Magnetothermoelectric properties of Bi₂Se₃

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We present a study of entropy transport in Bi₂Se₃ at low temperatures and high magnetic fields. In the zero-temperature limit, the magnitude of the Seebeck coefficient quantitatively tracks the Fermi temperature of the three-dimensional Fermi surface at the Γ point as the carrier concentration changes by two orders of magnitude (10¹⁷ to 10¹⁹ cm⁻³). In high magnetic fields, the Nernst response displays giant quantum oscillations indicating that this feature is not exclusive to compensated semimetals. A comprehensive analysis of the Landau level spectrum firmly establishes a large g factor in this material and a substantial decrease of the Fermi energy with increasing magnetic field across the quantum limit. Thus, the presence of bulk carriers significantly affects the spectrum of the intensively debated surface states in Bi₂Se₃ and related materials.

I. INTRODUCTION

The Bi₂X₃ family (X = Se, Te) is attracting tremendous attention as a topological insulator (TI). Recently, the existence of this class of bulk insulators was predicted and confirmed.¹ In a TI, the bulk energy gap is traversed by spin polarized surface states. Therefore, the electrical conduction is expected to occur only at the surface. In practice, however, these materials are often low-density bulk metals. Interestingly, many of the TI of the first and the second generation are well-known thermoelectric materials² and present a sizable thermoelectric figure of merit. This quantity, ZT=(S²ρ)/κT (here S is the Seebeck coefficient, κ is thermal conductivity, and ρ is resistivity) characterizes the thermoelectric efficiency of a material. To this date, the largest thermoelectric figure of merit in a bulk material at room temperature has been reported in Bi₂Te₃ (ZT=0.8 at T=300 K³). In high magnetic fields, the Nernst response displays giant quantum oscillations (in the case of A1 they dominate the signal). The surprising sensitivity of the Nernst effect to quantum oscillations in Bi₂Se₃ is indeed reminiscent of those reported in bismuth⁹ and graphite.¹⁰ The origin of these giant quantum oscillations is the subject of ongoing theoretical research.¹¹–¹³ These two elemental semimetals are, however, compensated systems, whereas the (bulk) Fermi surface of Bi₂Se₃, as we will see below is a single band at the Γ point. Therefore, the observation

II. QUANTUM OSCILLATIONS IN SEEBECK AND NERNST EFFECT

Measurements of longitudinal (S=E/νxT) and the transverse (N=E/νxT) thermoelectric response were performed on a standard one-heater-two-thermometers setup. For all samples, the electric current and the thermal gradient were applied in the plane of the quintuple layers and the magnetic field was oriented along the trigonal direction (perpendicular to the layers). The samples used in this study are similar to those previously studied by longitudinal and transverse magnetoresistance experiments⁸ and described there.

In Fig. 1, we compare the electrical transport and the entropy transport in two samples A₁ and B₁ with typical bulk concentrations of 10¹⁹ cm⁻³ to 10¹⁷ cm⁻³. For both samples, ρₓₓ and ρᵧᵧ display quantum oscillations on top of an almost linear monotonic base as previously reported and discussed.⁵–⁸ In the presence of a field of about 10 T, we find for the two concentrations that ρₓₓ ≈ ρᵧᵧ whereas S ≫ N (the typical ratios are 100 for A₁ and 10 for B₁). As in the case of the electrical transport, S and N display quantum oscillations with a phase shift of π. The oscillations are particularly pronounced in N (in the case of A₁ they dominate the signal). The surprising sensitivity of the Nernst effect to quantum oscillations in Bi₂Se₃ is indeed reminiscent of those reported in bismuth⁹ and graphite.¹⁰ The origin of these giant quantum oscillations is the subject of ongoing theoretical research.¹¹–¹³ These two elemental semimetals are, however, compensated systems, whereas the (bulk) Fermi surface of Bi₂Se₃, as we will see below is a single band at the Γ point. Therefore, the observation

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reported here establishes that this effect is not exclusive to compensated systems. We note that there are several reports on Nernst quantum oscillations with a large amplitude and a small frequency in metals like zinc\textsuperscript{14} and aluminum\textsuperscript{15} or in doped semiconductors such as \textit{n}-type InAs\textsuperscript{16} and iron-doped HgSe.\textsuperscript{17}

