High magnetic field corrections to resistance thermometers at low temperatures

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We present a reliable method to account for the magnetoresistance of resistance sensors which are used as thermometers in many low temperature ($T \leqslant 20$ K) experiments carried out in high magnetic fields (to 31 T). To apply the method, a set of isothermal magnetoresistance data, and a zero magnetic field temperature calibration are first necessary. A simple algorithm, which uses this data set, can then be applied to compute the temperature from the measured resistance at any field. The method is particularly useful for temperature dependent measurements at fixed field, or where, in cases where the temperature may change unpredictably during a change in magnetic field. We apply this method to the treatment of data in two separate experiments with the two different thermometers, RuO_2 (below 1 K) and Cernox (above 1 K) sensors, respectively. © *1999 American Institute of Physics.* [S0034-6748(99)02404-1]

I. INTRODUCTION

Resistance sensors such as RuO_2 (Refs. 1–7) and Cernox (Refs. 8–11) have been widely used as thermometers at low temperatures due to their small size, their simple instrumentation, and their relatively small magnetoresistance. However, with the expanding range of available magnetic fields and temperatures at high magnetic facilities worldwide, temperature errors due to magnetoresistance effects may occur in certain limits, if based on the zero field calibration. Specifically, at low temperatures disorder effects (weak localization[12]) may produce a large change in the resistance in the low field region. And at higher temperatures, where the sensitivity can be orders of magnitude less, small changes due to magnetoresistance become quite significant. Typically, the errors can be of the order of 10%–20% of the actual temperature in fields in the range 10–30 T. There are several alternatives to adjusting for these high field magnetoresistance effects, as recently described by Goodrich *et al.*[13] for RuO_2 in the 0.03–0.6 K range up to 32 T, and by Brandt, Liu, and Rubin[11] for Cernox sensors in the range 2–286 K also up to 32 T as applied to calibrated commercial sensors. In this article we show how, with a simple calibration procedure, one can obtain reliable values of the temperature from resistive sensors, based only on the value of the resistance and magnetic field for any specific data point. The method is general, and in principle can be applied to any experimental parameter which varies with temperature and magnetic field. Single valuedness of the parameter versus field and temperature is preferred, but not required. We provide two examples where the method has been successfully implemented.

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II. PRINCIPLE OF METHOD

A. Calibration

It is necessary to first establish a calibration curve of resistance versus temperature at zero field, as shown for instance in Fig. 1(a) for the RuO_2 sensor[14] used in this article. This calibration may be obtained from the manufacturer, or done by the experimenter at zero magnetic field by standard primary or secondary thermometry methods.[15] We have found that a polynomial expansion of the log of the temperature in terms of the log of the resistance provides an excellent fit to the resistive sensors we have studied. The second, crucial step is to obtain a set of (ideally) isothermal curves[16] of the resistance of the sensor versus magnetic field. We have used polynomial fits in our application, with as many terms as are needed to describe the data. This is to be distinguished from approaches where specific functions which describe physical models are used. In Fig. 1(b) we show the magnetoresistance data set for the RuO_2 sensor. The extent to which the temperature remains constant depends on the details of the experiment. In the cases studied in this article (see Sec. III) the magnetic field was swept very slowly, the thermometers were immersed in the cryogenic coolant, and the value of the resistance at the beginning and the end of the magnetic field sweep were nearly identical. In experimental situations where this is not possible, one may always use a transfer standard such as a capacitive thermometer[11] to electronically regulate the temperature during the magnetic field

