Dynamic feature linking in stochastic networks with short range interactions

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Abstract. It is well established that cortical neurons display synchronous firing for some stimuli and not for others. The resulting synchronous subpopulation of neurons is thought to form the basis of object perception. In this paper this 'dynamic linking' phenomenon is demonstrated in networks of binary neurons with stochastic dynamics. Feed-forward connections implement feature detectors and lateral connections implement memory traces or cell assemblies.

1 Introduction

It is well established that cortical neurons display synchronous firing for some stimuli and not for others [1, 2]. In particular, it has been shown experimentally, that correlations depend on the amount of conflict in the stimulus presented [3].

The resulting synchronous subpopulations of neurons (cell assemblies) are thought to form the basis of segmentation and object perception [4, 5]. The role of individual cells is to represent important 'atomic' visual features, such as edges, corners, velocities, colors, etc. Objects can be defined as a collection of these features: The cell assembly is a neural representation of the entire object. Thus if the local features are part of a *coherent* global stimulus, the corresponding neurons synchronize. If the same local features are not part of a global stimulus, no such synchronization occurs.

The observed synchronous firing in animal experiments has in fact two components: one is the presence or absense of an oscillatory component in the auto- and cross-correllograms. The second phenomenon is the presence or absence of a central peak in the cross-correllograms. We will refer to these phenomena as oscillations and correlations, respectively. Models for feature linking have been proposed for feature linking by various authors, either based on oscillating neurons [6, 7] or integrate-and-fire or bursting neurons [8, 9, 10].

In this paper we want to study stimulus dependent assembly formation in networks of stochastic binary neurons. Feed-forward connections implement feature detectors and symmetric lateral connections implement memory traces. We define correlation as the presence of a central peak in the cross-correllograms. Neurons that are synaptically connected will display correlated fire under almost all stimulus conditions. Therefore, we use a network with short-range connections and make use of the effect of long range correlations that are present in spin systems near the critical temperture.

These networks provide an attractive model to study feature linking for two reasons. The stochastic dynamics of these networks lead asymptotically to the Boltzmann-Gibbs distributions, which gives insight in the conditions under which long-range correlations occur. In this way the correlated firing can be related to well know equilibrium properties of spin systems. Secondly, these networks offer an immediate solution to learning based in correlated activity using the Boltzmann Machine learning paradigm [11].

2 The model

The basic architecture that we consider is given in Fig. 1. A sheet of sensory

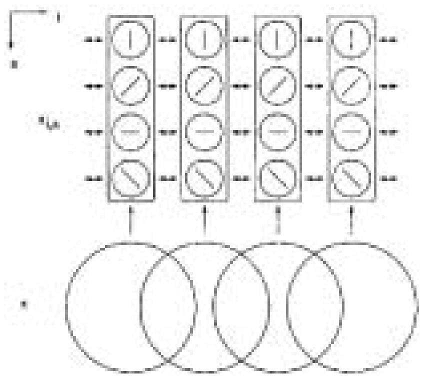

Fig. 1. Boltzmann Machine architecture with feed-forward and lateral connections. Input is provided by $x_1, \ldots, x_n$, with x_i real-valued or binary valued. Hidden units are $s_1, \ldots, s_h$, with $s_j = \pm 1$.

neurons in the visual cortex is modeled as a two-dimensional grid. At each grid location, a column of neurons with identical receptive fields is present. Neurons in one column respond optimally to different features in the receptive field.

The equilibrium distribution of the sensory neurons s *given* a stimulus x is given by

$$p(s|x) = \frac{1}{Z(x)} \exp\{w \sum_{(i,j),\alpha} s_{i,\alpha} s_{j,\alpha} + v \sum_{i,\alpha,\alpha'} s_{i,\alpha} s_{i,\alpha'} + \sum_{i,\alpha} h_{i,\alpha}(x) s_{i,\alpha}\} \quad (1)$$

$s_{i,\alpha} = \pm 1, i = 1, \ldots, n, \alpha = 1, \ldots, m$ denote the firing of the neuron with feature preference α at grid location i. $s_{i,\alpha}$ is a stochastic variable subject to Glauber dynamics, x denotes the external stimulus. It consists of a two-dimensional array of feature values $x_i = 1, \ldots, m, i = 1, \ldots, n$. w and v denote the strength of the lateral nearest neighbor interaction and the intra-column interaction, respectively.