\section*{A. Effective masse and Dingle temperature}

Next, we focus our attention on the temperature dependence of the quantum oscillations in the thermoelectric properties, which have been so far poorly studied. In Fig. 2, is plotted as a function of $B^{-1}$ for various temperature. In both cases, the signal is periodic in $B^{-1}$. The difference of the period between the two samples simply reflects the difference in the carrier concentrations. In both cases, the oscillations disappear at a typical temperature of 20 K suggesting rather similar low effective mass for both concentrations. Following the standard Lifshitz-Kosevich theory,\textsuperscript{18} we fit the temperature dependence of the oscillating part of $\frac{RT}{X}$ to:

\begin{equation}
\ln\left(\frac{A}{RT}\right) = -\alpha T_D m^* \times \frac{1}{B}.
\end{equation}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2}
\caption{(Color online) (a) and (b) $\frac{S_T}{T} = \frac{N}{RT}$ as a function of $B^{-1}$ for the sample $A_1$ and $B_2$ (sample different of $B_1$ with a similar bulk concentration) for the various temperatures explored. (c) Temperature dependence of the amplitude of the oscillations of the sample $A_1$ (in red) and $B_2$ (in blue). The line corresponds to a fit using the Lifshitz-Kosevich formula (see the text).}
\end{figure}

For the four temperatures, the field dependence of $\ln\left(\frac{A}{RT}\right)$ is linear. From Eq. (1), we found a Dingle temperature $T_D = 10 \pm 1$ K in good agreement with Shubnikov-de Haas measurements.\textsuperscript{5–8} The remarkable simplicity of our analysis of the quantum oscillations $\frac{S_T}{T}$ is related to the absence of a phonon drag contribution in the transverse thermoelectric response (contrary to its significant contribution in the Seebeck effect). This point is consistent with the early analysis of Tieke \textit{et al.}\textsuperscript{17} in the case of iron-doped HgSe.

\section*{B. Magnitude of the Seebeck and Nernst effect}

The magnitude of the thermoelectric response is dramatically affected by the change in the carrier concentration. Figures 4(a) and 4(b) present $S_T$ and $\nu_T$ as a function of temperature for the four samples studied. As the doping passes

\begin{figure}[h]
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\includegraphics[width=\textwidth]{figure3}
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\end{figure}
from $10^{19} \text{cm}^{-3}$ to $10^{17} \text{cm}^{-3}$, $\frac{S}{T}$ increases by one order of magnitude and $\frac{\nu}{T}$ increases by two orders of magnitude. In the Fermi liquid picture for a one-band system, the diffusive Seebeck is expected to be $T$ linear in the zero-temperature limit with a magnitude proportional to $1/n_F$. In the case of bismuth, for example, when the field is oriented along the trigonal direction, the $g$ factor of the hole pocket is as large as 62. This corresponds to a ratio of the Zeeman energy ($E_Z$) to the cyclotron energy ($\hbar \omega_c$) labeled $M = \frac{E_Z}{\hbar \omega_c}$ slightly larger than 2. As a consequence, the chemical potential starts to move significantly with increasing magnetic field, dramatically affecting the Landau level spectrum.

In the case of Bi$_2$Se$_3$, the data of sample B$_2$ down to $T = 0.38 \text{mK}$ and up to $B = 17 \text{T}$ does not reveal any Zeeman splitting of the peaks like the one found in bismuth. This suggests that the Zeeman energy and the cyclotron energy are commensurate (i.e., that $M$ is closed to an integer in the limit of the peak width) as originally suggested by Köhler et al. In order to differentiate between $M = 0, 1, 2, 3, \ldots$, we used a model similar to the one used by Köhler et al. that determines the Landau level spectrum and the field dependence of the Fermi energy. The calculation of $E_F$ is performed with the assumption that the carrier concentration (noted $n$) is independent of the magnetic field (see Appendix A for a justification): $n = \int_{-\infty}^{E_F} D(\epsilon) d\epsilon$ where $D(\epsilon)$ is the density of state, equal to the sum of the density of state of each Landau level (noted LL) depending on their position relative to the Fermi energy (see Appendix C for more details).