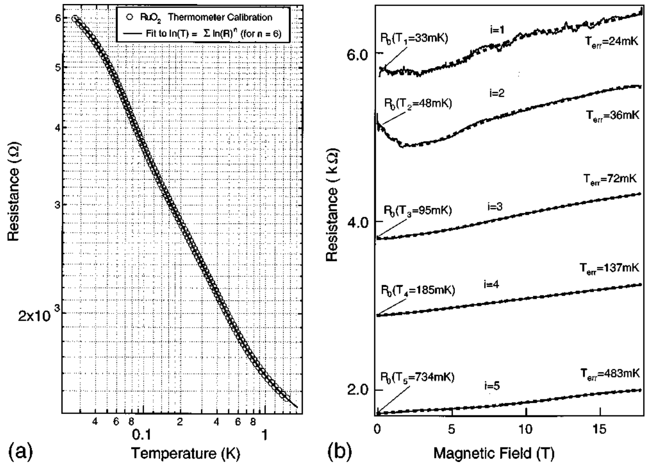

FIG. 1. (a) Resistance vs temperature calibration (Ref. 14) (open circles) for a RuO_2 1000 Ω resistance thermometer. Solid line, the result of a fit for ln(T) expanded as a sixth order polynomial in ln(R). (b) The magnetic field dependent resistance, for selected isotherms, of the RuO_2 resistance thermometer. Labels refer directly to terms associated with Eqs. (1) and (2). Solid lines, fifth order polynomial fits in B for each isotherm which determine the coefficients $b_n[R_0(T_i)]$. T_{err} refers to the apparent temperature at high field (17 T) based on the zero field calibration.

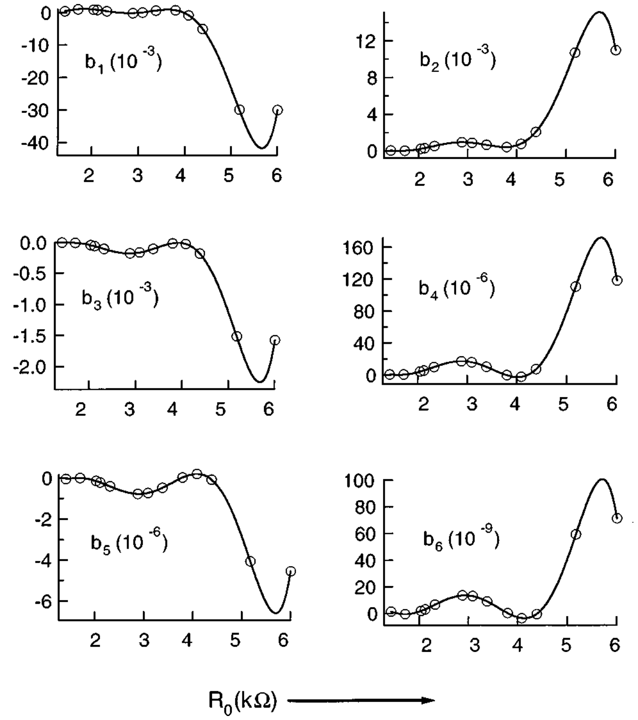

FIG. 2. Dependence of the coefficients b_n on R_0. Solid lines, sixth order polynomial fits in R_0 to determine the coefficients d_{nm}. The units of the coefficients are ΩT^{-n}. See the text for discussion.

sweep. Naturally, it is important to obtain a data set which covers in some reasonable mesh the range of temperature and magnetic field to be used in the experiments.

B. Parameterization and algorithm

The key feature of our method is to compute, based on a single value of the resistance at a particular field, the corresponding value of resistance at zero field. From this the temperature may be easily computed from the zero field calibration. Although the parameters in the computation depend on isothermal data initially, the resulting method relies only on the value of the resistance and the magnetic field at a single point, regardless of the details of any subsequent experiments. We demonstrate the method based on the RuO_2 data from Fig. 1. To accomplish this the isotherms (indexed by i) of Fig. 1(b) must first be accurately parameterized (i.e., with a sufficient term to represent nonlinearity in the field dependence). We therefore consider the relations

$$R_i(B)=\sum_n b_{ni}[R_0(T_i)]B^n \tag{1}$$

and

$$b_n[R_0(T_i)]=\sum_m [d_{nm}R_0(T_i)^m]. \tag{2}$$

In the above expressions we note that temperature is implicit in the zero field resistance according to $R_0(T_i)$. The notation in Eqs. (1) and (2) is further described as follows.