$h_{i,\alpha}(x)$ is the external field (stimulus) component for neuron $s_{i,\alpha}$ in the presence of stimulus x. We assume that $h_{i,\alpha}(x)$ only depends on the local stimulus value x_i, i.e. $h_{i,\alpha}(x) = h_\alpha(x_i)$. By definition, neuron $s_{i,\alpha}$ has a prefered stimulus value α but it is usually also activated with nearby feature values. Here we will assume $h_\alpha(x_i) = h\delta_{\alpha,x_i} + h_0$. h is the overal strength of the stimulus and h_0 is a neuron threshold, which are free parameters of our model.

In the case that the intra-column interaction $v = 0$, Eq. 1 becomes a product of independent models, one for each feature value α:

$$p(s|x) = \Pi_\alpha p_\alpha(s_\alpha|x)$$
$$p_\alpha(s|x) = \frac{1}{Z(x)} \exp\{w \sum_{(i,j)} s_i s_j + \sum_i (h\delta_{\alpha,x_i} + h_0) s_i\}$$

Thus we can study the behaviour for one value of α.

3 Correlation lengths

Consider a visual stimulus x. The external input to neuron i in layer α is either h or 0 depending on whether $x_i = \alpha$. The task of the network is to represent these inputs in the various feature layers, such that 1) neurons locally represent the presence or absence of a feature value and 2) the activity between the neurons that encode one stimulus are correlated in regions where the stimulus is coherent.

A convenient quantity that expresses correlation between neurons j and k is the correlation function:

$$\Gamma_{jk}(t) = \langle s_j(0) s_k(t) \rangle - \langle s_j(0) \rangle \langle s_j(0) \rangle \quad (2)$$

The correlation function depends on the temperature β and on the connectivity of the network. For instance, in a d-dimensional Ising spin system, the connections are only between nearest neurons in a d-dimensional grid. Γ_r can be calculated in the Landau approximation and takes the form

$$\Gamma_r \propto r^{2-d} \exp(-r/\xi)$$

with r the distance in the grid. ξ depends on the temperature of the system. Around the critical temperature T_c, $\xi \propto |T - T_c|^{-\frac{1}{2}}$.

In the absence of a stimulus to layer α, we want to have low firing rates of the neurons. Therefore, we require $h_0 < 0$. In the presence of a coherent stimulus, we would like long range correlations. Correlations are maximal when $h_0 + h = 0$, i.e. when the total external field is absent. This is because for $h_0 + h \neq 0$, the second term in Eq. 2 strongly increases, making the correlation function effectively zero.

In Fig. 2 we show the auto-correlations $\langle s_i(0) s_i(t) \rangle$ as a function of time t for 9 neurons in a 3×3 sub-grid of a 10×10 grid of neurons for various coupling strengths w and coherent stimulus. Note that $\langle s_i s_i \rangle$ approaches the squared mean firing rate $\langle s_i \rangle^2$ for large times. $\langle s_i \rangle = 0$ for small coupling w. There exists a critical coupling above which there is a coexistence of two phases, each with non-zero $\langle s_i \rangle$. The critical coupling is approximately $w = 0.4$.

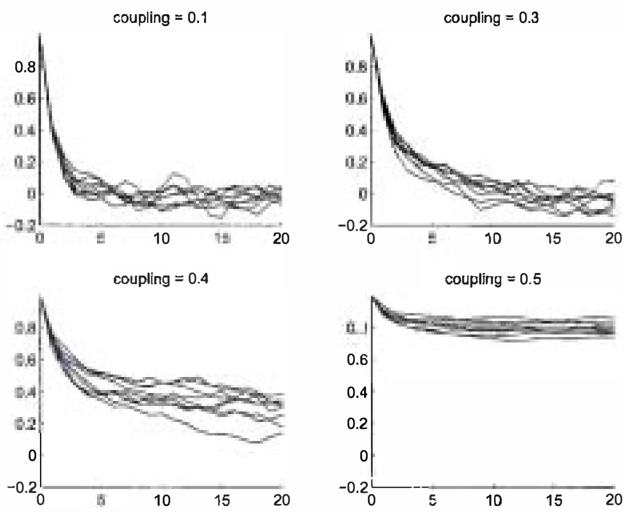

Fig. 2. Auto-correlations as a function of time for different coupling strength for coherent stimulation. The network consists of a 10×10 grid of neurons with periodic boundary conditions. External field is zero.