### A. $\alpha_{xy}$ or $\alpha^*$?

One complication arises in comparing the experimental Nernst peaks and the theoretical Landau spectrum. Several theories were proposed to explain the observation of giant quantum oscillations in bismuth and graphite. Two of these theories focus on the off-diagonal thermoelectric

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Table I. Properties of Bi$_2$Se$_3$.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$\mu$ (cm$^2$ V$^{-1}$ s$^{-1}$)</th>
<th>$T_F$ (K)</th>
<th>$\nu$ (μV K$^{-2}$)</th>
<th>$\nu$ (μV K$^{-2}$ T$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A$_1^a$</td>
<td>500</td>
<td>1150</td>
<td>-0.4</td>
<td>0.009</td>
</tr>
<tr>
<td>A$_2^b$</td>
<td>800</td>
<td>900</td>
<td>-0.45</td>
<td>0.012</td>
</tr>
<tr>
<td>B$_1^b$</td>
<td>4500</td>
<td>370</td>
<td>-6.1</td>
<td>1.24</td>
</tr>
<tr>
<td>B$_2^a$</td>
<td>3000</td>
<td>100</td>
<td>-6.5</td>
<td>2.4</td>
</tr>
</tbody>
</table>

$^a$Measured down to 2 K.
$^b$Measured down to 0.15 K.
```
conductivity $\alpha_{xy}$, another one is based on a semiclassical description of the Nernst effect $(N)$. In the case of bismuth and graphite, these two descriptions are equivalent because $\rho_{xx} \gg \rho_{yx}$ and $S \ll N$. However, in the case of Bi$_2$Se$_3$ $\rho_{xx}$ and $\rho_{yx}$ become comparable in amplitude and therefore $\alpha_{xy} \times B$ and $N$ do not peak exactly at the same magnetic field. We report in Fig. 5(a) the field dependence of the four quantities: $\alpha_{xy} \times B$, $N$, $\Delta \rho_{xx}$, and $\Delta \sigma_{xx} = \sigma_{xx}(0.25 K) - \sigma_{xx}(4.2 K)$ (where $\sigma_{xx} = \frac{\rho_{xx}}{\rho_{yx} + \rho_{xx}^*}$) as a function of the magnetic field. Several conclusions can be drawn from this plot.

First, $\alpha_{xy} \times B$ and the Nernst effect do not peak exactly at the same magnetic field. Figure 5(b) shows the $B^{-1}$ position of the Landau levels vs the index number. As seen in Fig. 5(a), the difference between the peak positions in $\alpha_{xy} \times B$ and in $N$ is very small compared to the periodicity of the signals. Thus, our conclusions are not affected by the choice of $\alpha_{xy} \times B$ or the Nernst effect. Further theoretical investigations are needed to clarify this issue.

Second, the maxima of $\Delta \sigma_{xx}$ are concomitant with the peak positions in $\alpha_{xy}$ and the Nernst effect, whereas the maxima in $\Delta \rho_{xx}$ are not. The origin of this difference is again related to the similar amplitudes of $\rho_{xx}$ and $\rho_{yx}$ above a few Tesla. Thus it appears that the enhancement of the conductivity generated by the crossing of an LL (Landau level) and the Fermi energy leads to a maximum in the Nernst response of the case of Bi$_2$Se$_3$, as it is the case for graphite and bismuth.

In Fig. 5(b) (Landau level fan), the intercepts of $\alpha_{xy} \times B$ and the Nernst effect are very close to 0, which is reminiscent of the nontrivial Berry phase as observed in LaRhIn$_5$. In the case of Bi$_2$Se$_3$, which is characterized by a parabolic dispersion, the large spin orbit interaction generates this peculiar behavior. The occurrence of a vanishing intercept points out to an odd value for the ratio of the Zeeman energy and the cyclotron energy (labeled M). This value is different from that deduced from the analysis of the peak position of $\Delta \rho_{xx}$ by Köhler et al. ($M = 2$). However, as discussed previously, it has been shown that the peak position in $\Delta \rho_{xx}$ differs from the Nernst response (and also differs from $\sigma_{xx}$). In addition, as we approach the quantum limit, the spectrum analysis becomes more difficult because of the field dependence of the Fermi energy. In order to clarify the possible values of $M$ we proposed in the next section a detail analysis of the Nernst peaks using the simple model introduced in Appendix A.

B. $g$ factor of the bulk states of Bi$_2$Se$_3$

The last peak resolved in the Nernst effect in sample B$_1$ occurs at 10.4 T. We have adjusted the carrier concentration in order to find the best agreement between the position of this peak and the crosssing points between a Landau level and the Fermi energy for different possible values of $M$. Figures 6(a)–6(d) show the Nernst effect (red lines) as a function of $B^{-1}$, superimposed with calculated Landau levels and Fermi energy $E_F$ (black points) with various parameters: $M = 0, 1, 2, 3$.