(a) Equation (1) expresses the resistance on an *isotherm* T_i as a polynomial in magnetic field. A fifth order polynomial in magnetic field (B) is generally sufficient. Referring to Fig. 2, we fit each of the isotherms to Eq. (1), thereby generating a set of coefficients $b_n[R_0(T_i)]$ for *each* isotherm.

(b) Equation (2) states that the coefficients of the polynomial expansion in Eq. (1) are themselves explicit functions of $R_0(T)$ (implicit functions of T). We may therefore expand each coefficient $b_n[R_0(T_i)]$ in terms of a polynomial in $R_0(T)$ to obtain the final set of coefficients d_{mn}. The first coefficient in Eq. (1) for each isotherm, namely $b_0[R_0(T_i)]$, is equal to $R_0(T_i)$, i.e., $d_{00}=1$, and $d_{0n}=0$ for $n\neq 0$. The result of the procedure for determining the coefficients d_{mn} is illustrated in Fig. 2. Here a sixth order polynomial in $R_0(T)$ is sufficient to accurately determine the d_{nm} coefficients. We note that at low temperatures the resistance, and also the magnetoresistance, changes significantly for small changes in temperature (of the order of tens of millidegrees Kelvin). Hence a finer mesh of data between 80 and 30 mK would be needed to reduce the tendency of the polynomial to oscillate in the low temperature tail. Nevertheless Fig. 2 shows a detailed balance of odd and even terms of b_n and the solution of Eq. (3) is well behaved, as the results of Fig. 3 will show.

Once all coefficients are determined, Eq. (1) may be expressed as more general function of B and T:

$$R(B)=\sum_{n,m} d_{nm}R_0(T)^m B^n. \tag{3}$$

Equation (3) may then be solved for $R_0(T)$ by standard root search methods, given $R(B)$ and B at any data point.[17] With the value of $R_0(T)$ which corresponds to $R(B)$ determined, the temperature may be obtained from the original calibration relation. Examples of the program code and coefficients are available for application on the internet.[18] In the event that the $R-T-B$ relationship for a particular thermometer is double valued somewhere, the root search program may be modified to place the initial guess for $R_0(T)$ in closer proximity to the solution.

