deliver the value of the firm in the case without debt and taxes. So we have:

$$(0.3.19) \quad R_u = \frac{E}{E+B+S} \cdot R_e + \frac{B}{E+B+S} \cdot R_f + \frac{S}{E+B+S} \cdot R_{ov} .$$

Now the problem can be seen easily. The fact is that the value of the government transfers is not necessarily under the control of the consumer. When consumers look upon the gifts from government as falling from heaven, the "required rate of return" of the third (imaginary) asset could be the risk-free rate of return. If holders of debt and equity look upon the firm after taxes as being exposed to the same risk as the firm before taxes, the required rate of return on the portfolio (consisting of debt and equity only) would be equal to the symbol R_f . Making assumptions on behalf of these matters solves the problem in principle. This seems to be a matter of arbitrary choice open to the designer of the model.⁶

Though I believe several "plausible" strategies are open to the specification of the complete macroeconomic model of chapter 4, one of the main concerns is "reproducing" standard Modigliani-Miller results now and in chapter 4. When the consumer sees through the model, he can easily detect that the gifts from the government are taken away from "his" firm. When the net operating income of the firm drops as a result of depressing economic factors, the amount of governmental transfers will drop too. As a result, it seems very reasonable to discount the transfers of government

⁶ The following illustrates our point: "The Internal Revenue Service can be considered as just another security holder, whose claim is essentially an equity one in the normal sense of events (but which can also take on some of the characteristics of secured debt when things go badly and back taxes are owed). Securities, after all, are just ways of partitioning the firm's earnings; the MM propositions assert only that the sum of the values of all the claims is independent of the number and the shapes of the separate partitions." (Miller(1988), p.111)

at the same rate of return as the stream of dividends. This insight adds a constraint to our problem:

$$(0.3.20) \quad R_e = R_{ov} .$$

The latter condition combined with condition (0.3.19) delivers the famous MM-relation for the required rate of return for equity as described in (0.3.10). Another way of writing the "required rate of return" for transfers is:

$$(0.3.21) \quad R_{ov} = R_u + (R_u - R_f) \cdot \tau \cdot \frac{B}{S} .$$

The latter is written down to show the similarity of the MM relation expressing the required rate of return for equity. The risk associated with the "holding" of the transfer-asset depends on the degree of leverage of the firm. Since the risks attached to holding equity and the transfer-asset are the same, the values of both differ only because of the fact that different portions of the net operating income of the firm are expected.

The assumption of equal return on equity and transfers implies that the return of what is commonly called the consumer's portfolio (equity and debt) is less than the return of the unlevered firm. For the numerical example we see that the return on the portfolio equals:

$$(0.3.22) \quad R_p = \frac{48}{68} \cdot 0.1125 + \frac{20}{68} \cdot 0.05 = \frac{64}{680} < 0.10 .$$

This phenomenon is quite logical as the risk of the standard portfolio is not as high as the risk of the unlevered firm. The total of risk is shared between equity and government transfers.

The results so far are used in the complete macroeconomic model of chapter 4. The consequence is that the budget constraint of the model of consumers has to be written in terms of the total "enlarged" portfolio, being the addition of debt, equity and transfer-assets. The present value of the tax-shield as stated by expression (0.3.14) can be derived given the relations so far. Though utterly irrelevant, the expression for the present value of the tax-shield could as well be written as:

$$(0.3.23) \quad \frac{R_u \cdot B \cdot \tau}{R_u} = \tau \cdot B .$$

This relation is derived for the macroeconomic model in chapter 4.

To illustrate the far-reaching consequences of assuming that consumers see (unjustly) upon the transfers as appearing out of thin air in a random fashion (not in any way connected with the firm), we continue the numerical example. The macroeconomic model requires for the stationary state that the return on the portfolio equals the exogenous rate of time preference (symbol: ν) (see relevant chapters). Assuming that only the amounts of debt and equity are under the control of consumers, we want the system to generate:

$$(0.3.24) \quad R_p = R_u = \nu = 0.10 .$$

In that case the correct present value of the tax shield is:

$$(0.3.25) \text{ present value of tax-shield} = \frac{R_f \cdot \tau \cdot B}{R_u} .$$

In that case the required rate of return for equity reads:

$$(0.3.26) R_e = R_u + (R_u - R_f) \cdot \frac{B}{E} .$$

This relation holds strong whether there are taxes or not.

We use this relation for the required rate of return on equity to revisit the numerical example of this section. Assuming that the cost of capital of the unlevered firm has to be equal to the return on the portfolio, we have:

$$(0.3.27) R_e = 0.10 + (0.10 - 0.05) \cdot \frac{2,000}{E} .$$

The other relation for equity is:

$$(0.3.28) E = \frac{D}{R_e} = \frac{(1 - \tau) \cdot NOI}{R_e} = \frac{540}{R_e} .$$

Combining these two relations delivers:

$$(0.3.29) R_e = \frac{54}{440} , \quad E = \frac{540}{\frac{54}{440}} = 4,400 .$$

The value of the levered firm becomes:

$$(0.3.30) \quad V_L = E + B = 6,400 \text{ .}$$

The present value of the tax shield now is:

$$(0.3.31) \quad V_L - V_U = 0.4 \cdot \frac{0.05}{0.10} \cdot 2,000 = 400 \text{ .}$$

It can now easily be checked that the rate of return on the (levered) portfolio stays the same at a rate of 10 percent:

$$(0.3.32) \quad R_P = \frac{44}{64} \cdot \frac{54}{440} + \frac{20}{64} \cdot 0.05 = 0.10 \text{ .}$$

The weighted average cost of capital is:

$$(0.3.33) \quad WACC = \frac{44}{64} \cdot \frac{54}{440} + 0.6 \cdot \frac{20}{64} \cdot 0.05 = 0.09375 \text{ .}$$

As before, it remains true that the value of the levered firm equals the net operating income after taxes divided by the WACC.

$$(0.3.34) \quad V_L = \frac{(1 - \tau) \cdot NOI}{WACC} = \frac{600}{0.09375} = 6,400 \text{ .}$$

The value of the transfer-asset is 3,600 (total value of assets equalling 10,000). The computed required rate of return on the transfer-asset is therefore 10%. The consumer attaches a risk to the transfers equal to the

risk belonging to the levered firm.⁷ The one-period tax-shields from leverage must be discounted at the portfolio's total return. The interpretation of this result could be that the extra money gain from leveraging can be reinvested at the rate of return on the portfolio (consisting of debt and equity only).

We originally stated the model of chapter 4 in terms of the assumption that the return on the portfolio (debt and equity) equals the cost of equity in the unlevered case. It served to show that for numerical simulation it does not matter that much. It seems to me however that the MM-version is the version that presupposes the most of rationality and "see-through" of agents. For that reason, equations (0.3.24) to (0.3.34) must be considered irrelevant for the chapters to come.

⁷ Now it is about time to quote the other half of the MM-couple. It was only after I had written this chapter that I came across an article of Modigliani(1988) (thanks to B. Hasselman).

"In other words, if one accepts the reasonable notion that the appropriate discount rate for $\tau \cdot R_f \cdot B$ is R_u rather than R_f , then the correction paper and its "definitive" corrections need never have been written: Personal taxation aside, the definitive truth was all in MM (though the original way of establishing the result was defective). Of course this is somewhat of an exaggeration since it would be foolhardy to claim that R_u is the appropriate way to discount the tax saving under all circumstances and tax regimes." (Modigliani(1988), p.153) I have taken the liberty to change the symbols used by Modigliani into (I hope) corresponding symbols of this chapter.

INTRODUCING FINANCIAL ASSETS IN CRUSOE AND BARTER ECONOMIES

1 Introduction

The role and impact of the introduction of two financial assets is studied in an infinite horizon general equilibrium model. The modelling is done in the tradition of intertemporal optimization of firm and household problems in a deterministic setting under the assumption that expectations are formed rationally. A representative firm and household are assumed to avoid any serious aggregation problems. The chapter evolves from the simplest barter economy without borrowing and lending (the Robinson Crusoe model) to a full-fledged monetary economy, this idea being derived from a textbook on macroeconomic theory (Hadjimichalakis(1982a)). Clearing markets are assumed throughout, while the impact of the same technological shock is demonstrated in the models with the help of numerical simulation.

The case of introducing borrowing and lending is demonstrated in the Fisher separation theorem. A two-period case is used as a starting point for our more complex infinite horizon model. The separation theorem concludes that the objective rate of interest, determined by demand for and

supply of loans, determines the range of possibilities when maximizing net worth and utility. The main drawback of the Fisher model is that the determination of the rate of interest at the loanable funds market is left out of the analysis. We repeat the Fisher analysis at a general equilibrium level by the introduction of a financial asset in a Crusoe economy. In this situation a firm becomes a net borrower of funds and a household becomes a net lender of funds, while the rate of interest clears the financial asset market.

The next step is the introduction of money in the barter economy with borrowing and lending. Money is looked at as a financial asset with its own characteristics. The necessity of money in the optimal portfolio of households and firms is underpinned with a liquidity constraint on certain transactions (transactions demand for money) and with Keynesian motives (precautionary/speculative demand for money). Money supply is assumed exogenous and constant. The monetary economy can be compared with the barter models when money demand resulting from liquidity preference is denied. In that case we also obtain a version of the famous quantity theory of money.

2 The separation theorem of Fisher

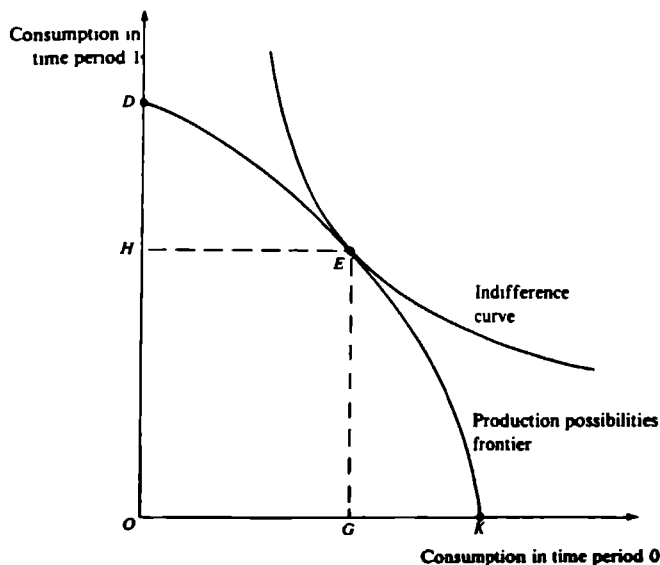
Before discussing the model under consideration it is instructive to look at a simpler case. The plot of the story is well-known and can be found at several places in the literature (see for instance Hirshleifer(1958) and Sutcliffe and Bromwich(1986)). Assume Robinson Crusoe lives for two peri-

ods in a deterministic environment. When Robinson starts to live he is endowed with a certain bundle of commodities, say coconuts. Now Robinson is free to choose how many of the coconuts he wants to consume today (period 0) and how many he wants to consume tomorrow (period 1). Just laying the nuts aside is always inferior to putting the nuts in a hole in the ground and reaping the fruits next period. Non-consumption of period 0 can be converted to consumption of period 1 along a production possibilities frontier (PPF) as shown in the figure. Goal of Robinson is maximizing intertemporal utility, some function of consumption in periods 0 and 1. Suppose that the highest attainable level of utility is reached at point *E* in the figure.

Now it is straightforward to see that Robinson will consume *OG* coconuts in the first period, while putting *GK* of them in the ground. The investment of *GK* coconuts brings him a consumption of *OH* coconuts in the second and last period.

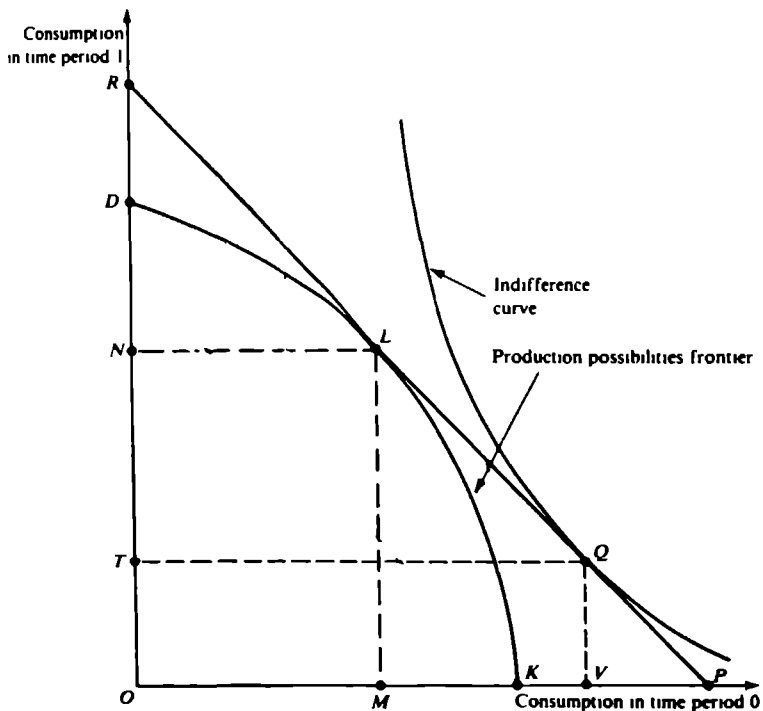
Now let us introduce a financial market. Robinson faces a market for borrowing and lending, that is, he can trade any bundle of commodities (consisting of today's and tomorrow's coconuts) against time. Given an *exogenous* interest rate, the value of his coconut-plant is known with certainty. Every point along the PPF brings along a different value of his plant. A wise thing for Robinson to do seems to be to maximize the value of his plant in the first place. The line *RP* in the next figure indicates the highest attainable net present value of "coconut incorporated".

Optimal consumption is somewhere along this line and guarantees that the intertemporal budget constraint is met. Now Robinson maximizes his utility by consuming *OV* coconuts today and *OT* coconuts tomorrow. A quantity of *MK* coconuts is put in holes, so there is a shortage of *MV* coconuts



today. The latter quantity can be lent at the inter-isle coconut market when he promises to pay back $(1 + R)$ times MV coconuts of tomorrow. The process of lending and borrowing can be envisaged here by the introduction of bonds made payable in coconuts.

Now at least two conclusions can be drawn. In the first place Robinson gets better off when borrowing and lending is possible. In the second place we have seen that Robinson derives his optimal plans in a two-step procedure: Robinson as a producer maximizes the net present value of his firm and Robinson as a consumer maximizes his intertemporal utility. The two decisions are completely separated. The latter result is referred to as the



Fisher separation theorem.

Our intention now is to generalize these results for an infinite horizon general equilibrium setting. The main drawback of the Fisher-model is the exogeneity of the interest rate.

3 The Crusoe model

There is only one person, even Friday is missing (in a very unfriendly way Friday can be seen as capital stock for Crusoe). There is absolutely no market where exchange takes place. Now Robinson likes two things: consumption of coconuts and (afterwards) doing nothing. To obtain the desired coconuts Robinson must stroll along the beach and this (production) process takes time (and consequently: leisure). Robinson has an continuous endowment of leisure of l_m units of time. Spending l units of time searching for coconuts leaves $(l_m - l)$ units of free time. Robinson's preferences can be described by an intertemporal utility function in consumption (c) and leisure $(l_m - l)$:

$$(1.3.1) \quad U = \int_t^{\infty} u(c, l_m - l) \cdot e^{-v \cdot (z-t)} dz ,$$

where v denotes the exogenous rate of time preference.

The output of the production process (y), the nuts found at the beach, can be either consumed immediately (c) or put in holes in the ground, that is invested (j) :

$$(1.3.2) \quad y = c + j .$$

This equation denotes equality of supply and demand for coconuts.

Just building an inventory of coconuts is not allowed; putting them into the ground is more profitable anyway. The more nuts in the ground, the more likely it is to find nuts (falling from the new trees) in the next peri-

od(s). The search- or production-process can then be modelled by a production function:

$$(1.3.3) \quad y = f(l, k) ,$$

where k denotes the stock of accumulated investments. This physical capital stock depreciates at the constant rate of δ . The capital accumulation can be described by:

$$(1.3.4) \quad \dot{k} = i - \delta \cdot k ,$$

where i denotes investment net of installation costs.

Total investment can be decomposed into investment net of installation costs and installation costs,¹ where the latter can be described by a function $h(i, k)$:

$$(1.3.5) \quad j = i + h(i, k) .$$

Installation costs are introduced to derive a well-behaved investment function, this method being standard by now. Total life-time utility is maximized with respect to consumption and leisure, subject to the accumulation equation and the condition that guarantees that total production equals consumption plus investment, using Pontryagin's maximum principle. The following first order conditions for an optimal solution can be found:

¹ This is a form of the adjustment costs mentioned in chapter 0.

$$(1.3.6) \quad u_c = x ,$$

$$(1.3.7) \quad u_{(l_m - l)} = x \cdot f_l ,$$

$$(1.3.8) \quad q = x \cdot (1 + h_i) ,$$

$$(1.3.9) \quad \dot{q} = (v + \delta) \cdot q - x \cdot (f_k - h_k) .$$

The symbol x is a Lagrange multiplier associated with equality constraint (1.3.2). The symbol q stands for the costate variable associated with the accumulation equation (1.3.4) and can be interpreted as the marginal utility of capital.

The real wealth of Crusoe (a) equals his capital stock (k) , the only asset in his portfolio:

$$(1.3.10) \quad a = k .$$

In contrast with the barter economy of the next section, there is no market-value for the (coconut)-firm of Crusoe. The reason is pretty obvious: Crusoe is the sole owner of the firm and there is no market for equity. When some unexpected technological innovation is discovered by Crusoe, he will probably feel richer because he has got a greater earning capacity now and in the future. His real wealth however is determined by history and consists of the existing physical capital stock. Crusoe cannot trade the coconuts over time and his prosperous new situation can be checked by

looking at his life-time utility, which will be higher when innovation takes place.

It is interesting to look at a numerical simulation of such a technological shock. In the first place the stationary state of the economy is determined, preferably in a non-numerical way. The stationary state of the model exhibits saddlepoint stability for the chosen parameter values. Stationary state and parameter values can be found in the appendix. The following specifications for the utility function, the production function and the function that describes the installation costs are used:

$$(1.3.11) \quad u = \frac{\gamma_c}{\gamma_c + \gamma_l} \cdot \ln(c) + \frac{\gamma_l}{\gamma_c + \gamma_l} \cdot \ln(l_m - l) ,$$

$$(1.3.12) \quad f(l, k) = \varepsilon \cdot \left\{ \alpha \cdot k^{\frac{\sigma-1}{\sigma}} + (1 - \alpha) \cdot l^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}} ,$$

$$(1.3.13) \quad h(i, k) = \frac{(i - \delta \cdot k)^2}{2 \cdot \psi \cdot k} .$$

The latter two function specifications are homogenous of degree one, which implies:

$$(1.3.14) \quad f_l \cdot l + f_k \cdot k = f(l, k) ,$$

$$(1.3.15) \quad h_i \cdot i + h_k \cdot k = h(i, k) .$$

The technological innovation is symbolized by a shift in the parameter ε from 0.25 to 0.26. Table 1 shows the outcome of the simulation of the Crusoe economy. The evolution from the old to the new stationary state is shown by the time paths of the percentage deviations of all variables from their old stationary state values.

Table 1 A technological shock in a Crusoe economy

period → variable ↓	0	1	2	5	10	stationary state
k (=a)	0	0.32	0.62	1.35	2.18	3.50
q	-1.38	-1.86	-2.29	-3.35	-4.54	-6.36
U	41.85	42.71	43.48	45.39	47.54	50.88
c	4.16	4.41	4.63	5.19	5.81	6.79
i	3.40	3.42	3.43	3.45	3.48	3.50
j	3.45	3.45	3.46	3.47	3.48	3.50
l	-0.03	0.00	0.03	0.11	0.19	0.32
y	3.98	4.17	4.34	4.76	5.23	5.96
x	-4.00	-4.22	-4.43	-4.93	-5.49	-6.36

As one can see from the table, the capital stock is the only variable that is determined by history. Optimal capital accumulation takes time as a result of the introduction of installation costs. Welfare (as indicated by life-time utility) jumps instantaneously to a higher level, which is due to a higher consumption pattern over time. Initially the more productive economy offers more leisure for Crusoe, but later on Crusoe works more than before

and is compensated for this disutility through consumption of extra nuts. Part of the extra production is used for investment and capital stock gradually builds up over time.

To ensure comparability with the macro-economy to be described in the next section, the Crusoe-economy must be thought of as scaled up to a many-individuals-economy, where each individual lives on his isle and is totally isolated from the other isles.

4 The barter economy with shares as a financial asset

We introduce the possibility of trade over time: one individual can move goods over time in order to get an optimal intertemporal consumption planning. Intertemporal trade is carried out not in terms of money but in terms of goods now against bonds or shares denominated in future goods. This possibility is introduced by making a distinction between households and firms. There is no money and consequently there are no nominal prices. For simplicity, we call the new financial asset a share of the firm. Ownership of a share guarantees real dividend payments. Consumers/workers are owners of the firm and have all existing shares in portfolio. No new shares are issued, which means that in case the firm faces negative cashflows dividend payments are of negative sign. Workers get paid a real wage w . Maximum consumption for an individual consumer/worker is:

$$(1.4.1) \quad c = d + l \cdot w + \text{net sales of shares} ,$$

where the symbol d stands for real dividend payments.

For the economy as a whole it holds true that net sales of shares is zero. For the representative consumer net sales of shares must therefore be zero. The representative firm is a net lender of goods, the representative consumer is a net borrower of goods. The interest rate guarantees that all shares are willingly held by consumers. In other words the share-market does "work" in such a way that the interest rate brings ex ante equilibrium at this market. The interest rate can be said to be determined on a market for "loanable funds".

This time there is a labour market and it is assumed that according to classical doctrines the real wage rate clears the labour market at all times. Two prices, the interest rate and the real wage rate, are sufficient to clear three markets, labour market, goods market and financial market, this being an application of Walras' law.

It is not straightforward to compute the rate of interest that clears the financial market, because the interest rate lurks in the background of the economy. The real rate of interest is not paid to anyone in a literal sense. As said above, the rate of interest is the rate of return that is required to keep consumers willing to hold the existing shares in portfolio. The return of holding shares consists of dividend payment and the increase in the market value of shares:

$$(1.4.2) \quad r = \frac{d + \dot{e}}{e} ,$$

where e stands for the market value of shares and \dot{e} for the increase of the market value of shares.

The value of the consumer's portfolio (a) consists of the value of shares only, capital stock being one of the underlying elements of the value of shares:

$$(1.4.3) \quad a = e .$$

Now from (1.4.1) to (1.4.3) we derive the intertemporal budget constraint for consumers:

$$(1.4.4) \quad \dot{a} = r \cdot a + l \cdot w - c .$$

The consumer's optimization problem can be stated as follows. Maximize intertemporal utility as stated in (1.3.1) with respect to consumption and leisure under consideration of the intertemporal budget constraint (1.4.4). The wage rate and interest rate are treated as given for the representative consumer. By forming the Hamiltonian function of this problem and using Pontryagin's maximum principle, we get:

$$(1.4.5) \quad u_c = x ,$$

$$(1.4.6) \quad u_{(l_m - l)} = w \cdot x ,$$

$$(1.4.7) \quad \dot{x} = (v - r) \cdot x .$$

The symbol x stands for the costate variable associated with the dynamic budget constraint and can be interpreted as the marginal utility of a unit of real wealth. Though not easy to detect, this variable contains the same

information as the symbol x in the Crusoe model.

The producer's problem is to maximize market value of shares, which equals the net present value of dividend payments:

$$(1.4.8) \quad e = \int_t^{\infty} \left\{ d \cdot e^{-\int_t^z r(s) ds} \right\} dz .$$

Dividend equals total output of the firm minus wages paid to workers minus total investment:

$$(1.4.9) \quad d = y - w \cdot l - j .$$

Maximization of (1.4.8) takes place with respect to employment and investment under consideration of accumulation of capital according to (1.3.4). By applying the same principle as above, we find the following necessary conditions:

$$(1.4.10) \quad f_l = w ,$$

$$(1.4.11) \quad \dot{q} = 1 + h_i ,$$

$$(1.4.12) \quad \dot{q} = (r + \delta) \cdot q - f_k + h_k .$$

The symbol q again stands for the costate variable associated with the accumulation equation and can be interpreted as the marginal (real) profit of investment (note the difference with the symbol q in the section on Cru-

soe).

The real wage rate can be solved by equating labour supply from the consumer's problem and labour demand from the producer's problem. Using the fact that total demand for goods must equal total supply of goods according to (1.3.2), the computation of the interest rate becomes feasible as it clears the market for shares. Loosely speaking, the firm faces the dilemma of investment (retained earnings) versus dividend payment, whereas the household faces the dilemma of consumption versus savings. The famous equality of savings and investment is thus the outcome of opposing forces in the economy. Now we have formulated the complete system of a barter economy with shares as a financial asset.

It is interesting to see how this economy responds to the same technological shock as was imputed on the Crusoe model. The variable q now has a clear interpretation as the ratio of market value of shares to the value of capital stock, which equals Tobin's average q (see for instance Haya-shi(1982) and Precious(1987)).² The latter statement implies that the time paths of financial wealth (=value of equity) and capital stock go their own way, this in contrast with the Crusoe model. Using (1.4.11) and the specification of installation costs, the investment function can be written as a function of q :

$$(1.4.13) \quad i = \delta \cdot k + k \cdot \psi \cdot (q - 1) \ .$$

² Marginal and average q coincide in this classical market-clearing model. In the next section, after introduction of money in the classical model, we have a problem here.