In Fig. 3 we show the equal time cross correlation function $\Gamma_r(0)$ for various coupling strengths w and coherent stimulus as a function of distance in the network.

In [12], we applied this mechanism to a stimulus that consists of 2 objects. It was shown, that all cells belonging to the same object are highly correlated, whereas cells belonging to different objects are not correlated.

4 Discussion

We have shown how long range correlations in spin models can be used to signal coherence in stimuli. In particular, we have proposed a two dimensional Ising model for feature linking in a visual task. This model allows to study the response of a network to stimuli with varying degree of coherence.

Whereas the mean firing rate indicates the local evidence for a stimulus feature, the correlations signal whether these local features are part of a coherent

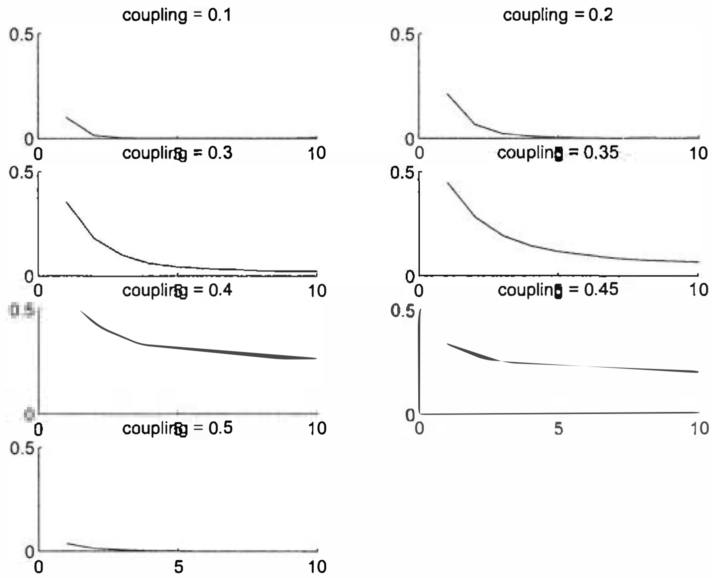

Fig. 3. Equal time cross-correlation function as a function of distance in the grid for different coupling streng th. The network consists of a 10×10 grid of neurons with periodic boundary conditions. External field is zero. The network was iterated 10000 times to remove transient effects. Correlation function was calculated over 3000 τ times. Results are averaged over all neuron pairs with equal distance.

stimulus or not. In this sense, stochastic networks can be used to solve the dynamic linking problem as stated in the introduction.

This model contains a sensory layer and a hidden layer. The hidden layer represents an interpretation of the sensory input. Sensory input provides local evidence. The lateral connections in the hidden layer provide global correlations between features that belong to the same stimulus and no correlations between features from different stimuli.

In this paper, we have only studied correlations at 0 time delay. In [13], delayed correlations were studied in networks composed of fully connected subpopulations, but the issue of dynamic linking was not addressed there. Nevertheless, one expects similar time-delayed correlation as were reported there.

Clearly, we are not proposing the Ising model as a serious computational model for the cortex and it should be investigated whether and how this mechanism can be extended to other network architectures. In the present model, neighboring cells with identical receptive fields are connected. In a more realistic network, the lateral connectivity would arise from learning and more complex connectivity patterns arrise. When learning results in a combination of excitatory and inhibitory connections, the resulting network will contain frustration, and may behave more like a spin glass than like ferromagnetic model. It is known, that long range correlations also exist in such frustrated systems.

A straightforward way to learn the lateral connections strengths from an environment is given by the Boltzmann Machine learning framework [11]. It is interesting to note that this rule is based on correlated activity $\langle s_i s_j \rangle$ instead of mean field activity $\langle s_i \rangle \langle s_j \rangle$.

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