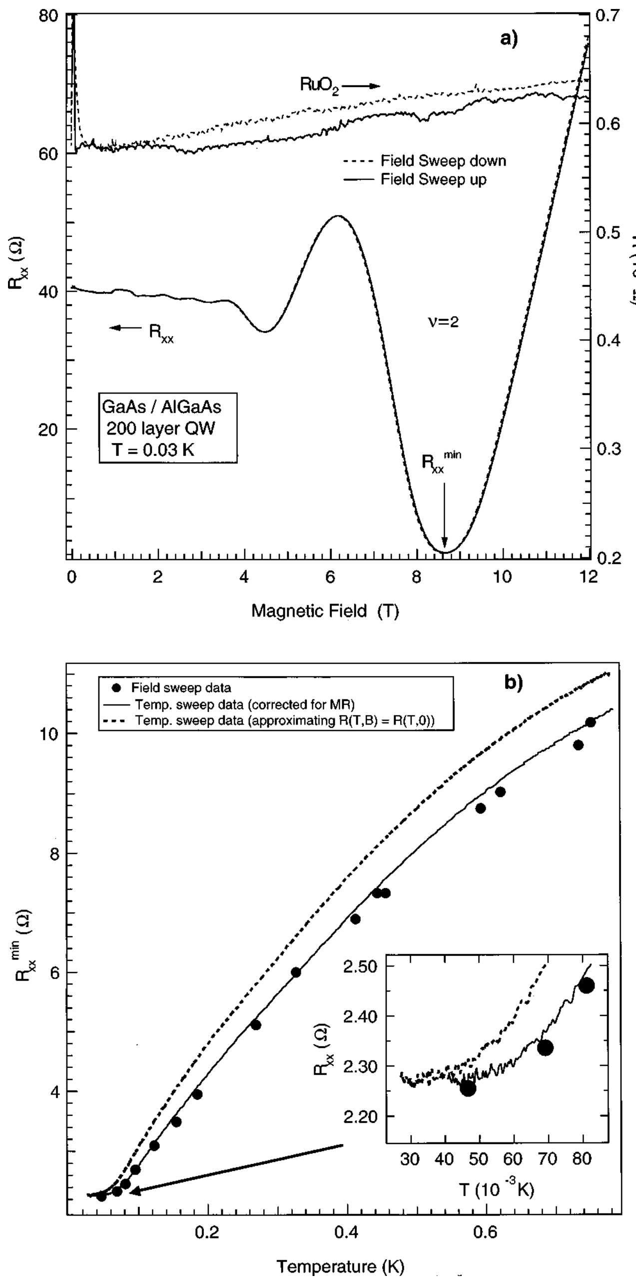

FIG. 3. (a) Isothermal field sweep at 0.03 K showing transport data, resistance thermometer, and position of the field and temperature dependent feature (R_{xx}) in the $\nu=2$ quantum Hall state in a 200 layer quantum well structure. The transients in the resistance thermometer were due to trapped flux in the superconducting magnet, which caused a magnetic field reversal near zero field. (b) Temperature dependence of R_{xx} vs temperature. Symbols—data from isothermal field sweeps; solid line—data from a temperature sweep at constant field (8.6 T) with temperature corrected for magnetoresistance; dashed line—same data without correction. Inset: expanded view of low temperature region.

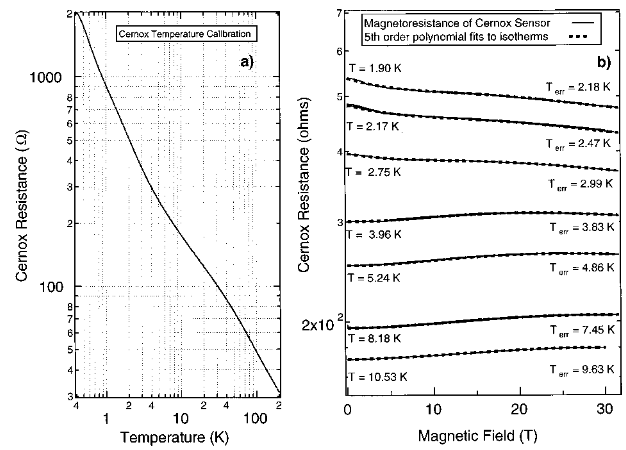

FIG. 4. (a) Zero field temperature calibration (Ref. 10) [for $\ln(T)$ to eighth order in $\ln(R)$] of the Cernox thermometer used in the second example. (b) The magnetic field dependence of the resistance for isotherms of the Cernox sensor. Where taken, up and down field sweeps overlap very well. Solid lines—fourth order polynomial fits in B to the data. T_{err} refers to the apparent temperature at high field (30 T) based on the zero field calibration.