It can be seen that a value of q other than 1 causes net (dis-)investment. Now it is also clear that in the absence of adjustment costs on investment (ψ tends to infinity) the rate of adjustment from the old to the new optimal capital stock is infinitely fast, whereas in the case of infinitely high adjustment costs ($\psi = 0$) net investment is always zero.

Two new variables, reflecting the existence of new markets, are the real wage rate and the real rate of interest. Table 2 shows the numerical simulation of a technological innovation in the barter economy with equity as a financial asset. The first striking result is that all variables but q and a (for reasons mentioned above) have exactly the same value as in table 1. Just generalizing the Fisher result for the infinite horizon general equilibrium case clearly gives false insights. It is not the rate of interest that determines the consumption possibilities but the other way round: the rate of time preference determines the rate of interest. It is quite easy to see that one can tie together the producer's and consumer's problems of the barter economy with the financial asset to obtain results that are in line with the Crusoe results.

Table 2 describes an economic process altogether different from the one of table 1. The technological innovation spurs investment activity as can be seen from Tobin's q . The value of equity during adjustment is greater than the value of capital stock. The value of equity immediately jumps to a higher value, because of a more profitable future. Equity is therefore an important forward looking indicator of the economic system. Firms struggle for means to invest (retained earnings) and this can be seen as the main reason for higher interest rates over time. Real wages are in line with higher productivity of labour that accompanies the higher capital stock. The real wage rate always equals the marginal product of labour, while it

Table 2 A technological shock in a barter economy

period → variable ↓	0	1	2	5	10	stationary state
k	0	0.32	0.62	1.35	2.18	3.50
q	2.72	2.47	2.23	1.66	1.01	0.00
U	41.85	42.71	43.48	45.39	47.54	50.88
a	2.72	2.80	2.87	3.03	3.22	3.50
c	4.16	4.41	4.63	5.19	5.81	6.79
i	3.40	3.42	3.43	3.45	3.48	3.50
j	3.45	3.45	3.46	3.47	3.48	3.50
l	-0.03	0.00	0.03	0.11	0.19	0.32
y	3.98	4.17	4.34	4.76	5.23	5.96
r	2.49	2.25	2.04	1.51	0.91	0.00
w	4.04	4.42	4.76	5.62	6.58	8.10
x	-4.00	-4.22	-4.43	-4.93	-5.49	-6.36

is only in the new stationary state that the marginal product of capital (net of depreciation) equals the real rate of interest.

5 A monetary economy

We introduce a second financial asset called money. But from what sources can supply and demand for money be derived? One could argue on sound reasons that in deterministic models of the kind presented above there is no need for money at all. Of course we can apologize by saying that economic agents carry out plans by acting in some respects as if there is uncertainty. Money does facilitate trade and is a way of reducing the (implicit) costs of walking to the barter markets and search for the right partner to trade with. In order to keep the model simple and in line with historical development of economic theory, the supply of money is treated here as exogenous. It is the subject of chapters 2 and 3 to derive an endogenous supply of money by the introduction of a value-maximizing bank.

Demand for money springs mainly from two motives: a transactions demand for money and a demand that embodies all (other) kinds of uncertainty and is usually called precautionary and/or speculative demand for money. The first motive has a classical origin, whereas the second motive is more or less of "Keynesian" nature. A straightforward and simple manner to introduce money in a barter economy would be to use some version of the well-known quantity theory of money. The volume of output is determined as in the barter economy, the income velocity of money is assumed to be constant and hence a one-to-one relation exists between money and prices. The problem with this approach is that the quantity of money is held by no one, it just is there when needed. This can be circumvented by imposing a liquidity constraint upon households and firms.

The money market is assumed to be in a state of equilibrium all the time, which implies the equality of money supply (\bar{M}) and money demand, which consists of transactions demand for money (M_t) and precautionary/speculative demand for money (M_v) :

$$(1.5.1) \quad \bar{M} = M_t + M_v$$

The precautionary/speculative demand for money is modelled by assuming that money used for the purpose (expressed in real terms) gives direct utility to consumers. Consumers maximize intertemporal utility, which now is:

$$(1.5.2) \quad U = \int_t^{\infty} u(c, l_m - l, \frac{M_v}{P_y}) \cdot e^{-v \cdot (z-t)} dz ,$$

where P_y denotes the price level of the homogenous goods in this economy. For instantaneous utility (u) a more general version of (1.3.11) is used for numerical simulation:

$$(1.5.3) \quad u = \frac{\gamma_c}{\gamma_c + \gamma_l + \gamma_m} \cdot \ln(c) + \frac{\gamma_l}{\gamma_c + \gamma_l + \gamma_m} \cdot \ln(l_m - l) + \frac{\gamma_m}{\gamma_c + \gamma_l + \gamma_m} \cdot \ln(\frac{M_v}{P_y}) .$$

When the parameter γ_m is set to zero, we have (1.3.11) again.

The transactions demand for money can be modelled by imposing a cash-in-advance constraint upon households and firms. The idea of cash-in-advance was originally motivated by the fact that government-issued cur-

rency (which we assume is the case with \overline{M}) must be acquired prior to purchasing goods. The latter suggests that a discrete time model best fits the bill and this tradition was started by the work of Lucas(1984), who meant to embody a version of the constraint on transactions recommended by Clower(1967). Essential is to capture the notion that in monetary economies money buys goods and goods buy money. As we use a continuous-time model we propose a continuous-time version of the Lucas cash-in-advance constraint, which implies that some cash must be held in "advance" all the time because transactions are carried out continuously.³ In this way we hope to cater for both mainstream approaches to modelling

- ³ It is a relevant question whether transactions are meant to denote goods transactions or some broader transactions concept (including exchange of financial assets). The idea that money is needed to carry out financial transactions can be implemented in the model by adding constraints such as:

$$\eta_1 \cdot c \cdot P_y \leq M_{t-real} ,$$

$$\eta_2 \cdot E \leq M_{t-financial} .$$

Now there are different velocities of circulation for different "monies". Two interesting optimality conditions for the problem are:

$$u_c = \left(1 + \frac{\eta_1}{1 + \eta_2} \cdot R \right) \cdot P_y \cdot X ,$$

money demand: the currency in the utility function approach and the cash-in-advance approach (see Sargent(1987) and Blanchard/Fisher(1989)).⁴ Total nominal wealth of households (A) consists of the nominal value of shares (E) and total money holdings ($M_v + M_{th}$):

$$(1.5.4) \quad A = E + M_v + M_{th} .$$

We state the consumer's problem. Maximize intertemporal utility (1.5.2), subject to the intertemporal budget constraint (1.5.5) (stated in nominal terms this time) and the cash-in-advance constraint (1.5.6):

$$(1.5.5) \quad \dot{A} = R \cdot (A - M_v - M_{th}) + l \cdot P_l - c \cdot P_y ,$$

$$(1.5.6) \quad c \cdot P_y \leq M_{th} ,$$

$$\dot{X} = \left(v - \frac{1}{1 + \eta_2} \cdot R \right) \cdot X .$$

⁴ The reader who is interested in the differences between cash-in-advance and money-in-the-utility-function has to compare table 5 of chapter 2 and table 14 of chapter 5. The models used are exactly the same for the two tables, except for the specification of the demand for money on behalf of consumers. There are absolutely no differences to be detected that are worth mentioning. It does not seem relevant to argue much about which of the two methods to choose for numerical simulation.

with respect to consumption, leisure and real balances. Upper case symbols are used to express their nominal character, i.e. they are stated in money terms (the exception to this is U). New symbols are P_t , which is the nominal wage rate, and M_{th} , which is the transactions demand for money by households. Households take the time paths of good prices, nominal wage rates and the nominal interest rates as given. We derive the following conditions in the same manner as before:

$$(1.5.7) \quad u_c = (1 + R) \cdot P_y \cdot X ,$$

$$(1.5.8) \quad u_{(l-h)} = P_l \cdot X ,$$

$$(1.5.9) \quad u_{M_y/P_y} = R \cdot P_y \cdot X ,$$

$$(1.5.10) \quad M_{th} = c \cdot P_y ,$$

$$(1.5.11) \quad \dot{X} = (\nu - R) \cdot X .$$

Note that the cash-in-advance constraint is always binding. Comparing (1.5.7), (1.5.8) and (1.5.11) with the results of the analogous barter problem (1.4.5) to (1.4.7), shows that, apart from nominal changes, the interest rate now figures in the expression for the marginal utility of consumption due to the extra cash-in-advance constraint. The idea behind this is that a consumer loses the amount of money he spends on goods, which is of course always the case, but additionally he foregoes a return on the amount of money he has to keep in cash.

The producer's problem is to maximize nominal market value of shares, the equivalent of expression (1.4.8):

$$(1.5.12) \quad E = \int_t^{\infty} \left\{ D \cdot e^{-\int_t^z R(s) ds} \right\} dz .$$

Dividend equals total receipts of money minus total cash-outlays:

$$(1.5.13) \quad D = y \cdot P_y - l \cdot P_l - j \cdot P_y - Z ,$$

where Z is the change in cash-holdings:

$$(1.5.14) \quad \dot{M}_{tf} = Z .$$

M_{tf} represents the amount of money the firm holds for transactions purposes.

The firm faces a cash-in-advance constraint with respect to the purchase of investment goods:

$$(1.5.15) \quad j \cdot P_y \leq M_{tf} .$$

The value of the firm as expressed by (1.5.12) is maximized with respect to employment, investment and cash-holdings, subject to accumulation of capital (1.3.4), accumulation of cash-holdings (1.5.14) and the liquidity constraint (1.5.15). This leads to the following insights:

$$(1.5.16) \quad f_l = \frac{P_l}{P_y} ,$$

$$(1.5.17) \quad Q = P_y \cdot (1 + R) \cdot (1 + h_i) ,$$

$$(1.5.18) \quad \dot{Q} = (R + \delta) \cdot Q - P_y \cdot (f_k - (1 + R) \cdot h_k) ,$$

$$(1.5.19) \quad M_{if} = j \cdot P_y ,$$

Comparing the results with (1.4.10) to (1.4.12), the main difference we see, again ignoring the nominal differences, is the effect of the cash-in-advance constraint, which is always binding, and the fact that a rate of return is foregone on idle balances.

To obtain the same investment function as described by (1.4.13), we have to redefine our uppercase Q as follows:

$$(1.5.20) \quad q = \frac{Q}{P_y \cdot (1 + R)} .$$

Average and marginal q no longer coincide in our market-clearing monetary economy. The reason for this result is that the liquidity constraint is always binding for the firm. For the barter economy, the marginal productivity of capital, f_k , equals $r + \delta$ in the stationary state, while the value of marginal and average q equal 1. As these results are standard, it is interesting to see how they change in case of a liquidity constraint. From (1.5.17) and (1.5.18), we have for the stationary state of the cash-in-advance econ-

omy:

$$(1.5.21) f_k = (R + \delta) \cdot (1 + R) \cdot$$

In general a relationship exists between average and marginal q that states that marginal q equals average q minus the discounted value of all costs and benefits associated with the constraints (see for instance Precious(1987), p.63). Worked out for the stationary state of the cash-in-advance model, we obtain:

$$(1.5.22) \text{ average } q = \text{marginal } q + \frac{R}{1 + \delta} \cdot$$

Another way of writing this is:

$$(1.5.23) \frac{E}{k \cdot P_y + M_{t,f}} = 1 + \frac{R}{1 + \delta} \cdot$$

The result that the constrained firm is associated with a higher average q than the unconstrained firm, is most peculiar. This result can also be found in Meijdam(1991a) for the more intelligible case of a price setting firm that faces a demand constraint.

The nominal wage rate is derived from equating labour supply and labour demand. The price of goods is determined to clear the market for goods. Now we have two markets left: the market for shares and the "new" money market. We only need one to determine the rate of interest. According to historical doctrine it is the loanable funds theory that considers the rate

of interest to be the price of loans, which is determined by the demand for and supply of loans, whereas it is the liquidity preference theory that considers the rate of interest to be the price of money, determined by the demand for and supply of money. We support the vision that the two theories are not opposed to one another but merely say the same thing in other words. In the end it is misleading to speak of different prices determined at different markets as the economic system consists of a set of markets that interact continuously. It is true that *ex ante* reasoning forces to mention the driving forces at separate markets.

Furthermore it is confusing to discriminate between stock- and flow-approaches to the problem. There are no separate markets for old and new bonds. A correct statement of the problem seems to be that the single demand curve together with the single supply curve determine the price of bonds, be it that these demand and supply curves contain all relevant information of past, present and future (realisations and expectations). A final relation necessary to solve the complete monetary model is that total demand for transactions-money consists of consumers' and producers' demand, so that money market equilibrium reads:

$$(1.5.24) \quad \bar{M} = M_{th} + M_{tf} + M_v .$$

In the classical case that $\gamma_m = 0$, together with the optimal cash-in-advance results (1.5.10) and (1.5.19) and good-market clearing, we obtain a quantity-theoretic result:

$$(1.5.25) \quad \bar{M} = y \cdot P_y .$$

Table 3 A technological shock in a monetary economy ($\gamma_m = 0$)

period → variable ↓	0	1	2	5	10	stationary state
k	0	0.33	0.62	1.37	2.24	3.70
Q	-1.16	-1.59	-1.97	-2.93	-4.02	-5.81
U	64.56	65.96	67.22	70.37	73.99	79.96
A	-0.84	-0.92	-0.99	-1.16	-1.36	-1.69
X	-0.21	-0.26	-0.30	-0.41	-0.53	-0.74
c	4.15	4.40	4.63	5.20	5.86	6.95
P _y	-3.83	-4.01	-4.18	-4.58	-5.05	-5.81
j	3.46	3.48	3.50	3.55	3.61	3.70
l	-0.03	0.00	0.03	0.10	0.19	0.32
y	3.98	4.18	4.36	4.80	5.31	6.17
R	0.51	0.46	0.42	0.31	0.19	0.00
P _l	0.06	0.26	0.44	0.90	1.43	2.31
E	-1.11	-1.22	-1.31	-1.56	-1.85	-2.33
M _{th}	0.16	0.21	0.26	0.38	0.52	0.74
M _{tf}	-0.51	-0.67	-0.82	-1.19	-1.62	-2.33

By this time we have formulated the complete monetary economy which is an intertemporal, micro-founded version of the IS-LM model with full-employment.

Differences between the numerical stationary states for the monetary economy and the barter economy can be found in the appendix. This com-

Table 4 A technological shock in a monetary economy ($\gamma_m=0.05$)

period → variable ↓	0	1	2	5	10	stationary state
k	0	0.33	0.63	1.38	2.24	3.70
Q	-1.14	-1.59	-2.00	-3.01	-4.16	-6.04
U	96.50	98.61	100.53	105.30	110.77	119.75
A	-0.73	-0.81	-0.89	-1.07	-1.29	-1.64
X	-0.21	-0.24	-0.26	-0.32	-0.38	-0.49
c	4.15	4.40	4.63	5.20	5.86	6.95
P_y	-3.81	-4.01	-4.20	-4.66	-5.18	-6.04
j	3.47	3.50	3.52	3.56	3.62	3.70
l	-0.03	0.00	0.03	0.11	0.19	0.32
y	3.99	4.18	4.36	4.81	5.32	6.17
R	0.27	0.25	0.22	0.17	0.10	0.00
P_l	0.08	0.26	0.42	0.83	1.29	2.06
E	-1.09	-1.22	-1.34	-1.64	-1.99	-2.56
M_t	0.03	0.00	-0.02	-0.07	-0.14	-0.24
M_v	-0.06	-0.01	0.04	0.15	0.28	0.50

parison cannot be made when the parameter γ_m has a value other than zero, because different utility functions apply in these cases. The monetary economy is clearly inferior to the barter economy as utility is lower. This result is not surprising: part of portfolio consists of money, a non-interest bearing asset. Consumption, production and employment are lower in the

monetary case. The monetary economy is money-neutral, no real effects occur at either short or long term. We start with the classical case (no precautionary/speculative money demand) of the monetary economy, which is best suited to be compared with the barter economy. Table 3 describes the effects of a technological shock in the monetary economy. Global effects are not different from tables 1 and 2. We see that the price of goods falls in order to take away the (ex ante) excess supply of goods. Total money stock is constant over time, which means that fluctuations in output are counterbalanced by fluctuations in price. Household balances increase over time as total consumption outlays increase, despite lower price of consumption. Nominal investment outlays decrease over time, which means less cash-in-advance will be held by firms (excess cash is paid out as dividend). In effect investment is (more or less) rationed because idle cash must be held by firms. Table 4 describes the effect of a technological shock when part of money demand springs from Keynesian liquidity preference. As said before, comparison with other tables must be done with care. Global effects are again roughly the same. Most important is that extra money demand out of liquidity preference leaves less money for transaction purposes. This implies that either output or prices (or the combination) must decline, compared to the case without precautionary/speculative money demand.

6 Conclusions

The purpose of this chapter was to develop some basic models that could serve as references for the chapters to come. The simplest model is the Crusoe-model, where lending and borrowing is not possible. As a consequence, there are no financial assets in a Crusoe-economy. The next model contained one financial asset, called a share. At this point, we made the distinction between producers or firms on the one hand and consumers or households on the other hand. Consumers are in possession of the shares of the firm. The goal of the firm is to maximize the value of equity or shareholders' wealth. Households keep maximizing intertemporal utility. Households offer labour to the firm and sell shares when not satisfied with the firm's performance. While the decisions in the Crusoe-economy were in one hand, they are not anymore in the shares-economy. Introducing borrowing and lending in a Crusoe-model makes no difference however for the time-paths of the variables, be it that extra (market)-information is reflected in the real wage rate and the real rate of interest. This may be so from a formal point of view, economic reasoning behind the models with and without shares as a financial asset is completely different. A share market introduces the possibility of obtaining Tobin's q and deriving a sensible investment function. We saw that the subjective rate of time preference is more likely to be responsible for the determination of the (objective) interest rate than the other way round, as was claimed to be the case in the Fisher model.

Finally, we introduced money which led to the formation of nominal prices. In effect the real wage rate is split into a nominal wage rate and a nominal good price, while the real rate of interest "vanishes" to make way

for a nominal rate of interest. Whether the rate of interest is determined at the market for loanable funds or at the money market (loanable funds versus liquidity preference) does not seem to be a relevant question. A quantity theory of money is obtained by imposing a liquidity constraint on both households and firms. It is interesting to see that all simulations of this chapter show the same picture of an economy that is hit by an unexpected technological shock. It is fair to assume that roughly the same results are generated by a fairly broad range of models and parameter sets.

APPENDIX

The Crusoe model, table 1

c		$u_c = x$
l		$u_{l_t-l} = f_l$
y		$y = f(l, k)$
i		$q = x \cdot (1 + h_i)$
j		$j = i + h(i, k)$
x		$y = c + j$
u		$u = u(c, l_m - l)$
a		$a = k$
k		$\dot{k} = i - \delta \cdot k$
q		$\dot{q} = (\nu + \delta) \cdot q - x \cdot (f_k - h_k)$
U		$\dot{U} = \nu \cdot U - u$

Specifications used in simulation: (1.3.11), (1.3.12) and (1.3.13).

The barter model, table 2

c		$u_c = x$
l_s		$u_{l_s-l} = w \cdot x$
l_d		$f_l = w$
w		$l_d - l_s = l$
y		$y = f(l, k)$
i		$q = 1 + h_i$
j		$j = i + h(i, k)$
r		$y = c + j$

$$\begin{array}{ll}
u & | \quad u = u(c, l_m - l) \\
e & | \quad e = a \\
d & | \quad d = y - w \cdot l - j \\
k & | \quad \dot{k} = i - \delta \cdot k \\
q & | \quad \dot{q} = (r + \delta) \cdot q - f_k + h_k \\
a & | \quad \dot{a} = r \cdot a + l \cdot w - c - r \cdot a - d \\
x & | \quad \dot{x} = (v - r) \cdot x \\
U & | \quad \dot{U} = v \cdot U - u
\end{array}$$

Specifications used in simulation: (1.3.11), (1.3.12) and (1.3.13).

The monetary model, tables 3 and 4

$$\begin{array}{ll}
c & | \quad u_c = (1 + R) \cdot P_y \cdot X \\
l_s & | \quad u_{l_s - l} = P_l \cdot X \\
l_d & | \quad f_l = \frac{P_l}{P_y} \\
P_l & | \quad l_d - l_s = l \\
y & | \quad y = f(l, k) \\
i & | \quad Q = P_y \cdot (1 + R) \cdot (1 + h_i) \\
j & | \quad j = i + h(i, k) \\
P_y & | \quad y = c + j \\
M_v & | \quad u_{\frac{M_v}{P_y}} = R \cdot P_y \cdot X \\
M_{th} & | \quad M_{th} = c \cdot P_y \\
M_{tf} & | \quad M_{tf} = j \cdot P_y \\
M_t & | \quad M_t = M_{th} + M_{tf} \\
R & | \quad \bar{M} = M_v + M_t
\end{array}$$

$$\begin{array}{lcl}
u & | & \dot{u} = u(c, l_m - l, \frac{M_v}{P_y}) \\
E & | & \dot{E} = A - M_v - M_{th} \\
k & | & \dot{k} = i - \delta \cdot k \\
Q & | & \dot{Q} = (R + \delta) \cdot Q - P_y \cdot (f_k - (1 + R) \cdot h_k) \\
A & | & \dot{A} = R \cdot (A - M_v - M_{th}) + l \cdot P_l - c \cdot P_y \\
X & | & \dot{X} = (v - R) \cdot X \\
U & | & \dot{U} = v \cdot U - u
\end{array}$$

Specifications used in simulations: (1.5.3), (1.3.12) and (1.3.13).

Parameter values:

$$\begin{array}{lll}
\alpha & = 0.25 & \gamma_c = 0.85 & l_m = 9.0 \\
\varepsilon & = 0.25 & M = 1.00 & \sigma = 0.40 \\
\gamma_m & = 0/0.05 & \delta = 0.10 & v = 0.10 \\
\psi & = 0.125 & \gamma_l = 0.10 &
\end{array}$$

Stationary state:

Crusoe/barter		Monetary economy (if $\gamma_m = 0$)		Monetary economy (if $\gamma_m = 0.05$)	
k	= 3.397	k	= 3.284	k	= 3.284
q	= 0.839/1.0	Q	= 0.810	Q	= 0.543
U	= 1.061	U	= 0.732	U	= 0.492
a	= 3.397	A	= 3.660	A	= 2.784
c	= 1.013	X	= 1.073	X	= 1.519
j	= 0.340	c	= 1.030	c	= 1.030
l	= 7.112	P_y	= 0.736	P_y	= 0.494
y	= 1.352	j	= 0.328	j	= 0.329
r	= --/0.10	l	= 7.440	l	= 7.440
w	= --/0.095	y	= 1.358	y	= 1.359
x	= 0.839	R	= 0.10	R	= 0.10
		P_l	= 0.063	P_l	= 0.042
		E	= 2.902	E	= 1.947
		M_t	= 1.000	M_t	= 0.671
		M_v	= 0.000	M_v	= 0.329

THE BANKING FIRM: THE CASE OF PERFECT COMPETITION

1 Introduction

In chapter 1 we introduced two financial assets, equity and money. The relevant distinction that can be made between equity and debt at the level of firms is only relevant when uncertainty is taken into account. The trade-off between risky equity and riskless debt will be discussed in chapter 4. In chapter 1 we discussed two approaches of deriving a demand for the financial asset money. We did not bother too much about the very crude manner of modelling the demand for money, since this topic is not discussed in great detail in the area of managerial finance either. The supply of money was assumed to be exogenous to the model. It seems quite logical to use the finance framework to derive an endogenous supply of money, however. It is a surprising thing to see that not much work has been done on the matter. We formulate the requirements of the model to fit into this study as follows.

There is some institution (or behavioral entity) that has the (technical) ability to produce a very liquid asset. Let us call this very institution a bank. Just as we did not bother about the question why a firm could produce goods or why it had access to some production function, we do

not bother about the question why a bank has access to a banking technology. Given this technology, it is clear that the relevant goal of the bank is the maximization of the value of bank-shares.¹ What really matters is how to specify the production technology and the banking environment. Just assuming a traditional production function with labour and capital as inputs and (real) money-supply as output, gives the well-known problem that the price-level of the economy is indeterminate. In order to get rid of this problem, we arrange diminishing returns to scale in one of the inputs. Assuming labour and capital are accepted inputs to the banking technology, we want capital to denote financial capital. What physical capital is to the firm that produces goods, financial capital is to the firm that produces money. When banks are forced to buy a financial asset (called a banking licence) in order to start banking, we have a perfect analogy to the physical capital of other firms. Decreasing returns to scale in labour are obtained by stating that the government, the implicit issuer of the licences, has issued a fixed number of licences once in history.²

¹ It should be clear that agency problems are circumvented in our model. For a dynamic theory of the banking firm that emphasizes agency problems see O'Hara(1983). The latter states that a theory of the banking firm should incorporate the roles of a bank as a financial intermediary, a firm (presumably run to benefit its stockholders) and a regulated enterprise. Though I am not convinced that these requirements are necessary and/or sufficient, our model incorporates these elements to a certain extent.

² Baltensperger(1980) gives an interesting overview of theories of the

On behalf of the demand for money we simplify matters by adding a cash-in-advance constraint to the consumer's problem only.³

The market where money demand and money supply meet will always be cleared. The representative bank faces a given rate of interest and is a price-taker consequently. In chapter 3, we look at the same model under a money-market regime of monopolistic competition.

Many ideas implicitly or explicitly used in the model of this chapter can be found in the economic literature. None of the literature, as far as we know, fulfils all of the requirements set out above. In constructing the model, we benefited notably from Saving(1977), Niehans(1978), Fama(1980), Hadjimichalakis(1982b) and Santomero(1984).

banking firm. He distinguishes between "partial" banking models (in the sense that the size of the bank's portfolio is exogenous) and "complete" models. Our model is of the latter type and could be sub-labelled as a "real-resource" model. The prominent role of the "real-production" aspects of the banking process is seen to be an important feature of a banking model since the amount of real resources absorbed by the banking industry is of quite substantial order of magnitude (see Baltensperger(1980), p.2).