a. $M = 0$. In Fig. 6(a), we adjust the carrier concentration to match the periodicity of the Nernst response $(n = 1.08 \times 10^{17} \text{ cm}^{-3})$. In this case, the crossings of the LL and the Fermi energy correspond to minima and not maxima. We cannot reproduce the lowest minimum of the Nernst effect. In Fig. 7(a), we try to adjust the carrier concentration to match the last resolved peaks at 10.4 T and the LL $1^{-2}$. However, with this carrier concentration $(n = 1.8 \times 10^{17} \text{ cm}^{-3})$, the low-field peaks cannot be reproduced at the right positions.

b. $M = 1$. Figure 6(b) shows the Landau level spectrum for a bulk carrier concentration of $n = 1.28 \times 10^{17} \text{ cm}^{-3}$. In this case, there is not a complete agreement for the two last peaks, but it is rather good for the other peaks. In the case of $M = 1$, the LL $n + 1^{-2}$ and $n^+$ are degenerate except for the last ones: $0^-$ in orange on Fig. 6(b). The last observed peak at 10.4 T is attributed to the Landau levels $0^+/1^-$. Above this field all the electrons are in the lowest Landau level $0^-$. 

c. $M = 2$. Figure 6(c) shows the Landau level spectrum for a bulk carrier concentration of $n = 1.08 \times 10^{17} \text{ cm}^{-3}$. We find a rather good agreement for all the peaks. Note that the agreement is slightly better for $M = 2$ than for $M = 1$. In the case of $M = 2$, the LL $n + 2^{-}$ and $n^+$ are degenerate except for the two last ones $1^-$ [in green on Fig. 6(c)] and $0^-$ [in orange on Fig. 6(c)]. The last observed peak at 10.4 T
is then attributed to the Landau level 1− and the Landau levels n− are degenerate with n-2−. This result is compatible with the conclusion of the early Shubnikov-de Haas analysis.\textsuperscript{6}

d. \(M = 3\) (and above). We also investigated higher values for \(M\). On Fig. 6(d) \((M = 3)\), we report the Landau level spectrum for a carrier concentration of \(n = 1.08 \times 10^{17} \text{cm}^{-3}\). The lowest observed peaks match well the 1− LL. This result

FIG. 6. (Color online) Nernst effect (red line) of the sample B\(_1\) at \(T = 3\) K on top of calculated Landau level energies and Fermi energy \(E_F\) (black open circle) with different parameters: \(M = 0, 1, 2,\) and 3 which, respectively, correspond to (a), (b), (c), and (d). The calculation was performed with a carrier concentration of \(n = 1.08 \times 10^{17} \text{cm}^{-3}\) for (a), (b), and (d) and \(n = 1.25 \times 10^{17} \text{cm}^{-3}\) for (b), a Dingle temperature of \(T_D = 8\) K and \(m_1 = m_2 = 0.14 m_0\) and \(m_3 = 0.24 m_0\).

FIG. 7. (Color online) \(N\) (in red lines) of the sample B\(_1\) at \(T = 3\) K on top of a calculated Landau level energies and Fermi energy \(E_F\) (black open circle) with \(M = 0\) for (a) and \(M = 3\) for (b). In a case of (a) the carrier concentration has been chosen in order that the last resolved peak corresponds to the LL 1\(^{±}\) \((n = 1.8 \times 10^{17} \text{cm}^{-3})\). For (b), the carrier concentration has been chosen in order that the last resolved peak matches the LL 2\(^−\) \((n = 2.9 \times 10^{17} \text{cm}^{-3})\). The calculation was performed with a Dingle temperature \(T_D = 8\) K and \(m_1 = m_2 = 0.14 m_0\) and \(m_3 = 0.24 m_0\).

FIG. 8. (Color online) In red line is the Nernst effect, \(N\), as a function of \(B^{-1}\) at \(T = 5\) K for the sample B\(_3\) (same as B\(_1\) few months after the (a) experiment) on (a) and (b). The evolution of the Fermi energy \(E_F\) is plotted in black points. Each time that the chemical potential is crossing a Landau level (colors lines), there is a kink due to the carrier variation. The best parameter to describe the peak position as measured in \(N\) is found to be for \(M = 1\) (with a bulk carrier concentration \(n = 1.8 \times 10^{17} \text{cm}^{-3}\)) on (a) and \(M = 2\) (with a bulk carrier concentration \(n = 1.5 \times 10^{17} \text{cm}^{-3}\)) on (b).
is rather surprising as we would have naively expected that
the field scale associated with the $1^-$ LL would continuously
increase with the value of $M$. In fact, even if the two lowest
LL $1^-, 0^-$ and the chemical potential are decreasing faster
for $M=3$ than for $M=2$, the crossing point between the LL
$1^-$ and the chemical potential occurs at the same field for