III. APPLICATION TO EXPERIMENT

We treat two experimental cases. (1) The first is an experiment on a 200 layer GaAs/AlGaAs structure.[19] Here the calibrated 1000 Ω RuO_2 sensor (Fig. 1) in the mixing chamber of an Oxford Instruments[14] top-loading dilution refrigerator was used to monitor the temperature during field sweeps to determine the magnetoresistance R_{xx} in a standard transport measurement. For each temperature at which a field sweep was made, a heater was used to warm the mixing chamber above the base temperature of about 0.03 K. About 1 h was needed for the system to come into thermal equilibrium after each new heater setting. The maximum heat applied was 6 mW at 0.7 K. To avoid heating due to eddy current effects, the field was swept at 0.032 T/min, and thermal drift was always checked by comparing the value of the RuO_2 sensor at the beginning and end of each field cycle—typically between 0 and 17.5 T. In Fig. 3(a) we show a trace of R_{xx} along with the temperature sensor for a field cycle at 0.03 K. Of interest in this particular experiment was the temperature dependence of the R_{xx} minimum at the $\nu=2$ filling factor which occurs at about 8.6 T. This may be accomplished in two ways—either by systematic isothermal field sweeps to pick off the R_{xx} minimum, or more conveniently, by fixing the field at the minimum, and changing the temperature. In the latter case however, there will be considerable error in the temperature if the magnetoresistance is not accounted for, since it too is temperature dependent. In Fig. 3(b) a comparison of the two ways of data acquisition are shown. The discrete points are the field sweep data where R_{xx} is picked off of each isothermal magnetoresistance curve, and the continuous data are R_{xx} versus the temperature, as determined from our method which accounts for the magnetoresistance of the thermometer. Also shown is the same data, without the magnetoresistance correction. Although such errors are only about 10% in magnitude, they may seriously compromise data analysis where it is necessary to distinguish, say, between activated, power law, or variable range hopping models for the transport data.

(2) The second example used a Cernox sensor.[10] Here the temperature dependence of the resistance of an organic conductor $(BEDT\text{-}TTF)_2KHg(SCN)_4$ has been investigated for different fixed magnetic fields orientation.[20] The characterization of the Cernox sensor with temperature and field is given in Fig. 4. For situations where the thermometer is in direct contact with helium exchange gas or liquid, the isothermal condition is less of a problem for field sweeps (typically 3 T/min). Of note here is that the sign of the magne-

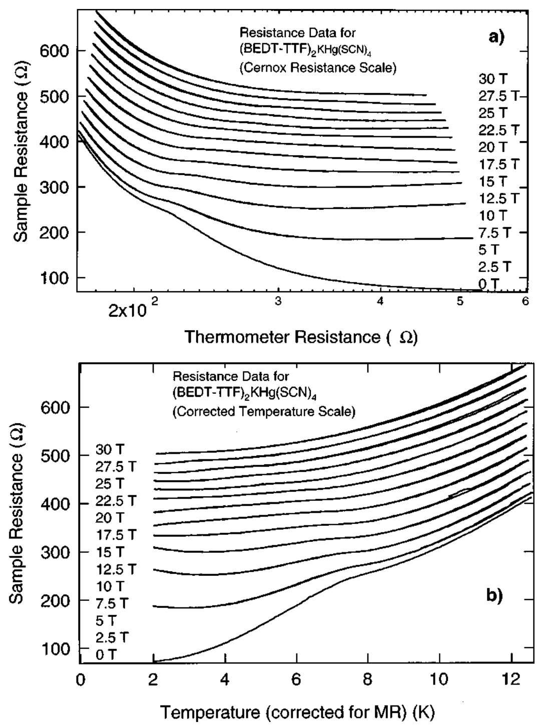

FIG. 5. (a) Resistance data on an organic conducting crystal vs thermometer resistance at different magnetic fields. (b) The same data vs temperature as corrected for magnetoresistance. See the text for discussion.

toresistance changes over the range of temperature in the experiment. The magnetic field was supplied by a 31 T resistive magnet,[21] with a vacuum-jacketed probe placed in a pumped He-4 dewar set at 2 K. Heat was applied at a fixed value to the sample platform to raise it just above 12 K, and when equilibrium was attained, the system was allowed to cool back to the base temperature. Hence, for this systematic study with steady state cryogenic cooling, fixed fields, and fixed heat during warming, the initial and final temperature values ($\approx$12.5 and 2 K) were always reached within an uncertainty of a degree. The data as taken versus the Cernox sensor are shown in Fig. 5(a). Here, although the initial and final temperatures are always nearly the same values, one can see a dramatic effect due to the magnetoresistance of the Cernox sensor. The same data versus temperature, after the magnetoresistance correction has been applied, is shown in Fig. 5(b). The slight variation starting (high) temperature above 12 K from sweep to sweep is real. It is due to the variable duration of the heater, which caused the cooling curves to start from slightly different temperatures. The base temperature of 2 K was always reached, however. Figure 5 demonstrates that our method can effectively recover the accurate temperature dependence of the sample in all ranges of field.