³ This chapter is based on an earlier paper by Meijdam and myself (see Van Stratum(1989)). At that time, we had non-clearing at the market for goods by the assumption of sticky prices. The modelling of the demand for money by putting it into the utility function gave rise to a number of problems associated with the determination of the rate of interest.

2 The banking firm

A bank is just like any other firm, be it that the produced output has a number of special features. The product supplied by the banking firm is called credit and supplied only to consumers. So, consumers and producers alike accept this indebtedness of banks as an ultimate means of payment and as such, credit belongs to the total stock of money. In order to produce these banking services one needs some ingredients. Just as a normal firm needs (for instance) capital (k) and labour (l) as inputs to produce goods (y), the institution "bank" needs labour and a banking licence to produce credit in our case. The rationale behind the need for labour is that every consumer who wants credit has to travel to the bank in order to arrange things, talk to officials, sign contracts etcetera. People that work at the bank to serve clients act as labour input in the bank's production function. The other input, the banking licence, can be compared with capital as input in the case of a standard firm. There are a few important differences however. In the first place, capital can be accumulated over time as a result of the investment decisions of the firm. It is assumed that the licences to bank are available in a strict limited and fixed quantity. The reason is that money as a product is something special related to such phrases as reliability and trustworthiness. Another reason is that the government as a(n) (implicit) supplier of the licences, can control the money supply to a certain extent by adding some specific requirements to the possession of the banking licence (such as a cash-reserve requirement).

A second difference with capital is that we assume that one and only one banking licence is needed to start banking. Earlier on capital and labour could be substituted for one another according to a CES specification.

Now we have for the banking process:

$$(2.2.1) \quad \frac{M_c}{P_y} = \begin{cases} g(l_b) & \text{when banking licence} \\ 0 & \text{when no banking licence} \end{cases}$$

where M_c denotes the nominal supply of credit and $g(\cdot)$ is the production function with banking labour (l_b) as input.

The availability of a banking licence thus defines which firm is a bank. Because one of the inputs is of fixed magnitude, it is assumed that the production of the real supply of credit shows diminishing returns to scale in labour. Of course, the product of the bank is not for free. The price of a unit of credit is the nominal rate of interest. Labour hired by the bank is paid the nominal wage rate, assumed to be uniform across the economy. Furthermore, there is (indeed) a cash-reserve requirement for banking business, dictated by the government. As was the case with standard firms, the banking firm's goal is to maximize the shareholder's wealth. The licences to bank are distributed for free once in history and are valuable hereafter.

Now the banker's problem can be stated as maximizing the following objective function:

$$(2.2.2) \quad E_b = \int_t^{\infty} \{ R \cdot M_c - l_b \cdot P_l - Z \} \cdot e^{-\int_t^z R(s) ds} dz ,$$

subject to the following constraints:

$$(2.2.3) \quad \dot{M}_{ob} = Z ,$$

$$(2.2.4) \quad M_c \leq \phi \cdot M_{ob} ,$$

$$(2.2.5) \quad \frac{M_c}{P_y} \leq g(l_b) .$$

The value of the bank-shares is E_b and is equal to the discounted flow of dividends. The first condition represents a condition for cash-accumulation, while the second tells that a fraction $1/\phi$ of credit supplied must be held at the bank in the form of cash (M_{ob}). Cash at the bank is part of the total amount of base money, notes issued by the government. Part of the (assumed) fixed amount of government money is held by banks (as a cash-reserve requirement) whereas the other part is held by consumers. Furthermore it is assumed that the cash-reserve parameter ϕ is greater than 1, while the first and second derivatives of the production function have properties $g' > 0$, $g'' < 0$. The following first-order conditions can be obtained after some substitution:

$$(2.2.6) \quad g_{l_b} = \frac{P_l / P_y}{R \cdot \frac{\phi - 1}{\phi}} ,$$

$$(2.2.7) \quad M_c = \phi \cdot M_{ob} ,$$

$$(2.2.8) \quad \frac{M_c}{P_y} = g(l_b) .$$

The amount of labour hired by the bank, and thus the supply of credit, depends negatively on the real wage rate, and positively on the nominal rate of interest.⁴ The two inequality conditions turn out to be always binding. Now the worth of the licence (GW) can be computed as follows:

$$(2.2.9) \quad GW = E_b - M_{ob} ,$$

which represents the abbreviated version of the balance-sheet of the banking firm. The worth of licences is indicated here by the term "goodwill" since this worth is created "out of nothing" it seems. It is interesting to draw a parallel with the standard firm here. The balance-sheet for the goods-producing firm in the stationary state can be represented by the equality of the (value of) the capital stock and the value of equity. The goods-producing firm shows no sign of goodwill or surplus value in the stationary state. The reason is that the investment decision of the firm depends on the ratio of equity and capital stock, the so-called q-ratio. If this ratio is greater than 1 for some reason, it shows that there are advantages to be got from extra investment. The marginal cost of one unit of

⁴ The supply of demand deposits by the banking firm is more commonly found to be dependent on the rate of interest and some technological parameters of the banking process (see for instance Niehans(1978), chapter 9).

investment is less than the marginal revenue of investment in such a case. So, capital accumulation stops when the q-ratio equals 1, so there is no goodwill left.

How does this relate to the banking firm? A crucial assumption is that there is no such thing in our model as "licences accumulation", the equivalent of capital accumulation. So the limited amount of licences brings forth a "first owner" surplus value of licences. It can be clarifying to compute the q-ratio of the banking firm for the stationary state of the model.

We define the q-ratio for banking as follows:

$$(2.2.10) \quad Q_b = \frac{E_b}{M_{ob}} .$$

We know that:

$$(2.2.11) \quad D_b = \frac{E_b}{R} = M_c - \frac{l_b \cdot P_l}{R} ,$$

where D_b represents the dividend of the bank (see equation 2.2.2). Together with equations (2.2.6) to (2.2.8) and the specification of the production function as follows:

$$(2.2.12) \quad \frac{M_c}{P_y} = g(l_b) = \varepsilon_b \cdot l_b^{\alpha_b} ,$$

$$(2.2.13) \quad g_{l_b} = \frac{\alpha_b \cdot g(l_b)}{l_b} ,$$

we get:

$$(2.2.14) \quad Q_b = \phi - (\phi - 1) \cdot \alpha_b .$$

Now it is clear that when $\alpha_b = 1$ (constant returns to scale in labour) we have a q-ratio of 1 and consequently no goodwill. In the relevant case of $0 < \alpha_b < 1$ we obtain a q-ratio between 1 and ϕ . The resulting goodwill or surplus value accrues to the first owners of licences.

Defining the return on a banking licence as:

$$(2.2.15) \quad R_b = \frac{D_b}{M_{ob}} = \{ \phi - (\phi - 1) \cdot \alpha_b \} \cdot R ,$$

the relevant range of rate of returns is:

$$(2.2.16) \quad R < R_b < \phi \cdot R ,$$

where the maximum possible return is determined by the cash-reserve requirement parameter ϕ .

3 The goods-producing firm

As it is not the goal of this chapter to emphasize the producer's problem, we repeat the standard conundrum of before. There are some minor adjustments to be made in context of the introduction of a banking sector. Production plans are made in order to maximize shareholders' wealth:

$$(2.3.1) \quad E_f = \int_t^{\infty} (y \cdot P_y - l_f \cdot P_l - j \cdot P_j) \cdot e^{-\int_t^z R(s) ds} dz .$$

Clearly, the subscript f denotes that the variable is associated with the (goods-producing) firm. Dividend payout is equal to the expression between the brackets (\cdot), so, in contrast to the previous chapter, no money balances are held by the firm. All specifications are exactly as before.

Investment (j) includes installation costs in order to obtain a well-behaved investment function. Maximization of the value of shares is done subject to the accumulation of capital stock, the only constraint in this case:

$$(2.3.2) \quad \dot{k} = i - \delta \cdot k .$$

The first-order conditions of the problem read:

$$(2.3.3) \quad f_{l_f} = \frac{P_l}{P_y} ,$$

$$(2.3.4) \quad \dot{Q} = P_y \cdot (1 + h_i) ,$$

$$(2.3.5) \quad \dot{Q} = (R + \delta) \cdot Q - P_y \cdot (f_k - h_k) .$$

The firm hires labour up to the point where the marginal product of labour equals the real (uniform) wage rate. The real q-value (defined as Q / P_y) is the relevant indicator for investment activity. A q-value greater than 1 leads to net capital accumulation, whereas a ratio between zero and 1 leads to net decumulation (under the chosen specifications). Assuming clearing market conditions (no effective rationing of the firm) the q-ratio expresses the ratio of the value of shares (E_f) and the value of existing capital stock ($k \cdot P_y$) :

$$(2.3.6) \quad q = \frac{Q}{P_y} = \frac{E_f}{k \cdot P_y} .$$

A stationary state is reached when the q-ratio equals 1 and no further incentives exist to change the stock of capital.

4 The representative consumer

Consumers maximize intertemporal utility, which is a function of consumption of goods and leisure. Consumers face two constraints. The first is the intertemporal budget constraint, while the second is the cash-in-advance constraint.

Cash must be taken here in a broad sense. Base money held by consumers of course is cash. Moreover, credit facilities of consumers are seen as a perfect substitute for base money. Producers accept both kinds of "monies" as a definitive means of payment when selling consumption goods. The reason why producers in the end have no cash (no base money and no account at the bank) is that all money left is paid to stockholders as dividend. The payment of dividend can either be done in the form of visible base money or by clearing the accounts at the bank. The same holds true for the payment of wage-income to workers. One could say that the goods-producing firms have an account at the bank only for infinitely short moments of time.

The intertemporal budget constraint for consumers is:

$$(2.4.1) \quad \dot{A} = R \cdot (A - M_1) + l \cdot P_l - c \cdot P_y \ .$$

A again is the symbol for financial wealth, M_1 denotes the total stock of money (by definition in possession of consumers) and l stands for the total amount of labour sold to firms and banks together. The total amount of labour used for the production of goods and (bank) services is:

$$(2.4.2) \quad l = l_f + l_b \ .$$

Base money is either held by the banking sector (as a reserve requirement, M_{ob}) or by the consumers (M_{oh}) :

$$(2.4.3) \quad \bar{M}_0 = M_{ob} + M_{oh} .$$

The total stock of base money is treated as a parameter to the model (\bar{M}_0) . The total amount of money (including the credit facilities at banks) is:

$$(2.4.4) \quad M_1 = M_{oh} + M_c .$$

The other constraint that the households face is the cash-in-advance constraint:

$$(2.4.5) \quad M_1 \geq c \cdot P_y ,$$

which states that the total amount of money at hand must be as great as the nominal consumption expenditure. It is interesting to quote from Stiglitz(1987):

"...in modern economies money provides a way of keeping score, but one which has increasingly being found to be inconvenient. In a world in which there is no way of peering into the future, to see whether individuals will be receiving income, and hence will be able to meet any promises, money might be required for transactions (other than barter) to occur. The fact that the individual has money ensures that the individual is not attempting to commandeer more resources than his life-time budget con-

straint allows. Conventional macro-economic models, relying on the cash-in-advance constraint, are not only ad hoc, in not explaining the source of this constraint, but plainly wrong.

*The growth of cash management accounts, in which individuals can write checks (on bank accounts in which funds are instantaneously deposited and withdrawn, generating an infinite velocity) against the value of their portfolio, has simply verified what is crucial for facilitating economic transactions is not money, but credit."*⁵

The first-order conditions of the problem are:

$$(2.4.6) \quad u_c = X \cdot P_y \cdot (1 + R) ,$$

$$(2.4.7) \quad u_{l_m - l} = X \cdot P_l ,$$

$$(2.4.8) \quad M_1 = c \cdot P_y ,$$

$$(2.4.9) \quad \dot{X} = (\nu - R) \cdot X .$$

As before, $u(\cdot)$ is used as instantaneous utility function, the parameter ν as exogenous rate of time preference and X as the shadowprice associated with the intertemporal budget constraint. The cash-in-advance constraint

⁵ Though I am sure that Stiglitz will not have any sympathy with the way we model the demand for money (because the demand for money springs from all kinds of uncertainty, informational asymmetries etc.), our cash-in-advance constraint reckons with credit too.

is always binding.

A final remark can be made about the components of financial wealth of consumers. The total of financial wealth is made up according to:

$$(2.4.10) \quad A = E_f + E_b + M_{oh} .$$

Shares of banks and firms are part of financial wealth. That part of total money stock that consists of credit supplied by banks does not belong to financial wealth at an aggregate level since it concerns inside money. The total stock of outside money (\overline{M}_0) belongs to financial wealth, be it that part of outside money (cash held at the bank) is hidden from equation (2.4.10). Because the value of bank-shares consists of the value of the banking licences (goodwill) and the stock of base-money to fulfil reserve-requirements, we have:

$$(2.4.11) \quad A = E_f + GW + M_{ob} + M_{oh}$$

and this in turn can be rewritten as:

$$(2.4.12) \quad A = E_f + GW + \overline{M}_0 .$$

Without a banking sector, financial wealth in the economy's stationary state consists of the value of capital stock and the stock of outside money. Now there is an additional element to financial wealth, namely the value

of the banking licences, generated by the banking business.⁶

5 The clearing of markets

The banking economy incorporates four markets: goods market, labour market, market for shares and market for credit facilities. The three prices around are assumed to clear all four markets immediately all the time. The clearing condition for the goods market is:

$$(2.5.1) \quad y = c + j .$$

The clearing condition for the labour market:

$$(2.5.2) \quad l_f + l_b = l_d = l_s = l .$$

The clearing condition for the market for credit:

$$(2.5.3) \quad M_c^d = M_c^s .$$

⁶ It should be quite clear from the exposition above what belongs to wealth and what does not. The formulation of a well-specified macro-economic model serves at least the purpose to skip (some confusing) discussions on this matter as can be found for instance in Niehans(1978), pp.193-194).

6 Simulation of a technological shock

We have formulated a complete model at an aggregate level with a banking sector and a clearing credit market. The main difference with earlier models is that the money supply becomes endogenous. Clearly there are incentives for an individual bank to sell more credit to the public when interest rates are high and/or real wages are low. As the main purpose of the model is to highlight an endogenous money supply, we did not focus on other functions that banks perform. One unsatisfactory element of the model is that still only one uniform rate of interest across the economy exists. It seems more realistic to incorporate the assumption that banks realize profits because the cost on liabilities (exclusive of bank capital) is lower than the yield on earning assets. Models that focus on the fact that banks earn their money thanks to the existence of this spread in yields in general fail to cater for the fact that banks produce their own earning assets in the form of credit supplied to customers. Our model focuses on the differences in quantities at both sides of the bank's balance-sheet in order to grasp some mechanisms behind an endogenous supply of money. The micro-foundation of different rates of interest is a different matter altogether and has got a great deal of attention in the literature over the past twenty years or so.⁷ A possible explanation for the existence of dif-

⁷ A great deal of the relevant literature is covered in Stiglitz(1987). Problems of adverse selection, moral hazard, asymmetric information, the principal versus the agent, are very fine concepts in explaining the existence of money, credit rationing, equity rationing and so on. At this stage of research we have not succeeded in leaving the very essential

ferent rates of interest can be found in the differences in the risks attached to the different assets which consumers can hold. This idea, which will suit another purpose in due course, is used in the chapter on the choice between debt and equity of firms. It is possible all right to use the latter framework for the modelling of the banking sector in order to highlight the bank's ability to reduce risk by asset diversification or their ability to use the law of large numbers. As said before, banks make a profit in our model thanks to the special character of their produced output. Producing credit in a profitable way can be thought of as stemming from managerial capacities, specialized knowledge, the implicit reduction in informational and transaction costs, the credibility and trustworthiness associated with the banking sector etcetera. In this manner the existence of commercial banks is localized in the existence of sort of monopolistic characteristics inherent to banking activity.

A further point to keep in mind is that all banking activity in the end runs through the relation with consumers of goods. Because firms do not use cash (or: use cash infinitely fast) they do not face a direct confrontation with banks. The need for money stems exclusively from the cash-in-advance constraint that consumers face. Again we do not mind using this somewhat "forced" assumption to derive a demand for money. Excluding firms from holding money is a simplifying assumption not harmful to the purpose of the model. Assuming cash-in-advance may be an assumption just as arbitrary as deriving a demand for money along the lines of money-

assumption of the representative agent behind, a prerequisite for implementing the lemon-approach. In this respect we stick to the "conventional" paradigm (Stiglitz/Weiss(1988)).

in-the-utility-function. The lack of some well-founded demand for money is regrettable but not killing for the study at hand.

To have a plain reference simulation we start with a model that is exactly the same as used in this chapter, be it that a banking sector is left out altogether (instead using an exogenous supply of money). The simulations of chapter 1 are not ideally suited for this purpose since firms hold cash in all variants presented there. The reference simulation for the banking model is shown in table 5. Again the economy is confronted with a technological shock of the same size. The discussion of this numerical simulation will be kept short as the results are in no important way different from earlier simulations.

Since the stock of money is fixed at a value of one and the only demand for money springs from consumer transaction motives, nominal consumption expenditure ($c \cdot P_y$) will remain constant over time. Because the economy can produce technologically more efficient, the supply of goods rises. To encourage sufficient demand for goods, prices go down. Extra demand comes from both consumption and investment. Investment demand rises because new profitable opportunities are at hand, while consumption demand rises due to additional real wealth of consumers. Initially less labour is used in production due to more efficient production, but as capital stock rises over time extra labour comes in. For that reason the nominal wage rate jumps downwards to rise from that moment on. Share prices go down since nominal revenues of firms fall. A share of the firm can buy more goods though.

Now we turn to the stationary state of the same economy including a banking sector, comparing it to the same without one. The stock of money now is endogenous to the model, since banks supply additional money to

Table 5 A technological shock in reference model

period → variable ↓	0	1	2	5	10	stationary state
k	0	0.32	0.62	1.34	2.16	3.42
Q	-1.39	-1.87	-2.31	-3.36	-4.53	-6.29
U	45.37	46.30	47.14	49.21	51.52	55.01
A	-1.07	-1.20	-1.31	-1.59	-1.90	-2.37
X	0.00	0.00	0.00	0.00	0.00	0.00
c	4.17	4.41	4.63	5.18	5.79	6.71
P _y	-4.00	-4.23	-4.43	-4.92	-5.47	-6.29
j	3.44	3.44	3.44	3.44	3.43	3.42
l	-0.03	0.00	0.03	0.08	0.15	0.25
y	3.99	4.17	4.33	4.74	5.19	5.89
R	0.00	0.00	0.00	0.00	0.00	0.00
P _l	-0.13	0.01	0.13	0.44	0.78	1.30
E	-1.39	-1.56	-1.70	-2.06	-2.47	-3.08

the consumers depending on the rate of interest and the real wage rate. The stock of base money is held by consumers for about 83.4%, the remaining part is used as working capital at the banks. The economy including the banking sector faces a higher stock of money and consequently a higher price level. Since banking activities use labour in producing their banking services, less labour is available to do real activities. It is not so that all labour used in banking activities is crowding out the other labour. Total

employment in the banking economy is higher at the expense of leisure of consumers. The real wage rate in the banking economy is the same as in the standard economy and determined by the following expression.

$$(2.6.1) \quad \frac{P_l}{P_y} = (1 - \alpha) \cdot \epsilon \cdot \left[(1 - \alpha) + \frac{(1 - \alpha) \cdot \alpha}{\left(\frac{\nu + \delta}{\alpha_f \cdot \epsilon_f} \right)^{\sigma - 1} - \alpha_f} \right]^{\frac{1}{\sigma - 1}}$$

(see Meijdam and van Stratum (1990))

Real consumption and investment are lower in the banking economy, due to lower output of goods. The level of utility is lower when a banking sector is introduced. This result is very misleading however. One can certainly not conclude that the banking sector is superfluous or unwanted. One can circumvent the utility-lowering status of banks quite easily by taking banking-services (providing ease of liquidity) into the utility functions of households. In the same manner one cannot say that "production" in the banking economy is lower, because there exists another basket of goods altogether. Output of the banking economy consists of a basket of industry-type goods and liquidity services. Consequently, the way one compares the results of both economies with respect to utility and production is completely arbitrary. Whether the banking sector is productive is a matter that cannot be resolved by the model. The purpose here is to show the process of adjustments in monetary aggregates with and without banks. It is possible however to use the model to simulate a more efficient process of producing liquidity services. When utility rises in such a case, one can conclude that such a financial innovation is desirable for the economy.

Comparing the simulation of both economies facing the same technological shock leads to the following insights. Broadly speaking one can say that the time-paths of the non-monetary variables (such as consumption, production, investment) give the same picture. Interesting differences arise when comparing the time-paths of the monetary variables. Prices go down after a technological shock because of the extra produced output coming to the market. A lower level of prices has consequences for the banking sector. There is less need for transaction money since nominal consumption expenditure goes down. As a direct result, banks see a loss in their output of credit to households. Some curtailing of the loss of output is reached by the lowering of the rate of interest (as a result of forces at the market for credit). Banks hold (government) money themselves for cash-reserve requirements. Less produced output in the form of credit means less need for government money at the bank. The surplus of visible cash at the bank is paid out to the holders of bank-equity in the form of extra dividends. We see that the total stock of money shifts in composition towards more visible government money at the expense of invisible bank-money. Banks become smaller in terms of output and employment. Extra labour is available for production in the real sector. As in the no-banking case the picture remains that there is initially a loss of total employment which is offset later on in time.

Table 6 A technological shock in banking model

period → variable ↓	0	1	2	5	10	stationary state
k	0	0.34	0.65	1.43	2.31	3.72
Q	-3.61	-4.22	-4.76	-6.09	-7.58	-9.86
U	53.44	54.59	55.64	58.23	61.15	65.72
A	-3.06	-3.28	-3.48	-3.97	-4.52	-5.38
X	2.49	2.61	2.71	2.96	3.23	3.66
c	4.23	4.49	4.73	5.31	5.97	7.02
P _y	-6.30	-6.64	-6.95	-7.71	-8.56	-9.86
j	3.63	3.64	3.65	3.67	3.69	3.72
l	-0.06	-0.04	-0.02	0.04	0.10	0.19
l _f	0.16	0.20	0.23	0.31	0.40	0.54
l _b	-15.02	-15.77	-16.45	-18.12	-19.99	-22.87
y	4.08	4.28	4.46	4.90	5.40	6.19
R	-1.16	-1.04	-0.94	-0.69	-0.41	0.00
P _l	-2.75	-2.73	-2.72	-2.68	-2.64	-2.56
E _f	-3.61	-3.89	-4.14	-4.75	-5.44	-6.51
E _b	-17.37	-18.10	-18.76	-20.35	-22.13	-24.84
GW	-20.66	-21.06	-21.43	-22.31	-23.31	-24.84
M _c	-16.39	-17.21	-17.96	-19.77	-21.77	-24.84
M _{0h}	3.26	3.42	3.57	3.93	4.33	4.94
M ₁	-2.33	-2.45	-2.55	-2.81	-3.10	-3.53

7 An increase in the required-reserve ratio

Some short comments are made about the (long-run) simulation results of an increase in the bank's required reserve ratio. It is a legitimate question to ask why the government does not carry out the task of the banking sector in the model. The government (only implicit in the model) issues pieces of paper money in the famous helicopter fashion. Why then is it not possible to vary the stock of paper money continuously to ensure that the (endogenous) demand for money always equals the supply of base money:

$$(2.7.1) \quad \overline{M} = c \cdot P_y .$$

Of course, the government in its most primitive form is not interested in creating extra inflation. All price movements in the model are a consequence of real shocks and disturbances that smooth out over time. The assumption then is that the government has perfect information about the workings of the model. This is not a strange assumption since every agent of the model is perfectly informed in this respect. Nevertheless, the continuous change in the stock of money that has to be accomplished is a cumbersome affair. It takes labour and trouble to monitor a number of relevant aggregates. Accepting the fact that it takes time and money to perform these monitoring activities, why not pass the job to private enterprise? Banks are specialized in performing these tasks, and probably more efficient too. It probably would not be a wise thing to carry over all monetary matters to private enterprise. Government and law can restrict all too greedy actions of private monetary business. Starting a monetary enterprise in our model is only possible when some requirements are met. The

fact that it belongs to the possibilities to set the required-reserve ratio to the banking sector, gives the government an instrument in controlling the total stock of money without many costs.

What roughly are the consequences of changing the required reserve ratio in the model? Let us take an increase of the ratio into account ($\phi = 1.9909$ instead of 2, see table 7). Given some stationary state of the model, banks must either pay less dividend to shareholders in order to re-balance their cash-holdings or shrink the production of credit. The first option has consequences in the form of a higher rate of interest and less (ex ante) supply of credit, so effectively works out the same as the second option. Less credit means a smaller banking sector and less labour working at the bank. Employment and output of the goods industry grows at the expense of banking activities. The total stock of money declines and as a consequence prices fall too.

8 Financial innovations in the banking model

The model is suited to simulate the consequences of some forms of financial innovation. In dealing with the subject of financial innovation we take the formulation of Silber (1983) as starting point:

"The main hypothesis is quite straightforward: new financial instruments or practices are innovated to lessen the financial constraints imposed on firms".