$M=2$ and $M=3$. This conclusion is true for all the values
$M>2$. The situation is, however, different for the other LL,
where the crossing points differ for $M=2$ and for $M=3$.
In other words, for $M=3$ we can find a carrier concentration
where the $10.4T$ peak matches the LL $1^-$, however, the peak
positions at lower field cannot be explained. In Fig. 7(b), we
investigate the possibility that the lowest observable peak is
not associated with the $1^-$ LL but with the $2^-$ LL. We found
a carrier concentration of $n = 3 \times 10^{17}$ cm$^{-3}$. However,
the peak positions at lower fields cannot be explained.

In conclusion, the $M=1$ and $M=2$ hypothesis yields the
best agreement between the peak positions in the Nernst effect
and the crossing of Landau levels and the Fermi energy.
Remarkably, in both cases above $10.4$ T, all bulk carriers are
spin polarized. For $M=1$ and $M=2$ and with a cyclotronic mass of $0.14m_0$, we find, respectively, that $g = 14.3$ and
$g = 28.6$.

C. Nernst effect at high field

Sample B$_3$ was studied up to $32$ T. In this case, the last
resolved peak in the Nernst effect occurs at $11.4$ T because of
a slightly higher carrier concentration. Interestingly, between
$17$ and $32$ T, an unexpected increase of the Nernst effect was
observed. At first glance, one could explain this increase by
the presence of additional Landau levels which are expected in the
case of $M > 2$. For instance, for $M=3$, the last observed peak
could be attributed to the LL $2^-$. However, this scenario
does not reproduce the low field peak positions. Moreover, it
would yield a chemical potential crossing for the $1^-$ LL at
$19$ T, in contrast with our experimental data.

The increase in the Nernst effect observed at high field calls
for an alternative scenario. As reported in Fig. 8(b), once the
$1^-$ LL crosses the chemical potential, the enhanced shift in
chemical potential pushes it closer towards the $0^-$ LL. In the
low field regime the Nernst response increases as the distance
between the highest filled Landau level and the chemical
potential becomes shorter and attains a maximum when the
two levels coincide. Here if the chemical potential approaches
the $0^-$ LL without crossing it in order to keep charge neutrality,
one expects it to peak without attaining a peak. This scenario
may qualitatively explain the experimentally observed increase
in the Nernst effect.

IV. CONCLUSION

In conclusion, the overall thermoelectric and thermomag-
etic response in Bi$_2$Se$_3$ can be quantitatively understood in
the context of a single three-dimensional band in the presence
of large Zeeman splitting. Our investigation reveals a large
variation of the thermoelectric coefficients with the bulk carrier concentration which can be quantitatively
understood as a result of a change in the mobility and the bulk
Fermi energy. We resolved large quantum oscillations in the
Nernst coefficient suggesting that such an effect is a common
feature to both compensated and uncompensated low carrier
systems. For the lowest carrier concentrations investigated,
we found that the Zeeman energy is either equal to one or two
times the cyclotron energy. In both cases, a strong variation
of the chemical potential above the quantum limit is expected,
which can naturally explain the observed increase in the Nernst
response. Finally we would like to point out that our results
could impact the physics of the surface states. In the presence
of a metallic bulk coupled with surface states, we can expect
that the field-induced shift of the chemical potential of the
bulk state could also generate a variation of the Fermi energy
of the surface states. Such a situation will affect, for example,
the Landau level indexing and the determination of the Berry
phase of the surface state.

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APPENDIX A: HALL EFFECT AND CARRIER
CONCENTRATION

Figure 9 shows the field dependence of the Hall resistivity
for five samples. Up to $32$ T and down to the lowest carrier
concentration investigated, the Hall effect was found to be
linear in magnetic field suggesting that the carrier concen-
tration remains constant and does not vary with the field.
From the slope, we can determine the carrier concentration,
labeled $n^H$. These values can be directly compared with the