IV. UNCERTAINTIES IN THE PROCEDURE

The present work uses a zero field calibration of a sensor (either calibrated by a researcher, or by a manufacturer) as the reference point for the high field calibration procedure. Hence the accuracy to which the final, absolute temperature value at high fields is accurate depends on the precision to which the calibration has been carried out, and the care the experimenter exercises in properly measuring the resistance (i.e., self-heating effects, etc.). The ultimate accuracy further depends on the degree to which the field sweeps in the field calibration are truly isothermal, and how faithfully the parameterization represents the magnetoresistance. In the work presented here we have focused on the process of going from the zero field calibration to the high field calibration. Hence we only consider the uncertainties due to nonisothermal conditions and imprecise parameterizations for the field calibration. First we have shown in the isothermal data the errors in temperature which will result if the magnetoresistance is not accounted for [Figs. 1(b) and 4(b)]. Second, to indicate in a practical manner how the zero field and high field calibrations correspond, we have presented two experimental examples where the procedure has been carried out. In the case of the low temperature example for the RuO_2 sensor [Fig. 3(b)], we may take 70 mK as a reference point. Here, without the field calibration, the error in temperature would be about 18 mK, i.e., the R_{xx} value at 70 mK would appear at 52 mK, an error of about 25%. In the inset of Fig. 3(b), we note that ideally the discrete field sweep data should fall exactly on the continuous temperature sweep data. Again, taking the 70 mK point as reference, we see there is a shift of up to 5 mK between the two data sets, which results in an error of 7% for this point. Inspection of the full data set in Fig. 3(b) shows a similar uncertainty, some of which is attributable to the R_{xx} measurement, but the field corrected scale is clearly superior to the zero field calibration in all cases. In the second experiment where a Cernox sensor was used, we may take the results of Fig. 5(b) to serve as a scale of the uncertainties. At low temperatures, if we assume the same base temperature is reached at the end of each sweep, then the maximum deviation is about 0.1 K from 0 to 30 T, or 5%. At the high temperature end, again if we assume the starting temperatures are the same, the maximum deviation is about 0.2 K from 0 to 30 T, or about 2%. Since the heating and cooling times were not always precisely the same, the actual uncertainties are less (we estimate 2.5% at 2 K and 1% at 12 K). Again, with no correction for the field dependence, we see by inspection of Figs. 4 and 5(a) that the errors would be as high as 14% at 2 K and 9% at 10 K.

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[15] See for instance the recent review article by L. Rubin, Cryogenics **37**, 341 (1997).

[16] We use the term ''isotherm'' for convenience to describe field sweeps where the temperature did not drift significantly over the period of the data acquisition. Some drift and/or noise in the data are inevitable in many cases of high field experiments.

[17] There are, in general dozens of polynomial terms, each of which must be computed many times to obtain a solution for $R_0(T)$. Again, this computation must be performed for perhaps thousands of data points from an experiment. However, current desk-top computers running at over 300 MHz can produce a corrected temperature scale in seconds or less.

[18] Examples of the program are available through the NHMFL (Ref. 21) web page, http://theory.magnet.fsu.edu/btt/btt.html. They are written for RuO_2 sensor ''R4'' in the NHMFL Millikelvin Facility top-loading dilution refrigerator. The application program is Igor Pro (WaveMetrics, Inc., P.O. Box 2088, Lake Oswego, OR 97035).

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