We are dealing here with the constraints imposed on banking firms and their technological possibilities. The banker's model has two constraints of

Table 7 Stationary state effects of 3 shocks

	$\phi = 1.9909$	$\varepsilon_b = 1.414$	$\eta = 0.99$
k	0.04	-0.04	0.01
A	-0.43	0.49	0.94
c	0.04	-0.04	0.01
P _y	-0.54	0.61	1.16
j	0.04	-0.04	0.01
l	-0.01	0.01	0.01
l _f	0.04	-0.04	0.01
l _b	-3.00	3.37	0.00
y	0.04	-0.04	0.01
P _l	-0.54	0.61	1.16
GW	-3.52	4.01	1.16
M _c	-3.08	4.01	1.16
M _{0h}	0.52	-0.80	-0.23
M _l	-0.50	0.57	0.17

importance to the problem of financial innovation:

$$(2.8.1) \quad M_c \leq \phi \cdot M_{0b} ,$$

$$(2.8.2) \quad \frac{M_c}{P_y} \leq g(l_b) = \varepsilon_b \cdot l_b^{\alpha_b}$$

The effects of tightening the first constraint have already been discussed in paragraph 7. A change in one of the parameters of the second constraint (ϵ_b, α_b) can be interpreted as a change in the efficiency of the banking process. A cost-reducing financial innovation should have the same consequences as simulated by the model when the technological parameters ϵ_b or α_b are changed. The long run consequences of a change in the parameter ϵ_b have been indicated in table 7.

The profit maximizing condition for the bank is:

$$(2.8.3) \quad g_{l_b} = \alpha_b \cdot \epsilon_b \cdot l_b^{\alpha_b - 1} = \frac{P_l / P_y}{\frac{\phi - 1}{\phi} \cdot R}.$$

Since the stationary state value of the real wage is independent of the technological parameter ϵ_b and it is true that $R = v$, a higher value of ϵ_b must have as a consequence higher employment in banking (since $(\alpha_b - 1) < 0$). So better technological possibilities lead to higher bank-employment and a higher amount of credit-supply. The consumer consumes a greater part of his wealth in the form of liquidity services at the expense of normal consumption. For the greater stock of money to be absorbed, the price level rises a great deal since real consumption expenditure declines ($M_1 = c \cdot P_y$).

The two constraint related innovations discussed so far, are related to the supply-side of credit. One can imagine that for some reason or another consumers have less need for cash to complete their planned transactions. One can imagine a demand-reducing innovation in the way that goods-producing firms accept other means as definitive payment. For instance,

consumers and firms deal with each other directly without intermediation of banks. Firms can make their own means of settlement and create what can be called "disintermediation". When the stock of money falls in such a case, it must be noted that the "old" stock of money does fall indeed, being the sum of government notes and credit supplied by banks, but as there is an additional form of money in existence it is impossible to conclude that the stock of what is properly called money falls. Firms take over part of traditional banking activities and in doing so they have an advantage compared with traditional banks. Banks are regulated by all kinds of governmental rules, whereas firms are not. A demand-for-(traditional)money reducing innovation is embarrassing for the government's money-regulating power.

The demand-reducing innovation can be incorporated into the model in the following way. The consumers' problem is modified slightly into:

Maximize intertemporal utility, subject to

$$(2.8.4) \quad \dot{A} = R \cdot (A - M_1) + l \cdot P_l - c \cdot P_y ,$$

$$(2.8.5) \quad M_1 \geq \eta \cdot c \cdot P_y .$$

The new parameter η indicates to what extent traditional money is used in trade between firms and consumers. The most sensible range of values of η is between zero and 1. The new set-up of the problem changes a number of relations used before:

$$(2.8.6) \quad u_c = (1 + \eta \cdot R) \cdot X \cdot P_y ,$$

$$(2.8.7) \quad M_1 = \eta \cdot c \cdot P_y \cdot$$

The first relation expresses the fact that a return is lost over a fraction η of total consumption expenditure. The second condition states that the new cash-in-advance constraint is always binding.

The long run consequences of a demand-reducing financial innovation are shown in table 7.

The innovation pushes up real consumption. A smaller part of income has to be spent on liquidity services, because the cash-in-advance constraint is relevant only for part of consumption expenditure. Consumers can buy more goods as a consequence. An upward pressure in prices results. The banking firm faces two opposite forces: on the one hand output diminishes because consumers demand less money in the form of banking services, on the other hand, since more goods are sold and prices rise along with this, consumers have to return to the banking industry. In the end, the banking sector is better off in the presence of the financial innovation. Both industrial firms and consumers are better off too. The stock of money is greater than before, while there are other forms of money or means of payment in the economy as well. The reason for this global prosperity is pretty obvious: the greater the part of the stock of money that can be used in carrying out transactions without losing interest, the better for the economy as a whole. A more sophisticated monetary economy (in terms of the amount of traditional cash in use) will benefit its inhabitants.

9 Conclusions

In the value-maximizing tradition of finance, we have formulated a macroeconomic model including a banking sector and an endogenous money supply. Banks maximize the value of outstanding shares and use labour and banking-licences as input in producing lines of credit. It turns out that the supply of credit is determined by the real wage rate and the nominal rate of interest, representing the costs and benefits of the bank. Furthermore, the supply of money is determined by the parameters of the banker's production function and the cash-reserve requirement parameter. In our model, banking licences have a net worth due to their cash-flow generating character. The stationary state return on these banking licences is higher than the market rate of interest due to the monopoly supply of the licences.

An economy that includes a banking sector (the way we modelled it) faces a higher stock of money and a higher price-level, compared with an economy that lacks this banking sector. Total employment is higher, while the real wage rate is the same. Utility comparisons cannot be made since our banking model is not apt to answer these kind of questions. The banking model seems to be able to generate reasonable simulations of some forms of financial innovation.

APPENDIX

Reference model, table 5

c		$u_c = (1 + R) \cdot P_y \cdot X$
l_s		$u_{l_s-l} = P_l \cdot X$
l_d		$f_l = \frac{P_l}{P_y}$
P_l		$l_d - l_s = l$
y		$y = f(l, k)$
i		$Q = P_y \cdot (1 + h_i)$
j		$j = i + h(i, k)$
P_y		$y = c + j$
M_{th}		$M_{th} = c \cdot P_y$
R		$\bar{M} = M_{th}$
u		$u = u(c, l_m - l)$
E		$E = A - \bar{M}$
k		$\dot{k} = i - \delta \cdot k$
Q		$\dot{Q} = (R + \delta) \cdot Q - P_y \cdot (f_k - h_k)$
A		$\dot{A} = R \cdot (A - \bar{M}) + l \cdot P_l - c \cdot P_y$
X		$\dot{X} = (\nu - R) \cdot X$
U		$\dot{U} = \nu \cdot U - u$

Specifications used in simulations: (1.3.11), (1.3.12) and (1.3.13).

Banking model, tables 6 and 7

$$l_b \quad | \quad g_l = \frac{P_l / P_y}{R \cdot \frac{\phi - 1}{\phi}}$$

$$l_f \quad | \quad f_l = \frac{P_l}{P_y}$$

$$c \quad | \quad u_c = (1 + \eta \cdot R) \cdot P_y \cdot X$$

$$l_s \quad | \quad u_{l_s-1} = P_l \cdot X$$

$$M_c^d \quad | \quad M_c^d = M_1 - M_{0h}$$

$$M_1 \quad | \quad M_1 = \eta \cdot c \cdot P_y$$

$$M_{0h} \quad | \quad M_{0h} = \bar{M}_0 - M_{0b}$$

$$M_{0b} \quad | \quad M_{0b} = \frac{M_c}{\phi}$$

$$M_c^s \quad | \quad \frac{M_c^s}{P_y} = g(l_b)$$

$$P_l \quad | \quad l_b + l_f = l_s = l$$

$$y \quad | \quad y = f(l_f, k)$$

$$i \quad | \quad Q = P_y \cdot (1 + h_i)$$

$$j \quad | \quad j = i + h(i, k)$$

$$P_y \quad | \quad y = c + j$$

$$R \quad | \quad M_c^d = M_c^s = M_c$$

$$E_b \quad | \quad E_b = A - E_f - M_{0h}$$

$$GW \quad | \quad GW = E_b - M_{0b}$$

$$D_f \quad | \quad D_f = y \cdot P_y - l_f \cdot P_l - j \cdot P_y$$

$$u \quad | \quad u = u(c, l_m - l)$$

$$k \quad | \quad \dot{k} = i - \delta \cdot k$$

$$Q \quad | \quad \dot{Q} = (R + \delta) \cdot Q - P_y \cdot (f_k - h_k)$$

$$A \quad | \quad \dot{A} = R \cdot (A - M_1) + l \cdot P_l - c \cdot P_y$$

$$X \quad | \quad \dot{X} = (\nu - R) \cdot X$$

$$U \quad | \quad \dot{U} = \nu \cdot U - u$$

$$E_t \quad | \quad \dot{E}_t = R \cdot E_t - D_t$$

Specifications used in simulations: (2.2.12), (1.3.11), (1.3.12) and (1.3.13).

Parameter values:

$$\alpha_f = 0.25$$

$$\alpha_b = 0.70$$

$$\gamma_c = 0.85$$

$$l_m = 9.0$$

$$\varepsilon_f = 0.25$$

$$\varepsilon_b = 1.40$$

$$\overline{M} = 1.00$$

$$M_0 = 1.00$$

$$\sigma = 0.40$$

$$\delta = 0.10$$

$$\nu = 0.10$$

$$\psi = 0.125$$

$$\phi = 2.00$$

$$\eta = 1.00$$

$$\gamma_l = 0.10$$

Stationary state:**Reference**

k	= 3.599
Q	= 0.932
U	= 1.032
A	= 4.355
X	= 0.813
c	= 1.073
P _y	= 0.932
j	= 0.360
l	= 7.533
y	= 1.433
R	= 0.10
P _l	= 0.088
E	= 3.355

Banking

k	= 3.554
Q	= 1.100
U	= 0.907
A	= 4.960
X	= 0.698
c	= 1.059
P _y	= 1.100
j	= 0.355
l _f	= 7.440
l _b	= 0.111
y	= 1.415
R	= 0.10
P _l	= 0.104
E _f	= 3.912
E _b	= 0.216
GW	= 0.050
M _c	= 0.332
M _{0h}	= 0.834
M ₁	= 1.166
M _{0b}	= 0.166

THE BANKING FIRM: THE CASE OF MONOPOLISTIC COMPETITION

1 Introduction

In this chapter the consequences are studied of changing the perfect competition setting of chapter 2 into a setting of monopolistic competition. We have dealt explicitly with the credit market as a product market. Introducing a market imperfection at the credit market can be accorded with the basic philosophy of this study. The starting point for capturing the idea of monopolistic competition is the famous study of Chamberlin(1933). A modern restatement of the Chamberlin-notion in a macroeconomic model is found in Meijdam(1991a). We have studied this paper and the references therein in order to reformulate the model in terms of the banking model of chapter 2. The idea that banks sell slightly different types of credit (or think they do) with their own fringe of buyers makes banks think they do have price-setting power. Our intention is to study the impact of this price-setting power (where the price is "the" rate of interest) on the main banking variables, such as the value of bank-shares, the value of licences and the rate of interest. Since the banking model of chapter 2 is exactly the same in all other respects, a comparison between the two banking models

is legitimate. In fact, the full-competition case of chapter 2 is a special version of the model of this chapter (the difference springs from changing the value of one parameter).

There are a few important consequences of the imperfect competition set-up. In the first place there is no market-clearing as a rule. It is up to the banking decision to deviate from (ex ante) demand. Secondly, though the model exhibits perfect foresight for most variables, the bank actually clings to the wrong model. Bankers think they can set a rate of interest of their own (vis-a-vis the market rate of interest), but since all banks do the same thing this appears to be impossible ex-post. In this respect, the model struggles with the notions of rational expectations and perfect foresight. A nice comparison can be made with chapter 18 of Barro(1984).

Barro uses a many-local-markets set-up. In this model, agents know the local price of goods, but are less sure about the general or average price. Because there is an implicit process of obtaining costly information, sellers and buyers make do with incomplete knowledge about the prices of other-than-their-own-market goods. In this manner buyers and sellers alike may (mistakenly) think they are situated in a favourable market. The Barro-model is consistent with the formation of rational expectations (assuming imperfect information). As a consequence it is very unlikely that people keep making the same mistakes in estimating the general price-level for very long by not receiving enough information in due time. These kinds of models (including ours) are clearly best suited for studying short-term behaviour.

In contrast with the Meijdam(1991a)-paper, we start our simulations in absence of adjustment costs of changing the rate of interest. It turns out that it is optimal for banks to "over-supply" the market for credit. In order

to gain further insights into spill-over effects from the financial sector to the real sector, we simulate the model with the inclusion of adjustment costs thereafter.

2 The banking firm

The difference with the banking model of the chapter before is that each bank is capable of setting a rate of interest of its own. When banks are perfectly equal in all respects and are rate-of-interest takers, it is a standard practice to assume clearing money markets.¹ In case of price-setting behaviour, markets generally do not clear. We will discuss a variant of the model in which the regime of credit-rationing is a rational possibility open to banking decisions. We will discuss the new banking environment first. A crucial feature is that we keep assuming a representative banker, who thinks (in this chapter wrongly so) that he is different from other bankers. Suppose the market for credit is divided into many small local submarkets. For some reason, customers like to go to their own local banks in order to obtain credit-lines. Reasons for this behaviour can be manifold. Maybe some time and effort is spent on building a strong customer-bank relationship. Search-costs can be taken into account: when a customer wants to visit another, hopefully cheaper, bank, he has to forego some search costs.

¹ It is possible to have representative banks being rate-of-interest takers and a non-clearing money market by adding a relation to the model that describes some arbitrary process for sluggish rate-of-interest formation.

Customers may dislike travelling to banks that are further away, they may (wrongly) think that their own bank is cheaper, better, more trustworthy etc. The problem with all these motivations is the specification of the costs and benefits of deviating behaviour (compared to standard models on the matter): the behaviour has to be accounted for in the model explicitly. Another way round the problem is to assume that banks just have some price-setting behaviour. So, banks act as if they have some price-setting behaviour. The problem now is that as time evolves it must come out that the model the bank has in mind when formulating pricing policy is systematically wrong. Clinging to the wrong model for a long time clearly contradicts the rationality postulate used throughout.

So, a satisfactory answer of why markets are segmentated or why banks act as if they have price-setting power will not be given here. The set-up is that banks just think they face a market for credit that is locally separated to a certain extent to be specified. In fact all banks make different products with their own circle of customers around it. It comes to the same thing as assuming that all products are alike except for their colour. Every colour however has its own special set of customers (see Barro(1984)).

We assume that total demand over all existing products is divided into such a way that the banking firm faces the following (perceived) demand for his type of credit:

$$(3.2.1) \quad M_c^d = \frac{\bar{M}_c^d}{N} \cdot \left\{ \frac{r}{R} \right\}^{-\zeta}, \quad \zeta > 0.$$

N indicates the large number of banks and types of credit, \bar{M}_c^d is the total demand for credit (given to the individual bank), M_c^d is the demand for the

n^{th} type of credit ($1 \leq n \leq N$), R is a weighted average of all rates of interest,² while r is the n^{th} product specific rate of interest. The parameter ξ indicates to what extent total demand comes to the specific bank in case the specific rate of interest differs from the market rate of interest.

In case ξ approaches infinity, we obtain the case of price-taking banks again, discussed in chapter 2. The indicator of the monopolistic power of the individual bank is therefore ξ .

Another point is that we need some sort of adjustment costs to obtain the mere possibility of credit rationing or credit absorption. We use the symbol s as defining the change of interest related to the market rate of interest:

$$(3.2.2) \quad s = \frac{\dot{r}}{R} .$$

We define the real costs of change as a function $z(s)$ to be specified for numerical simulation as:

$$(3.2.3) \quad z(s) = \frac{s^2}{2 \cdot \beta} ,$$

where β indicates some measure of the penalty of adjustment. In case β approaches infinity there are no costs of change. In the case of β equalling zero, the costs of change are infinitely large and no change of the rate of

² A further elaboration on this matter can be found in Meijdam(1991a), which deals with monopolistic competition at the market for goods.

interest will ever occur. Infinitely large costs of adjustment prevent rational bankers from deviating their price of credit from the general price of credit.

Apart from the issues discussed so far, the banking environment is exactly as described in the previous chapter.

The parameters ζ and β can be used to summarize the three possible banking models. There is either price-taking behaviour or some degree of monopolistic competition and, in the latter case only, there are adjustment costs or not:

Banking model 1 (chapter 2): ζ tends to infinity.

Banking model 2 (this chapter): $\zeta = 4$ and β tends to infinity (no costs of change).

Banking model 3 (this chapter): $\zeta = 4$ and $\beta = 4$ (costs of change).

The objective function, to be maximized by the individual bank, can be stated as follows:

$$(3.2.4) \quad E_b = \int_t^{\infty} \{ r \cdot M_c - l_b \cdot P_l - Z - z(s) \cdot P_y \} \cdot e^{-\int_t^s R(s) ds} dz .$$

Maximization must be done considering the following five constraints:

$$(3.2.5) \quad \dot{M}_{ob} = Z ,$$

$$(3.2.6) \quad \frac{\dot{r}}{R} = s ,$$

$$(3.2.7) \quad M_c \leq M_c^d = \frac{\overline{M}_c^d}{N} \cdot \left\{ \frac{r}{R} \right\}^{-\tau} ,$$

$$(3.2.8) \quad M_c \leq \phi \cdot M_{ob} ,$$

$$(3.2.9) \quad \frac{M_c}{P_y} \leq g(l_b) .$$

A number of remarks are in order here. Shareholders of each type of bank require the market rate of interest. No specific risk is attached to one bank vis-a-vis another one, or alternatively, the average shareholder consumes a basket of credit-types. Costs of change are modelled as expenses that must be financed out of retained earnings. We suppose that costs of change come about by buying goods from the industry-type firms. Costs of change are therefore part of the total demand for goods as can be seen from the clearing condition for the goods market (section 4). Everything is, as before, formulated in terms of cash-flows.

The following first-order conditions result from the above formulated maximization problem:

$$(3.2.10) \quad g_{l_b} = \frac{P_l / P_y}{r - \lambda_r - \frac{R}{\phi}} ,$$

$$(3.2.11) \quad \frac{M_c}{P_y} = g(l_b) ,$$

$$(3.2.12) \quad z_s = \frac{\mu \cdot R}{P_y} ,$$

$$(3.2.13) \quad \dot{\mu} = R \cdot \mu - M_c + \frac{\xi}{r} \cdot \lambda_r \cdot M_c^d ,$$

$$(3.2.14) \quad M_c = \phi \cdot M_{ob} .$$

μ is the shadowprice of the rate of change of the interest rate, λ_r is the Lagrange-multiplier of the demand-constraint. Two of these equations are the same as before (chapter 2) and do not call for further explanation. Equation (3.2.10) indicates that banks go on hiring labour to produce credit until marginal costs equal marginal revenues of labour. Equation (3.2.12) represents the same thing with respect to changing the own rate of interest. Equation (3.2.13) describes the time path of the shadowprice of the rate of change of the rate of interest.

As we introduced a representative bank from the outset, it must be so that in the end all banks end up doing the same thing (without their knowing this beforehand). The macroeconomic consequence is that in the end there will be only one rate of interest for all types of credit. The relevant relations above can therefore be simplified by inserting $R = r$ into equations (3.2.10) and (3.2.13):

$$(3.2.15) \quad g_{I_b} = \frac{P_I / P_Y}{\frac{\phi - 1}{\phi} \cdot R - \lambda_r} ,$$

$$(3.2.16) \quad \dot{\mu} = R \cdot \mu - \frac{M_c}{R} \cdot (R - \xi \cdot \lambda_r) .$$

The latter equation represents the motion of the shadowprice of the change in the rate of interest all right, since:

$$M_c < M_c^d \rightarrow \lambda_r = 0 ,$$

$$M_c = M_c^d \rightarrow \lambda_r > 0 .$$

Now it is about time to define what we called earlier the regime of credit rationing and the regime of credit absorption:

$$\lambda_r = 0 \longleftrightarrow \text{credit rationing} ,$$

$$\lambda_r > 0 \longleftrightarrow \text{credit absorption} .$$

In case consumers are credit-rationed, repercussions are to be expected at other markets. The credit-rationing regime shows the interesting phenomenon of spill-over effects from financial to so-called real activities.

3 The representative consumer

The representative consumer faces a slightly different environment as described in chapter two due to the monopolistic behaviour of bankers. The consumer has to reckon with the possibility that he cannot buy all the goods he wants due to the inavailability of credit lines from the banking sector. Vis-a-vis the previous chapter, the consumer faces one extra constraint. Now the consumer problem is as follows:

Maximize intertemporal utility $U(c, l_m - l)$

subject to the constraints:

$$(3.3.1) \quad \dot{A} = R \cdot (A - M_1) + l \cdot P_t - c \cdot P_y ,$$

$$(3.3.2) \quad M_1 \geq c \cdot P_y ,$$

$$(3.3.3) \quad M_1 \leq \bar{M}_1^s .$$

The representative consumer considers the relevant stock of money (including credit facilities) as given. First order conditions for the problem at hand are:

$$(3.3.4) \quad u_c = P_y \cdot \{ X \cdot (1 + R) + \lambda_c \} ,$$

$$(3.3.5) \quad u_{l_m - l} = X \cdot P_l ,$$

$$(3.3.6) \quad M_1 = c \cdot P_y .$$

Only the first of these three equations contains a new element. The fact that the consumer can be credit-rationed is discounted for in the first marginality rule. The Lagrange-multiplier λ_c represents the same regimes as mentioned earlier, credit rationing and credit absorption. It is therefore true that λ_r of the bankers problem and λ_c of the consumers problem contain the same information:

$$\lambda_r = 0 \wedge \lambda_c > 0 \quad \longleftrightarrow \quad \text{credit rationing: } M_c < M_c^d ,$$

$$\lambda_r > 0 \wedge \lambda_c = 0 \quad \longleftrightarrow \quad \text{credit absorption: } M_c = M_c^d .$$

The model of the goods-producing firm is exactly the same as used (and discussed) in chapter 2, section 3. Therefore, we will start with the section on markets immediately.

4 The markets

The market for credit is dealt with via a standard rationing scheme in the following way:

$$(3.4.1) \quad M_c = \min (M_c^d, M_c^s) ,$$

where M_c is the actual amount of credit supplied, determined by the minimum of ex ante supply and demand for credit.

The clearing condition for the goods market must meet the requirement of adjustment costs, since we assumed that the adjustment was serviced by production of goods:

$$(3.4.2) \quad y = c + j + z(s) .$$

The clearing condition for the labour market stays as it was before:

$$(3.4.3) \quad l_f + l_b = l_d = l_s = l .$$

5 Some remarks about the stationary state

The stationary state of the economy is the same whether there are adjustment costs or not. Of course, this characteristic is due to the chosen specification of adjustment costs. These are treated in the same way as the

installation costs in the case of investment.³ In the stationary state there are no adjustment costs, since:

$$(3.5.1) \quad z_s = \frac{s}{\beta} = \frac{\dot{R}/R}{\beta} ,$$

and:

$$(3.5.2) \quad \dot{R} = 0 .$$

However, it is only in the case of positive adjustment costs that the possibility of two credit regimes opens up during adjustment from one old stationary state to an other new one. When no costs are associated with altering the own rate of interest vis-a-vis the market rate of interest, the bank will always charge a rate of interest to its clients in a way that the demand for credit is never binding. This can be seen as follows. In the case of no adjustment costs, we have $\beta \rightarrow \infty$ and therefore:

³ We introduced installation costs as:

$$h(.) = \frac{k^2}{2 \cdot \beta} .$$

Applying this idea to rate-of-interest-adjustment-costs:

$$z(.) = \frac{(\dot{r}/R)^2}{2 \cdot \beta} .$$

$$(3.5.3) \quad z(s) = \frac{s^2}{2 \cdot \beta} = 0 .$$

The optimal adjustment rule (3.2.12) reads in this case:

$$(3.5.4) \quad z_s = \frac{s}{\beta} = 0 = \frac{\mu \cdot R}{P_y} .$$

As both the rate of interest and the price level of goods are assumed to be of positive value (which is not an unreasonable assumption after all), we conclude that $\mu = 0$. This value of μ is always relevant in the case without adjustment costs (not only in the stationary state). Therefore, from (3.2.16):

$$(3.5.5) \quad \dot{\mu} = 0 = - \frac{M_c}{R} \cdot (R - \zeta \cdot \lambda_r) .$$

Assuming only positive levels of credit supply, the shadowprice of the demand-constraint facing banks is equal to:

$$(3.5.6) \quad \lambda_r = \frac{R}{\zeta} > 0 ,$$

which implies that there will be credit absorption all of the time, while the regime of credit rationing is not possible.

The information so far about the possibilities of rationing can be summarized into the following scheme.

Monopolistic competition in the banking sector ($\xi = 4$ in simulations to come):

- Positive adjustment costs ($\beta = 4$ in simulation):

$\lambda_r = 0$ if $M_c < M_c^d$ (credit rationing).

$\lambda_r > 0$ if $M_c = M_c^d$ (credit absorption).

- No adjustment costs ($\beta \rightarrow \infty$):

$\lambda_r = \frac{R}{\xi} > 0$ (no credit rationing possible).