carrier concentration deduced from the period of the quantum
oscillations labeled $n^F$. The conduction-band structure in
this range of carrier densities is approximately parabolic.
The Fermi energy is then determined by the period of the
oscillation through the Onsager relation. Assuming a mass
anisotropy independent of the doping ($m_1 = m_2 = 0.14m_0$
and $m_3 = 1.7m_1$) one can determine the carrier concentration
of the samples. For all samples, within a few percent, we found
a good agreement between $n^F$ and $n^H$.

FIG. 9. (Color online) (a) Field dependance $\rho_{xy}$ for $B_1$ ($T=1.5$ K
up to $12$ T), $B_2$ ($T=0.35$ K up to $17$ T), and $B_3$ ($T=1.6$ K up to $30$ T).
(b) Field dependance $\rho_{xy}$ for $A_1$ and $A_2$ at $T=1.6$ K up to $12$ T.
APPENDIX B: ELECTRONIC MEAN FREE PATH AND IMPURITY SCATTERING

In the case of Bi$_2$Se$_3$ it is believed that Se vacancies are the charge dopants responsible for the $n$-type nature of the system. Here we propose to make a simple discussion to know if the vacancies are the main source of scattering of the electrons at low temperature. In other words we propose to compare the electronic mean free path and the distance between vacancies ($d_v$) for the two extreme concentrations that we studied, we can determine the electronic mean free path $l_e$ using the knowledge of the mobility and the Fermi velocity $v_F$.

$$l_e = \frac{\hbar v_F}{e B}$$

Each Se atom can contribute to two electrons. For a given bulk carrier concentration $n$, the number of lacuna per unit cell, $N_i$ is given by $N_i = \frac{2N}{V}$ where $V$ is the unit volume. If we assume a homogeneous distribution of the lacuna, the typical distance between vacancies sites can be estimated: $\frac{4}{3} \pi d_v^3 = \frac{N_i}{V} = \frac{2N}{V}$. Table II yields values of $d_v$ for the two carrier concentrations. For both samples, the mean free path is longer than the typical distance between impurity sites, suggesting that the scattering probability between a traveling electron and a point defect is much smaller than unity.

### Table II. Comparison of the electronic mean free path and the distance between vacancy sites.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$n$ (cm$^{-3}$)</th>
<th>$l_e$ (nm)</th>
<th>$d_v$ (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A$_1$</td>
<td>$3 \times 10^{19}$</td>
<td>23</td>
<td>2</td>
</tr>
<tr>
<td>B$_1$</td>
<td>$2.5 \times 10^{17}$</td>
<td>550</td>
<td>10</td>
</tr>
</tbody>
</table>

APPENDIX C: FIELD DEPENDENCE OF THE FERMI ENERGY

In the presence of a large $g$ factor ($M \geq 1$) for one single band, the Landau spectrum is modified in two ways:

(i) The quantum limit is reached at a higher magnetic field. For example, when $M = 2$, the quantum limit is approximately one period above the quantum limit attained than in the case of $M = 0$.

(ii) The lowest LL $0^-$ is going down with increasing magnetic field. This pulls down the chemical potential near the quantum limit regime. In order to quantify these effects in the case of Bi$_2$Se$_3$, we compute the field dependence of the Fermi energy for various values of $M$. As discussed in Appendix A, we assumed that the carrier concentration is independent of the magnetic field and is given by $n = \int_{E_F}^{E_{F+}} D(\epsilon) d\epsilon$ where $D(\epsilon)$ is the density of state. In the presence of a magnetic field and in the absence of a spin splitting, the density of state $D_0(\epsilon)$ can be written as

$$D_0(\epsilon) = \frac{\sqrt{\pi} m_0}{2\pi^2 h^2} e B \sum_{n=0}^{n_{\text{max}}} \left[ \frac{\epsilon - \epsilon_n + \sqrt{(\epsilon - \epsilon_n)^2 + \frac{\hbar^2 c^2}{m_n l_e^2}}}{\epsilon - \epsilon_n} \right]^{1/2} \left[ \frac{\epsilon - \epsilon_n - \frac{\hbar c^2}{2m_n l_e^2}}{\epsilon - \epsilon_n} \right]^2 \bar{m}_e$$

where $\epsilon_n = (n + \frac{1}{2}) \hbar c^2 / 2m_n l_e$, $n_{\text{max}}$ is the highest value of $n$ yielding a positive value for $\epsilon - \epsilon_n$ and $\Gamma$ is the broadening of the Landau levels ($\Gamma = \frac{\pi \hbar c^2}{m_n l_e}$). In the presence of a spin splitting $\epsilon_n = (n + \frac{1}{2}) \pm \frac{\hbar c^2}{2m_n l_e}$, the density of state is given by $D(\epsilon) = \frac{1}{2} (D_0(\epsilon - \frac{\hbar c^2}{2m_n l_e}) + D_0(\epsilon + \frac{\hbar c^2}{2m_n l_e}))$. 

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