As before, we can derive the so-called "banking Q" for the case of monopolistic banking for the stationary state (see chapter 2). Remembering that we defined this ratio as:

$$(3.5.7) \quad Q_b = \frac{E_b}{M_{ob}} .$$

A banking Q greater than one implies that a windfall gain can be derived from trading in the sum of money M_{ob} (necessary to start a banking business) for a new share (worth E_b) just by starting to produce credit. In the previous section a banking Q different from 1 (greater than 1) could exist in the stationary state because an additional licence to bank was needed to start banking. The number of licences was given and therefore we could

assume decreasing returns to scale in banking. The difference $E_b - M_{ob}$ describes the worth of the licence and as such represents a windfall profit for the very owner of the license (or for that matter: the issuer of the licence). Using the right relations again in case of the monopolistic banking case, we obtain for the banking Q :

$$(3.5.8) \quad Q_b = \phi - (\phi - 1) \cdot \alpha_b \cdot \left\{ \frac{P_l \cdot l_b}{P_l \cdot l_b + \lambda_r \cdot \alpha_b \cdot \phi \cdot M_{ob}} \right\}$$

In the case of credit rationing ($\lambda_r = 0$) we have (2.2.14) back again. That is to say: the rate of return on a banking licence is the same as the market rate of interest in the "normal" case of constant returns to scale in licences ($\alpha_b = 1$). In the case of a fixed number of licences ($0 < \alpha_b < 1$ together with $\lambda_r = 0$) a higher rate of return is realized, bounded from above by the cash-reserve parameter ϕ . When the other regime prevails ($\lambda_r > 0$) the interesting parameter determining the height of Q_b and consequently the rate of return on a banking licence is λ_r itself. In the stationary state it is true that:

$$(3.5.9) \quad \lambda_r = \frac{R}{\zeta} > 0 .$$

Now, we see from equation (3.5.8) that the greater λ_r , the greater the return on a banking licence. A starting point in understanding this is that the parameter ζ is crucial here. The parameter ζ expresses the relative power of the type of credit vis-a-vis other types of credit. In case ζ tends to infinity we have a homogenous type of credit and perfect competition in this

market. So, the lower ζ , the greater the relative power or attractiveness of the type of credit. The lower the parameter ζ the higher the Lagrange-multiplier λ_r for the stationary state, the higher the monopolistic rent for banking. We can summarize this discussion by stating that the lower the demand elasticity ζ for the type of credit, the higher the rate of return on a banking licence. In this way equation (3.5.8) seems to be a banking variant of the well-known Amoroso-Robinson formula.

In the appendix the complete macroeconomic model is summarized. Because of the possibility of credit-rationing, a number of ex ante variables show up. Bankers have an ex ante demand for labour based on the real wage and rate of interest, not reckoning with the sales constraint (assuming $\lambda_r = 0$ in equation (3.2.15)). Consumers have an ex ante demand for consumption goods based upon (among other things) the price level of goods and the rate of interest, not reckoning with the eventuality that the credit constraint can be binding (assuming $\lambda_c = 0$ in equation (3.3.4)). Given the ex ante demand for consumption goods there is an ex ante demand for credit. The ex ante supply of credit is computed by inserting the ex ante demand for labour in the bankers production function. The actual level of credit is then computed as the minimum of ex ante supply and demand for credit. Given this new information (will there be rationing or absorption) actual (ex post) levels of consumption and labour can be computed. This in turn influences the sales of goods relevant for the goods producing industry. If credit is rationed, consumers cannot buy the goods they want and firms consequently cannot sell as much. Consumers then face the fact that less labour demand springs from industrial firms. This affects the real wage rate, the demand for goods and so on.

6 Simulation of a technological shock

Because of the identical stationary states of the monopolistic banking model with or without adjustment costs, we will discuss the differences between the stationary states of the banking model with full competition and monopolistic competition. The first important difference is, as said before, that the return to first owners of a banking licence rises from 13 percent in the case of perfect competition to 16.5 percent in the case of monopolistic competition. Due to monopolistic banking powers there is an Amoroso-Robinson-like extra-rent above what can be called a normal rent. So the trajectory of rates of return starts with a normal return equal to the (exogenous) rate of time preference ν with a chosen value of 10 percent. In the case of decreasing returns to scale in banking technology (due to the limited number of licences) the return on a banking licence rises to 13 percent in the case of full-competition-banking. The final rise in return, from 13 percent to 16.5 percent, springs from monopolistic market characteristics in banking business. A second important difference compared to the full-competition case is the fact that the banking-sector becomes smaller in size. The labour-hiring rule for full competition was:

$$(3.6.1) \quad g_{l_b} = \frac{P_l/P_y}{\frac{\phi-1}{\phi} \cdot R} \cdot$$

The macro-rule for the monopolistic competition case reads:

$$(3.6.2) \quad g_{l_b} = \frac{P_l / P_y}{\frac{\phi - 1}{\phi} \cdot R - \lambda_r} .$$

Since the real wage rate and rate of interest are the same in both cases, a positive value of λ_r implies that the marginal productivity of a unit of banking labour must be higher. Less employment in the banking sector results in less credit supply and a higher return on supplying credit (comparable to the standard case of monopoly on the goods market). Therefore the stock of money will be lower. Greater part of labour supply is used in producing goods. Consumption, investment and the stationary stock of capital are higher. The smaller amount of money therefore does not result in smaller real magnitudes but in a lower general price level.

Simulation assuming no adjustment costs

As said before, without adjustment costs a regime of credit rationing is not possible. The shadowprice of the constraint is always equal to $\frac{R}{\xi}$. In the table this is confirmed by the fact that the percentual deviation of the shadowprice λ_r always equals the percentual deviation of the rate of interest. The general idea of the simulation of a technological shock is the same as before. There is extra produced output alongside a lower price level. Transaction demand for money goes down and the banking sector sells less credit to consumers. To regain some of the lost credit-output, the rate

Table 8 A technological shock, no adjustment costs

period → variable ↓	0	1	2	5	10	stationary state
k	0	0.32	0.62	1.35	2.18	3.45
Q	-1.76	-2.26	-2.71	-3.82	-5.05	-6.90
U	46.07	47.02	47.88	49.59	52.35	55.95
A	-1.43	-1.58	-1.71	-2.03	-2.39	-2.94
c	4.18	4.42	4.64	5.19	5.80	6.74
P _y	-4.36	-4.61	-4.84	-5.38	-5.99	-6.90
i	3.41	3.42	3.43	3.44	3.45	3.45
l _f	-0.01	0.02	0.05	0.11	0.17	0.28
l _b	-13.01	-14.00	-14.90	-17.05	-19.41	-22.87
l	-0.03	0.00	0.02	0.08	0.15	0.24
y	4.00	4.18	4.35	4.76	5.21	5.92
R	-0.25	-0.22	-0.20	-0.14	-0.08	0.00
P _l	-0.53	-0.42	-0.32	-0.07	0.21	0.64
E _f	-1.76	-1.94	-2.11	-2.52	-2.99	-3.69
E _b	-15.09	-15.82	-16.48	-18.07	-19.81	-22.37
GW	-17.92	-18.37	-18.76	-19.73	-20.79	-22.37
M _c	-13.25	-14.17	-15.00	-16.99	-19.17	-22.37
M _{0h}	0.39	0.42	0.44	0.50	0.57	0.66
M ₁	-0.37	-0.40	-0.42	-0.47	-0.53	-0.62
λ _r	-0.25	-0.22	-0.20	-0.14	-0.08	0.00

of interest is lower during adjustment. The composition of the total stock of money shifts from credit to government paper-money.

Comparing the precise results with the full-competition case shows that banks in the case of some monopolistic power can cope better with the adverse effects of the technological shock. The price of credit does not fall as much as was the case before. Consequently one can see some of the beneficial effects in the table by looking at the time paths of the volume of credit, the worth of licences and the price of a bank-share.

Simulation in the case of adjustment costs

What we are most interested in is the simulation of the regime of credit rationing and the spill-over effects of the monetary to the real sector. As a matter of fact it proved practically impossible to generate this regime by simulation, whatever the set of parameters chosen. The shocks have to be so big to possibly generate credit-rationing during a short interval of time (by forcing the shadow-price λ_r down to the value zero), that the regime of credit absorption practically always prevails during adjustment.

The main point to focus on is that due to the adjustment costs of changing the rate of interest, the latter is predetermined. That is to say that the rate of interest cannot react immediately in response to changing market conditions. Though it is optimal for banks to lower the rate of interest in response to the technological shock, they can only do this gradually because the existence of adjustment costs. More correctly: by definition the response of banks is optimal, but in the case of absence of adjustment

Table 9 A technological shock, adjustment costs included

period → variable ↓	0	1	2	5	10	stationary state
k	0	0.33	0.62	1.35	2.18	3.45
U	46.07	47.03	47.89	50.00	52.36	55.95
A	-1.48	-1.61	-1.73	-2.04	-2.39	-2.94
c	4.17	4.42	4.64	5.19	5.80	6.74
P _y	-4.44	-4.66	-4.87	-5.39	-5.99	-6.90
i	3.42	3.43	3.43	3.44	3.45	3.45
l _f	-0.01	0.02	0.05	0.11	0.17	0.28
l _b	-17.10	-16.73	-16.67	-17.43	-19.28	-22.87
l	-0.03	0.00	0.02	0.08	0.15	0.24
y	4.00	4.18	4.35	4.76	5.22	5.92
R	0	-0.05	-0.08	-0.11	-0.08	0.00
P _l	-0.61	-0.47	-0.35	-0.08	0.21	0.64
E _f	-1.82	-1.99	-2.14	-2.53	-2.98	-3.69
E _b	-17.02	-17.09	-17.30	-18.22	-19.73	-22.37
GW	-18.29	-18.58	-18.87	-19.70	-20.73	-22.37
M _c ^s	-12.81	-13.88	-14.80	-16.93	-19.17	-22.37
M _c ^d	-16.20	-16.13	-16.27	-17.26	-19.08	-22.37
M _{0h}	0.48	0.48	0.48	0.51	0.56	0.66
M ₁	-0.45	-0.45	-0.45	-0.48	-0.53	-0.62
λ _r	1.68	1.08	0.66	0.06	-0.13	0.00

costs it is optimal to lower the rate of interest immediately, whereas with these costs it is not.

As a result of the relatively high rates of interest, banks will loose more customers and sell less credit than could have been the case without costs of adjustment.

7 Conclusions

The perfect competition bank-model of chapter 2 is modified to show the characteristics of monopolistic competition at the market for credit. In line with the literature (see Meijdam(1991a), Barro(1984)) some degree of imperfect knowledge (or myopicity) is introduced on behalf of the bankers. When banks are thought to have price-setting power, it appears that in the absence of adjustment costs the regime of credit-absorption is always relevant, that is to say that banks always find it optimal to over-supply the market for credit. Introducing adjustment costs regarding the change of the rate of interest (in the same way as installation costs for capital-stock were dealt with) shows that both regimes, credit absorption and credit rationing, can be optimal outcomes depending on the value of the relevant shadow-price. Simulation shows that it is virtually impossible to generate the regime of credit-rationing. Very strong adjustment costs and relatively big shocks to the economy should give scope for simulating the regime of credit rationing, though computational problems arise in these cases.

The stationary state rate of return on a banking licence is determined by the attractiveness of credit. This can be read from the Amoroso-Robinson

formula for the banking-model. The lower the demand elasticity for credit, the higher the rate of return on a banking licence.

Simulation results show that the monopolistic bank can cope better with adverse shocks than its full-competition brother or sister. Introducing adjustment costs decreases the advantages for the monopolistic bank to some degree.

Banking model, tables 8 and 9

$$l_b^d \quad | \quad g_l = \frac{P_l / P_y}{R \cdot \frac{\phi - 1}{\phi}}$$

$$l_f \quad | \quad f_l = \frac{P_l}{P_y}$$

$$c_d \quad | \quad u_c = (1 + R) \cdot P_y \cdot X$$

$$l_s \quad | \quad u_{l-s} = P_l \cdot X$$

$$M_c^d \quad | \quad M_c^d = (c_d \cdot P_y - \bar{M}_0) \cdot \frac{\phi}{\phi - 1}$$

$$M_c^s \quad | \quad \frac{M_c^s}{P_y} = g(l_b^d)$$

$$M_c \quad | \quad M_c = \min(M_c^d, M_c^s)$$

$$c \quad | \quad c \cdot P_y = \bar{M}_0 + \frac{\phi - 1}{\phi} \cdot M_c$$

$$M_1 \quad | \quad M_1 = c \cdot P_y$$

$$M_{0b} \quad | \quad M_{0b} = \bar{M}_0 - M_{0s}$$

$$M_{0b} \quad | \quad M_{0b} = \frac{M_c}{\phi}$$

$$l_b \quad | \quad \frac{M_c}{P_y} = g(l_b)$$

$$P_l \quad | \quad l_b + l_f = l_s = l$$

$$y \quad | \quad y = f(l_f, k)$$

$$i \quad | \quad Q = P_y \cdot (1 + h_i)$$

$$j \quad | \quad j = i + h(i, k)$$

$$P_y \quad | \quad y = c + j + z(s)$$

$$\begin{array}{lcl}
s & | & \dot{s} = \frac{\dot{R}}{R} \\
E_b & | & \dot{E}_b = A - E_f - M_{ob} \\
GW & | & \dot{GW} = E_b - M_{ob} \\
D_f & | & \dot{D}_f = y \cdot P_y - l_f \cdot P_l - j \cdot P_y \\
u & | & \dot{u} = u(c, l_m - l) \\
\lambda_r & | & \dot{\lambda}_r = R \cdot \frac{\phi - 1}{\phi} - \frac{P_l / P_y}{g_{l_i}} \\
k & | & \dot{k} = i - \delta \cdot k \\
Q & | & \dot{Q} = (R + \delta) \cdot Q - P_y \cdot (f_k - h_k) \\
R & | & \dot{R} = \beta \cdot \frac{\mu}{P_y} \cdot R^2 \\
A & | & \dot{A} = R \cdot (A - M_1) + l \cdot P_l - c \cdot P_y \\
X & | & \dot{X} = (v - R) \cdot X \\
U & | & \dot{U} = v \cdot U - u \\
E_f & | & \dot{E}_f = R \cdot E_f - D_f \\
\mu & | & \dot{\mu} = R \cdot \mu - \frac{M_\varepsilon}{R} \cdot (R - \zeta \cdot \lambda_r)
\end{array}$$

Specifications used in simulations: (2.2.12), (1.3.11), (1.3.12), (1.3.13) and (3.2.3).

Parameter values:

$\alpha_f = 0.25$	$\alpha_b = 0.70$	$\gamma_c = 0.85$
$l_m = 9.0$	$\varepsilon_f = 0.25$	$\varepsilon_b = 1.40$
$M_0 = 1.00$	$\sigma = 0.40$	$\delta = 0.10$
$\nu = 0.10$	$\psi = 0.125$	$\phi = 2.00$
$\beta = 4.00 / \infty$	$\zeta = 4.00$	$\gamma_l = 0.10$

Stationary state:

k = 3.594	R = 0.10
Q = 0.960	P _l = 0.091
U = 1.020	E _f = 3.451
A = 4.469	E _b = 0.047
X = 0.791	GW = 0.019
c = 1.071	M _c ^d = 0.057
P _y = 0.960	M _c ^s = 0.289
j = 0.339	M _c = 0.057
l = 7.535	M _{0h} = 0.971
l _f = 7.524	M ₁ = 1.029
l _b = 0.011	M _{0b} = 0.029
y = 1.431	$\lambda_r = 0.025$

THE CHOICE BETWEEN DEBT AND EQUITY

1 Introduction

We return to the basic model of chapter 1 to study the introduction of a risky asset. This makes it of interest to distinguish between shares and bonds (equity and debt) at the firm's level. The main point of this chapter is to re-discuss the Modigliani-Miller results of their original (1958)-paper and the (1963)-correction thereof. When dealing with matters associated with "risk", we want to analyse the issues of uncertainty without departing from the approach taken for deterministic problems. Uncertainty is just another dimension to the model. Different probabilistic "states of nature" are introduced to compute expected rates of return at any given moment. Now it is the cash-flow to the holder of the security that determines the value of that security, alongside the expected rate of return and the amount of risk. A project of which the returns are not the same in all states of nature is risky, in the sense that the investor's realized return cannot be predicted with certainty (see Auerbach(1983), p.931). Uncertainty is modelled following the Yaari-Blanchard life-time model. Any firm has a constant probability π at any moment of continuous time of stopping economic activity. The probability of failure at time t is $\pi \cdot e^{-\pi \cdot t}$. The probability

that a firm is "dead" before a fixed date T is $\int_0^T \pi \cdot e^{-\pi \cdot t} dt = 1 - e^{-\pi \cdot T}$. There-

fore, the probability that the firm is still alive at time T is $e^{-\pi \cdot T}$. The

expected life-time of a firm is $\int_0^{\infty} t \cdot \pi \cdot e^{-\pi \cdot t} dt = 1/\pi$. The reference model

of chapter 1 appears to be a special case of the model of this chapter: as π goes to zero, the expected life-time of the firm goes to infinity.

To finance their activities, firms have equity and debt outstanding. Consumers think they are small enough not to be able to completely diversify all risk of their portfolios at no cost. The choice between risk and return is being made continuously according to a CES-utility specification, assuming the degree of certainty and return to be inputs.

There are several ways out of the well-known problem that the optimal amount of equity is indeterminate. Most, if not all, of them are "deus ex machina" for explaining the coexistence of sources of finance with apparently different costs. Not satisfactory at all are approaches that assume some arbitrary maximum debt-equity ratio or that assume that the required rates of return on debt and equity increase (arbitrarily) with leverage, presumably because of some (undefined) factor of (most of the time: bankruptcy) risk.¹ Our approach is, given the framework of the thesis, less

¹ The fact that the firm in our model faces some probability of breakdown must not be confused with the notion of bankruptcy costs. The introduction of bankruptcy costs serves to make the sum of pay-offs discontinuous at the point of bankruptcy (so avoid MM-irrelevance) (see also Hayashi(1985)). Our probability of breakdown is connected with differ-

arbitrary to a greater extent, though (certainly) not free of caveats.

In dealing with the problem of optimal leverage, we impose an additional constraint upon the firm's problem. This capital-in-advance constraint guarantees the absolute risk-freeness of debt. We assume management to make the same decisions as equity holders would have made, so as to exclude agency problems. An agency problem arises when there is a conflict between the interests of several groups within the firm, to be seen as a "nexus of contracts" (See Bishop, Crapp and Twite(1983)). For example, as managers are also utility-maximizers, it is possible for them to direct resources to their own benefit rather than that of the equity holders. The costs of divergence between the interests of the equity holders and managers is referred to as an agency cost. We assume that the market for managers will be able to control deviant actions of managers, so that this kind of agency problem is non-existent. Since we assumed that no new equity is ever issued (that is to mean that no new equity holders come into play) there cannot be the question whether management should operate in the interest of existing or all equity holders. With debt securities outstanding, maximizing equity holders' wealth and maximizing the total value of the firm are not necessarily consistent objectives. Management could sell all the assets of the firm and distribute the proceeds to equity holders as dividend. This would leave the debt-holders with empty claims, the value of their claims being passed to the equity holders. In the model of this chapter, the objectives of maximizing the value of equity or the value of the firm turn out to be the same. A distortionary element is introduced in the chapter in the form of a profit tax with tax-deductible interest payments.

ent states of nature and exogenous to the model.

The latter is done to escape the irrelevance of financial structure of the MM-world without taxes.

Again we have a number of departures from the basic postulates of chapter 0. Because of the uncertainty structure of the model, there is no longer perfect foresight with respect to the moment of financial breakdown of the individual firm. Secondly, consumers think (mistakenly) that they can't diversify all risk (whereas they can by organizing an insurance company).² Thirdly, debt holders require a certain amount of real assets to be held by the firm to guarantee their claims to be of risk-free quality. In fact, debt is risk-free from the outset, since equity holders reckon with the fact that debt must be repaid at all times and this shows in the expected value of equity.

2 Firm behaviour

Every individual firm continuously faces the possibility of going bankrupt. In the case of bankruptcy, the firm ceases to exist. This sudden breakdown can be looked upon as caused by a local earthquake that destroys all managerial capabilities embodied in this particular firm. It is assumed that all

² One can imagine that consumers pay for the organization of an insurance company that reduces the risk of their portfolios. Then the question would be to what extent consumers would want to pay for letting their risk be reduced. Modelling these costs in the "right" way could deliver the same results as obtained in our model.

firms face a constant probability of managerial breakdown, π . This structure of breakdown-activity bears great resemblance with the age-structure of population assumed in Blanchard (1985).³ It is legitimate to say that the representative firm in our model is finitely lived. To finance its activities, the firm issues shares of equity. It is clear that the holder of equity bears the risk of breakdown of the firm. In order to supply a range of assets to the public, the firm also issues debt. Debt is assumed to be risk-free, so that the individual holder of assets can decide upon the optimal risk-structure of his portfolio by combining the appropriate amounts of equity and debt.

Let us assume for the moment that the firm's objective is to maximize shareholders' wealth, E . The value of equity consists of all dividend payments that are expected to be generated by the firm, discounted by the required rate of return on equity: R_e . The moment of breakdown is denoted by T and it is assumed that in the case of breakdown all outstanding debt, B , is repaid by the owners of the firm. Capital stock, k , is the only assumed physical asset at the firm's level. The stock of capital can be sold at price P_y per unit. Once sold, these units of capital are assumed to be of the same quality as produced output, y , and can consequently be used for consumption or investment purposes.

The probability that the firm breaks down at moment T is: $\pi \cdot e^{-\pi \cdot T}$. We have to integrate the (certain) value of equity when the firm stops at (a certain) moment T over all possible moments of breakdown in order to obtain

³ This implies that in order to obtain a constant number of firms at any moment of time, it has to be assumed that at each moment of time, a certain amount of entrepreneurs starts a new firm.

the expected value of equity. When doing this, the certain values have to be weighted with the probability that the firm breaks down exactly at moment T . Hence, the expected value of equity of the representative firm should be:

$$(4.2.1) \quad E = \int_0^{\infty} \left\{ \int_0^T D \cdot e^{-\int_0^z R_s ds} dz + \{ k \cdot P_y - B \} \cdot e^{-\int_0^T R_s ds} \right\} \cdot \pi \cdot e^{-\pi T} dT .$$

This expression can be rewritten as follows, by changing the order of integration (see Yaari(1965)):

$$(4.2.2) \quad E = \int_0^{\infty} \{ D + \pi \cdot (k \cdot P_y - B) \} \cdot e^{-\int_0^z (R_s + \pi) ds} dz .$$

Dividend payments, as used in the latter two expressions, consist of:

$$(4.2.3) \quad D = y \cdot P_y - l \cdot P_l - j \cdot P_y - R_f \cdot B + \dot{B} .$$

Another way of writing expression (4.2.2) is:

$$(4.2.4) \quad E = \int_0^{\infty} \{ D + \pi \cdot (k \cdot P_y - B) - \pi \cdot E \} \cdot e^{-\int_0^z R_s ds} dz .$$

The latter expression can be more insightful than the others because the value of equity is related directly to the required return on equity itself.

As expression (4.2.3) indicates, dividend payments consist of a series of cash-flows, springing from different sources: sales of produced output ($y \cdot P_y$) minus wages paid to hired labour force ($l \cdot P_l$) minus investment expenditure ($j \cdot P_y$) minus interest paid on the existing body of debt ($R_f \cdot B$) plus the additional funds available from issuing new debt (\dot{B}).⁴

The first three components of the right hand side of (4.2.3) are standard and can be found in earlier chapters too. It is assumed that output is a function of labour and capital inputs, $y = f(l, k)$, and that investment, j , includes installation costs, the latter a function of investment and the level of capital stock. The last two terms of the right hand side of (4.2.3) are due to the introduction of debt. The return on debt is the risk-free rate of interest, R_f . In maximizing the value of equity, the firm has three instruments at its disposal: labour, investment and debt accumulation. The producers' problem can now be solved, taking into account the constraints on capital and debt accumulation. Capital stock depreciates at a constant rate of δ , whereas only investment net of installation costs, i , is assumed to build up capital stock:

$$(4.2.5) \quad \dot{k} = i - \delta \cdot k .$$

It should be noted that this process of capital accumulation is relevant only for individual firms during their producing period, implying that capital accumulation at the aggregate level must take into account the conse-

⁴ In fact this can be redemption as well as issuance of debt.

quences of breakdown.

Debt accumulation is catered for by substituting the dummy variable Z in equation (4.2.3) for \dot{B} and adding the constraint on debt accumulation according to:

$$(4.2.6) \quad \dot{B} = Z .$$

Use of standard solution procedures delivers as one of the first-order conditions the well-known result that the required rate of return on equity should equal the risk-free rate of return:

$$(4.2.7) \quad R_e = R_f .$$

The interpretation of this statement is that the firm accumulates debt until the rate of return on debt and equity are the same. In a risk-neutral world, which implies that investors (in our case: consumers) do not require a risk-premium on top of the risk-free rate, the debt/equity-ratio would be indeterminate. In the more common case of risk-averse consumers (implying that $R_e > R_f$) the firm would accumulate debt infinitely.

This result is most problematic. We avoid this perfidious result by adding another constraint to the producers problem. While the return on debt is assumed to be risk-free, there is no guarantee whatsoever that debt is repaid by stockholders at the moment of bankruptcy. There is always some unspecified probability of debt not being totally risk-free. It does not seem unreasonable therefore that holders of debt require the firm to back the claims of debt by holding a certain amount of real (i.e. non-financial) assets. Let us therefore take it that holders of debt require the total amount

of nominal debt not to exceed the nominal sales-value of capital stock:

$$(4.2.8) \quad B \leq k \cdot P_y .$$

We shall refer to this important condition as the capital-in-advance condition.⁵ Adding this condition and solving the producers' problem once again, delivers the following first-order conditions:

$$(4.2.9) \quad f_l = \frac{P_l}{P_y} ,$$

$$(4.2.10) \quad Q = P_y \cdot (1 + h_l) ,$$

⁵ Here and before, we assumed that the scrap value of capital stock is the same as the market value of the other goods at that moment. In general the scrap value is some fraction of this amount. There is no problem in taking a more general relation into the model. It is quite possible however that, depending on a number of additional assumptions that have to be made, the variance of returns on shares changes from a simple and constant $\pi \cdot (1 - \pi)$ (see section 3), into a very complex expression, containing several variables of the model. We have actually run the model with a variable scrap value of capital stock. Since no additional benefits could be seen from this operation, we present a simplifying case.

$$(4.2.11) \quad \dot{Q} = (R_e + \pi + \delta) \cdot Q - P_y \cdot (f_k - h_k + \pi + \mu) ,$$

$$(4.2.12) \quad \mu = R_e - R_f \quad (> 0) ,$$

$$(4.2.13) \quad B = k \cdot P_y .$$

The function $h(i, k)$ describes, as before, installation costs, Q denotes the shadowprice of capital accumulation and μ is the Lagrange-parameter for the capital-in-advance condition. Assuming risk-averse consumer behaviour (see next section) it is clear that $R_e > R_f$. As a result, the shadowprice μ is positive and the capital-in-advance condition is always binding.⁶ This imposes an optimal debt/equity-ratio to the firm. However, as we will demonstrate below, the value of the firm, being the addition of equity and debt, is unaltered by shifting from equity to debt or vice versa.⁷

⁶ The fact that $B = k \cdot P_y$ will be used in deriving various expressions to come.

⁷ As the shadow-price μ is always positive, the firm wants to finance as much with debt as possible. Our model knows only one finance regime that is optimal at all times: debt finance. Hayashi(1985) deals with bankruptcy costs in order to make a distinction between three relevant financing-regimes: the regimes of retained profits, debt finance and (new) equity finance. In our model it should be quite easy to discrimi-

Now we wonder whether it is more suitable for the firm to maximize the value of the firm, this including the value of debt:

$$(4.2.14) \quad V = E + B .$$

We already have an expression for the value of equity in equations (4.2.1), (4.2.2) and (4.2.4). By definition it is true that:

$$(4.2.15) \quad B = \int_t^{\infty} \{ (R_e + \pi) \cdot B - \dot{B} \} \cdot e^{-\int_t^z (R_e + \pi) ds} dz .$$

By adding relations (4.2.2) and (4.2.15), we get for the total value of the firm at the micro-level:

nate between a number of different finance-regimes. Forgetting for a moment about the distinction between internal and external equity finance, we can discriminate between financing with retained profits and debt-finance as optimal regimes by the introduction of flotation costs associated with the issuance of debt. For example, introduce a function $z(s)$ to describe flotation costs, where: $s = \dot{B}/B$. The stationary state of the model does not change by adding these flotation costs. Solving the complete model of the firm shows that in the case of flotation costs the shadowprice μ no longer has to be of positive value all the time. In case the shadowprice is zero it is optimal for the firm to finance with retained profits. Moreover, the market value of debt no longer has to coincide with the value of the stock of capital.

$$(4.2.16) \quad V = \int_0^{\infty} \{y \cdot P_y - l \cdot P_l - j \cdot P_j + (R_e - R_f) \cdot B + \pi \cdot k \cdot P_y\} \cdot e^{-\int_0^z (R_e + \pi) ds} dz .$$

Now one can easily see that all actions taken by the firm under this alternative value-maximizing regime are exactly the same as derived above under the assumption of value maximization of shares. Because it is impossible to make debt-holders worse off by shifting risk from equity to debt, it is clear that the two seemingly different objective functions deliver the same results.

Another problem is whether the value of the firm depends on the amount of debt issued. In other words: does leveraging affect the value of the firm? To demonstrate the Modigliani-Miller proposition that the value of the firm does not alter with the debt-equity ratio, we introduce two variables at the micro-level:

V_L = value of the levered firm.

V_U = value of the unlevered firm.

Of course, the value of the levered firm corresponds with expression (4.2.16) above. We gain some insight when rewriting (4.2.16) in terms of the weighted average cost of capital (the so-called WACC), which is defined as:

$$(4.2.17) \quad WACC = R_f \cdot \frac{B}{V} + R_e \cdot \frac{E}{V} .$$

The value of the levered firm, using the definition of the WACC, reads:

$$(4.2.18) \quad V_L = \int_t^{\infty} \{ y \cdot P_y - l \cdot P_l - j \cdot P_y - \pi \cdot E \} \cdot e^{-\int_t^z WACC ds} dz .$$

If this firm is forbidden to issue debt ($B = 0$), we get the unlevered value of the firm. We know beforehand that the following conditions hold true for the unlevered firm:

$$(4.2.19) \quad V_U = E ,$$

$$(4.2.20) \quad WACC = R_e .$$

Reckoning with the continuous probability of managerial breakdown, we derive the following for the equity-only firm:

$$(4.2.21) \quad V_U = \int_t^{\infty} \{ y \cdot P_y - l \cdot P_l - j \cdot P_y \} \cdot e^{-\int_t^z (R_e + \pi) ds} dz .$$

Rewriting this in terms of the WACC:

$$(4.2.22) \quad V_U = \int_t^{\infty} \{ y \cdot P_y - l \cdot P_l - j \cdot P_y - \pi \cdot E \} \cdot e^{-\int_t^z WACC ds} dz ,$$

which is the same as expression (4.2.18) for the levered firm above. By now we have stated the Modigliani-Miller proposition at the firm's level in a world without taxes and a continuous probability of managerial breakdown:

$$(4.2.23) \quad V_L = V_U .$$

Staying on speaking terms with the finance literature, the expression $(y \cdot P_y - l \cdot P_l - j \cdot P_y)$ is called the net operating income of the firm (*NOI*). The *NOI* constitutes the cash-flow of the unlevered firm above. It is clear by now that even in the presence of uncertainty the choice of the debt-equity ratio has no effect on market-value (MM-theorem), since the cost of capital is (ultimately) independent from the firm's debt-equity ratio. We have reached two important conclusions of the analysis so far. Firstly, firm behaviour does not change whether the goal is maximizing the value of equity or maximizing the value of equity and bonds taken together. Secondly, the value of the firm is independent from the value of bonds (irrelevance of finance structure).

3 Consumer behaviour

In contrast with firm behaviour, we assume that each consumer is a representative one and infinitely lived. Consumers are very large in number and so small in terms of financial wealth that they cannot avoid (or they think they cannot) the risk of losing the value of equity in the case of a financial

breakdown of "their" firm. Of course, a collection of asset-holders could cancel out all existing risk by holding all equity together in one portfolio. Consumers are ignorant or myopic at this particular point. Financial wealth of consumers (A) can be stored in equity, bonds or (helicopter) money:

$$(4.3.1) \quad A = E + B + M .$$

Holding bonds promises to pay the risk-free rate of interest R_f , holding equity has an expected return R_e , whereas money does not deliver a nominal return. Consumers are supposed to choose in two stages. In stage one they decide how much to consume of the three "goods" from which (direct) utility can be derived: consumption of goods (c), consumption of leisure ($l_m - l$) and consumption of liquidity services by holding money. Clearly we assume a traditional intertemporal utility function with as inputs consumption, leisure and real cash-balances. In stage two, consumers decide every time again hereafter (continuously in fact) how to divide the non-monetary part of their financial wealth between bonds and equity. Equity promises to pay a higher return at a higher level of risk. The consumers are supposed to be risk-averse, a standard assumption in the theoretic financial literature.

To derive a capital market-line, representing all combinations of risk and return open to consumers, we need to know the risk of equity. Risk is looked upon by the consumer as the variance of returns. Holding a share of the firm gives a return of R_e^+ with probability $(1-\pi)$ and a return of R_e^- with probability π . R_e^+ and R_e^- denote the rate of return in case of success

and failure of the firm respectively:

$$(4.3.2) \quad R_e^+ = \frac{D + \dot{E}}{E} ,$$

$$(4.3.3) \quad R_e^- = \frac{D + \dot{E} - E + (k \cdot P_y - B)}{E} .$$

The first of these two expressions speaks for itself: in case of success the shareholder benefits from dividend payout and capital gains. In case of failure, according to (4.3.3), the normal return accrues to the shareholders for the last time, but in addition the shareholders guarantee the risk-freeness of debt (which means paying or receiving a sum of money equal to $k \cdot P_y - B$) and face the fact that equity is worthless from that moment on. The standard deviation of returns on equity is in this case:

$$(4.3.4) \quad \sigma_e = \sqrt{\pi \cdot (1 - \pi)} ,$$

whereas the expected return on equity should be:

$$(4.3.5) \quad R_e = \frac{D + \dot{E} - \pi \cdot E}{E} .$$

Now the consumer can choose the most desired portfolio of risk and return, according to the following relation:

$$(4.3.6) \quad R_p = R_f + \left\{ \frac{R_e - R_f}{\sigma_e} \right\} \cdot \sigma_p ,$$

$$0 \leq R_p \leq R_e , \quad 0 \leq \sigma_p \leq \sigma_e ,$$

where R_p and σ_p symbolize the amounts of (expected) return and risk to choose respectively. Expression (4.3.6) represents the capital-market-line. Which of these combinations of risk and return is chosen, depends on the degree of risk-aversion of consumers. For the sake of convenience, we suppose a simple CES-structure between risk and return:

$$(4.3.7) \quad u^* = \{ \gamma^* \cdot R_p^{-\rho^*} + (1 - \gamma^*) \cdot (\sigma_e - \sigma_p)^{-\rho^*} \}^{-\frac{1}{\rho^*}} .$$

The difference between σ_e and σ_p figures as a measure of portfolio-risk (σ_e being the maximum amount of risk available) while the parameter γ^* indicates the degree of risk-aversion.⁸ Part of the consumer's problem consists of maximizing the "risk-utility" subject to the possibilities offered by firms according to (4.3.6). Solving this part of the consumer's problem delivers:

⁸ We have also experimented with a quadratic utility function (following much of the literature on the topic) but met insurmountable stability problems when the results were inserted into the complete macroeconomic model and numerical simulations were carried out.

$$(4.3.8) \quad R_e - R_f = \left\{ \frac{\gamma^*}{1 - \gamma^*} \right\}^{\frac{1}{\rho^*}} \cdot \sigma_e \cdot \left\{ \frac{R_e}{R_p} - 1 \right\}^{\frac{\rho^* + 1}{\rho^*}},$$

To simplify matters still further we assume from now on that $\rho^* = 1$, and state the following properties, which can be derived from (4.3.8), verbally.

When the parameter for the degree of risk-aversion γ^* is increased the required return on the portfolio will be greater, while an increase in the maximum amount of risk available, σ_e , also increases the required rate of return on the portfolio.

Though the treatment of the risk-return trade-off falls completely within the textbook framework, note that our approach is more complicated because the return on equity is endogenized in the complete macroeconomic model, which implies that the capital-market line is on the move all the time during adjustment.

The last step in formulating the consumer model is the maximization of intertemporal utility (U) subject to the intertemporal budget constraint.

The objective function reads:

$$(4.3.9) \quad U = \int_t^{\infty} u(c, l_m - l, \frac{M}{P_y}) \cdot e^{-\rho \cdot (z-t)} dz,$$

while the intertemporal budget constraint is:

$$(4.3.10) \quad \dot{A} = R_p \cdot (A - M) - c \cdot P_y + l \cdot P_l + \pi \cdot B.$$

The latter equation can be derived using the aggregate values of equity (see next section) and bonds, assuming a constant nominal stock of money. The financial wealth of a representative consumer is determined by macroeconomic variables. It is therefore remarkable that for all consumers taken together financial wealth is deterministic in character, devoid of any risk. The reason is, as mentioned before, that on a macro-level it is known with perfect certainty how many firms go bankrupt and stop economic activity. It is therefore important to remember that the representative consumer in the model is too small to eliminate all risk of the portfolio at no cost. The behavioural equations of the consumer's model therefore reflect the riskiness of equity.

Solving the intertemporal maximization problem of the consumers subject to the budget constraint, delivers as first order conditions:

$$(4.3.11) \quad u_c = X \cdot P_y ,$$

$$(4.3.12) \quad u_{l_m - l} = X \cdot P_l ,$$

$$(4.3.13) \quad u_{\frac{M}{P_y}} = X \cdot P_y \cdot R_p ,$$

$$(4.3.14) \quad \dot{X} = (\nu - R_p) \cdot X ,$$

where X is the shadowprice of financial wealth, as it was before.

4 Aggregate behaviour

When turning to macro relations, we have to aggregate over all existing firms. This has implications for a number of relations derived previously. In order to avoid notational problems, we do not use subscripts to denote aggregate variables. It should be clear from the context whether micro or macro variables are meant. The value of equity aggregated over all existing firms is:

$$(4.4.1) \quad E = \int_t^{\infty} \{ y \cdot P_y - l \cdot P_l - j \cdot P_j - R_f \cdot B + \dot{B} \} \cdot e^{-\int_t^z R_e ds} dz .$$

Comparing this result with the value of equity at the firm's level shows that a fraction π of the value of aggregate equity is destroyed continuously. The value of all firms taken together is:

$$(4.4.2) \quad V = \int_t^{\infty} \{ y \cdot P_y - l \cdot P_l - j \cdot P_j \} \cdot e^{-\int_t^z R_p ds} dz .$$

The latter expression can be obtained by adding the value of equity and bonds at the macro level. The value of bonds can for that purpose be rewritten as:

$$(4.4.3) \quad B = \int_t^{\infty} \{ R_e \cdot B - \dot{B} \} \cdot e^{-\int_t^z R_e ds} dz .$$

The return on the consumer's total portfolio is determined by:

$$(4.4.4) \quad R_p \cdot V = R_f \cdot B + R_e \cdot E .$$

Physical capital stock at the firm's level depreciates at a constant rate δ . At the macro level a fraction π of the capital stock is sold as goods for consumption and (new) investment purposes. This implies for the macro-model:

$$(4.4.5) \quad \dot{k} = i - (\delta + \pi) \cdot k ,$$

$$(4.4.6) \quad y + \pi \cdot k = c + j .$$

The last equation describes the continuous goods-market clearance, which of course implies that total supply of goods ($y + \pi \cdot k$) equals total demand for goods ($c + j$).

Financial wealth of consumers consists of the total of aggregate equity, bonds and money:

$$(4.4.7) \quad A = E + B + M = V + M .$$

Given a constant stock of money, the budget constraint of all consumers together (which equals the budget constraint per individual consumer, forgetting about the different interpretation) is:

$$(4.4.8) \quad \dot{A} = \dot{V} = R_p \cdot V - y \cdot P_y + l \cdot P_l + j \cdot P_y .$$

Together with the expression for the market clearance of goods, this determines the expression for the dynamic budget constraint as used in the previous section on consumer behaviour.

All other relations derived in the micro sections can be used straightaway in the macro-model (be it that they must be interpreted as being macro relations). The model is completed by assuming market clearance both on the money and labour market.

5 Introducing a tax on profits

As one of our main concerns in this chapter is studying the impact of leveraging between equity and bonds, it is interesting to incorporate some form of tax on profits. We assume that a tax is levied on profits of firms. In order to avoid any problems associated with government behaviour, it is assumed that tax revenues are simply redistributed among consumers in a lump-sum fashion. To obtain a straight comparison with the model without taxes, we use the same structure as before in dealing with the producer problem, the consumer problem and the macro model respectively.

Firm behaviour

As before we assume the goal of the firm to be the maximization of the value of equity:

$$(4.5.1) \quad E = \int_0^{\infty} \{ (1-\tau) \cdot \{ NOI - R_f \cdot B \} + \dot{B} + \pi \cdot (k \cdot P_y - B) \} \cdot e^{-\int_0^z (R_e + \pi) ds} dz ,$$

where τ is a constant tax rate on profits. The government allows for complete deductibility of investment outlays and interest paid to debtholders. The maximization is done subject to the additional capital-in-advance condition. Should the maximization be done without the capital-in-advance condition (while reckoning with the conditions of capital and debt accumulation), one of the first order conditions turns out to be:

$$(4.5.2) \quad R_e = (1 - \tau) \cdot R_f .$$

Even in a risk-neutral world, the firm would go on accumulating debt endlessly, which is clearly not a very satisfactory starting point for numerical simulation.⁹ The benefits of accumulating debt would be even greater in the case of risk-averse consumers ($R_e > R_f$).

⁹ The deductibility of interest imparts a bias to the financing decision as firms have an incentive to finance their investment by borrowing. The firm and the household face different after-tax rates of interest. Therefore, the firm's owners, in order to finance investment, will maximize their wealth by borrowing at the firm's level, rather than at the personal level (see Auerbach(1983), p.919). Some constraint is needed to prevent endless borrowing at the firm's level.

The following results can be obtained, taking the capital-in-advance constraint into account:

$$(4.5.3) \quad f_l = \frac{P_l}{P_y} ,$$

$$(4.5.4) \quad Q = P_y \cdot (1 + h_i) \cdot (1 - \tau) ,$$

$$(4.5.5) \quad \dot{Q} = (R_e + \pi + \delta) \cdot Q - P_y \cdot \{ (1 - \tau) \cdot (f_k - h_k) + \pi + \mu \} ,$$

$$(4.5.6) \quad \mu = R_e - (1 - \tau) \cdot R_f \quad (> 0) ,$$

$$(4.5.7) \quad B = k \cdot P_y .$$

It is stated here without proof that the alternative goal of maximizing the value of the firm again leads to exactly the same behavioural results as derived in the case of equity maximization. The total value of the levered firm is:

$$(4.5.8) \quad V_L = \int_0^{\infty} \{ (1 - \tau) \cdot NOI + (R_e - (1 - \tau) \cdot R_f) \cdot B + \pi \cdot k \cdot P_y \} \cdot e^{-\int_0^z (R_e + \pi) ds} dz .$$

In the case of a profit tax, the WACC and the return on the consumer's portfolio are not the same anymore.

As before we have the definition of the return on the consumer's portfolio:¹⁰

$$(4.5.9) \quad R_p = R_e \cdot \frac{E}{V} + R_f \cdot \frac{B}{V} .$$

But now, since the firm reckons with the after tax cost of debt, we have for the weighted average cost of capital:

$$(4.5.10) \quad WACC = R_e \cdot \frac{E}{V} + R_f \cdot (1 - \tau) \cdot \frac{B}{V} .$$

Now it is time to refer to chapter 0. There we discussed how to obtain the results of MM(1958 and 1963) in the case of a corporate tax. Crucial is the introduction of a new financial asset S that represents the market value of all tax-payments to consumers. The relevant rate of discount is called R_{ov} . Rational consumers who "see through" the model recognize the fact that tax-redistributions are characterized by the same risk as the dividends they receive. The implication would be that consumers require the same rate of return on holding these two assets. We define the return on the "enlarged" portfolio of consumers:

$$(4.5.11) \quad R_u = \frac{E}{V+S} \cdot R_e + \frac{B}{V+S} \cdot R_f + \frac{S}{V+S} \cdot R_{ov} .$$

¹⁰ In this section V and V_L will be used interchangeably.

As can be seen in chapter 0 this leads to the relation expressing that the required rate of return on equity depends on the amount of leverage in the following manner:

$$(4.5.12) \quad R_e = R_u + (R_u - R_f) \cdot (1 - \tau) \cdot \frac{B}{E} .$$

It is now straightforward to rewrite the value of the levered firm in terms of the enlarged portfolio return:

$$(4.5.13) \quad V_L = \int_0^{\infty} \left\{ (1 - \tau) \cdot NOI + \tau \cdot R_u \cdot B - \pi \cdot E \right\} \cdot e^{-\int_0^z R_u ds} dz .$$

Let us look at the value of the firm when financed only with equity. In that case a number of conditions hold:

$$(4.5.14) \quad V_U = E ,$$

$$(4.5.15) \quad WACC = R_e ,$$

$$(4.5.16) \quad R_e = R_u .$$

We have for the equity-only firm:

$$(4.5.17) \quad V_U = \int_0^{\infty} \{ (1 - \tau) \cdot NOI \} \cdot e^{-\int_0^z (R_u + \pi) ds} dz .$$

Rewriting this in terms of R_u (or for that matter in terms of R_e or the WACC):

$$(4.5.18) \quad V_U = \int_0^{\infty} \{ (1 - \tau) \cdot NOI - \pi \cdot E \} \cdot e^{-\int_0^z R_u ds} dz .$$

The comparison of the value of financial wealth (equity plus bonds) between the levered and unlevered situation in the case of a profit tax with tax-deductible interest payments, shows:

$$(4.5.19) \quad V_L = V_U + \int_0^{\infty} \{ \tau \cdot R_u \cdot B \} \cdot e^{-\int_0^z R_u ds} dz .$$

This implies that MM-relation (0.3.13) of chapter 0 holds for the stationary state of our model:

$$(4.5.20) \quad V_L = V_U + \tau \cdot \frac{R_u}{R_u} \cdot B = V_U + \tau \cdot B .$$

Consumer behaviour

The government redistributes taxes levied on profits in a lump-sum manner to consumers:

$$(4.5.21) \quad \tau \cdot (y \cdot P_y - l \cdot P_l - j \cdot P_j - R_f \cdot B) = T ,$$

where T are lump-sum transfers to consumers.

The intertemporal budget constraint for consumers changes therefore into:

$$(4.5.22) \quad \dot{V} = R_p \cdot V - c \cdot P_y + l \cdot P_l + \pi \cdot B + T .$$

The trick now is to write the intertemporal budget constraint of consumers in terms of the enlarged portfolio (including the new transfer asset S). Financial wealth of the consumer must be written as:

$$(4.5.23) \quad A = E + B + S + \bar{M} .$$

From the definition of the market-value of transfers, we know that:

$$(4.5.24) \quad \dot{S} = R_{ov} \cdot S - T .$$

For the rewritten intertemporal budget constraint, we obtain:

$$(4.5.25) \quad \dot{A} = R_u \cdot (A - \bar{M}) - c \cdot P_y + l \cdot P_l + \pi \cdot B .$$

Solving the consumer's problem again, this time with the inclusion of taxes, leads to the same optimal strategies for consumers as derived in the section without taxes (see equations (4.3.11) to (4.3.14)). The one difference however is that the symbol R_p has to be replaced by the symbol R_u . Now it is not at all clear that the consumer can decide upon the amounts of all financial assets around. The addition of lump-sum transfers to the problem and letting the consumers optimize over the amounts of debt and equity (and the relevant portfolio return) does not lead to the standard MM-results however (see section 0 once again).

Aggregate behaviour

The value of equity aggregated over all firms is:

$$(4.5.26) \quad E = \int_0^{\infty} \{ (1-\tau) \cdot (NOI - R_f \cdot B) + \dot{B} + \pi \cdot (k \cdot P_y - B) \} \cdot e^{-\int_0^z R_s ds} dz$$

The aggregate value of firms can be expressed as follows:

$$(4.5.27) \quad V = \int_0^{\infty} \{ (1-\tau) \cdot (y \cdot P_y - l \cdot P_l - j \cdot P_j) \} \cdot e^{-\int_0^z WACC ds} dz ,$$

or alternatively as:

$$(4.5.28) \quad V = \int_0^{\infty} \{ y \cdot P_y - l \cdot P_l - j \cdot P_j - T \} \cdot e^{-\int_0^z R_p ds} dz .$$

The aggregate value of financial wealth is described by the following condition (a rewritten version of (4.5.25)):

$$(4.5.29) \quad \dot{A} = R_u \cdot (A - \bar{M}) - y \cdot P_y + l \cdot P_l + j \cdot P_j .$$

This completes the discussion on the change of the model on behalf of the introduction of a profit tax.

6 Some remarks about the numerical simulations

In order to simulate the effects of some disturbances hitting upon this economy, parameter values have to be chosen and some concrete specification of functions have to be introduced. As mentioned earlier, all specifications of functions and parameter sets are chosen uniformly across all chapters.

The computer program starts to compute the stationary state of the economy. Because we assume clearing markets throughout, we encounter the problem of solving a large number of non-linear relations simultaneously. A shock to the economy is modelled by a change in the parameter set. A new stationary state, belonging to the changed parameter set, is computed

by the program. The time paths between the two stationary states are found by utilizing the method of multiple shooting, using the relevant differential equations mentioned above, provided that the complete system exhibits the right stability properties.

The confrontation of producers' and consumers' plans leads to equilibrium outcomes of all prices. The nominal wage rate equilibrates labour supply and demand. The price of goods is found by equating total goods supply and total goods demand. The total return on the consumers portfolio turns out to be a constant due to the Cobb-Douglas intertemporal utility specification. The demand and supply for risky and riskless assets determines the level of the returns on equity and bonds. It is superfluous to say that the demand and supply at every market co-determines demand and supply at every other market. The capital market line, which normally represents a straight line between two fixed points, is itself determined by market forces (while remaining a straight line). The risk on bonds and equity is assumed to be constant and it is the return at a specific level of risk that is determined by market forces.

7 Some remarks about the stationary state

We do not intend to discuss the computation of the stationary state at length. There are important differences with the previous chapters however. The specification of the adjustment costs was chosen originally to ensure that the stationary state of the economy did not exhibit adjustment costs. In the model under consideration there would be no adjustment

costs in the stationary state if no firm was ever to break down ($\pi = 0$). Because this economy not only shows physical depreciation of capital goods at the firms level, but also shows capital destruction at the aggregate level ($\pi > 0$), this economy also faces adjustment costs in the so-called stationary state.

From $\dot{k} = 0$ it follows that:

$$(4.7.1) \quad i = (\delta + \pi) \cdot k .$$

Therefore, given the specification of adjustment costs:

$$(4.7.2) \quad h_i = \frac{\pi}{\psi} \quad , \quad h_k = \frac{2 \cdot \delta \cdot \pi - \pi^2}{2 \cdot \psi} .$$

Therefore:

$$(4.7.3) \quad f_k = (R_e + \delta + \pi) \cdot \left(1 + \frac{\pi}{\psi}\right) - \frac{\pi^2 + 2 \cdot \delta \cdot \pi}{2 \cdot \psi} - \frac{\mu + \pi}{1 - \tau} .$$

Inserting the fact that $\mu = R_e - (1 - \tau) \cdot R_f$ (according to equation (4.5.6)) and assuming for the moment that no taxes are levied, we have:

$$(4.7.4) \quad f_k = R_e \cdot \frac{\pi}{\psi} + R_f + \delta + \frac{\pi^2}{2 \cdot \psi} .$$

Now it is clear that the equilibrium capital stock in the stationary state of our economy depends critically on both the returns on equity and bonds, a

result that can be of great help when analyzing the simulation results.¹¹

For the average q of the economy, the following can be established:

$$(4.7.5) \text{ average } q = \frac{V}{k \cdot P_y} = 1 + \frac{\pi}{\psi} - \frac{\pi}{R_e} .$$

The marginal q for the stationary state of the debt-equity model is different, since:

$$(4.7.6) \text{ marginal } q = \frac{Q}{P_y} = 1 + \frac{\pi}{\psi} .$$

The difference between average q and marginal q must be attributed to the costs associated with the additional capital-in-advance constraint.

8 Numerical simulation results

¹¹ Note that in case there are no taxes and no probability of failure, we have a familiar equation (see chapter 1) back again:

$$f_k = R_f + \delta .$$

Consumers show greater risk-aversion

Greater risk-aversion ($\gamma^* = 0.45$ instead of 0.50) on behalf of consumers means that a greater weight is attached to certainty. In order to achieve more certainty, consumers want to take more bonds in portfolio at the expense of equity holdings. So, there is initially an excess supply of equity (as consumers want to get rid of it) and an excess demand for riskfree bonds. Prices of equity and bonds adjust to accomplish equilibrium at these two financial markets. The price of equity has to go down, whereas bond prices have to go up. The possibility that consumers have to buy equity at a lower price means that they realize a higher expected return on equity. The return on holding equity rises in order to restore equilibrium. Just the opposite can be said of bonds. In order to restore equilibrium at the bond market, the risk-free rate of interest goes down. Of course, both rates of return contribute to the firm's decision to invest in physical capital. The strong rise in the cost of equity finance dominates the fall in the cost of bond finance (see equation (4.7.3) by inserting the appropriate parameter values). The marginal product of capital can be raised by establishing a lower stock of capital, that is to say: investment falls. Some marginal profitable opportunities are no longer profitable.

The higher return on equity is not the only factor determining the value of equity. The holders of equity benefit from the fact that cost of debt diminishes. Furthermore, the fact that investment outlays and wage payments go down leads to higher dividends during a number of periods and higher value of equity. On the other hand, the smaller amount of debt can only be realized at the expense of dividend payments.

Table 10 Greater risk-aversion

period → variable ↓	0	1	2	5	10	stationary state
k	0	-0.09	-0.18	-0.39	-0.65	-1.12
Q	-0.92	-0.79	-0.68	-0.38	-0.03	0.62
U	-0.51	-1.20	-1.83	-3.42	-5.32	-8.91
A	1.07	1.11	1.15	1.25	1.37	1.59
X	0.00	0.00	0.00	0.00	0.00	0.00
c	0.37	0.29	0.21	0.02	-0.20	-0.62
P _y	-0.37	-0.29	-0.21	-0.02	0.20	0.62
j	-0.84	-0.86	-0.89	-0.94	-1.00	-1.12
l	-0.06	-0.07	-0.08	-0.10	-0.13	-0.18
y	-0.02	-0.08	-0.14	-0.27	-0.44	-0.75
R _p	0.00	0.00	0.00	0.00	0.00	0.00
R _f	-1.88	-1.92	-1.96	-2.07	-2.19	-2.43
R _e	7.95	7.97	8.00	8.06	8.12	8.26
P _l	-0.28	-0.32	-0.36	-0.46	-0.58	-0.80
E	26.83	27.86	28.79	31.17	34.00	39.31
B	-0.37	-0.38	-0.39	-0.42	-0.45	-0.51
V	1.27	1.32	1.37	1.49	1.63	1.89
μ	26.27	26.43	26.57	26.93	27.36	28.18

Consumers choose to consume more goods and leisure. Less labour supply combined with less labour demand (because of a shrinking capital stock)

apparently dictates a lower nominal wage rate for the labour market to clear. Realized consumption can rise at the expense of investment demand. Prices fall initially to guarantee an equality of demand and supply. After the jump, prices start to rise slowly due to the smaller capital stock and the effect on output.

Shifting from equity to debt does not alter the value of the firm in the absence of any tax rate. The rationale behind the smaller amount of debt is that the back-up of debt, being the value of capital stock, becomes smaller. It is amazing to see how holders of equity benefit from the new situation, because the firm shrinks down in terms of capital stock and production.

More risk-hating behaviour results in higher share-prices, a greater return for bearing risk of course, and, more importantly, to a smaller and less producing economy. Cautious investors cause unemployment at the aggregate.

Firms are faced with a smaller probability of managerial breakdown

This simulation is derived by introducing a change in the parameter π from the old value of 0.05 to the new value of 0.04. The news that firms are not as risky as before of course benefits the existing holders of equity. The required rate of return on the (less) risky asset declines instantaneously and the equity market shows a booming tenet. One would expect the promised rate of return on equity to go down as a consequence.

Table 11 Smaller objective risk

period → variable ↓	0	1	2	5	10	stationary state
k	0	0.50	0.96	2.14	3.55	6.14
Q	-1.82	-2.49	-3.11	-4.65	-6.43	-9.58
U	20.88	24.49	27.80	36.13	45.92	63.48
A	3.98	3.75	3.53	3.00	2.36	1.22
X	0.00	0.00	0.00	0.00	0.00	0.00
c	-0.93	-0.50	-0.10	0.91	2.10	4.28
P _y	0.94	0.50	0.10	-0.90	-2.06	-4.10
j	-4.12	-4.01	-3.91	-3.66	-3.38	-2.88
l	0.15	0.21	0.25	0.37	0.51	0.75
y	0.06	0.38	0.68	1.42	2.29	3.88
R _p	0.00	0.00	0.00	0.00	0.00	0.00
R _f	-2.81	-2.58	-2.37	-1.85	-1.24	-0.15
R _e	5.05	4.92	4.81	4.52	4.18	3.56
P _l	0.71	0.94	1.16	1.71	2.36	3.53
E	63.74	58.14	52.99	39.94	24.43	-3.89
B	0.94	1.00	1.07	1.22	1.42	1.79
V	4.72	4.44	4.19	3.55	2.80	1.45
μ	19.70	18.92	18.20	16.40	14.28	10.47

The representative consumer faces a sudden shift in his portfolio from debt to equity. Apparently, consumers want to restore portfolios in the

direction of bonds. The demand for portfolio compensating bonds is such that the return on these riskless assets goes down, whereas the return on equity goes up for that matter.

The effects of the lower probability of breakdown on dividends clearly dominates the effect of the higher cost of equity, so the net effect is higher stock prices initially. The effect of the higher cost of equity together with lower cost of debt is difficult to predict. The net effect of (good) news of lower risk is that firms establish a higher stock of capital. Concluding that this requires more aggregate investment is clearly false, since the aggregate capital stock depreciates more slowly as a consequence of more firms staying alive and kicking. A bigger stock of capital establishes itself in this case. Investment demand even goes down as the simulation shows. It is plausible to assume that this economy, which benefits at the aggregate, wants to consume more in terms of goods and leisure. However, prices must rise in order to bring down consumption demand sufficiently. Total supply of goods initially falls because less physical capital is coming free for consumption purposes. This is a consequence of our assumption that the capital stock of "dead" firms is sold as consumption in order to repay debt-holders. In due course the positive effect of a growing capital stock on output brings about greater consumption and lower prices. Firms want to hire more labour, consumers want to enjoy more leisure, so the wage rate has to rise. The amount of debt again rises along with the value of capital stock.

Firms are faced with a more efficient production process

This is the technological shock that dominates all chapters of the book. The value of the parameter ϵ is brought up from 0.25 to 0.26. The results of the simulation of the technological shock in the debt/equity economy are (happily) in line with results obtained before. Producing more efficiently means that given a certain amount of labour and capital (which are inputs to the technology) more output is possible. The extra supply of goods at the market leads to falling prices. The firm's cash-flow benefits from the gains in producing output more efficiently, but suffers from lower goods prices. It is clear that life-time utility benefits from the greater productivity. Households want to consume more goods together with more leisure. Firms need less labour for a given amount of output and therefore it is not clear beforehand whether wages will show tendencies in an upward or downward direction. In contrast with simulations presented in earlier chapters, the net effect is an increase in wages together with more employment in the economy.

Though a great number of variables affect the value of equity, it seems fair enough to expect a big upward jump in share-prices. Households want to reallocate their wealth in order to counterbalance the dominating position of equity in their portfolios. The return on bonds falls as a consequence of (ex ante) excess demand for bonds. The return on equity shows the mirror image. Firms supply less bonds because the scrap-value of capital stock is lower due to lower prices. New investment activity (financed out of retained earnings) dictates that some extra debt will be issued in the course of time. The nominal value of capital stock increases over time (after the initial fall) because additional investment dominates the ever falling price

Table 12 A technological shock

period → variable ↓	0	1	2	5	10	stationary state
k	0	0.36	0.70	1.54	2.55	4.39
Q	-1.52	-2.01	-2.45	-3.56	-4.85	-7.14
U	88.49	91.13	93.54	99.62	106.76	119.53
A	-0.62	-0.79	-0.94	-1.32	-1.77	-2.58
X	0.00	0.00	0.00	0.00	0.00	0.00
c	3.74	4.07	4.37	5.14	6.04	7.68
P _y	-3.61	-3.91	-4.19	-4.89	-5.70	-7.14
j	3.34	3.43	3.52	3.72	3.96	4.39
l	0.04	0.08	0.11	0.20	0.29	0.47
y	4.02	4.26	4.48	5.03	5.69	6.87
R _p	0.00	0.00	0.00	0.00	0.00	0.00
R _f	-1.74	-1.59	-1.45	-1.11	-0.71	0.00
R _e	0.97	0.89	0.82	0.63	0.40	0.00
P _l	0.19	0.36	0.51	0.90	1.36	2.18
E	44.00	40.07	36.46	27.33	16.53	-3.06
B	-3.61	-3.56	-3.52	-3.42	-3.30	-3.06
V	-0.74	-0.94	-1.12	-1.57	-2.10	-3.06
μ	6.03	5.52	5.05	3.87	2.48	0.00

of goods (the latter due to ever increasing supply of goods out of new production technology).

A tax on profit is levied on firms

The three simulations discussed so far assumed no tax on profits. Now a profit tax is introduced along the lines of section 5. In practical terms this means that the simulations so far have been carried out with the parameter τ set to zero. This simulation presents the effects of an introduction of a profit tax of value 0.025 (starting from a situation without taxes). An essential feature of the model is that interest on debt is tax deductible.

It is clear that this tax structure favours debt financing over equity financing (in the form of retained profits). It would be an understatement to say that this tax structure is not exempt from connections with real-world situations. It is stressed that from a macro-point of view, the behaviour of the tax-levying institute seems devoid of any economic sense. The government levies taxes on firms, which implies less dividend for holders of equity, in order to give the money back to households in a lump-sum fashion. But of course the introduced tax is distortionary in character. It proves very difficult to guess (on an intuitive basis) the impact of a tax on profits. What is most striking is that life-time utility jumps upwards after introducing a distortionary tax. In an all-clearing, almost standard neo-classical model the introduction of a nasty tax-levying government can improve welfare significantly. Let us try to interpret the simulation results.

It is very clear from the outset that the introduction of any tax at the firm's level causes a fall in share-prices. The reason is simply that a third party takes away cash, which could have been given to shareholders. Households, or equivalently shareholders, want to compensate portfolios by selling debt and buying equity. The result is a higher risk-free rate of interest

Table 13 Introduction of a profit tax

period → variable ↓	0	1	2	5	10	stationary state
k	0	0.14	0.27	0.59	0.98	1.72
Q	-1.14	-1.32	-1.50	-1.93	-2.44	-3.40
U	0.65	1.69	2.64	5.04	7.90	13.23
A	-1.64	-1.70	-1.76	-1.91	-2.09	-2.43
X	0.00	0.00	0.00	0.00	0.00	0.00
c	-0.56	-0.44	-0.33	-0.04	0.30	0.93
P _y	0.56	0.44	0.33	0.04	-0.30	-0.92
j	1.28	1.32	1.35	1.43	1.53	1.72
l	0.09	0.11	0.12	0.15	0.19	0.26
y	0.04	0.13	0.21	0.42	0.66	1.12
R _u	0.00	0.00	0.00	0.00	0.00	0.00
R _p	-0.04	-0.04	-0.04	-0.04	-0.04	-0.03
R _f	1.39	1.44	1.48	1.60	1.74	1.99
R _e	-0.90	-0.93	-0.96	-1.02	-1.10	-1.25
P _l	0.42	0.49	0.55	0.70	0.88	1.22
E	-42.47	-43.98	-45.37	-48.90	-53.11	-61.00
B	0.56	0.58	0.60	0.64	0.69	0.78
V	-2.03	-2.10	-2.17	-2.34	-2.55	-2.94
μ	-0.44	-0.61	-0.77	-1.17	-1.65	-2.54

and a lower risky rate. Firms have strong motives for issuing debt (extra

supply of bonds). Though the return on debt rises, it is still true that the cost of debt falls. The cost of debt equals $(1 - \tau) \cdot R_f$ and differs from the return on debt R_f . Indicative for investment behaviour is (among other things) the total cost of capital. The ultimate indicator for investment is marginal q which equals:

$$q = \frac{Q}{(1 - \tau) \cdot P_y} = 1 + h_i .$$

The upward jump in this variable¹² dictates higher investment activity, which in turn allows the firm to go heavier into debt. Strong investment demand forces lower (realized) consumption and higher prices initially. Capital accumulation during time delivers a higher potential for the production of goods. Consumption can rise after some periods of time. Strong investment also brings higher wages and more employment. A most remarkable insight is that introducing a distortionary tax can produce extra output, bring more employment and welfare to the economy. The reason is that the tax system forces firms to invest by introducing a subsidy on debt finance. Extra investment clearly dominates this scenario.

¹² The average q for this economy could be defined as the ratio between the market-value of the firm (debt plus equity) and the market-value of the stock of physical capital. See also formula (4.7.5) on page 153.

9 Conclusions

Risk is introduced by assuming a Yaari-Blanchard population structure. Riskless debt and risky equity can both be used to finance the firm's activities. Assuming a MM-world without taxes shows that the irrelevance-of-finance claim of MM holds in our macroeconomic model. Introducing a distortionary corporate profit tax shows that the levered firm has a higher value than the unlevered firm. The value of the tax-shield is higher as more debt is used. For practical purposes, the level of debt is bounded from above by the introduction of a capital-in-advance condition. In order to obtain the MM-results in a world with a corporate profit tax for our macroeconomic model, some kind of "super-rationality" had to be imposed on consumers. Consumers must look upon government transfers as being the same in nature as the dividend payments of firms. It is not so hard to see parallels between the Modigliani-Miller (ir-)relevance and the Ricardian (non-)equivalence.¹³

Simulation results show that more risk-hating behaviour results in higher share-prices, a greater return for bearing risk and a smaller and less producing economy. Introducing a corporate profit tax improves life-time utility of consumers. The tax-system subsidizes debt-finance. As firms want to go into debt more heavily, they meet the requirements of debt

¹³ Some inspiring discussion on the Ricardian equivalence doctrine can be found in the dissertation of Meijdam (1991b, chapter 4). It is left to the reader as a(n) (probably time-consuming) exercise to draw the parallels between the finance of firms and governments in the context of the kind of models used in both dissertations.

holders that more physical capital has to be installed to back up their claims. Extra investment and more output dominates the tax-scenario.

APPENDIX

Debt-Equity model, tables 10,11,12 and 13

$$c \quad | \quad u_c = P_y \cdot X$$

$$l_e \quad | \quad u_{l_e - l} = P_l \cdot X$$

$$l_d \quad | \quad f_l = \frac{P_l}{P_y}$$

$$P_l \quad | \quad l_d = l_e = l$$

$$y \quad | \quad y = f(l, k)$$

$$i \quad | \quad Q = P_y \cdot (1 + h_l) \cdot (1 - \tau)$$

$$j \quad | \quad j = i + h(i, k)$$

$$P_y \quad | \quad y + \pi \cdot k = c + j$$

$$M_d \quad | \quad u_{\frac{M}{P}} = R_u \cdot X \cdot P_y$$

$$R_u \quad | \quad M_d = \bar{M}$$

$$R_p \quad | \quad R_p = \beta \cdot R_f + (1 - \beta) \cdot R_e$$

$$R_f \quad | \quad R_u = \frac{E}{V + S} \cdot R_e + \frac{B}{V + S} \cdot R_f + \frac{S}{V + S} \cdot R_{ov}$$

$$R_{ov} \quad | \quad R_{ov} = R_u + (R_u - R_f) \cdot \tau \cdot \frac{B}{S}$$

$$R_e \quad | \quad \beta = \frac{1 - \gamma^*}{\gamma^*} \cdot (R_e - R_p) \cdot \frac{1}{(R_e / R_p - 1)^2} \cdot \frac{1}{\sqrt{\pi} \cdot (1 - \pi)}$$

$$\beta \quad | \quad \beta = \frac{B}{V}$$

$$\mu \quad | \quad \mu = R_e - (1 - \tau) \cdot R_f$$

$$B \quad | \quad B = k \cdot P_y$$

$$V \quad | \quad V = A - S - \bar{M}$$

$$\begin{aligned}
E & \quad | \quad E = V - B \\
u & \quad | \quad u = u(c, l_m - l, \frac{M}{P_y}) \\
T & \quad | \quad T = \tau \cdot (y \cdot P_y - l \cdot P_l - j \cdot P_y - R_f \cdot B) \\
k & \quad | \quad \dot{k} = i - (\delta + \pi) \cdot k \\
Q & \quad | \quad \dot{Q} = (R_e + \pi + \delta) \cdot Q - P_y \cdot \{ (1 - \tau) \cdot (f_k - h_k) + \mu + \pi \} \\
A & \quad | \quad \dot{A} = R_u \cdot (A - \bar{M}) - y \cdot P_y + l \cdot P_l + j \cdot P_y \\
X & \quad | \quad \dot{X} = (\nu - R_u) \cdot X \\
S & \quad | \quad \dot{S} = R_{ov} \cdot S - T \\
U & \quad | \quad \dot{U} = \nu \cdot U - u
\end{aligned}$$

Specifications used in simulations: (4.3.7), (1.5.3), (1.3.12) and (1.3.13).

Parameter values:

$$\begin{array}{lll}
\alpha = 0.25 & \gamma_c = 0.85 & l_m = 9.0 \\
\varepsilon = 0.25 & \bar{M} = 1.00 & \sigma = 0.40 \\
\gamma_m = 0.05 & \delta = 0.10 & \nu = 0.10 \\
\psi = 0.125 & \pi = 0.05 & \gamma^* = 0.50 \\
\rho^* = 1.00 & \tau = 0.00 & \gamma_l = 0.10
\end{array}$$

Stationary state:

k	= 2.732	y	= 1.220
Q	= 2.589	R _p	= 0.10
U	= -0.539	R _f	= 0.097
A	= 6.376	R _e	= 0.149
X	= 0.500	P _l	= 0.123
c	= 0.919	B	= 5.052
P _y	= 1.849	E	= 0.323
i	= 0.410	V	= 5.376
j	= 0.437	μ	= 0.052
l	= 7.378	R _u	= 0.10

AN OPEN ECONOMY: A FINANCIAL PERSPECTIVE ON DEBT ACCUMULATION

1 The closed model

This chapter draws heavily on Meijdam and Van Stratum(1990). The results of the paper will not be repeated here. Instead the focus will be on the financial implications of modelling a small open economy.

The model used here is exactly the same as used in the paper, with the exception of the inclusion of a clearing labour market, in order to bring this chapter in line with the other chapters. Equation (2.3.1) on page 7 of the paper describes sticky nominal wage formation:

$$(5.1.1) \quad \dot{P}_t = \theta \cdot (l_d - l_s) \cdot P_t ,$$

where actual employment is determined by the minimum of ex ante demand and supply of labour:

$$(5.1.2) \quad l = \min(l_d, l_s) .$$

These latter two equations regarding the labour-market will be replaced by the market-clearing conditions of earlier chapters:

$$(5.1.3) \quad P_l \mid l_d = l_s ,$$

$$(5.1.4) \quad l = l_d = l_s .$$

Actual employment equals ex ante demand and supply of labour, according to the last equation.

A closed economy version is used to have a point of reference in discussing the implications of opening up an economy. The model exhibits money in the utility function (see chapter 1) in the following manner:

$$(5.1.5) \quad u = u \left(c, l_m - l, \frac{M}{P_y} \right) .$$

The supply of money is exogenous (a parameter to the model). Combining this with the results of chapter 1 gives for the consumer's problem:

$$(5.1.6) \quad u_c = X \cdot P_y ,$$

$$(5.1.7) \quad u_{l_m - l} = X \cdot P_l ,$$

$$(5.1.8) \quad u_{\frac{M}{P_y}} = X \cdot P_y \cdot R ,$$

$$(5.1.9) \dot{A} = R \cdot (A - M) + I \cdot P_I - c \cdot P_y ,$$

$$(5.1.10) \dot{X} = (v - R) \cdot X .$$

The producer's problem will not be discussed here, as it is the same as used in chapter 1. The relevant equations will be repeated here:

$$(5.1.11) f_l = \frac{P_l}{P_y} ,$$

$$(5.1.12) Q = P_y \cdot (1 + h_i) ,$$

$$(5.1.13) j = i + h(i, k) ,$$

$$(5.1.14) \dot{k} = i - \delta \cdot k ,$$

$$(5.1.15) \dot{Q} = (R + \delta) \cdot Q - P_y \cdot (f_k - h_k) .$$

Finally, there are the three market clearing conditions. The clearing condition for the labour market has already been specified by (5.1.3). The remaining two conditions read:

$$(5.1.16) y = c + j ,$$

$$(5.1.17) \quad M_d = \bar{M} .$$

This model can be seen as a micro-founded intertemporal version of the standard IS-LM framework. A more simple micro-founded model in the dynamic tradition is hardly imaginable.

Table 14 shows the effects of a technological shock in the closed model. The results are in line with the relevant table of chapter 1 and will not be repeated here for that reason.

It is interesting to compare the results with the ones from the paper. Comparing table 1 from the paper (p.11) with table 14 shows the effects of introducing flexible wage-formation or a clearing market for labour. The stationary states of both models are the same, so the differences can be found in the adjustment processes. The effects of introducing flex-wages are:

- The stock of capital accumulates faster, due to higher levels of investment.
- The path of consumption is higher during all periods.
- The level of employment is higher during all periods.
- The nominal wage is not predetermined anymore. In the first period the wage rate declines but thereafter nominal wages rise. From period 1 onward the clearing economy shows higher nominal wages.
- Prices are lower during all periods.
- Utility levels are higher during all periods.

Note that the rate of interest is constant over time and equal to the exogenous rate of time preference, ν . This phenomenon is described by propo-

Table 14 A technological shock in reference closed model

period → variable ↓	0	1	2	5	10	stationary state
k	0	0.32	0.62	1.34	2.16	3.40
Q	-1.40	-1.88	-2.31	-3.36	-4.53	-6.27
U	58.53	59.73	60.81	63.46	66.42	70.86
A	-1.19	-1.33	-1.45	-1.76	-2.10	-2.62
X	0.00	0.00	0.00	0.00	0.00	0.00
c	4.17	4.41	4.63	5.17	5.78	6.69
P _y	-4.00	-4.23	-4.43	-4.92	-5.46	-6.27
j	3.44	3.44	3.44	3.43	3.42	3.40
l	-0.02	0.00	0.02	0.08	0.14	0.23
y	3.99	4.17	4.33	4.74	5.19	5.87
R	0.00	0.00	0.00	0.00	0.00	0.00
P _l	-0.13	0.01	0.13	0.45	0.79	1.32
E	-1.40	-1.56	-1.71	-2.07	-2.47	-3.08

sition I of the paper.

2 Turning the closed economy into a (small) open one.

The focus of our thesis is the modelling of financial assets in macroeconomic perspective. The starting point of the paper at that time was: why not see a (small) country as a big firm that pays dividends to the rest of the world. A country produces goods and uses goods and other means in producing them. Say, Americans own the small country called the Netherlands, what would they want to sell it for? In other words: what determines the market-value of the Netherlands in the world economy?

Assuming homogeneous, internationally traded, goods greatly simplifies matters. How do we define the dividend of the big firm "The Netherlands"? All goods that stream across the borders can be seen as net payments to the rest of the world, to the shareholders of the firm. So, export of goods is seen as the net production of a country. All production of course equals y but the amount of $c + j$ is "used" in producing this output. So, net output equals export of goods:

$$(5.2.1) \quad b = y - c - j .$$

The price the Americans want to give for this bundle of goods equals the world price of the good at the moment of arrival of the goods. Assuming purchasing power parity (in conjunction with homogeneous goods) brings:

$$(5.2.2) \quad P_y = P^* \cdot e ,$$

where P^* is the goods price in dollars (at moment t), P_y the goods price in guilders and e the (flexible and market clearing) rate of exchange. The net

output of "The Netherlands" measured in dollars, thus is $b \cdot P^\bullet$ at every moment of time.

At what rate must the levels of net output (=dividends to the owners) be discounted to obtain the market-value of "The Netherlands"? Assuming interest-rate parity again simplifies matters enormously:

$$(5.2.3) \quad R = R^\bullet + \frac{\dot{e}}{e} ,$$

where R^\bullet is the American rate of interest, R the Dutch rate of interest. The net worth, or market-value, of "The Netherlands" (vis-a-vis the rest of the world) should therefore be:

$$(5.2.4) \quad V_{NL} = \int_t^\infty \{ b \cdot P^\bullet \} \cdot e^{-R^\bullet \cdot (z-t)} dz ,$$

where R^\bullet is assumed to be a constant. The market value V_{NL} is measured in dollars (of moment t) and represents the amount of money Americans would want to sell or buy "The Netherlands" for at the beginning of time.¹ In the paper, the value of net foreign claims of the small country is indicated by the expression $(A^\bullet - E)$ and is denominated in guilders. The symbol A^\bullet stands for the total value of non-monetary financial assets (being Dutch

¹ The expression "at the beginning of time" is meant to denote the fact that no trade has ever taken place. All countries start with a zero debt position. This implies that $E_0 = A^\bullet_0$.

and foreign shares) that are in possession of inhabitants of the country, while E stands for the value of shares issued by Dutch firms.² The position of debt in guilders at any moment in time then is $(E - A^*)$, while the position of debt in dollars is: $(E - A^*)/e$. The debt position of the Netherlands exactly represents the market value as mentioned before:

$$(5.2.6) \quad V_{NL} = \frac{E - A^*}{e} .$$

According to (5.2.4):

$$(5.2.8) \quad \dot{V}_{NL} = R^* \cdot V_{NL} - b \cdot P^* ,$$

or, alternatively:

$$(5.2.9) \quad \frac{\dot{E} - \dot{A}^*}{e} - \frac{\dot{e}}{e} \cdot (E - A^*) = R^* \cdot \frac{E - A^*}{e} - b \cdot P^* .$$

Multiplying by e :

$$(5.2.10) \quad \dot{E} - \dot{A}^* = \left(R^* + \frac{\dot{e}}{e}\right) \cdot (E - A^*) - b \cdot P^* \cdot e .$$

Assuming interest rate parity and purchasing power parity:

² The total value of financial assets (including money) now is:

$$A = A^* + \bar{M}.$$

$$(5.2.11) \quad \dot{E} - \dot{A}^* = R \cdot (E - A^*) - b \cdot P_y .$$

Assuming a flexible exchange rate comes to the same thing as requiring the balance of payments to be in equilibrium all the time (assuming the appropriate stability requirements). The current account reads:

$$b \cdot P_y + R \cdot (A^* - E) ,$$

while the capital account is:

$$\dot{E} - \dot{A}^* .$$

The total balance of payments can therefore be written as:

$$(5.2.12) \quad S_b = b \cdot P_y + R \cdot (A^* - E) - (\dot{A}^* - \dot{E}) = 0 ,$$

which is equation (3.1.8) of the (1990)-paper.

It is clear by now that the equation of debt-accumulation (5.2.11) comes to the same thing as the balance-of-payments condition as stipulated in the (1990)-paper.

Now, returning to equation (5.2.4):

$$(5.2.13) \quad V_{NL} = \int_t^{\infty} b \cdot P^* \cdot e^{-R^* \cdot (z-t)} dz .$$

We can look upon this as being the life-time budget-constraint of a small country. A "rational" country will not allow a positive value of the present discounted value of current and future trade imbalances at the beginning of time. In the case of $V_{NL} > 0$ (at the beginning of time) the small country gives some amount of goods away to the rest of the world. In summary: the condition that the value of a country must be zero, $V_{NL} = 0$, can be seen as the no-Ponzi-game condition for a small open economy.³

The simulation results of a technological shock in a small open economy are presented in table 15.⁴ We have chosen to focus on a small open economy that looks like being closed before the technological shock arrives. Now it becomes possible to concentrate on the effects of being open as such. This implies that the stationary state of our small open economy displays no exports of goods and no position of debt. Whatever the shock to this economy, the position of debt remains zero at period zero (and a very short period after that). This is quite logical since the zero debt position does not allow for (unforeseen) changes in the value of debt. So it should be clear that the position of debt is not a predetermined variable per se. An economy that has some positive amount of debt for some reason faces a windfall profit in the case of an appreciating home currency. This situation can be compared with the case of a household that is in possession of shares of the firm. An unexpected shock to the economy causes a sudden

³ This of course implies that the market-value of any country (now defined as the addition of net exports and existing debt) must be zero at any moment of time.

⁴ The variables b and V_{NL} are shown in absolute values (x100).

jump in the value of shares. At this very moment the rate of return on shares is undefined and the household faces some windfall (say) profit. The American people in possession of "The Netherlands" may be confronted with surprises in the value of their small-country-shares in case the exchange rate jumps to other heights. The situation changes drastically in case the contracts of delivery to Americans are stated in terms of goods. Due to the Cobb-Douglas utility-specification there is the phenomenon of consumption-smoothing over time and a once-and-for-all appreciation of the guilder. Consumption smoothing can be attained by importing goods during a number of periods and by exporting goods, by the time the own production has reached appropriate heights. In this way the economy is taking a short-cut to future returns of the technological innovation. Maybe one would expect a diminishing position of debt after the moment that the economy starts exporting goods. Due to the accumulation of debt and the fact that a rate of return has to be paid over this debt, the absolute height of debt rises continuously (till the stationary state is reached).

Note that the stationary state of the small open economy after the technological shock is different for flexible and sticky wages. This is a special feature of the open economy as modelled here and must be explained by referring to the so-called hysteresis-phenomenon (implying that the equilibrium-position is path-dependent).

The main differences comparing the results with sticky and flexible wage formation in the small open economy can be summarized as follows (these being the effects of introducing flexible wages):

- Output of goods is higher in the short and middle-long run, lower in the long run.
- The same applies for the accumulation of capital.

Table 15 A technological shock in a small open economy

period → variable ↓	0	1	2	5	10	stationary state
k	0	0.40	0.76	1.61	2.51	3.61
Q	-1.95	-2.32	-2.64	-3.42	-4.22	-5.19
U	58.58	58.35	58.14	57.66	57.15	56.54
A	-1.95	-2.38	-2.77	-3.69	-4.65	-5.82
X	0.00	0.00	0.00	0.00	0.00	0.00
c	5.47	5.47	5.47	5.47	5.47	5.47
P _y	-5.19	-5.19	-5.19	-5.19	-5.19	-5.19
j	4.34	4.26	4.18	4.01	3.83	3.61
l	-0.20	-0.13	-0.07	0.09	0.24	0.43
y	3.89	4.14	4.36	4.88	5.42	6.08
R	0.00	0.00	0.00	0.00	0.00	0.00
P _l	-1.14	-0.73	-0.37	0.49	1.39	2.50
E	-1.95	-2.38	-2.77	-3.69	-4.65	-5.82
b	-1.88	-1.49	-1.14	-0.32	0.53	1.56
V _{NL}	0.00	1.77	3.34	7.05	10.92	15.62

- The utility-index is higher all the time.
- The flex-wage economy accumulates less debt.
- The fluctuation in the level of employment is less.
- There is less import of goods in the short run, while there is less export in the long run.

- The currency-appreciation is higher in case of flexwage, an indication for the fact that the flex-wage country is the economically stronger one.
- Consumption of goods is higher all the time.

Exactly the same model as presented here is dealt with in Meijdam(1991b), be it that the latter caters for spill-over effects due to the existence of non-clearing markets.

Reference model, table 14

c		$u_c = P_y \cdot X$
l_a		$u_{l_a-l} = P_l \cdot X$
l_d		$f_l = \frac{P_l}{P_y}$
P_l		$l_d = l_a - l$
y		$y = f(l, k)$
i		$Q = P_y \cdot (1 + h_i)$
j		$j = i + h(i, k)$
P_y		$y = c + j$
M_d		$u_{\frac{M}{P_y}} = R \cdot P_y \cdot X$
R		$\bar{M} = M_d$
u		$u = u(c, l_m - l, \frac{M}{P_y})$
D		$D = y \cdot P_y - l \cdot P_l - j \cdot P_y$
k		$\dot{k} = i - \delta \cdot k$
Q		$\dot{Q} = (R + \delta) \cdot Q - P_y \cdot (f_k - h_k)$
A		$\dot{A} = R \cdot (A - \bar{M}) + l \cdot P_l - c \cdot P_y$
X		$\dot{X} = (v - R) \cdot X$
U		$\dot{U} = v \cdot U - u$
E		$\dot{E} = R \cdot E - D$

Specifications used in simulations: (1.5.3), (1.3.12) and (1.3.13).

Model of small open economy, table 15

Replace the clearing condition for the goods market by:

$$b \quad | \quad b = y - c - j$$

Add the following three relations:

$$A^* \quad | \quad \dot{A}^* = b \cdot P_y + R \cdot A^* - D \quad (A^* = A - \overline{M})$$

$$P_y \quad | \quad \dot{P}_y = P_y \cdot (R - R^*)$$

$$V_{NL} \quad | \quad \dot{V}_{NL} = R^* \cdot V_{NL} - b \cdot P^*$$

Parameter values:

$$\alpha = 0.25$$

$$\gamma_c = 0.85$$

$$l_m = 9.0$$

$$\varepsilon = 0.25$$

$$\overline{M} = 1.00$$

$$\sigma = 0.40$$

$$\gamma_m = 0.05$$

$$\delta = 0.10$$

$$\nu = 0.10$$

$$\psi = 0.125$$

$$R^* = 0.10$$

$$P^* = 1.00$$

$$\gamma_l = 0.10$$

Stationary state:

$$k = 3.653$$

$$l = 7.646$$

$$Q = 1.561$$

$$y = 1.454$$

$$U = 0.804$$

$$R = 0.10$$

$$A = 6.703$$

$$P_l = 0.148$$

$$X = 0.500$$

$$E = 5.703$$

$$c = 1.089$$

$$b = 0$$

$$P_y = 1.561$$

$$V_{NL} = 0$$

$$j = 0.365$$

$$A^* = 5.703$$

SUMMARY

We took the Blanchard/Sachs(1982) and Peters/Van de Klundert(1986) models of the intertemporally optimizing consumer-producer variety as starting points for our study. We tried to capture several ideas of the finance literature into the macroeconomic framework of the kind mentioned. This synthesis of financial theory and macroeconomics is best seen as an asset-approach to macroeconomics. In principle, all markets are assumed to be cleared all the time in our study, this assumption being made to exclude matters of price-stickiness and disequilibrium. One of the drawbacks of the study (and the papers mentioned above) is that the models are virtually impossible to solve analytically. For that reason all chapters are illustrated with numerical simulations. The models are simulated with the same parameter-set and the same (technological) shock everywhere in the study, in order to make comparisons between the different chapters feasible. In order to get interesting dynamics (in the presence of clearing markets), adjustment costs are introduced at several places in the study. The focus of the study is mainly on the behaviour of firms and finance. One of the accepted elements of finance is the principle of value maximization. The firms in our study maximize the value of equity (shareholders' wealth) or the value of the firm (equity and debt taken together). Chapter 0 gives an outline of the scope and subject of the study and serves as an introduction, clarifying concepts and philosophy. It is already in

chapter 0 that a simple example of the MM-propositions is set out. The purpose of this exercise is that the reader can see the parallels of our chapter 4 and the standard textbook results quite easily. It hopefully stimulates reading of chapter 4 too, since it forms the heart of our study.

Chapter 1 is intended to build and discuss the ingredients of a micro-founded, intertemporal analogon of the clearing IS-LM model. Constructing this model is done in several steps by looking at Crusoe and barter economies first. The introduction of financial assets is the key to passing from the one model to the other in chapter 1.

The subject of chapter 2 and 3 is the modelling of the banking firm. According to financial theory, a bank is just like any other firm and strives for value maximization accordingly. Our bank produces lines of credit and maximizes shareholders' wealth. To cope with the problem of price-level indeterminacy, we introduced a banking licence, which forms a financial asset in its own right. The main difference between chapters 2 and 3 is the competition structure of the credit market. Chapter 2 assumes that banks take the rate of interest as given, while chapter 3 sees banks as having some degree of monopolistic power. The supply of credit is determined by the real wage rate and the nominal rate of interest (and some technological parameters). In the case of monopolistic competition the demand elasticity of credit is another element determining the supply of credit. An economy that includes a banking sector faces a higher stock of money and a higher price level, compared with an economy that lacks this banking sector. Simulation results show that the monopolistic bank can cope better with adverse shocks to the banking industry. Banks that have monopolistic power find it optimal to oversupply the market for credit when there are no adjustment costs in changing the rate of interest. The introduction of

adjustment costs shows the theoretical possibility of optimal credit rationing. Very strong adjustment costs and relatively big shocks are needed to generate the regime of credit rationing by numerical simulation however.

Chapter 4 introduces a risky asset by assuming a Yaari-Blanchard population structure. Assuming a MM-world without taxes shows that the irrelevance-of-finance claim of MM holds in the macroeconomic model. Introducing a distortionary corporate tax shows that the levered firm has a higher value than the unlevered firm in the macroeconomic model. The more debt is used the higher is the value of the tax-shield. The height of debt is bounded from above by the introduction of a so-called capital-in-advance condition. MM compute the value of the tax-shield by discounting one-period tax-shields at the risk-free rate of interest. Reproduction of this result in the macroeconomic model is judged to be possible only when consumers have perfect "see-through" on behalf of the character of the transfers they receive from government.

Chapter 5 focuses on a small open economy. A country is looked upon as being a big firm that pays dividend to the "shareholders", which are the people from the outside world. The phenomenon of debt accumulation can then be looked upon as being the mirror image of the formation of capital gains to the outside world. Assuming interest rate parity and purchasing power parity across the world, a life-time budget constraint of a small country is derived looking at the value of the small country. Of course, the "shareholders" and "employees" of the small country are quite different from the normal case of the value-maximizing firm. It is clear that the Dutch government cannot be looked upon as the management of the firm "The Netherlands" operating in the interests of the American people, being the shareholders(?).

REFERENCES

- Abel, A.B. and O.J. Blanchard (1983), "An intertemporal model of saving and investment", *Econometrica*, pp.675-692.
- Allen, D. (1983), "Finance, a theoretical introduction".
- Archer, S.H. and C.A. D'Ambrosio (1976), "The theory of business finance", second edition, MacMillan Publishing Company.
- Ascher, U.M., R.M.M. Mattheij and R.D. Russell (1988), "Numerical solution of boundary value problems for ordinary differential equations", Prentice Hall.
- Auerbach, A.J. (1983), "Taxation, corporate financial policy and the cost of capital", *Journal of Economic Literature*, pp. 905-940.
- Baltensperger, E. (1980), "Alternative approaches to the theory of the banking firm", *Journal of Monetary Economics*, pp. 1-37.
- Barro, R.J. (1984), "Macroeconomics", first edition, Wiley and sons.
- Bishop, S.R., H.R. Crapp and G.J. Twite (1984), "Corporate finance", Holt, Rinehart and Winston.

- Blanchard, O.J. (1985), "Debt, deficits and finite horizons", *Journal of Political Economy*, pp. 223-247.

- Blanchard, O.J. and S. Fisher (1989), "Lectures on macroeconomics", M.I.T. Press.

- Blanchard, O.J. and J. Sachs (1982), "Anticipations, recessions and policy; an intertemporal disequilibrium model", *Annales de l'Insee*, pp. 117-144.

- Chamberlin, E. (1933), "The theory of monopolistic competition", Harvard University Press.

- Clower, R.W. (1967), "A reconsideration of the microfoundations of monetary theory", *Western Economic Journal*, pp. 1-18.

- Fair, R.C. (1974), "A model of macroeconomic activity, volume 1: the theoretical model", Ballinger Publishing Company.

- Fama, E.F. (1980), "Banking in the theory of finance", *Journal of Monetary Economics*, pp. 39-57.

- Feenstra, R.C. (1986), "Functional equivalence between liquidity costs and the utility of money", *Journal of Monetary Economics*, pp. 271-291.

- Hadjimichalakis, M.G. (1982a), "Modern macroeconomics", Prentice Hall.

- Hadjimichalakis, M.G. (1982b), "Monetary policy and modern money markets", Lexington Books, Mass.

- Hayashi, F. (1982), "Tobin's marginal q and average q : a neoclassical interpretation", *Econometrica*, pp. 213-224.

- Hayashi, F. (1985), "Corporate finance side of the Q theory of investment", *Journal of Public Economics*, pp. 261-280.

- Hirshleifer, J. (1958), "On the theory of optimal investment decisions", *Journal of Political Economy*, pp. 329-372.

- Lucas, R.E. (1984), "Money in a theory of finance", in: K. Brunner and A.H. Meltzer (eds.), "Essays on macroeconomic implications of financial and labor markets and political processes", *Cornegie-Rochester Conference Series on Public Policy* (1984), pp. 9-46.

- Klundert, Th. van de and P. Peters (1986), "Tax incidence in a model with perfect foresight of agents and rationing in markets", *Journal of Public Economics*, pp. 37-59.

- Meijdam, L. (1986), "Simulation with models combining perfect foresight and rationing in markets, a sensitivity analysis", *Research Memorandum 8603*, University of Nijmegen, The Netherlands.

- Meijdam, L. (1991a), "Price-setting versus price-taking in a dynamic disequilibrium model with correct anticipation of agents" (in Dutch),

- Meijdam, L. (1991b), "Rational disequilibrium dynamics", dissertation Tilburg University.
- Meijdam, L. and R. van Stratum (1990), "Dynamic adjustment and debt accumulation in a small open economy", Journal of Economics, pp. 1-26.
- Miller, M.H. (1988), "The Modigliani-Miller proposition after thirty years", Journal of Economic Perspectives, pp. 99-120.
- Modigliani, F. (1988), "MM - Past, present, future", Journal of Economic Perspectives, pp 149-158.
- Modigliani, F. and M.H. Miller (1958), "The cost of capital, corporate finance, and the theory of investment", American Economic Review, pp. 261-297 (also to be found in the Archer/D'Ambrosio(1976)-volume).
- Modigliani, F. and M.H. Miller (1963), "Corporate income taxes and the cost of capital: a correction", American Economic Review, pp. 433-443 (also to be found in the Archer/D'Ambrosio(1976)-volume).
- Moyer, R.C., J.R. McGuigan and W.J. Kretlow (1987), "Contemporary financial management", third edition, West Publishing Company.
- Mussa, M. (1976), "A study in macroeconomics", North Holland Publishing Company.

- Niehans, J. (1978), "The theory of money", John Hopkins University Press.
- O'Hara, M. (1983), "A dynamic theory of the firm", The Journal of Finance, pp. 127-140.
- Precious, M. (1987), "Rational expectations, non-market clearing and investment theory", Oxford University Press.
- Ramsey, F.P. (1928), "A mathematical theory of saving", Economic Journal, pp. 543-549.
- Santomero, A.M. (1984), "Modeling the banking firm", Journal of Money, Credit and Banking, pp. 576-602.
- Sargent, Th.J. (1987), "Dynamic macroeconomic theory", Harvard University Press.
- Saving, T.R. (1977), "A theory of the money supply with competitive banking", Journal of Monetary Economics, pp. 289-303.
- Shell, K.(ed.) (1967), "Essays on the theory of optimal economic growth", M.I.T. Press.
- Silber, W.L. (1983), "The process of financial innovation", American Economic Review, Papers and Proceedings, pp.88-95.

- Stiglitz, J.E. (1987), "The new keynesian economics: money and credit", paper prepared for presentation of the meetings of the Econometric Society, Copenhagen, August 1987.
- Stiglitz, J.E. and A. Weiss (1988), "Banks as social accounts and screening devices for the allocation of credit", NBER working paper no. 2710.
- Stratum, R. van (1989), "Money and finance: intertwined in a macroeconomic model" (in Dutch), *Maandschrift Economie*, pp. 94-115.
- Sutcliffe, C. and M. Bromwich (1986), "Investment appraisal", in: M. Firth and S.M. Keane, "Issues in finance", Philip Allan.
- Yaari, M.E. (1965), "Uncertain lifetime, life insurance, and the theory of the consumer", *Review of Economic Studies*, pp. 137-150.

Modellering van financiële activa vanuit macro-economisch perspectief.

Onderwerp van deze studie is de vraag hoe financiële activa in een macro-economische context gemodelleerd kunnen worden. Als uitgangspunt daarbij dienen de modellen van Blanchard en Sachs uit 1982 en van Peters en Van de Klundert uit 1986. Deze modellen zijn macro-economisch van opzet en gaan uit van intertemporeel optimerend gedrag van consumenten en producenten en lijken op voorhand goede mogelijkheden te bieden aan te knopen bij gedachten uit de financieringstheorie. Met "macro-economisch" bedoelen we in deze studie eigenlijk alleen maar te zeggen dat de kringloopgedachte verwerkt is (de schuld van de een is het tegoed van de ander), een gedachte die de financieringsliteratuur ten ene male vreemd is. De echte aggregatie en dus het loslaten van het concept van de representatieve agent blijft in deze studie meestentijds achterwege. Bij de implementatie van de financiële activa proberen we in deze studie zoveel mogelijk aan te sluiten bij in de financierings-theorie gebruikelijke en beproefde concepten. Het principe van de waardemaximalisatie neemt een centrale plaats in in onze studie en hoe kan het ook anders.

Om de bestudering van de financiële markten te bevorderen nemen we aan dat alle markten ruimen, dit in tegenstelling tot de twee bovengenoemde artikelen die uitgaan van trage prijzen op de goederen- en arbeidsmarkt. Ruimende markten lijken het best aan te sluiten bij voor onze studie nuttige concepten als efficiënte markten en rationele verwachtingen. Numerie-

ke simulatie met behulp van de geformuleerde macro-economische modellen is de gebruikte methode om de werking van een en ander te illustreren. Steeds wordt daarbij uitgegaan van dezelfde technologische impuls en dezelfde parameter set teneinde een zinvolle vergelijking tussen modellen onderling te bevorderen.

In hoofdstuk 0 wordt een overzicht en introductie gegeven van wat gaat komen in de hoofdstukken 1 tot en met 5. Een eenvoudig rekenvoorbeeld dient als eerste kennismaking met de proposities van Modigliani en Miller. In hoofdstuk 1 wordt een Crusoe-model geformuleerd waarin geen enkel financieel activum aanwezig is. Het vormt het meest simpele model dat geformuleerd kan worden in de traditie van intertemporele optimalisering. Het eerste financiële activum dat aan deze economie wordt toegevoegd is een aandeel van de representatieve onderneming. Zo ontstaat een ruil-economie waarin lenen mogelijk is zonder de aanwezigheid van geld. De ondernemingsleiding maximaliseert de waarde van de uitstaande aandelen (oftewel de waarde van de onderneming) in opdracht van de aandeelhouders. Een volgende stap is de introductie van het activum geld. Het aanbod van geld wordt in hoofdstuk 1 stiefmoederlijk behandeld teneinde een intertemporele versie van het bekende IS-LM model te verkrijgen. De vraag naar geld wordt afgeleid volgens de methoden van "geld-in-de-nutsfunctie" en "cash-in-advance". We doen alsof deze methoden niet problematisch van aard zijn, daar de aandacht in deze studie uitgaat naar de aanbodzijde van de economie.

Onderwerp van de hoofdstukken 2 en 3 is het endogenizeren van het geldaanbod in de geest van de waardemaximaliserende gedachte uit de financieringstheorie. Een bank wordt ten tonele gevoerd. De bank heeft als taak de waarde van de bank-als-onderneming te maximaliseren. Het belang-

rijkste instrument van de bank is haar potentie om meer of minder krediet te verlenen aan kooplustige consumenten. Het verschil tussen de hoofdstukken 2 en 3 is gelegen in het feit dat hoofdstuk 2 de bank ziet als een rente-nemer en een hoeveelheidsaanpasser, terwijl hoofdstuk 3 de bank enige ruimte toebedeelt de rente te zetten in een omgeving van monopolistische concurrentie. In hoofdstuk 3 behoort rantsoenering van krediet als uitkomst van optimaliserend handelen tot de economische mogelijkheden.

Hoofdstuk 4 heeft tot onderwerp het onderscheiden van eigen en vreemd vermogen op het niveau van een onderneming. We nemen aan dat elke onderneming een kans heeft om failliet te gaan en ophoudt te bestaan. In zo'n geval zal de aandeelhouder de negatieve gevolgen van het faillissement voor zijn rekening nemen, terwijl de verschaffer van vreemd vermogen geen enkel risico loopt. Aangezien de beleggers een risico-aversie ten toon spreiden, zullen zij een hoger rendement eisen op aandelen dan op obligaties. De waarde van de onderneming blijkt in het macro-economische model niet te variëren als een andere financieringsstructuur wordt gekozen (MM-irrelevantie). De introductie van een winstbelasting verandert de zaak drastisch. De waarde van de onderneming zal toenemen naarmate meer met vreemd vermogen wordt gefinancierd, een resultaat dat geheel in overeenstemming is met de micro-georiënteerde financieringstheorie. Een opmerkelijk fenomeen is, en hier blijkt de meerwaarde van de macro-economische aanpak, dat simulatie toont dat een (verhoging van de) winstbelasting bij gelijktijdige lump-sum teruggave van de belastingopbrengsten aan de beleggers, nutsverhogende effecten kan hebben.

Hoofdstuk 5 vraagt zich af of het mogelijk is een kleine open economie te zien als een (grote) onderneming die in handen is van de buitenwereld. De waarde van zo'n klein land zou dan uitgedrukt kunnen worden in termen

van de opbrengsten die ze genereert voor haar buitenlandse "aandeelhouders".

Hoofdstuk 6 tenslotte is een samenvatting van de studie en, zo men wil, het Engelstalige equivalent van het hier geschrevene.

Curriculum Vitae

De schrijver van dit proefschrift werd geboren op 8 december 1960 te Den Helder. Na zijn middelbare school te Uden studeerde hij economie te Tilburg. In 1985 behaalde hij zijn doctoraal examen algemene economie "richting Schouten". Gedurende ruim anderhalf jaar startend in september 1985 verrichtte hij zijn vervangende militaire dienstplicht aan de Katholieke Universiteit Nijmegen. In de periode 1986-1988 volgde hij de post-doctorale opleiding "Bankwezen en Financiering" van het Tilburgs Instituut voor Akademische Studies. Van 1987 tot 1990 was hij werkzaam als toegevoegd onderzoeker bij de vakgroep Toegepaste Economie van de faculteit der beleidswetenschappen van de K.U. Nijmegen. Per 1 november 1990 is hij als wetenschappelijk medewerker verbonden aan het Centraal Planbureau te Den Haag.

